PHYSICAL REVIEW A VOLUME 60, NUMBER 4 OCTOBER 1999

Irrelevance of photon events distinguishability in a class of Bell experiments

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(Received 12 March 1999)

We show that the possibility of distinguishing between single- and two-photon detection events, usually not met in the actual experiments, is not a necessary requirement for proof that the experiments of Alley and Shih [Phys. Rev. Lett. **61**, 2921 (1988)] and Ou and Mandel [Phys. Rev. Lett. **61**, 50 (1988)] are modulo a fair sampling assumption, valid tests of local realism. We also give the critical parameters for the experiments to be unconditional tests of local realism, and show that some other interesting phenomena (involving bosonic-type particle indistinguishability) can be observed during such tests. [S1050-2947(99)50709-9]

PACS number(s): 3.65.Bz, 42.50.Dv, 89.70.+c

The first Bell-type experiments that employed the parametric down-conversion process as the source of entangled photons were those reported in Refs. [1] and [2]. However, the specific traits of those experiments have led to a protracted dispute on their validity as tests of local realism. In this case the issue was not the standard problem of detection efficiency (which up till now permits a local realistic interpretation of all performed experiments). The trait that distinguishes the experiments is that, even in the perfect *gedanken* situation (which assumes perfect detection), only in 50% of the detection events does each observer receive a photon; in the other 50% of events one observer receives both photons of a pair while the other observer receives none. The early ''pragmatic'' approach was to discuss only the events of the first type (as only such ones lead to spatially separated coincidences). Only those were used as the data input to the Bell inequalities in [1] and [2]. This procedure was soon challenged (see, e.g., [3,4], and especially the theoretical analysis of Ref. [5]), as it raises justified doubts as to whether such experiments could ever be genuine tests of local realism (as the effective overall collection efficiency of the photon pairs, 50% in the gedanken case, is much below what is usually required for tests of local realism). Ten years after the first experiments of this type were made, the dispute was finally resolved [6]. It was proposed, that those ''unfavorable'' cases also be taken into account and that the entire pattern of events be analyzed. In this way one can indeed show that the experiments are a true test of local realism [namely, that the CHSH inequalities are violated by quantum predictions for the idealized case]. The idea was based upon a specific value assignment for the ''wrong events'' (see further or [6] itself). However, the scheme presented by Popescu *et al.* [6] has one drawback. The authors assumed in their analysis that the detecting scheme employed in the experiment should be able to distinguish between two- and one-photon detections. This was not the case in the actual experiments. The aim of this work is to show that even this is unnecessary; all one needs is use of the specific value assignment procedure of [6].

What is perhaps even more important, problems similar to those sketched above are also shared by the new, potentially highly promising class of electron paramagnetic resonance–Bell-type experiments, which involves the entanglement swapping procedure [7]. Also in this case the first performed experiment did not employ detectors that were able to distinguish between firings caused by two photons and a single photon [8]. The entanglement swapping experiments thus far have not violated the visibility threshold for local realism (71%); however, in the future the problem of their relation to the Bell theorem will be of fundamental importance (as entanglement swapping may find application in future quantum communication schemes [9]). The analysis presented in [6] can be adapted to describe such experiments, clearly indicating the violation of local realism.

Finally, we shall also give a prediction of all effects occurring in the experiment. It is quite often overlooked that a kind of Hong-Ou-Mandel dip phenomenon [10] can be observed in the experiment.

