

## Determination of the Complex Permittivity of Each Layer for a Bi-layer Dielectric Material Using Transmission (ABCD) Matrix in Ku-Band Frequency

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### Abstract

A new technique is presented to determine the complex permittivity of each layer for a bi-layer dielectric material. The bi-layer material sample is loaded in a Ku-band rectangular waveguide WR62 and its two port S-parameters are measured as a function of frequency using the E8634A Network Analyzer. Also, by applying transmission (ABCD) matrix, expressions for the S-parameters of the bi-layer dielectric material as a function of complex permittivity of each layer are developed. To estimate the complex permittivity of each layer's dielectric material, the square sums of errors between the measured and calculated S-parameters are minimised using a nonlinear optimization algorithm. The complex permittivity of each layer for a bi-layer dielectric material such as FR4-Teflon, FR4-Delrin and Delrin-Teflon are determined at the Ku-band frequencies, and the average relative errors between the individual dielectric materials and those of each individual layer of the bi-layer dielectric materials are calculated.

**Keywords:** Complex permittivity, Microwave, transmission ABCD matrix, Dielectric material, Optimizations methods.

### 1. Introduction

The bi-layer dielectric materials are currently used in many practical applications such as microwave integrated circuits and monolithic microwave integrated circuits. By choosing the electromagnetic properties and appropriate thickness for each layer, it is possible to synthesize composite bi-layer materials with novel electromagnetic properties otherwise not found in a single material [1]. The choice of a technique of characterization is initially determined by the exploited frequency band, then by the physical properties of material. A number of techniques have been developed and used to determine the complex permittivity of single layer dielectric materials [2-3]. These include free space methods, cavity resonators techniques, and transmission line or waveguide techniques. Each technique has its distinct advantages and drawbacks. The free space methods are used when the material is available in a big sheet form [4] [5]. These measurements are less accurate because of unwanted reflections from surrounding objects. The resonant cavity measurement technique is more accurate [6]. The disadvantage of this technique is that it is narrowband. For measurements of complex permittivity of material over a wideband of frequencies, transmission line, or waveguide techniques are widely used [7-8].

In this paper, a waveguide measurement method is presented to estimate the complex permittivity  $\epsilon_r^*$  of each layer of a composite bi-layer material. This method requires associating an optimization program with dynamic electromagnetic analysis. The electromagnetic analysis is

based on the use of transmission ABCD matrix. The complex permittivity computed from a data-processing program, using a numerical optimization procedure to match calculated and measured values of the S-parameters. Lately, the same procedure has been used for the measurement of the complex permittivity of each layer's substrate, and good results were obtained in the Ku-band.

### 2. Theory

#### 2.1 Direct Problem

This section presents computation of the ABCD parameters of rectangular waveguide loaded with bi-layer dielectric sample presented in figure 1. The S-parameters is found from an analysis of the electric field at the sample interfaces.

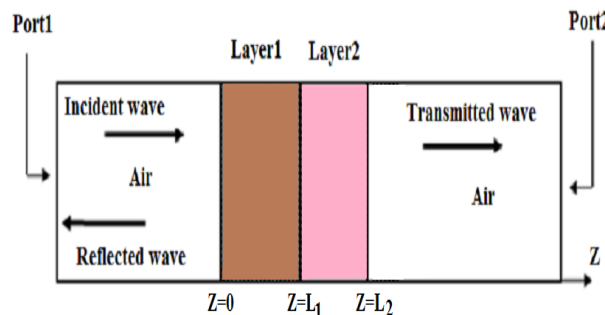


Fig.1 Rectangular waveguide loaded with bi-layer dielectric material.

