

### Comment on "Nonlocality of a Single Photon"

Tan, Walls, and Collett propose an experiment involving a single photon incident on a half-silvered mirror which may be detected in either of two homodyne detectors, each including a local oscillator [1]. They conclude that the quantum prediction for the intensity correlation coefficient involving four photodetectors "may not be explained classically using a particle, wave, or hidden-variables theory involving local causality." This conclusion, so sharply stated, is not true as the following counterexample shows.

The correlation coefficient is predicted to depend on the common intensity  $\alpha$  and the phases  $\theta_1$  and  $\theta_2$  of the local oscillators in the form

$$E(\theta_1, \theta_2) = \langle (I_{d1} - I_{c1})(I_{d2} - I_{c2}) \rangle \langle (I_{d1} + I_{c1})(I_{d2} + I_{c2}) \rangle^{-1} \\ = [1/(\alpha^2 + 1)] \sin(\theta_1 - \theta_2) \quad (1)$$

[see Eqs. (14) and (16) of Ref. [1]]. The averages  $\langle I_i I_j \rangle$ , where  $i$  and  $j$  stand for either  $c$  or  $d$ , are identified with coincidence probabilities. According to Bell, a local hidden-variables model should give the coincidence probability by means of an expression of the form

$$\langle I_i I_j \rangle = \int \rho(\lambda) P_i(\lambda, \alpha_1, \theta_1) P_j(\lambda, \alpha_2, \theta_2) d\lambda, \quad (2)$$

where  $\lambda$  is the set of hidden variables and  $\alpha_k, \theta_k$  are the parameters controlled in the experiment (for simplicity we shall follow Tan, Walls, and Collett, putting  $\alpha_1 = \alpha_2 = \alpha$ ). The density  $\rho$  should be positive and normalized and  $0 \leq P_k \leq 1$ .

Our model (similar to one recently proposed for experiments measuring the polarization correlation of optical photon pairs [2]) consists of choosing  $\lambda = \{\varphi_1, \varphi_2\}$  to be a pair of angles with domain  $\{0, 2\pi\}$  and assuming their joint distribution to be

$$\rho = (2\pi)^{-2} [1 + \sin(\varphi_1 - \varphi_2)]. \quad (3)$$

The effect of the beam splitter  $BS_k$  and the appropriate detector is given by the following detection probability of the signal going in mode  $c_k$  (putting  $j=c$  and  $\delta=0$  below) or in mode  $d_k$  ( $j=d$ ,  $\delta=\pi$ ):

$$P_j(\varphi_k, \alpha, \theta_k) = \begin{cases} \gamma & \text{if } |\varphi_k - \theta_k + \delta| < \varepsilon \pmod{2\pi}, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Here  $\gamma$  is a parameter, with values in the interval  $[0, 1]$ , which does not enter in the final expression, and  $\varepsilon$  (with

domain  $[0, \pi/2]$ ) is related to  $\alpha$  by

$$\alpha^2 = (\varepsilon/\sin\varepsilon)^2 - 1. \quad (5)$$

It is a simple matter to show that the model reproduces the quantum prediction (1).

It may be argued that the model could be easily disproved by additional experiments, which is true. Indeed, it does not even reproduce the quantum-mechanical prediction for single probabilities [Eq. (6) of Ref. [1]]. However, we do not pretend to give a theory for the interpretation of the experiment, but just show that the measurement of coincidence rates alone cannot prove rigorously the nonlocal properties of a single-photon field.

The reader may be surprised by the apparent contradiction between the existence of the model and the claimed violation of a Bell inequality by the quantum prediction for the experiment of Ref. [1]. Actually the violated inequality is not a general one, derived from local realism alone, but involves an untestable supplementary assumption about the behavior of the hidden variables, introduced by Grangier, Potasek, and Yurke (Ref. [12] of Tan, Walls, and Collett [1]). It is clear from the article (including title and abstract) that Tan, Walls, and Collett [1] believe that the assumption of Grangier, Potasek, and Yurke is much more plausible than local realism. However, they should have stated the correct conclusion of the paper, independently of beliefs, namely, that the quantum predictions for the proposed experiment contradict either local realism or Grangier, Potasek, and Yurke's hypothesis about the hidden variables.

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