In the class of experiments we consider (Fig. 1) [6] a type-I parametric down-conversion source [11] is used to generate pairs of photons that are degenerated in frequency and polarization (say $\hat{x}$) but propagate in two different directions. One of the photons passes through a wave plate (WP) that rotates its polarization by 90°. The two photons are then directed onto the two input ports of a (nonpolarizing) ''50-50'' beam splitter (BS). The observation stations are located in the exit beams of the beam splitter. Each local observer is equipped with a polarizing beam splitter [12] orientated along an arbitrary axis (which, in principle, can be randomly chosen, in the delayed-choice manner, just before the photons are supposed to arrive). Behind each polarizing beam splitter are two detectors, D_1^+, D_1^- and D_2^+,D_2^-, respectively, where the lower index indicates the corresponding

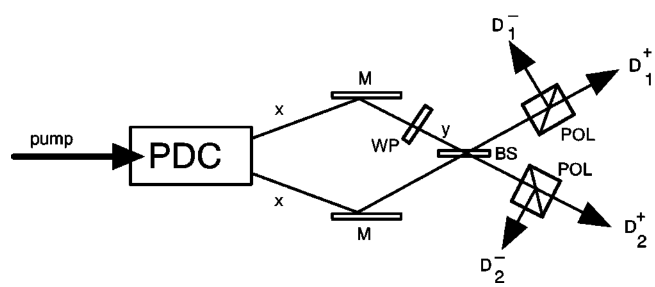

FIG. 1. Schematic of the setup. For explanations see the main text.

observer and the upper index the two exit ports of the polarized beam splitter (''+'' meaning parallel with the polarization axis of the beam splitter and ''−'' meaning orthogonal to this axis). All optical paths are assumed to be equal.

Let us calculate the quantum predictions for the experiment. We will use the second quantization formalism, which is very convenient here, since the whole phenomenon observed in the experiment rests upon the indistinguishability of photons.

After the action of the wave plate one can approximate the quantum mechanical state describing two photons emerging from a nonlinear crystal along the ''signal'' and the ''idler'' beam by

$$|\Psi_0\rangle = a^\dagger_{1\vec{x}} a^\dagger_{2\vec{y}} |0\rangle, \tag{1}$$

where $a^\dagger_{1\vec{x}}$ and $a^\dagger_{2\vec{y}}$ are creation operators and $|0\rangle$ denotes the vacuum state. Subscripts $\vec{x},\vec{y}$ decode the polarization of the photon (either along the $\vec{x}$ or the $\vec{y}$ axis). The beam-splitter action can be described by

$$a^\dagger_{1\vec{x}} = \frac{1}{\sqrt{2}}(ic^\dagger_{\vec{x}} + d^\dagger_{\vec{x}}), \quad a^\dagger_{2\vec{y}} = \frac{1}{\sqrt{2}}(c^\dagger_{\vec{y}} + id^\dagger_{\vec{y}}), \tag{2}$$

where $c^\dagger_{\vec{x}}, d^\dagger_{\vec{x}}, c^\dagger_{\vec{y}}, d^\dagger_{\vec{y}}$ are operators describing output modes of the beam splitter (c stands for the first observer and d for the second one). Thus our state $|\Psi_0\rangle$ changes to

$$|\Psi\rangle = \frac{1}{2}(ic^\dagger_{\vec{x}}c^\dagger_{\vec{y}} - c^\dagger_{\vec{x}}d^\dagger_{\vec{y}} + c^\dagger_{\vec{y}}d^\dagger_{\vec{x}} + id^\dagger_{\vec{x}}d^\dagger_{\vec{y}})|0\rangle. \tag{3}$$

Next comes the action of the polarizers in both beams, which can be described as

$$n^\dagger_{\vec{x}} = \cos(\theta_1)n^\dagger_\parallel + \sin(\theta_1)n^\dagger_\perp, \quad n^\dagger_{\vec{y}} = \sin(\theta_1)n^\dagger_\parallel - \cos(\theta_1)n^\dagger_\perp, \tag{4}$$

where $n^\dagger = c^\dagger$ or $d^\dagger$, $n^\dagger_\parallel$ describes the mode parallel to polarizer's axis, and $n^\dagger_\perp$ describes the mode perpendicular to polarizer's axis; θ is the angle between the $\vec{x}$ axis and polarizer's axis. Thus the final state reaching the detector reads