Assuming that only the dominant  $TE_{10}$  mode propagates in the loaded waveguide (see Fig. 1), the formulation of the  $S_{ij}$ -parameters can be expressed in terms of bi-layer dielectric material sample thickness ( $L_1, L_2$ ) and unknown permittivity ( $\epsilon_{r1}, \epsilon_{r2}$ ), using ABCD-parameters as follows [9-10]:

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$$\begin{bmatrix} A_m & B_m \\ C_m & D_m \end{bmatrix} = \begin{bmatrix} \cosh(\gamma_m L_m) & Z_m \sinh(\gamma_m L_m) \\ \frac{\sinh(\gamma_m L_m)}{Z_m} & \cosh(\gamma_m L_m) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{j=1}^m \begin{bmatrix} A_m & B_m \\ C_m & D_m \end{bmatrix} \quad (2)$$

Where  $\gamma_m$ ,  $Z_m$  and  $Z_0$  are the propagation constant, wave impedance of the material sample and empty waveguide impedance respectively, and given as:

$$\gamma = \sqrt{\left(\frac{\pi}{a}\right)^2 - \omega^2 \mu_r^* \epsilon_r^*} \quad (3)$$

$$Z_m = \frac{j\omega\mu}{\gamma_m} \quad (4)$$

$$Z_0 = \frac{j\omega\mu_0}{\gamma_0} \quad (5)$$

Where (a) is the appropriate (maximum) cross-sectional dimension of the waveguide, ( $\epsilon_{r1}$ ,  $\epsilon_{r2}$ ) are the complex permittivity relative of bi-layer dielectric material and  $\omega$  is the angular frequency.

The ABCD-parameters can be directly converted to S-parameters using the following equations:

$$S_{11} = \left(\frac{A + B/Z_0 - CZ_0 - D}{X}\right) \quad (6)$$

$$S_{12} = \left(\frac{2(AD - CB)}{X}\right) \quad (7)$$

$$S_{21} = \left(\frac{2}{X}\right) \quad (8)$$

$$S_{22} = \left(\frac{-A + B/Z_0 - CZ_0 + D}{X}\right) \quad (9)$$

$$X = (A + B/Z_0 + CZ_0 + D) \quad (10)$$

### 2.2 Inverse Problem

This section presents computation of the complex dielectric constant in a given sample with a specific prior knowledge of the thickness of each layer. For this an optimization method on MATLAB [11], by using `fminsearch` function which is based on the Nelder-Mead sequential simplex algorithm[12]. which finds the minimum of a scalar function of several variables from an initial guess of the complex permittivity was  $\epsilon_r' = 1.0$ ,  $\epsilon_r'' = 0.001$  and optimization settings by using maximum number of function evaluations = 1000, and termination tolerance on the function value =  $10^{-08}$  to find the complex permittivity of each layer of bi-layer dielectric sample to match the measured  $S_{ij}^m$ -parameters and calculated  $S_{ij}$ -parameters. The error function (objective function) we want to minimize is written as follows:

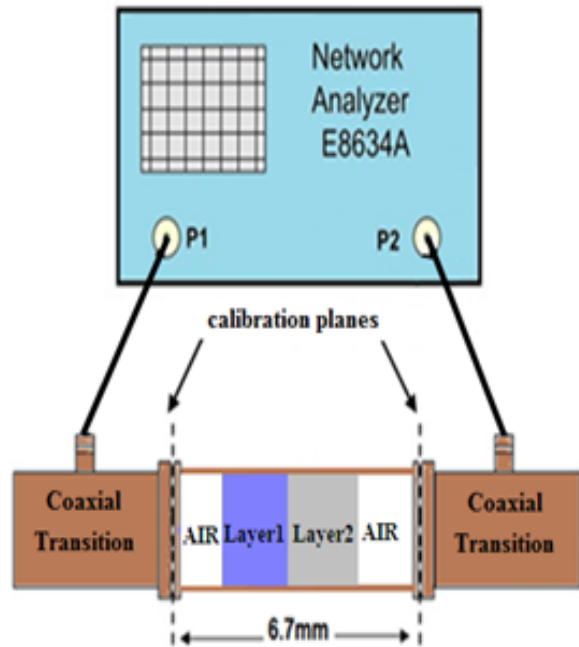
$$F = \sum_{i,j=1,2} ((\text{Real}(S_{ij}(\epsilon_r', \epsilon_r'') - S_{ij}^m))^2 + (\text{Imag}(S_{ij}(\epsilon_r', \epsilon_r'') - S_{ij}^m))^2) \quad (11)$$

