Focal modulation using rotating phase filters

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Abstract: We describe a simple method of refocusing optical systems that is based on the use of two identical phase filters. These filters are divided in annuli and each annulus is divided into sectors with a particular phase value. A controlled focus displacement is achieved by rotating one filter with respect to the other. This displacement is related with the filter parameters. Transverse responses are studied as a function of filters relative position. Furthermore, the experimental set up shows that theoretical prediction fit well with experimental results. The main advantage of this system is the ease of fabrication so that it could be useful in different applications requiring small size, light weight or thin systems, like mobile phone cameras, microscopy tomography, and others.

OCIS codes: (100.2980) Image enhancement; (120.2440) Filters.

References and links

1. N. A. Riza, "Axial scanning confocal microscopy with no moving parts," Opt. Photon. News **19**(12), 33 (2008).
2. E. J. Botcherby, R. Juskaitis, M. J. Booth, and T. Wilson, "An optical technique for remote focusing in microscopy," Opt. Commun. **281**(4), 880–887 (2008).
3. C. A. López, and A. H. Hirsa, "Fast focusing using a pinned-contact oscillating liquid lens," Nat. Photonics **2**(10), 610–613 (2008).
4. L. Dong, A. K. Agarwal, D. J. Beebe, and H. Jiang, "Adaptive liquid microlenses activated by stimuli-responsive hydrogels," Nature **442**(7102), 551–554 (2006).
5. S. Kuiper, and B. H. W. Hendriks, "Variable-focus liquid lens for miniature cameras," Appl. Phys. Lett. **85**(7), 1128–1130 (2004).
6. D. Graham-Rowe, "Liquid lenses make a splash," Nat. Photonics **sample**, 2–4 (2006).
7. H. Ren, S. Xu, Y. J. Lin, and S. T. Wu, "Adaptive-focus lenses," Opt. Photon. News **19**(10), 42–47 (2008).
8. L. W. Alvarez, "Two-element variable-power spherical lens," U.S. patent 3,305,294 (1967).
9. A. W. Lohmann, and D. P. Paris, "Variable fresnel zone pattern," Appl. Opt. **6**(9), 1567–1570 (1967).
10. S. Bara, Z. Jaroszewicz, A. Kolodziejczyk, and V. Moreno, "Determination of basic grids for subtractive moire patterns," Appl. Opt. **30**(10), 1258–1262 (1991).
11. S. Bernet, and M. Ritsch-Marte, "Adjustable refractive power from diffractive moiré elements," Appl. Opt. **47**(21), 3722–3730 (2008).
12. M. P. Cagigal, J. E. Oti, V. F. Canales, and P. J. Valle, "Analytical design of superresolving phase filters," Opt. Commun. **241**(4-6), 249–253 (2004).
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1. Introduction

Lenses able to vary their focal length are attractive because they can be used in many different fields. For example, they can be applied in microscopy to focus at different levels without perturbing the sample [1,2], in adaptive optics requiring a fast response to compensate defocus, which is the most important aberration besides tip and tilt [3], in human vision in order to compensate the presbyopia disease [4], and also in miniature cameras for portable applications [5,6]. Different types of microlenses have been developed for this task. Some of them are based on liquid crystals, other on elastic membranes and other in liquid-liquid or liquid-air interfaces [7]. In all cases these lenses have a reduced but non-negligible thickness and need an external electronic driver. As an alternative, there are devices with two components that attain focus change by the components displacement [8,9]. More recently, the use of two rotating filters has also been proposed. For example, Bara et al [10] proposed moiré based optical systems able to modify their focus, and Bernet et al [11] described

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diffractive moiré phase elements to obtain an adjustable focus. The obtained result is good but the phase map that composes the filter is difficult to be fabricated.

In this paper we propose a set up consisting on two identical phase filters placed beside a lens. The total phase shown by the two filters is the addition of the phases for each single filter. Since filters have a phase distribution that is a function of the position, by simply rotating one filter with respect to the other the total phase distribution can be modulated providing the required focus displacement. Here we propose a system based on rotating filters with the important advantage that it is very simple to be made: only discrete phase values (0, π/2, π and 3π/2) are used, instead of continuous phase values, and their spatial distribution is also very simple. For short displacements the focus moves in a continuous way from one focus to the next one making a zoom effect. In this travel the focus intensity suffers a small intensity modulation. When the displacement is large the intensity modulation is so strong that the zoom effect is lost and the system becomes a multifocal tunable optical system. The focus quality is high, and no additional aberrations are introduced in the initial and final positions, although for intermediate displacements the focus quality is slightly poor. The couple of filters is robust and its thickness is almost negligible. This allows its application to mobile phone cameras, portable reading systems, etc. In addition, the focusing mechanism is simple and easy to be performed. For example, if these filters were used in the human eye, the rotation of the filter could be performed by the ciliary muscle without the needing of external help.

