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Loophole-free test of the Bell inequality

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(Received 23 January 1995)

An atomic cascade experiment is proposed that includes the detection of a recoil atom after the emission of two photons. This would permit testing of the Bell inequality without the need for additional assumptions, provided that the quantum efficiency of single-photon detectors exceeds a threshold of 0.92. The atomic source consists of calcium atoms, first trapped and then accelerated by gravity, whose velocity is controlled before they reach the interaction region with the two counterpropagating laser beams. The procedure allows a relatively high background in the photodetectors.

PACS number(s): 03.65.Bz, 42.50.Wm

Theoretical arguments [1–3] have shown that some quantum-mechanical predictions are incompatible with the requirements of locality and realism. The discussion about Bell's inequality has lasted almost 30 years, and today it appears that the question will be better settled by proposing and performing more refined experiments than by further theoretical discussions. To date, the best experiments performed to rule out local hidden variable (LHV) theories, atomic cascade experiments [4–6], have three loopholes, making their results inconclusive: (i) The inefficiency of the available photodetectors and other nonidealities of measuring devices, (ii) the poor angular correlation between the directions of the photon momenta, and (iii) the possible (casual) connection between different parts of the apparatus due to no-space-like separation.

The so-called low-efficiency loophole has been extensively studied in the literature [7] and we will not elaborate on it here. The second difficulty has gone almost unnoticed for the last 20 years (see, for instance, Refs. [8] and [9]) and has its origin in the three-body character of the process. The angle between the wave vectors of a given photon pair can differ from π by an amount large enough for the polarization correlation to be appreciably decreased. This fact, clearly pointed out by Clauser and Horne [2], precludes any discrepancy between quantum mechanics and LHV theories for atomic cascade experiments, and it has been used to exhibit LHV models for all previously performed polarization correlation tests, in perfect agreement with quantum mechanics for all measurable quantities, even with ideal measuring devices [10]. The mere existence of the models proves that these experiments cannot discriminate between quantum mechanics and the whole family of LHV theories but

only between quantum mechanics and the restricted family of LHV theories, fulfilling the well-known "additional assumptions" used to circumvent loopholes (i) and (ii) quoted above.

Therefore, it can be stated that no incontrovertible violation of Bell's inequality has been observed, which suggests a need for additional experiments. With this motivation we propose an atomic cascade experiment that blocks loopholes (i) and (ii) without using additional assumptions, provided that the quantum efficiency of the photodetectors is higher than 92%, something that seems achievable in the near future [11]. This atomic cascade experimental scheme closes the detection and angular correlation loopholes simultaneously. Other attempts at a loophole-free test have been proposed recently in very different contexts [12,13]. The proposal by Kwiat *et al.* [12] constitutes an attempt at a loophole-free test; that is, a test without the implementation of non-enhancement-like assumptions, using pairs of photons created by parametric down-conversion (PDC). All other PDC tests performed until now have suffered from loopholes and may be criticized at least as much as atomic cascade tests. Indeed, local realistic models can be exhibited that are in perfect agreement with all available experimental data [14]. However, as we shall see afterwards, the loophole-free proposal [12] using down-converted photon pairs is extremely demanding as far as background noise is concerned, which constitutes a serious disadvantage in order for the test to be feasible in the near future.

The crucial point of our proposal is the operational definition of the ensemble of photon pairs involved in the test. Such an ensemble will consist of photon pairs that leave the atom with a linear momentum between certain bounds. This definition enables an event-ready detection

scheme [3], in the sense that the detection of the corresponding recoil atom determines whether or not an emission belongs to the ensemble of interest. Moreover, the additional requirement of detecting the recoil atom allows us to select photon pairs with a good angular correlation.

A test of local realism involves checking whether or not the quantum-mechanical predictions for single and coincidence detection probabilities violate the Bell inequality:

$$-1 \leq P_{12}(a,b) - P_{12}(a,b') + P_{12}(a',b) + P_{12}(a',b') - P_1(a') - P_2(b) \leq 0. \quad (1)$$

For the ensemble of photon pairs previously defined, the probabilities involved in (1) can be written as

$$\begin{aligned} P_1(a) &= \frac{1}{2} \xi_1 \left[\frac{N(1 \otimes 2)}{N} + \frac{N(1)}{N} \right], \\ P_2(b) &= \frac{1}{2} \xi_2 \left[\frac{N(1 \otimes 2)}{N} + \frac{N(2)}{N} \right], \\ P_{12}(a,b) &= \frac{1}{4} \xi_{12} \frac{N(1 \otimes 2)}{N} [1 + \varepsilon \cos(2a - 2b)], \end{aligned} \quad (2)$$

where a and b are the angles of the polarizers with respect to a given plane; ξ_{12} , ξ_1 , and ξ_2 are overall efficiency parameters; ε is a polarization correlation factor [3], which is a function of the half angle ϑ subtended by the lens system and the practical inefficiencies of the analyzers; N is the number of atoms that have actually decayed in the source; and $N(1 \otimes 2)$ [$N(i)$, $i=1,2$] refers to the number of events such that, provided that the corresponding atom is detected, both emissions are (only one of the emissions is) collected by the lens system.