$$\begin{aligned}|\psi_{final}\rangle = \frac{1}{2}\Bigg[& \sin(\theta_1-\theta_2)|c_\parallel, d_\parallel\rangle + \cos(\theta_1-\theta_2)|c_\parallel d_\perp\rangle \\ & -\cos(\theta_1-\theta_2)|c_\perp, d_\parallel\rangle + \sin(\theta_1-\theta_2)|c_\perp, d_\perp\rangle \\ & + i\frac{1}{\sqrt{2}}\sin(2\theta_1)|2c_\parallel\rangle + i\frac{1}{\sqrt{2}}\sin(2\theta_1)|2c_\perp\rangle \\ & - i\cos(2\theta_1)|c_\perp, c_\parallel\rangle + i\frac{1}{\sqrt{2}}\sin(2\theta_2)|2d_\parallel\rangle \\ & + i\frac{1}{\sqrt{2}}\sin(2\theta_2)|2d_\perp\rangle - i\cos(2\theta_2)|d_\parallel, d_\perp\rangle\Bigg], \end{aligned} \tag{5}$$

where, e.g., $|c_\parallel, d_\parallel\rangle$ denotes one photon in the mode $c_\parallel$ and one in $d_\parallel$, whereas $|2c_\parallel\rangle = (1/\sqrt{2})c_\parallel^{\dagger 2}|0\rangle$ denotes two photons in the mode $c_\parallel$.

Let us denote by $P(i,\theta_1;j,\theta_2)$ the joint probability for the outcome i to be registered by observer 1 when her polarizer is oriented along the direction that makes an angle θ_1 with the $\vec{x}$ direction and the outcome j to be registered by observer 2 when her polarizer is oriented along the direction that makes an angle θ_2 with the $\vec{x}$ direction. Here $i,j = 1-6$ and have the following meaning [6]: 1=one photon in D^-, no photon in D^+; 2=one photon in D^+, no photon in D^-; 3=no photons; 4=one photon in D^+ and one photon in D^-; 5=two photons in D^+, no photon in D^-; 6=two photons in D^-, no photons in D^+.

The quantum predictions for joint probabilities of those events are given by

$$P(1,\theta_1;1,\theta_2) = P(2,\theta_1;2,\theta_2) = \tfrac{1}{8}[1-\cos 2(\theta_1-\theta_2)], \tag{6}$$

$$P(2,\theta_1;1,\theta_2) = P(1,\theta_1;2,\theta_2) = \tfrac{1}{8}[1+\cos 2(\theta_1-\theta_2)], \tag{7}$$

$$P(5,\theta_1;3,\theta_2) = P(6,\theta_1;3,\theta_2) = \tfrac{1}{8}\sin^2(2\theta_1), \tag{8}$$

$$P(3,\theta_1;5,\theta_2) = P(3,\theta_1;6,\theta_2) = \tfrac{1}{8}\sin^2(2\theta_2), \tag{9}$$

$$P(4,\theta_1;3,\theta_2) = \tfrac{1}{4}\cos^2(2\theta_1), \tag{10}$$

$$P(3,\theta_1;4,\theta_2) = \tfrac{1}{4}\cos^2(2\theta_2). \tag{11}$$

Following [6] we associate with each outcome registered by the observers 1 and 2 a corresponding value a_i and b_j, respectively, where $a_1 = b_1 = -1$ while all the other values are equal to 1. Let us denote by $E(\theta_1,\theta_2)$ the expectation value of their product,

$$E(\theta_1,\theta_2) = \sum_{i,j} a_i b_j P(i,\theta_1;j,\theta_2). \tag{12}$$

After simple calculations one has

$$E(\psi_1,\psi_2) = -\tfrac{1}{2}\cos(\psi_1+\psi_2) + \tfrac{1}{2}, \tag{13}$$

where we have put $2\theta_k = (-1)^{k-1}\psi_k$.

