Hypercyclicity for the elements of the commutant of an operator

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Abstract

Given a bounded linear operator T acting on a complex Banach space, we obtain a spectral condition implying that each non-scalar operator in the commutant of T has a hypercyclic multiple, and we show several examples of operators satisfying this condition. We emphasize that for some of these examples we do not have a description of the commutant of T.

Key words: Hypercyclic operator, Hypercyclic commutant, Cesàro operator, Composition operator
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1 Introduction

A bounded linear operator T on a Banach space is called *hypercyclic* if it has a dense orbit. A vector x for which the orbit $\{T^n x\}$ is dense is called a *hypercyclic vector* for T. We refer to the books [2] and [9] for information on hypercyclic operators.

When we sometimes abusively say that an operator T has a *hypercyclic commutant*, what we really mean is that each non-scalar operator in the commutant has a hypercyclic multiple (a non-scalar operator is an operator which is not a multiple of the identity). The study of operators with hypercyclic commutant was originated by Godefroy and Shapiro in their seminal 1991 paper [7]. Their work suggests that operators behaving like backward shifts tend to transfer hypercyclicity to "appropiate" operators in their commutant. The word "appropiated" means that we must discard from the commutant some operator that can never be hypercyclic, like the multiples of the identity or the contractions.

In their work, Godefroy and Shapiro use some descriptions of the commutants of the involved operators acting on Hilbert spaces which are not valid in the Banach space setting, and they also consider some bilateral weighted shifts and composition operators. Later, Herzog and Schmoeger [11] extended Godefroy and Shapiro's results to general operators T on Banach spaces. Using the holomorphic functional calculus, they transfer the hypercyclicity to operators $\varphi(T)$ under certain spectral conditions on T. This work was improved by Bermúdez and Miller [2]. Since the holomorphic functional calculus does not cover the whole commutant, the following general question arises: *Given a hypercyclic operator T, when T has a hypercyclic commutant?* Recently, V. Müller [13] improved the results of [11,2], and discovered that these results become stronger if we consider operators with closed range. His study allows to prove that any non-scalar operator in the commutant of a generalized backward shift (acting on certain Banach spaces) is supercyclic, solving a longstanding question posed by Godefroy and Shapiro.

In this paper we study Banach space operators T with hypercyclic commutant. To characterize the commutant of T is a really complicated problem even for well known operators in Hilbert spaces. However we show that, under certain spectral conditions and without a description of the commutant of T, it is possible to ensure that T has a hypercyclic conmutant.

The paper is structured as follows. In Section 2 we prove our main result (Theorem 2.1) that gives a sufficient spectral condition for an operator T to have a "hypercyclic commutant". Section 3 is devoted to give examples of operators satisfying that spectral condition: Bilateral weighted shifts, unilateral weighted backward shifts, the adjoint of the discrete Cesàro operator acting on the sequence spaces ℓ^p, the continuous Cesàro operator acting on $L_p(0,1)$, and the adjoint of the Cesàro operator acting on the Hardy spaces $H^p(\mathbb{D})$. We also show that some composition operators acting on the Hardy spaces, which are induced by linear fractional self-maps of the unit disk $\mathbb{D}$, fail both the hypothesis and the thesis of Theorem 2.1. The results of the paper suggest some questions which will be formulated in the last section.

2   Main Theorem

In this section we give a general theorem for Banach space operators T asserting that, under certain spectral conditions, each operator in the commutant of T different from λI has a hypercyclic multiple.

Theorem 2.1 *Let T be a bounded operator on a (complex) Banach space X. Suppose that the spectrum of T contains a non-empty connected open subset U such that the following conditions are satisfied:*

(1) Every $\lambda \in U$ is a simple eigenvalue of T,
(2) $\mathrm{span}\{\mathrm{Ker}(T - \lambda I) \ : \ \lambda \in U\}$ is dense in X,
(3) there exists a holomorphic function $x : \lambda \in U \to \hat{x}(\lambda) = x_\lambda \in X$ such that $0 \neq x_\lambda \in \ker(T - \lambda I)$.

Then every operator in the commutant of T different from λI has a hypercyclic multiple.