The scheme of the paper is as follows. First, the basic theory of our filters is outlined. Secondly, the role of the different filter parameters is explained. Then, the focus displacement that can be attained with the filters is analyzed. In the next section, the transverse behavior of the PSF as the focus changes is studied. Finally, the experimental checking and the most relevant conclusions are presented.

2. Basic theory of segmented filters

To obtain an optical system able to produce focus displacement we propose the use of a couple of identical phase-only filters. Both filters are composed of a series of regions as shown in Fig. 1. These regions are organized in annuli and sectors. The number of annuli and sectors can be modified as a function of the displacement goal. The filters are used in cascade, one behind the other, so that the total phase introduced by the couple of filters is the addition of phases of both filters. Since the phase value in one filter depends on the position, the resulting phase value on a specific region, obtained as the addition of phases, will also depend on the region position.

The procedure proposed to obtain focus displacement is to rotate one filter with respect to the other. At the initial position, the two filters are placed with the same orientation. In this case, we choose not to introduce focus displacement and, hence, the resulting phases have to be as similar to zero as possible. On the other hand, when one filter is rotated a particular angle with respect to the other one, the resulting radial phase distribution has to produce a focus displacement. It is well known that a focus displacement is easily obtained when a phase screen whose phase evolves proportionally to the square of the radius is introduced. This defocus phase screen can be described by the Z_4 Zernike polynomial, given by $Z_4(\rho) = 2\rho^2-1$, where ρ is the normalized radial coordinate. Hence, our goal is to obtain a final phase distribution as similar as possible to the phase described by the Z_4 Zernike polynomial. Obviously, since we can only create annuli, we have to approximate the Z_4 Zernike polynomial using a stepped function. There are different possibilities for the value of the step height. The step value has to take values in the interval (0, 2π). It has been shown that step values that are not a multiple of π produce focus displacement [12]. Hence, the only possible step values for the total system to avoid focus displacement for the initial position are $n\pi$ where $n = 1, 2\ldots$ Then, in each filter we use a step value of π/2.

As an example we will analyze the case of the filter shown in Fig. 1. Figure 1a shows a phase distribution for the case of three annuli and four quadrants. The inner circle is composed of four sectors of phase 0. The intermediate annulus has alternatively phases of π/2

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and 0. The outer annulus has alternatively phases of π and 0. When the two filters, like that shown in Fig. 1a, are placed one beside the other the resulting phase is the addition of the two filter phases as shown in Fig. 1b. If one is rotated an angle of π/2 with respect to the other the radial phase obtained is that shown in Fig. 1c. This is a stepped version of the Z_4 Zernike polynomial provided the radii of the annuli are $\rho_i = \sqrt{i/3}$, with i = 1, 2, 3. A drawback of this procedure is that with no rotation it yields a central focus (formed by two opposite quadrants with null phase) and two smaller spots in front and after the central spot (focused by the 0-π Fresnel lens in the two other quadrants). This disadvantage is overcome in the device shown in Fig. 2.

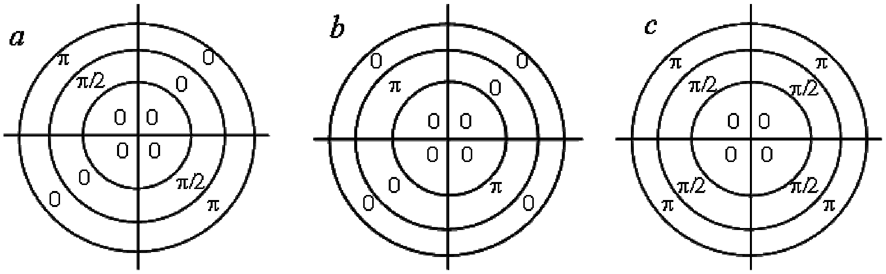

Fig. 1. Phase distribution for a single phase filter (*a*), for two filters superimposed (*b*) and for two filters superimposed but one rotated π/2 with respect to the other (*c*).