The above formula for the correlation function is valid if one assumes that it is possible to distinguish between single- and double-photon detection. This is usually not the case. Thus it is convenient to have a parameter α that measures the distinguishability of the double and single detection at one detector ($0 \leqslant \alpha \leqslant 1$, and gives the probability of distinguishing by the employed detecting scheme of the double counts). The partial distinguishability blurs the distinction between events 1 and 6 (2 and 5) and thus part of the events of type 6 is interpreted as being of type 1 and is ascribed by the local observer a wrong value; e.g., an event of type 6, if both photons go to the ''−'' exit of the polarizer, can be interpreted as a firing due to a single photon and is ascribed a -1 value. Please note that events like 1 or 2 in station 1 accompanied by 3 (no photon) at station 2 do not contribute to the correlation function because for any $\alpha P(1,\theta_1;3,\theta_2) = P(2,\theta_1;3,\theta_2)$.

If the parameter α is taken into account, the correlation function acquires the following form:

$$E(\psi_1,\psi_2;\alpha)=-\tfrac{1}{2}\cos(\psi_1+\psi_2)+\tfrac{1}{2}\alpha+\tfrac{1}{4}(1-\alpha)(\cos^2\psi_1+\cos^2\psi_2). \quad (14)$$

In this case after the insertion of the quantum correlation function (14) into the CHSH inequality,

$$-2\leq E(\psi_1,\psi_2;\alpha)+E(\psi_1',\psi_2;\alpha)+E(\psi_1,\psi_2';\alpha)-E(\psi_1',\psi_2';\alpha)\leq 2,$$

one obtains

$$-2\leq-\tfrac{1}{2}[\cos(\psi_1+\psi_2)+\cos(\psi_1'+\psi_2)+\cos(\psi_1+\psi_2')-\cos(\psi_1'+\psi_2')]+\alpha+\tfrac{1}{2}(1-\alpha)(\cos^2\psi_1+\cos^2\psi_2)\leq 2. \quad (15)$$

The interesting feature of this inequality is that it can be violated for all values of α. What is perhaps even more important, it can be robustly violated even when one is not able to distinguish between single and double clicks at all ($\alpha=0$). The actual value of the CHSH expression can reach in this case 2.337 12 (a numerical result), which is only slightly less than the maximal value for $\alpha=1$, which is $\sqrt{2}+1\approx 2.414\,21$. Therefore we conclude that in the experiment *one can observe violations of local realism even if one is not able to distinguish between the double and single counts at one detector*. That is, the essential feature of the method of [6] to reveal violations of local realism in the experiment of this type is the specific value assignment scheme and not the double-single photon counts distinguishability.

The specific angles at which the maximum violation of the CHSH inequality is achieved for $\alpha=0$ differ very much from those for $\alpha=1$ (for which the standard result is reproduced), and they read (in radians) $\psi_1=2.937\,98$, $\psi_1'=4.255\,13$, $\psi_2=-0.202\,41$, and $\psi_2'=1.117\,08$.

Let us notice that with the setup of Fig. 1 one is able to observe effects of similar nature to the famous Hong-Ou-Mandel dip [10]. These are revealed by the probabilities pertaining to the wrong events (8)–(11). Simply, for certain orientations of the polarizers, if the two photons emerge on one side of the experiment only, then they must exit the polarizing beam splitter via a single output port (this effect is due to the bosonic-type indistinguishability of photons; see [10]).