Proof. First let us see that given a non-empty open subset $V \subset U$, the set

$$\mathrm{span}\{\mathrm{Ker}(T - \lambda I) \ : \ \lambda \in V\}$$

remains dense in X. Otherwise, there exists $0 \neq x^\star \in X^\star$ such that $x^\star(\hat{x}(\lambda)) = 0$ for all $\lambda \in V$. By the analyticity condition (3) we have that $x^\star(\hat{x}(\lambda)) = 0$ for all $\lambda \in U$, and by (1) and (2) $x^\star = 0$, a contradiction.

Let $A \in L(X)$ in the commutant of T with $A \neq \mu I$. For each $\lambda \in U$ we have $Tx_\lambda = \lambda x_\lambda$, hence $TAx_\lambda = \lambda Ax_\lambda$. Thus by condition (1) we have $Ax_\lambda = a(\lambda)x_\lambda$ for some complex number $a(\lambda)$. So we get a function $a : U \to \mathbb{D}$. Boundedness of the operator A provides boundedness of $a(\lambda)$ on U, and $A \neq \mu I$ implies that a is non-constant. Let us see that $a : U \to \mathbb{D}$ is a holomorphic function.

For each $y^\star \in X^\star$, our function a satisfies

$$a(\lambda) = \frac{y^\star(Ax_\lambda)}{y^\star(x_\lambda)}.$$

Thus it is holomorphic on $U \setminus Z$, where $Z = \{\lambda \ : \ y^\star(x_\lambda) = 0\}$ is a discrete subset of U. But for $\lambda_0 \in Z$ we can take $u^\star \in X^\star$ such that $u^\star(x_{\lambda_0}) \neq 0$. This implies that our function

$$a(\lambda) = \frac{u^\star(Ax_\lambda)}{u^\star(x_\lambda)}$$

is holomorphic also on λ_0, hence $a(\lambda)$ is holomorphic on U.

Since $a(\lambda)$ is non-constant, $a(U)$ is an open subset of the complex plane. Then for some $m \in \mathbb{C}$ the set $m \cdot a(U)$ intersects the unit circle. To prove that mA

is hypercyclic, let us see that the subsets

$$\mathrm{span}\{\ker\,(\mu A - \lambda I)\,:\,|\lambda| < 1\}\ \text{ and }\ \mathrm{span}\{\ker\,(\mu A - \lambda I)\,:\,|\lambda| > 1\}$$

are dense in X, and the hypercyclicity of mA will follow from the Godefroy-Shapiro Criterion (see Corollary 1.10 in [2]).

Indeed, $\mathrm{span}\{\ker\,(\mu A - \lambda I)\,:\,|\lambda| < 1\} = \mathrm{span}\{\ker(T - \lambda I)\,:\,\lambda \in (m \cdot a)^{-1}(\mathbb{D})\}$ is dense by the considerations at the beginning of the proof, and a similar argument is valid for the other set. □

Remarks 2.2 *The hypothesis of Theorem 2.1 implies that*

(a) the space X is infinite dimensional and separable, and
(b) the operator T fails the single valued extension property (see [1]).

3   Some applications

In this section we show that several concrete examples of operators satisfy the hypothesis of Theorem 2.1.

3.1   Bilateral weighted shifts.

Let us consider a bilateral weighted shift W on the complex Hilbert space of sequences $\ell_2(\mathbb{Z})$, that is defined by

$$W e_n = w_n e_{n+1},$$

where n runs through the set of all integers $\mathbb{Z}$, $\{e_n\,:\,n \in \mathbb{Z}\}$ is the natural orthonormal basis of $\ell_2(\mathbb{Z})$, and (w_n) is a bounded sequence of strictly positive numbers, the sequence of weights. We follow the notation used in the survey by Allen L. Shields [18].

