Five-Zero Texture non-Fritzsch like Quark Mass Matrices in the Standard Model

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We will consider a five-zero texture non-Fritzsch like quark mass matrices that is completely valid and generates all the physical quantities involved, including the quark masses, the Jarlskog invariant quantity and the inner angles of the Cabibbo-Kobayashi-Maskawa unitarity triangle, and explaining the charge parity violation phenomenon at 1σ confidence level. To achieve this, non-physical phases must be included in the unitary matrices used to diagonalize the quark mass matrices, in order to put the Cabibbo-Kobayashi-Maskawa matrix in standard form. Besides, these phases can be rotated away so they do not have any physical meaning. Thus, the model has a total of nine parameters to reproduce ten physical quantities, which implies physical relationships between the quark masses and/or mixings.

I. INTRODUCTION

In the Standard Model (SM), the quark mass matrices are introduced into the Lagrangian by means of Higgs-fermion couplings

$$-\mathcal{L}_M = \bar{u}_R M_u u_L + \bar{d}_R M_d d_L + h.c. \qquad (1.1)$$

Here, the three-dimensional complex quark mass matrices M_u and M_d are arbitrary, and contain 36 real parameters, which are larger than the ten physical observables to describe: six quark masses, three flavor mixing angles and one charge parity (CP) violating phase. However, models like the SM or its extensions, where the right-handed fields are $SU(2)$ singlets, it is always possible to choose a suitable basis for the right-handed quarks by using the unitary matrix coming from the *polar decomposition theorem* of matrix algebra, such that the resultant up- and down-type mass matrices become hermitian. For this reason, without losing generality, the quark mass matrices M_u and M_d can be assumed to be hermitian [11, 13, 16, 19, 22, 28, 32].

$$M_u^\dagger = M_u, \quad \text{and} \quad M_d^\dagger = M_d. \qquad (1.2)$$

Another consequence for models like the SM is that one has the freedom to make a unitary transformation for left- and right-handed quarks under which the gauge currents are invariant, and as a result the mass matrices transform to new equivalent mass matrix pictures. It consists basically of a common unitary transformation applied to M_u and M_d known as a "Weak Basis" (WB) Transformation [1, 11, 33, 36], as follows

$$M_u \to M_u' = U^\dagger M_u U, \quad M_d \to M_d' = U^\dagger M_d U, \qquad (1.3)$$

where U is an arbitrary unitary matrix which preserves the hermiticity of the mass matrices, Eqs. (1.2), and leaves the physics invariant, including the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. Additionally, an important result established by me [16, 18] is that the WB transformations not only generates new equivalent mas matrix pictures but also these transformations are "complete" in the sense that any physical

viable mass matrices can be derived from specific quark mass matrices. So WB transformations have been used in quark mass matrices to obtain either non-physical zero textures [1] or physical zero textures by using experimental data [16].

1. Fritzsch six-zero textures

The hermiticity of quark mass matrices M_u and M_d brings down the number of free parameters from 36 to 18, which however, is still a large number compared to the number of observables. With the idea of reduce the number of free parameters Weinberg and Fritzsch [9, 10, 30] initiated the idea of texture specific mass matrices which on the one hand can result in self-consistent and experimentally-favored relations between quark masses and flavor mixing parameters, while on the other hand, the discrete flavor symmetries hidden in such mass matrix textures might finally provide useful hints towards the dynamics of quark mass generation and CP violation in a more fundamental theoretical framework. To define the various texture specific cases, we present the typical Fritzsch-like texture hermitian quark mass matrices with six-zero textures [10], e.g.,

$$M_u = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & 0 & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, M_d = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & 0 & B_D \\ 0 & B_D^* & C_D \end{pmatrix}, \qquad (1.4)$$

where both matrices have the up-down parallelism and each one is a three-zero texture type. This Fritzsch ansätze were ruled out because of the large value of 't' quark mass, such that the predicted $|V_{cb}|$ is far off from the experimental data [7, 19]. Additionally, the predicted magnitude of $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}$ is too low ($V_{ub}/V_{cb} \approx 0.05$ or smaller for reasonable values of the quark masses m_u and m_c [13]) to agree with the actual experimental result ($|V_{ub}/V_{cb}|_{\text{ex}} \approx 0.09$ [24]).

As a result, the next case to be considered is the five-zero textures, which has been extensively studied by researchers, but results have been mixed.

2. Five-zero textures

In recent papers, several authors have explored the five-zero texture quark mass matrices [1, 6, 8, 19, 22, 23, 25, 26, 28, 29]. A general conclusion given by them is that five-zero textures are not viable models or they have a limited viability. However, numerical examples showing the viability of the model are given by me in paper [16], where its validity is checked by adding non-physical phases to the diagonalization matrices [17].