Now, we will analyze the case shown in Fig. 2. If we take two filters like that shown in Fig. 2a, and the second one is reversed (i.e., flipped upside down), the resulting global phase corresponds to a radial decreasing phase (Fig. 2b). This is a stepped version of the Z_4 Zernike polynomial, provided the correct radii ($\rho_i = \sqrt{i/3}$). If the first filter is rotated an angle of π/2 with respect to the second one, a constant phase (modulus 2π) is obtained (Fig. 2c). Finally, if the first filter is rotated an angle of π we obtain a stepped version of the $-Z_4$ (Fig. 2d). In this case the focus displacement provided by the resulting filter is the opposite of that obtained from Fig. 2b.

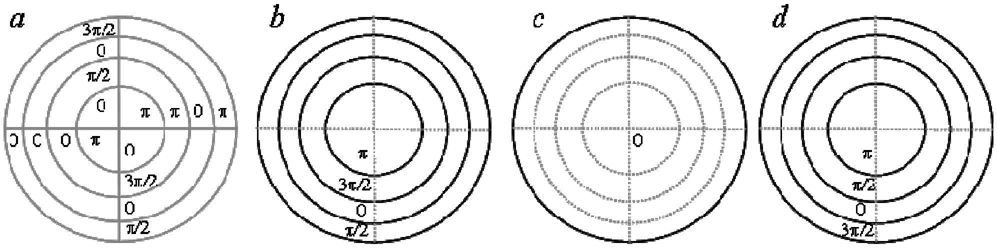

Fig. 2. Phase distribution single phase filter (*a*), two filters superimposed (*b*), two filters superimposed, the first one rotated π/2 with respect to the second one, (*c*) and two filters superimposed, the first one rotated π with respect to the other (*d*). In all cases, the values are modulus 2π. Note that in such case 3π/2 = -π /2 and π = -π.

3. Design of the filter

The two parameters that define the filter structure are the number of sectors and the number of annuli. To analyze the effect of varying these parameters we will consider the set-up shown in Fig. 3. A collimated beam illuminates a lens L, then it traverses phase filters F1(ρ,θ) and F2(ρ,θ), placed very close to the lens, and, finally, focuses in a certain spatial light distribution at the focal region. To estimate the effect produced by the filters we analyze the axial behavior of the Point Spread Function (PSF) of the optical system.

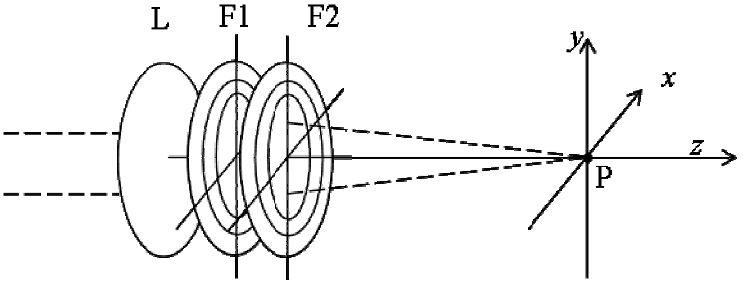

Fig. 3. Optical system consisting on a lens (L) and two identical phase filters (F1 and F2). Analysis will be performed axially or transversally around the lens focal point P.

The first parameter to be analyzed is the number of annuli in filters F1(ρ,θ) and F2(ρ,θ). In fact, there are two questions to solve: the number of annuli and the area of every annulus. In order to approximate the defocus Zernike mode, we have selected annuli with the same area (although other discretization alternatives could be used). Hence, if N is the number of annuli the radii will be: $\rho_i = \sqrt{i/N}$. The number of annuli is directly related to the maximum defocus value we desire to introduce. Obviously, if we increase the number of annuli maintaining the phase step equal to $\pi/2$, for a constant pupil radius, the result is an increase of the curvature of the phase screen introduced by the pair of filters. This will produce a larger focus displacement. As an example, Fig. 4 shows two possible schemes of annulus distribution for a pupil of radius equal to 1. Case *a* corresponds to a four annuli couple of filters and case *b* corresponds to a eight annuli couple of filters and describes a phase screen with a curvature radius twice that of case *a*.