We consider the quantities

$$r_3^+(W) := \limsup_{n \to \infty}(w_0 \cdots w_{n-1})^{1/n}$$

and

$$r_2^-(W) := \liminf_{n \to \infty}(w_{-1} \cdots w_{-n})^{1/n}.$$

It turns out (see [18] Theorem 9) that when $r_3^+(W) < r_2^-(W)$ the set

$$U := \{\lambda \in \mathbb{C}\,:\,r_3^+(W) < |\lambda| < r_2^-(W)\}$$

is contained in the point spectrum of W, that every $\lambda \in U$ is a simple eigen-value of W and that a corresponding eigenvector is given by the following expression:

$$x_\lambda = e_0 + \sum_{n=1}^{\infty} \frac{w_0 \cdots w_{n-1}}{\lambda^n} e_n + \sum_{n=0}^{\infty} \frac{\lambda^n}{w_{-1} \cdots w_{-n}} e_{-n}. \qquad (1)$$

Clearly the function $f(\lambda) = x_\lambda$ is analytic on U. Let us see that the set $\text{span}\{x_\lambda : \lambda \in U\}$ is dense in $\ell_2(\mathbb{Z})$. Indeed, given $(y_i) \in \ell_2(\mathbb{Z})$ we have

$$\langle x_\lambda, (y_i) \rangle = y_0 + \sum_{n=1}^{\infty} \frac{w_0 \cdots w_{n-1}}{\lambda^n} y_n + \sum_{n=0}^{\infty} \frac{\lambda^n}{w_{-1} \cdots w_{-n}} y_{-n}.$$

Therefore if $\langle x_\lambda, (y_i) \rangle = 0$ for all $\lambda \in U$, then $y_i = 0$ for all $i \in \mathbb{Z}$. Thus W satisfies the conditions of Theorem 2.1 and we have the following result.

Theorem 3.1 *Let W be an injective bilateral shift with $r_3^+(W) < r_2^-(W)$. Then every operator in the commutant of W different from λI has a hypercyclic multiple.*

Remarks 3.2 *It is not difficult to see that Theorem 3.1 remains valid for bilateral weighted shifts acting on $\ell_p(\mathbb{Z})$, $1 \leq p < \infty$.*

3.2 The unilateral weighted backward shift.

Here we consider a unilateral weighted shift U on the complex Hilbert space of sequences ℓ_2, that is defined by

$$U e_n = w_n e_{n-1}, \, n \geq 2 \qquad U e_1 = 0$$

where n runs through the set of all positive integers $\mathbb{N}$, $\{e_n : n \in \mathbb{N}\}$ is the natural orthonormal basis of ℓ_2, and (w_n) is a bounded sequence of strictly positive numbers, the sequence of weights. The relevant quantity to describe the spectrum of U is

$$r_2(U) = \liminf_{n \to \infty} (w_1 \cdot \cdots \cdot w_n)^{1/n}.$$

If $r_2(U) > 0$ then $U_p = \{\lambda \, |\lambda| < r_2(U)\} \subset \sigma_p(U)$ and consists of simple eigenvalues (see [18] Theorem 8).

A similar argument to the one given in Subsection 3.1 provides the following result.

Theorem 3.3 *Let U be an unilateral weighted backward shift with $r_2(U) > 0$. Then every operator in the commutant of U different from λI has a hypercyclic multiple.*

3.3 The adjoint of the Cesàro operator on the sequences spaces

The Cesàro operator $C : \ell_p \to \ell_p$ $(1 < p < \infty)$ is given by

$$C((x_n)) = \left(\frac{x_1 + \cdots + x_n}{n} \right)$$

and has norm equal to q, where $1/p + 1/q = 1$.

The adjoint of the Cesàro operator $C^\star : \ell_q \to \ell_q$ is given by

$$C^\star e_n = \frac{e_1 + \cdots + e_n}{n}.$$

Let us denote $U_q = \{\lambda \in \mathbb{C} : |\lambda - q/2| < q/2\}$, and let us consider the sequences $\hat{x}(\lambda) = (x_n(\lambda))_{n=1}^{\infty}$, where

$$x_1(\lambda) = 1 \quad \text{and} \quad x_n(\lambda) = \prod_{j=1}^{n-1} \left(1 - \frac{1}{j\lambda} \right) \quad n > 1.$$

Then $\hat{x}(\lambda) \in \mathrm{Ker}(C^\star - \lambda I)$ for $\lambda \in U_q$ and U_q consists of simple eigenvalues of $C^\star$ (see [8] for proper references).