With

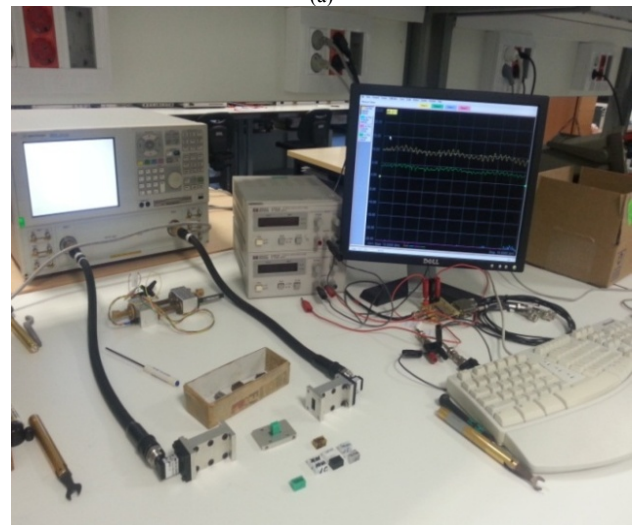
$$F = F(\epsilon_{r1}', \epsilon_{r1}'', \epsilon_{r2}', \epsilon_{r2}'')$$

### 3. Experimental Setup and Results

We consider the measurement setup shown in figure 2. The  $S_{ij}$ -parameters at references plane of a Ku-band rectangular waveguide WR62 (with dimensions:  $a=15.8\text{mm}$  and  $b=7.9\text{mm}$ ) loaded by a bi-layer dielectric material was measured using Network Analyzer. Before the measurements, a calibration using a standard waveguide calibration kit is done to measure the  $S_{ij}$ -parameters at references plane. The material sample is sufficiently machined, in order to minimize the effect of the gap between conductive walls and the sample material to be characterized. However, the sample occupies a part of the cutting plane of the waveguide. All measurements are performed at Ku- band with 401 frequency points.



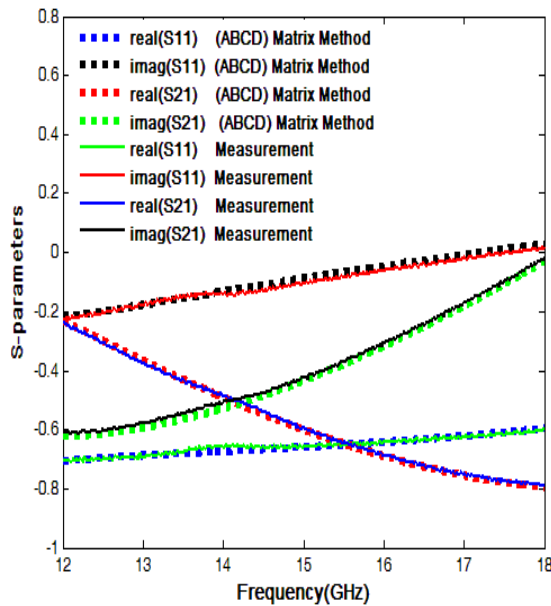
(a)



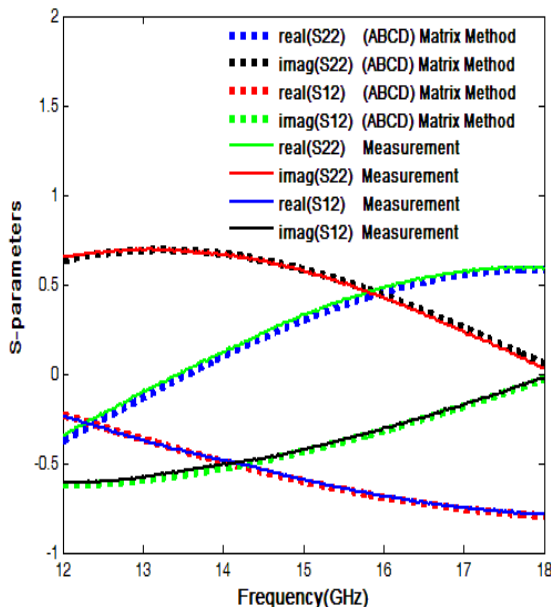
(b)

**Fig. 2** Rectangular waveguide measurement setup loaded by a bi-layer dielectric material

To validate the direct problem, the  $S_{ij}$ -parameters of rectangular waveguide loaded by the bi-layer dielectric material FR4 ( $\epsilon_r' = 4.4, \epsilon_r'' = 0.08$  and thickness  $D1 = 1.5\text{mm}$ ) with Teflon ( $\epsilon_r' = 2.08, \epsilon_r'' = 0.002$  and thickness  $D2 = 1.9\text{mm}$ ) are calculated using the procedure described in section (2.1) and measured with E8634A Network Analyzer as shown in figure 3. It is seen, from these results, an excellent agreement between measured and calculated  $S_{ij}$ -parameters.