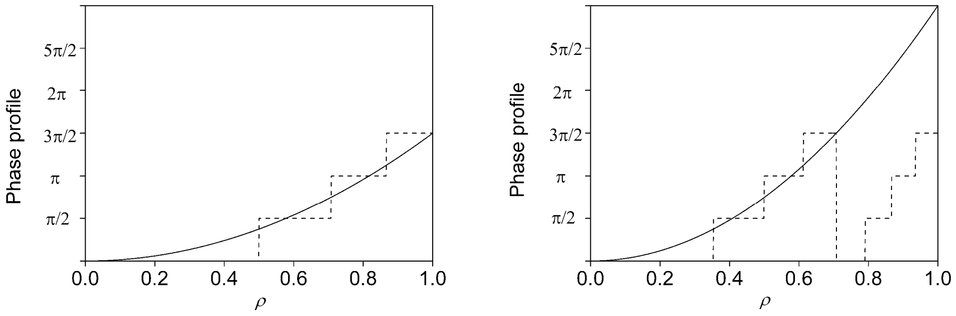

Fig. 4. Phase annulus value as a function of the pupil radius for a four annuli filter (*a*) and a eight annuli filter (*b*).

Now we will consider the effect of varying the number of sectors. Since the phase distribution along an annulus changes from sector to sector, a transversal cut of the PSF will not show rotational symmetry. This effect is negligible when we analyze the PSF in the best focus plane as we will see below. Nevertheless, this effect can be further reduced by increasing the number of sectors. On the other hand, if the number of sector increases the rotation angle needed to obtain the maximum focus displacement decreases, so in those cases where a small rotation is required a good strategy may be to increase the number of sectors. In this work we will consider only four sectors since it produces a PSF of fairly good quality, as we will see further.

4. Analysis of focus displacement

The goal of the proposed set-up is to produce a focus displacement. In fact, a continuous displacement from zero to its maximum value along the z axis, producing a zoom effect, would be interesting. This ideal behavior can be attained but with some limitations. We will work within the framework of the Debye scalar diffraction theory. Let us consider a general complex pupil function $P(\rho,\theta) = \exp[j\ \phi(\rho,\theta)]$, where j is $\sqrt{-1}$, ρ is the normalized radial coordinate over the pupil plane, θ is the azimuthal angle and $\phi(\rho,\theta)$ is the phase function of the filter. For a converging monochromatic spherical wave front passing through the center of the pupil, the normalized field amplitude U in the focal region may be written as

$$U(v,\psi,u) = 2\int_0^{2\pi} d\theta \int_0^1 \mathrm{d}\rho\, P(\rho,\theta)\rho \exp(-jv\rho\cos(\theta-\psi))\ \exp(ju\,\rho^2/2)\ , \qquad (1)$$

where v and u are radial and axial dimensionless optical coordinates with origin at the geometrical focus, given by $v = k\ NA\ r$ and $u = k\ NA^2\ z$. NA is the numerical aperture, $k = 2\pi/\lambda$ and r and z are the usual radial and axial distances. The pupil function is $P(\rho,\theta) = \mathrm{F1}(\rho,\theta)\mathrm{F2}(\rho,\theta + \theta_0)$. Hence, the pupil function will vary as a function of the relative angular position (θ_0) between both filters. The PSF will be obtained as the squared modulus of Eq. (1).

Under this mathematical framework we have estimated the PSF for different filters and their performance when one filter is rotated with respect to the other. We have checked that as the filters are rotated the axial PSF displaces but maintains its shape, with a Strehl modulation (the Strehl ratio is defined as the ratio between the maximum intensity of the PSF with filters and that corresponding to the unobstructed pupil).

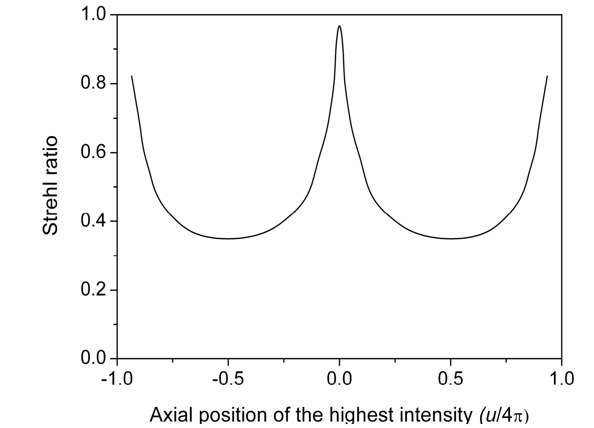

Fig. 5. Strehl ratio as a function of the displacement produced by rotating the filters in Fig. 2.