Note that that $x_n(1/k) \neq 0$ if and only if $n \leq k$, and that the sequence $(1/k)$ is contained in U_q. Hence $\mathrm{span}\{\mathrm{Ker}(C^\star - \lambda I) : \lambda \in U_q\}$ is dense in ℓ_q. Moreover it is not difficult to show that the map $\hat{x} : \lambda \in U_q \to \hat{x}(\lambda) \in \ell_q$ is analytic.

Therefore the conditions of Theorem 2.1 are fully satisfied by $C^\star$, and as in the previous examples we conclude that every operator in the commutant of C different from λI has a hypercyclic multiple.

Remarks 3.4 *The Cesàro operator C is also bounded in the space c_0 of sequences converging to 0. In this case the adjoint $C^\star : \ell_1 \to \ell_1$ satisfies the same conditions that $C^\star$ on ℓ_q with $U_1 = \{\lambda \in \mathbb{C} : |\lambda - 1/2| < 1/2\}$. Thus it satisfies the hypothesis of Theorem 2.1.*

3.4   The Cesàro operator on spaces of integrable functions

The Cesàro operator $C : L_p(0,1) \to: L_p(0,1)$ $(1 < p < \infty)$ is given by

$$Cf(x) = \frac{1}{x} \int_0^1 f(s) \, ds$$

and has norm equal to q, where $1/p + 1/q = 1$. Moreover, it was proved by Leibowitz (see [12]) that the point spectrum of C is the set

$$U_p = \{\lambda \in \mathbb{C} \ : \ |\lambda - q/2| < q/2\} = \{\lambda \in \mathbb{C} \ : \ \mathrm{Re}(1/\lambda) > 1/q\},$$

that U_p consists of simple eigenvalues of C, and that for every $\lambda \in U_p$, the function $g_\lambda(x) = x^{-1+1/\lambda}$ satisfies $Cg_\lambda = \lambda g_\lambda$.

Obviously, the function $\lambda \in U_p \to g_\lambda \in L_p(0,1)$ is analytic. Moreover, since $f_{1/n}(x) = x^{n-1}$ for all $n \geq 1$, the set $\mathrm{span}\{f_\lambda \ : \ \lambda \in U_p\}$ is dense in $L_p(0,1)$. Therefore the conditions of Theorem 2.1 are fully satisfied by C, and we can conclude that every operator in the commutant of C different from λI has a hypercyclic multiple.

3.5   The adjoint of the Cesàro operator on Hardy spaces of analytic functions

The Cesàro operator acting on the space $H(\mathbb{D})$ of analytic functions on the unit disk $\mathbb{D}$ is defined formally by

$$Cf(z) = \sum_{n=0}^{\infty} \left(\frac{1}{n+1} \sum_{k=0}^{n} a_k \right) z^n$$

where $f(z) = \sum_{k=0}^{\infty} a_k z^k$. The operator C is bounded on several Banach spaces of analytic functions on $\mathbb{D}$. Boundedness of C on the Hardy space $H^p(\mathbb{D})$ was proved by Hardy for $p = 2$, and by A. Siskakis for $1 \leq p < \infty$ ([19]). It is not bounded acting on $H^\infty(\mathbb{D})$. For further examples we refer to [14].

We will need the following properties provided by Persson [14, Theorem B].

Theorem 3.5 *Let $1 < p < \infty$, and let C be the Cesàro operator acting on the Hardy space $H^p(\mathbb{D})$.*

(1) $\sigma(C) = \{\lambda \in \mathbb{C} \ : \ |\lambda - p/2| \leq p/2\}$

(2) For $|\lambda - p/2| < p/2$, the range of $C - \lambda I$ is a closed subspace of $H^p(\mathbb{D})$ of codimension 1.

Let us denote $U_p := \{\lambda \in \mathbb{C} : |\lambda - p/2| < p/2\} = \{\lambda \in \mathbb{C} : \mathrm{Re}(1/\lambda) > 1/p\}$. It follows from part (2) of Theorem 3.5 that each $\lambda \in U_p$ is a simple eigenvalue of the adjoint operator $C^\star$.