Finally let us discuss what the critical efficiency of the detection of experiments of this type is. To this end, in our calculations we will use a very simple model of imperfect detections: we insert a beam splitter with reflectivity $\sqrt{1-\eta}$, in front of an ideal detector, which observes only the transmitted light. This results in the system behaving just like a detector of efficiency η. If we assume that the incoming light is described by a creation operator $a^\dagger$, then the transmitted mode is denoted as $t_a^\dagger$ whereas reflected mode is denoted as $r_a^\dagger$ and one has

$$a^\dagger=\sqrt{1-\eta}\,r_{a^\dagger}^\dagger+\sqrt{\eta}\,t_{a^\dagger}^\dagger. \quad (16)$$

For instance, if one takes the following part of the state vector (5)

$$c_{||}^\dagger d_{||}^\dagger|0\rangle, \quad (17)$$

the beam-splitter model of an imperfect detector transforms this term into

$$[(1-\eta)r_{c_{||}}^\dagger r_{d_{||}}^\dagger+\sqrt{\eta(1-\eta)}\,r_{c_{||}}^\dagger t_{d_{||}}^\dagger+\sqrt{\eta(1-\eta)}\,t_{c_{||}}^\dagger r_{d_{||}}^\dagger+\eta t_{c_{||}}^\dagger t_{d_{||}}^\dagger]|0\rangle. \quad (18)$$

The probabilities now read

$$P(3,\theta_1;2,\theta_2)=P(2,\theta_1;3,\theta_2),$$
$$P(1,\theta_1;3,\theta_2)=P(3,\theta_1;1,\theta_2)=\eta(1-\eta), \quad (19)$$

$$P(1,\theta_1;1,\theta_2)=P(2,\theta_1;2,\theta_2)=\tfrac{1}{4}\eta^2[\sin(\theta_1-\theta_2)]^2, \quad (20)$$

$$P(2,\theta_1;1,\theta_2)=P(1,\theta;2,\theta_2)=\tfrac{1}{4}\eta^2[\cos(\theta_1-\theta_2)]^2, \quad (21)$$

$$P(5,\theta_1;3,\theta_2)=P(6,\theta_1;3,\theta_2)=\tfrac{1}{8}\eta^2[\sin(2\theta_1)]^2, \quad (22)$$

$$P(3,\theta_1;5,\theta_2)=P(3,\theta_1;6,\theta_2)=\tfrac{1}{8}\eta^2[\sin(2\theta_2)]^2, \quad (23)$$

$$P(4,\theta_1;3,\theta_2)=\tfrac{1}{4}\eta^2[\cos(2\theta_1)]^2, \quad (24)$$

$$P(3,\theta_1;4,\theta_2)=\tfrac{1}{4}\eta^2[\cos(2\theta_2)]^2. \quad (25)$$

The correlation function, which includes the inefficiency of the detection, reads

$$E(\psi_1,\psi_2;\eta,\alpha)=\eta^2E(\psi_1,\psi_2;\alpha)+(1-\eta)^2, \quad (26)$$

where $E(\psi_1,\psi_2;\alpha)$ is given by Eq. (14). We have tacitly assumed here that the parameters α and η are independent of each other (this assumption may not hold for specific technical arrangements). Putting this prediction into the CHSH inequality, assuming that $\alpha=1$ (full distinguishability), we obtain a minimum quantum efficiency needed for violation of local realism equal to 0.91, whereas for other values of α we have for $\alpha=0,\eta=0.926$; for $\alpha=0.5,\eta=0.92$; for $\alpha=0.75,\eta=0.92$; and for $\alpha=0.875,\eta=0.91$. One should note here that the method of value assignment of [6] is in accordance with the method given by Garg and Mermin [13] for the optimal estimation of required detector quantum efficiency to violate local realism in a Bell test. Thus the obtained efficiencies are indeed the lowest possible, and show that experiments of this type are not good candidates for a ''loophole-free'' Bell test [14]; nevertheless, due to the fact that the whole observable effect is a consequence of quantum principle of particle indistinguishability, such tests are very interesting in themselves—they reveal the entanglement inherently associated with this principle.

M.Z. was supported by the University of Gdansk under Grant No. BW/5400-5-0202-8. D.K. was supported by the KBN under Grant No. 2 P03B 096 15.

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