Note that with the present scheme, the Bell inequality is directly testable in terms of coincidences between an atom and two photons (coincidence photon counts, supplied only by events of type $1 \otimes 2$) and coincidences between an atom and a single photon [single-photon counts, supplied by both events $1 \otimes 2$ and events i ($i=1,2$)]. It is easy to check that (1) will be violated only if

$$\frac{2\xi_{12}}{\xi_1 + \xi_2} (1 + \sqrt{2}\varepsilon) - 2 \geq t, \quad t := \frac{N(1) + N(2)}{N(1 \otimes 2)}. \quad (3)$$

Considering the precise expressions for the overall efficiencies and the polarization correlation factor, the condition (3) for the violation of Bell's inequality can be expressed in a form explicitly dependent on the quantum efficiency of single-photon detectors η , considered to be the same in both sides of the apparatus, as

$$\eta \geq \frac{t+2}{\xi(\vartheta, \varepsilon)}, \quad (4)$$

which clearly shows the existence of a detection efficiency cutoff in the experiment proposed here. The threshold efficiency required for demonstrating the violation of Bell's inequality decreases with t . By means of a Monte Carlo simulation we have found that this ratio takes its minimum value when the atomic detector is placed in a

region such that the number of events where only one of the members of a photon pair is collected by the corresponding lens system, $N(1) + N(2)$, is minimized. Such a location will be denoted as the optimal region (OR), and its precise characteristics will be described in the context of a concrete experimental arrangement.

In order to make the above scheme feasible, the following requirements have to be satisfied:

(i) The mean velocity of the atoms has to be slowed down from typical thermal values in order to increase the recoil effect [15].

(ii) The transverse velocity spread in the incoming beam should be very narrow (ultimately limited by the Heisenberg relation). Otherwise, the inequality (4) is not satisfied, even with a quantum efficiency equal to 1, owing to the fact that the value of the ratio t increases very quickly with the transverse dispersion.

(iii) The atomic beam should have a high degree of monochromaticity. This condition has no strong influence on the precise value of the threshold but fixes the time window w required for the atomic detector. Hence, it determines the background r in the photodetectors that can be tolerated in order for the production of spurious single counts, proportional to the product rw , to be negligible.

We have recently proposed an experiment of this kind with a thermal source of calcium atoms [16]. There we envisaged a collimation and velocity selection of the atomic beam by purely mechanical procedures. In the end the experiment seemed feasible but rather difficult. Here we propose a very different experimental technique, where slow atoms are obtained after a trapping process. This will result in a much simpler experiment and, more importantly, it will allow a substantial increase in the photodetector background noise that can be tolerated. As will be detailed later, with an atomic detector window w of the order of microseconds, it will be possible to fix the admissible background in the photodetectors to a range that is likely to be compatible with the high efficiency required for a conclusive test.

The proposed experimental setup is sketched in Fig. 1. The atomic source consists of calcium atoms coming

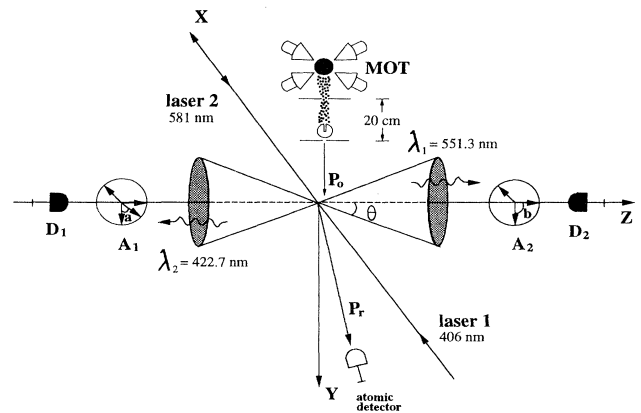


FIG. 1. Proposed experimental setup.

from a magneto-optical trap (MOT). The three components of the velocity have zero mean value, and the spread can be reduced to 5 cm/s [17]. After the trapping process, the slow cloud is dropped through a diaphragm and accelerated by gravity to 2 m/s after 20 cm of free fall. Before the atoms reach the interaction region, where the cascade is produced by two-photon absorption from two counterpropagating laser beams, it is still necessary to complete the collimation using a second diaphragm and a longitudinal selector of velocities with the aim of guaranteeing conditions (ii) and (iii).

The collimation has been envisaged by means of diaphragms of dimension x - z equal to 1.4 and 0.042 mm, respectively, which results in semidivergences smaller than 3.5 and 0.1 mrad in directions x and z , respectively.

Under these conditions, and with a Fizeau velocity filter in the range 2 ± 10^{-4} m/s, the optimal region is defined by a solid angle of 8×10^{-7} sr whose axis has polar angles $(\theta, \varphi): \pi/2 - \theta = 2.3$ mrad and $\pi/2 - \varphi = 3.6$ mrad.