(a)



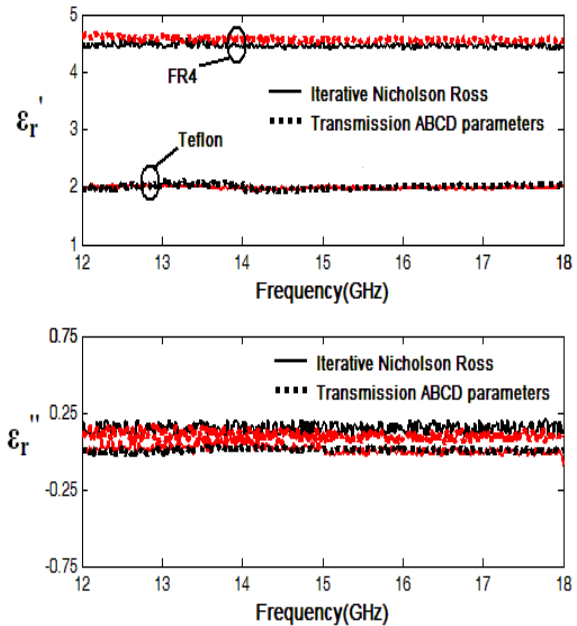
(b)

**Fig.3** Measured and calculated  $S_{ij}$ -parameters of rectangular waveguide loaded by the bi-layer dielectric sample FR4 ( $\epsilon_r' = 4.4, \epsilon_r'' = 0.08$ ) with Teflon ( $\epsilon_r' = 2.08, \epsilon_r'' = 0.002$ ).

For the inverse problem, using the procedure described in sections (2.1) and (2.2), the complex permittivity ( $\epsilon_r', \epsilon_r''$ ) of each layer for bi-layer dielectric material was determined in the Ku-band frequencies. In the first estimate for the starting frequency, the initial guess of the complex permittivity was

$\epsilon_r' = 1.0$  and  $\epsilon_r'' = 0.001$ . To validate our method we characterize a single layer dielectric material using iterative Nicholson Ross method [13]. This method allows to determine the complex permittivity of single layer dielectric material, but it does not allow to determine the complex permittivity of each layer for bi-layer material. The values of complex permittivity of FR4 with thickness  $D1 = 1.5\text{mm}$  and Teflon with thickness  $D2 = 1.9\text{mm}$  for bi-layer material FR4-Teflon are determined and plotted in fig.4.

The results obtained for the complex permittivity of each layer for bi-layer FR4-Teflon by using the procedure described in this work are in good agreement with the results obtained of the same single layer of Teflon and FR4 using iterative Nicholson Ross method [13].



**Fig.4** Complex relative permittivity of each layer for bi-layer material FR4-Teflon by Transmission ABCD parameters and Iterative Nicholson Ross

To validate this technique, the complex permittivity ( $\epsilon_r', \epsilon_r''$ ) of Delrin with thickness  $D1 = 2\text{mm}$  and Teflon with thickness  $D2 = 1.9\text{mm}$  for bi-layer Delrin-Teflon were determined in the Ku-band frequencies and presented in figure 5. From this figure, it is seen that the results obtained agree quite well with the results obtained with iterative Nicholson Ross method [13].

Using the same procedure as described above, the complex permittivity ( $\epsilon_r', \epsilon_r''$ ) of FR4 with thickness  $D1 = 1.5\text{mm}$  and Delrin with thickness  $D2 = 2\text{mm}$  for bi-layer FR4-Delrin were determined in the Ku-band frequencies and shown in fig.6. The results in this figure are compared very well with the results obtained with iterative Nicholson Ross method [13], which agree well with the published data imply that this technique is able to provide acceptable accuracy in real applications.