For each $\lambda \in U_p$ we consider the function f_λ defined on the unit disk $\mathbb{D}$ by

$$f_\lambda(z) = (1-z)^{\frac{1}{\lambda}-1}.$$

Proposition 3.6 *Let $1 < p < \infty$ and let q such that $1/p + 1/q = 1$.*

(1) For each $\lambda \in U_p$, $f_\lambda \in H^q(\mathbb{D})$.

(2) The function $\lambda \in U_p \to f_\lambda \in H^q(\mathbb{D})$ is analytic.

Proof. Once we establish (1), part (2) is immediate.

As usual, we identify $H^q(\mathbb{D})$ with the subspace of functions in $L^q(\mathbb{T})$ whose Fourier coefficients satisfy $\hat{f}(n) = 0$ for $n < 0$.

To prove (1), it is enough to note that $\lambda \in U_p$ implies $\mathrm{Re}(1/\lambda) - 1 > -1/q$; therefore $|f_\lambda(z)|^q \leq \mathrm{const}|1-z|^\alpha$ with $\alpha > -1$, hence $f_\lambda \in H^q(\mathbb{D})$. $\square$

Recall that for $1 < p < \infty$, $H^p(\mathbb{D})$ is a complemented subspace of $L^p(\mathbb{T})$. As a consequence, the dual space $H^p(\mathbb{D})^\star$ is isomorphic to $H^q(\mathbb{D})$. In fact, for every $G \in H^p(\mathbb{D})^\star$ there exists $g \in H^q(\mathbb{D})$ such that for each $f \in H^p(\mathbb{D})$,

$$\langle G, f \rangle = \frac{1}{2\pi} \int_{\mathbb{T}} \overline{g(t)} f(t) \, dt.$$

In this way, given the Cesàro operator C acting on $H^p(\mathbb{D})$ $(1 < p < \infty)$, we can see $C^\star$ as an operator acting on $H^q(\mathbb{D})$.

With this identification, each $f \in H^p(\mathbb{D})$ has the form $f(\theta) = \sum_{n=0}^\infty a_n e^{in\theta}$, the action of C is given by

$$(Cf)(\theta) = \sum_{n=0}^\infty \frac{1}{n+1} \left(\sum_{k=0}^n a_k \right) e^{in\theta},$$

and it easy to check that the action of the adjoint $C^\star$ on the elements $e^{in\theta}$ of $H^q(\mathbb{D})$ is given by $C^\star e^{in\theta} = (n+1)^{-1} \sum_{k=0}^n e^{ik\theta}$.

Therefore, if $\sum_{n=0}^\infty b_n e^{in\theta} \in H^q(\mathbb{D})$, then

$$C^\star \left(\sum_{n=0}^\infty b_n e^{in\theta} \right) = \sum_{n=0}^\infty \frac{b_n}{n+1} \left(\sum_{k=0}^n e^{ik\theta} \right) = \sum_{k=0}^\infty \left(\sum_{n=k}^\infty \frac{b_n}{n+1} \right) e^{ik\theta}.$$

Proposition 3.7 *As an operator acting on $H^q(\mathbb{D})$, $C^\star$ satisfies $C^\star f_\lambda = \lambda f_\lambda$ for each $\lambda \in U_p$.*

Proof. It is easy to check that for each $\lambda \in U_p \setminus \{1/n : n \in \mathbb{N}\}$, the derivatives of f_λ satisfy

$$\frac{f_\lambda^{(n)}(0)}{n!} = \prod_{j=1}^{n}\left(1 - \frac{1}{j\lambda}\right).$$

Proceeding as in the proof of Theorem 2 in [5], we get $C^\star f_\lambda = \lambda f_\lambda$.

The case $\lambda \in \{1/n : n \in \mathbb{N}\}$ is easy. In fact, denoting F_n the subspace generated by $\{e^{ik\theta} : 0 \leq k \leq n\}$, we have that $C^\star F_n \subset F_n$ for $n \in \mathbb{N} \cup \{0\}$, hence $\{f_{1/n} : n \in \mathbb{N}\}$ generates a dense subspace of $H^q(\mathbb{D})$. $\square$

As a consequence we obtain the following result.