Locating the atomic detector 1.5 m away from the interaction region [18], its acceptance slit would have dimensions x - z equal to 9.1 and 0.2 mm, respectively, being separated from the incident direction 3.3 mm above the xy plane. The resulting value for the ratio t is 0.052. In the numerical calculation, the transverse velocity components are assumed to have Gaussian shapes with standard deviations equal to the corresponding maximum semidivergence multiplied by the mean velocity of the beam. With this procedure the resulting OR is symmetrical with respect to the x axis. However, this symmetry breaks when the two-photon absorption process, which causes an overall displacement of the v_x distribution, is taken into account. Hence, the center of the optimal region has to be translated in the positive x direction, and it is this new location that defines the polar angles (θ, φ) quoted above.

With the above value of t we get, via Eq. (4), that the quantum efficiency of the photodetectors must satisfy $\eta \geq 91.9\%$ for quantum-mechanical predictions to violate Bell's inequality. In order to make an estimation of the statistics achievable with this proposal, we have assumed that 10^7 atoms can be stored in the trap in a volume of 1 mm^3 . This will produce a flux of 8.3×10^8 atoms/s for a characteristic storage time of about 12 ms. However, the real flux reaching the interaction region is seven orders of magnitude smaller due to the collimation and velocity selection required. The production rate in the source can be obtained as the product of the atomic flux times the excitation probability, P_{fe} , of an atom in the source. The length of the interaction region is given by the y dimension of the laser beams and has been taken to be 0.1 mm. With 0.1 W from each laser, $P_{fe} = 0.3$ [19], and the number of cascades is estimated to be about 25 s^{-1} , which results in a photon coincidence rate of 20 per hour. We must stress that, with the definition considered for the ensemble of interest, the duration of the experiment does not pose any difficulty, in contrast with previous atomic cascade tests, where it was necessary to ensure a constant production rate. Here, since the ensemble of photon

pairs is monitored by the atomic detector, the decay rate in the source is not required to be a constant.

As far as spurious events are concerned, the main source of noise is due to false single-photon counts produced by background counts r paired with atoms that recoil through the OR with none of the corresponding emissions captured by the pertinent lens system. Setting the product rw to 0.03, and taking into account that the time window has to accommodate different propagation times due to the imperfect monochromaticity as well as the finite length of interaction region [20], the background of the photodetectors cannot exceed the value of 6×10^2 counts/s, in order that the false single counts remain by one order of magnitude below the true coincidence counts. We must stress that, in this sense, our proposal is much less sensitive to noise effects than the loophole-free proposal using parametric down-converted photon pairs [12], a fact that can be decisive in the feasibility of the test. The efficiency needed in the PDC test is lower than ours, the threshold being 86%, but, as the authors clearly point out, a background noise above 1% (that is, of the order of 1 count/s) would make the test extremely difficult. In contrast, the background admissible in the present scheme can be two orders of magnitude greater than this value. Hence, the requirement of high efficiency is combined with a less demanding requirement for the admissible background. Other sources of noise, due, for example, to the wrong pairing of a recoil atom of the type described above with the emissions of a different atom that does not recoil through the OR are three orders of magnitude smaller than true coincidences and hence negligible.

In summary, on the basis that single photodetectors in the range of 90% efficiency and maximum background of 6×10^2 counts/s may be available, the experiment proposed here provides a suitable scenario for simultaneously closing the detection and poor angular correlation loopholes. Consequently, no LHV model in agreement with quantum mechanics (QM) would be tenable in the high-efficiency domain. Only the so-called spacelike separation loophole would remain. It could be thought that due to the enormous difference in the propagation velocity of atoms and photons, it will be impossible to prevent connections between the different measuring devices implied in the test. However, with atomic detectors being almost ideal this loophole might be blocked, preventing connections between the measuring devices concerning photon pairs with the implementation of a nonstatic scheme similar to the one employed by Aspect, Dalibard, and Roger in their third experiment [6].

We are very grateful to F. Riehle for a valuable critical reading of the manuscript. One of us (S.F.H.) also acknowledges J. Helmcke, F. Riehle, and K. D. Stock for their hospitality and valuable discussions during a short stay at PTB (Braunschweig) and J. Dalibard for discussing the reliability of laser cooling techniques in a preliminary stage of this paper. This work has been supported by DGICYT Project No. PB-92-0507 (Spain) and Universidad de Oviedo Project No. DF-94-214-3 (Spain).

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- [18] A vacuum of less than 10^{-8} Torr is required to reduce to 10^{-4} the probability of scattering of one calcium atom by background gas.
- [19] Note that this value cannot be arbitrarily increased, otherwise spurious events might be generated from atoms repeatedly excited.
- [20] For the limits set in the filter and the longitudinal dimension of $100 \mu\text{m}$ taken for the interaction region, the detection window is required to be about $50 \mu\text{s}$.