The table 1 shows that the error in the real part of complex permittivity is small (within 1.96%), but in the imaginary part can be large ( $\leq 3.7\%$ ) for the studies materials. The results show a good agreement between the average values of the relative complex permittivity of individual dielectric materials and those for each layer of the bi-layer dielectric materials.

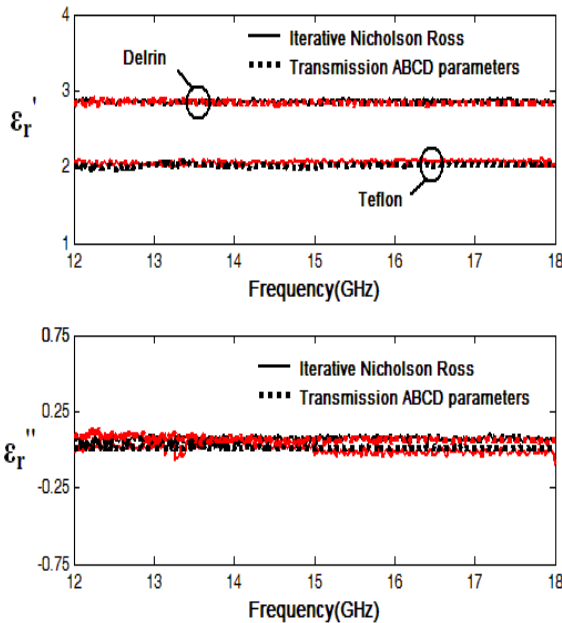


Fig. 5 Complex relative permittivity of each layer for bi-layer material Delrin-Teflon by Transmission ABCD parameters and Iterative Nicholson Ross

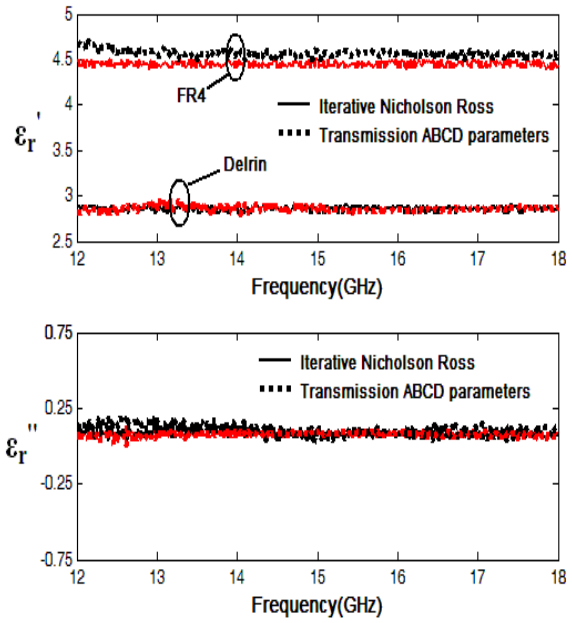


Fig 6 Complex relative permittivity of each layer for bi-layer material FR4-Delrin by Transmission ABCD parameters and Iterative Nicholson Ross

**Table 1** Average relative complex permittivity and the average relative error percentage on the real and imaginary parts of the complex permittivity at Ku-band frequencies

Bi-layer materials	individual layer	$\langle \epsilon_r^* \rangle$		$\langle \%Error \epsilon_r' \rangle$	$\langle \%Error \epsilon_r'' \rangle$
		Bi-layer measurement (Transmission ABCD parameters)	Iterative Nicholson Ross		
FR4-Teflon	FR4	4.4670 - j0.0918	4.4089 - j0.0889	≤1.96%	≤3.7%
	Teflon	2.0331 - j0.0073	2.0802 - j0.0037		
FR4-Delrin	FR4	4.4988 - j0.0971	4.4089 - j0.0889		
	Delrin	2.8553 - j0.0756	2.9023 - j0.0643		
Delrin-Teflon	Delrin	2.8423 - j0.0704	2.9023 - j0.0643		
	Teflon	2.0203 - j0.0104	2.0802 - j0.0037		

The complex permittivity values estimated for FR4, Teflon and Delrin by this technique described in this work with those obtained by another method of single layer material are similar. These results show that the technique developed in this work allows determining the complex permittivity of each layer for a bi-layer dielectric material.