Theorem 3.8 *Let C be the Cesàro operator acting on $H^p(\mathbb{D})$, $1 < p < \infty$. Then every operator in the commutant of $C^\star$ different from λI has a hypercyclic multiple.*

3.6   Composition operators without a "hypercyclic commutant"

Here we analyze some examples of operators T which do not satisfy the hypothesis of Theorem 2.1, and also fail to have a hypercyclic commutant.

(a) In the absence of an open set contained in the point spectrum, an operator does not satisfy the hypothesis of Theorem 2.1. This is the case for the following example studied by Godefroy and Shapiro (see [7] Section 3.7).

Let B be a compact unilateral weighted backward shift on the sequence space ℓ^p, $1 \leq p < \infty$. By a result of Salas (see [15]), $I + B$ is hypercyclic. However λB is hypercyclic for no $\lambda \in \mathbb{C}$ because λB is quasinilpotent and the orbit of each vector is bounded.

(b) Next we consider some examples of operators for which the eigenvalues are not simple, and they do not have hypercyclic commutant.

Let φ denote a linear fractional selfmap of the unit disc, and let C_φ be the composition operator induced by φ acting on the Hardy space $H^2(\mathbb{D})$. Its action is given by $C_\varphi f = f \circ \varphi$. The linear dynamics of such composition operators was studied by Bourdon and Shapiro ([4]). A comprehensive treatment on dynamics of composition operators is carried out in the books [4] and [16].

Theorem 3.9 *Let φ be a linear fractional self map of the unit disk, and let C_φ denote the associated composition operator. Then there exists an operator T in the commutant of C_φ such that λT is hypercyclic for no $\lambda \in \mathbb{C}$.*

Proof. The only cases which is necessary to check are those in which C_φ is hypercyclic, namely, when φ is a hyperbolic or parabolic automorphism (see [16] Chapter 7). In these cases, the point spectrum of C_φ contains the point 1 (see [6] Chapter 7, or [17] Theorems 4.1.1 (ii), 4.1.2 and 4.1.3).

Let f be a non-zero function such that $C_\varphi f = f$. The the analytic Toeplitz operator T_f commutes with C_φ and T_f has not a hypercyclic multiple because $\sigma_p(T_f^\star)$ is non-empty. $\square$

4 Concluding remarks

To understand the influence of hypercyclic operators in linear dynamics it would be desirable to characterize operators with a *hypercyclic commutant*.

Let us denote by H an infinite dimensional complex Hilbert space. Using the ideas of D. A. Herrero in [10] (see also [13] Section 4) together with Theorem 2.1 we obtain the following result.

Corollary 4.1 *Let us denote by $\mathcal{R}(H)$ the set of hypercyclic operators acting on H with a hypercyclic commutant. Then the closure of $\mathcal{R}(H)$ in the norm topology is equal to the norm-closure of the set of all hypercyclic operators.*

Thus, although the set of operators with a *hypercyclic commutant* is invariant under similarity, it does not seem that this class can be characterized in spectral terms. To understand the nature of this phenomenon we suggest to analyze the following question.

Problem 4.2 *Let T be a unilateral (respectively bilateral) weighted backward shift with weight sequence $\{w_n\}$ defined on ℓ^p, (respectively $\ell_p(\mathbb{Z})$) $1 \le p < \infty$.*

Is it be possible to characterize when T has a hypercyclic commutant in terms of the sequence $\{w_n\}$?

The Cesàro operator is also bounded on other spaces of analytic functions on the unit disc (see [14]) such as the Bergman spaces L_a^p, $p > 1$, weighted Bergman spaces $L_a^{p,\alpha}$ and Dirichlet spaces $D^{2,\alpha}$. From the results of Persson it follows that the adjoint $C^\star$ of the Cesàro operator on these spaces has an open set of set of simple eigenvalues. However we do not know a good representation on the dual space of such spaces.

Conjecture 4.3 *The adjoint $C^\star$ of the Cesàro operator C acting on L_a^p, $L_a^{p,\alpha}$ or $D^{2,\alpha}$ has a hypercyclic commutant.*

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