As an example, Fig. 5 shows a plot of the Strehl ratio as a function of the displacement produced by rotating two filters like those shown in Fig. 2. Since the Strehl ratio never falls under 0.35 and in every case the axial PSF maintains its shape (with a core much higher than any sidelobe), this couple of filters can be considered as an optical system with adjustable focal length. Finally, it is important to note that the focus shift changes in a continuous but not linear way with rotation angle.

5. Transversal behavior

In the previous section we have analyzed the PSF axial behavior. However, an important issue is to determine the quality of the transversal PSF produced by this kind of optical system (lens plus two phase filters). Hence, we will perform a qualitative analysis of the transversal light distribution at the plane x-y shown in Fig. 3. Following the previous scheme we will work within the framework of the scalar diffraction theory, using Eq. (1). We analyze the transversal behavior of the filter described in Fig. 1. Figure 6a shows the transversal PSF for a single lens (lens L in Fig. 3) at its focal plane. Figure 6b shows the transversal PSF at the lens focal plane when the two filters are introduced with no rotation between them. It can be seen that the PSF maintains the same axial position and width as for the case without filter. When a rotation angle $\theta_0 = \pi/2$ is introduced between the two filters, the focus is displaced producing a

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transversal PSF as that shown in Fig. 6c. This PSF is very similar to the previous ones, with a slight resolution loss. If a rotation $\theta_0 = \pi$ is performed (Fig. 6d) the same result as for a rotation $\theta_0 = 0$ is obtained.

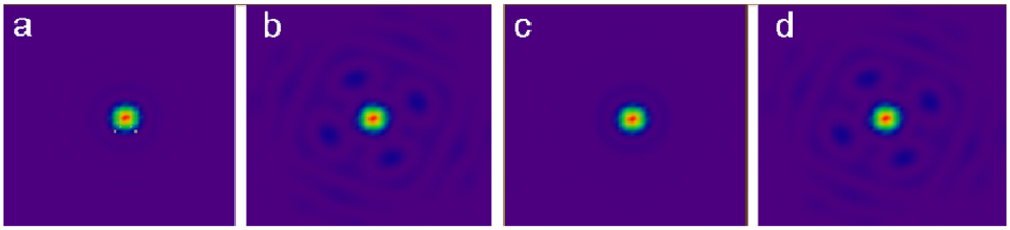

Fig. 6. Picture of the transversal PSF (*a*) for the clear pupil, (*b*) for the pupil covered with two three annuli filters without rotation, (*c*) once one filter has been rotated π/2 with respect to the other and (*d*) once one filter has been rotated π with respect to the other.

6. Experimental checking

An experimental checking was carried out using the set up shown in Fig. 7. A solid state laser provides a light beam of wavelength 532 nm, which is spatially filtered with the help of a pinhole (PH) and collimated using a lens (CL). After a pupil (P) of 8 mm of radius, two identical filters F1 and F2 are placed together behind the imaging lens IL of 20 cm of focal length. The phase filters were made at the ITME [13] using an electron beam writer and reactive ion etching to attain the phase map required. They present five annuli and four sectors. The focal region is axially scanned and transversally imaged by a back-illuminated CCD camera. The scanning device (SD) allows a step as small as 2.5 µm.

The phase values of the filter sectors have been chosen so that the phase distribution is that shown in Fig. 8. Figure 8a shows the distribution with no rotation, which presents zero phase in the whole pupil except in four sectors (in red) of phase π. Figure 8b shows the phase obtained for a rotation angle $\theta_0 = \pi/2$. In this case the phase grows from zero to 2π (from the center to the outside) with steps of π/2. With no rotation, there is no focus displacement, while with π/2 rotation the focus is displaced at a distance of 775µm from the first one, with a high Strehl value. Figure 9 shows the transversal light distribution of both foci. It can be seen that a slight pattern appears as a consequence of the filter structure. Furthermore, with no rotation the two spots focused by the 0-π Fresnel lens in opposite quadrants yield the side-lobes in the Fig. 9a.