Fig.7 presents the complex relative permittivity of each individual layer for bi-layer dielectric material FR4-Teflon with and without air-gap, by different air-gap measurement, we can say that the average relative error percentage on the real and imaginary parts of the complex permittivity of Teflon and FR4 without and with air gap within 1.7%, 3%, 7% for 0.1mm, 0.15mm, 0.20mm respectively at Ku-band frequencies. Therefore it can be concluded that the air gaps around samples has a negligible effect on the measured value of permittivity of low loss materials provided that the air-gap length is less or equal 0.1mm.

#### 4. Conclusion

In this paper, the transmission (ABCD) matrix method combined with the fminsearch MatLab optimization search function has been presented to determine the complex permittivity of each layer for bi-layer dielectric material at the Ku-band using rectangular waveguide measurements WR62. The S-parameters are measured using the E8634A Network Analyzer and calculated using transmission (ABCD) matrix. The technique developed in this work makes it possible to determine the complex permittivity of each layer for bi-layer dielectric material. By matching the calculated value with the measured value of  $S_{ij}$ -parameters of a Ku-band rectangular waveguide, loaded by bi-layer dielectric material. The results obtained using this method are in good agreement with values of complex permittivity obtained with iterative Nicholson Ross method.

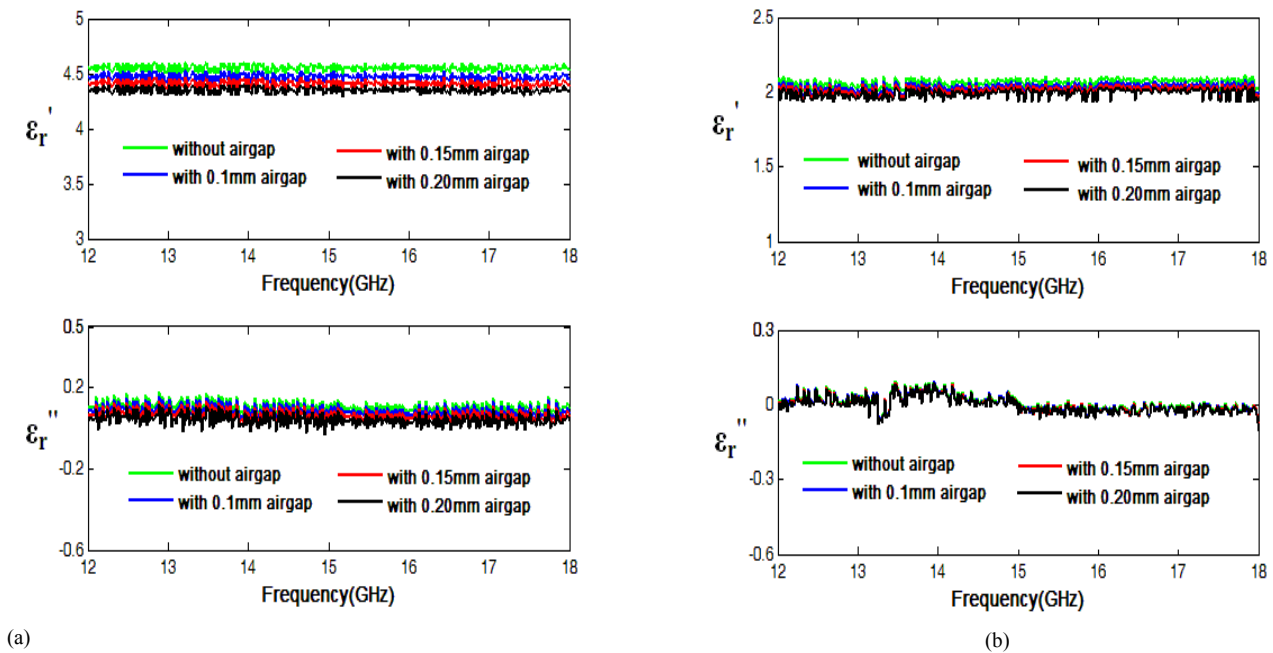


Fig. 7 Complex relative permittivity of each layer for bi-layer material a)FR4-b)Teflon with and without air-gap.

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