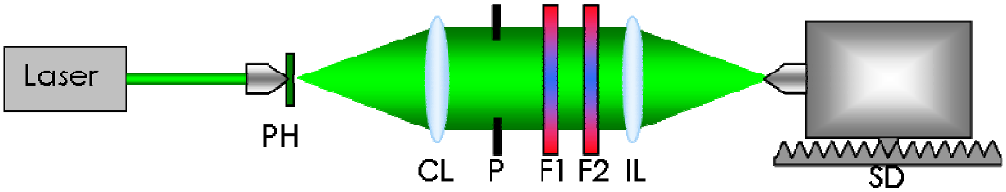

Fig. 7. Experimental setup: Laser beam (532 nm), pin hole (PH), collimating lens (CL), pupil (P), two filters F1 and F2, imaging lens IL (focal length: 20 cm), CCD camera and scanning device (SD).

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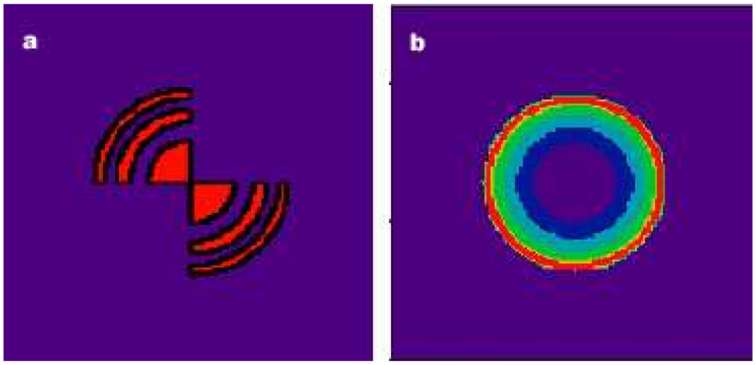

Fig. 8. Phase distribution of the couple of filters with no rotation (*a*), composed of four sectors (in red) of phase π over a zero phase background. Phase distribution for a rotation angle $\theta_0 = \pi/2$, where the phase grows from zero to 2π with steps of π/2 (*b*).

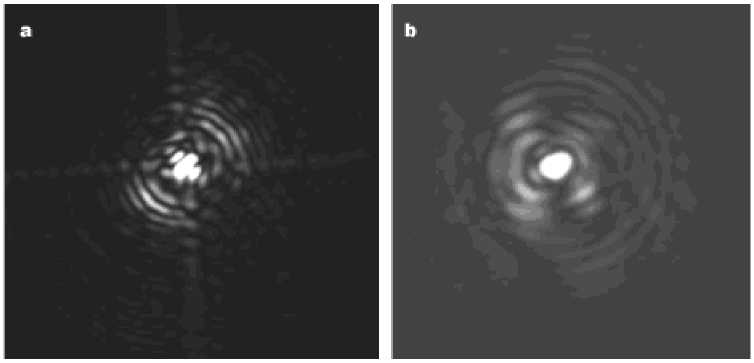

Fig. 9. Transversal light distribution at the focus for the configuration shown in Fig. 8, with no rotation (*a*) and for a rotation angle $\theta_0 = \pi/2$ (*b*).

The experiment was repeated using a different couple of filters with five annuli and four sectors. These filters have been designed in order to produce three foci with high Strehl value, one corresponding to a positive defocus, one to no defocus and the third corresponding to a negative defocus. In the experiment we obtained three different foci, as expected, with an average distance between them of 855 μm. The focus quality is similar to that shown in Fig. 9.

7. Conclusions

We have proposed a device able to obtain focus displacement in an optical system. It is composed of phase filters which are divided in sectors and annuli, and focus displacement is achieved by rotating one filter with respect to the other. Annulus radii and phase values have been selected to enable the process. The magnitude of the focus displacement depends on the number of annuli. The transverse PSF shape obtained for the whole axial displacement range is of good quality, very similar to that of the clear pupil but with Strehl modulation.

The focus displacement obtained at the experiment confirms the theoretical predictions. The main advantage of this system is that it is thin and light, but it provides the same results as other heavier systems. There are other combinations to be explored as combinations of more than two filters, filters with different phase distribution, etc. Hence, this is a promising way to produce a cheap (and possibly fast) focus modulation with an extremely thin optical device.

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