This paper is organized as follows: in Sect. II we give the recent experimental data for quark masses and mixing angles, we stablish the initial quark mass matrices for the SM, and we give a better justification to assume only a negative eigenvalue in each quark mass matrix. So in Sect. III we explore all possible three-zero textures in each quark mass matrix, such that using the WB transformation we obtain numerical five-zero texture non-Fritzsch like quark mass matrices. A first analytical study for a five-zero texture will be carried out in Sect. IV, and our conclusions are presented in Sect. V. An appendix at the end improves the mathematical tools used.

II. DATA AND INITIAL CONDITIONS

Before getting numerical five-zero textures for quark mass matrices, we need to give the recent experimental data, like quark masses and CKM mixing angles. We need to give the initial quark mass matrices on which the WB transformation is applied. And we give a brief reasoning about to assuming only a negative eigenvalue in each quark mass matrix, in order to simplify calculations without losing generality.

A. Quark masses and CKM

The observed CKM matrix parameters $|V_{ij}|$ are given at $\mu = m_Z$, because of that, we summarize quark masses (in MeV units) at $\mu = m_Z$ [13, 24, 34, 35].

$$
m_u = 1.38^{+0.42}_{-0.41}, \ m_c = 638^{+43}_{-84}, \ m_t = 172100 \pm 1200,
$$
$$
m_d = 2.82 \pm 0.48, \ m_s = 57^{+18}_{-12}, \ m_b = 2860^{+160}_{-60}. \tag{2.1}
$$

The CKM mixing matrix [3, 21, 24] is a 3×3 unitary matrix, which can be parametrized by three mixing angles and the CP-violating Kobayashi-Maskawa (KM) phase [21]. Usually it has the following standard choice [5]

$$
V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix}, \tag{2.2}
$$

where $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$, with the angles lying in the first quadrant, so $s_{ij}, c_{ij} \geq 0$. And δ is the phase responsible for all CP-violating phenomena in flavor-changing processes in the SM.

The Wolfenstein parametrization [2, 4, 31]

$$
s_{12} = \lambda, \qquad s_{23} = A\,\lambda^2,
$$
$$
s_{13}\,e^{i\delta} = \frac{A\,\lambda^3(\bar\rho + i\,\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2}\,[1 - A^2\,\lambda^4(\bar\rho + i\,\bar\eta)]}, \tag{2.3}
$$

exhibits the experimental hierarchy $s_{13} \ll s_{23} \ll s_{12} \ll 1$. The small range allowed for some of the CKM elements

due to the unitarity of the three generation CKM matrix, permits to fit, using the method of Refs. [2, 20, 31], the Wolfenstein parameters defined in Eq. (2.3), giving

$$
\lambda = 0.22537 \pm 0.00061, \quad A = 0.814^{+0.023}_{-0.024},
$$
$$
\bar\rho = 0.117 \pm 0.021, \quad \bar\eta = 0.353 \pm 0.013. \tag{2.4}
$$

The fit results for the values of all nine CKM elements are

$$
V = \begin{pmatrix} 0.974267 & 0.225369 & 0.00355431\,e^{-i\,1.25135} \\ 0.225222\,e^{-i\,3.14099} & 0.97343\,e^{-i\,3.23011\times10^{-5}} & 0.0413441 \\ 0.00886248\,e^{-i\,0.379708} & 0.0405392\,e^{-i\,3.12285} & 0.999139 \end{pmatrix}, \tag{2.5}
$$

with magnitudes

$$
|V| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}, \tag{2.6}
$$

and the Jarlskog invariant is

$$
J = \left(3.06^{+0.21}_{-0.20}\right) \times 10^{-5}. \tag{2.7}
$$

B. The initial quark mass matrices

In the quark-family basis, it is more convenient to use either

$$M_u = D_u = \begin{pmatrix} \lambda_{1u} & 0 & 0 \\ 0 & \lambda_{2u} & 0 \\ 0 & 0 & \lambda_{3u} \end{pmatrix}, \qquad (2.8a)$$

$$M_d = V D_d V^\dagger,$$

or

$$M_u = V^\dagger D_u V,$$

$$M_d = D_d = \begin{pmatrix} \lambda_{1d} & 0 & 0 \\ 0 & \lambda_{2d} & 0 \\ 0 & 0 & \lambda_{3d} \end{pmatrix}, \qquad (2.8b)$$

as the initial general quark mass matrix bases [1, 13, 16–18]; where V is the CKM mixing matrix, and the quark mass eigenvalues $|\lambda_{iq}|$ $(i = 1, 2, 3)$ for up- $(q = u)$ and down- $(q = d)$ satisfy

$$|\lambda_{1u}| = m_u, |\lambda_{2u}| = m_c, |\lambda_{3u}| = m_t,$$
$$|\lambda_{1d}| = m_d, |\lambda_{2d}| = m_s, |\lambda_{3d}| = m_b. \qquad (2.9)$$

Thus, λ_{iq} may be either positive or negative and obey the hierarchy

$$|\lambda_{1q}| \ll |\lambda_{2q}| \ll |\lambda_{3q}|. \qquad (2.10)$$

The bases (2.8a) or (2.8b) are identified as the *diagonal-up (quark) basis* or the *diagonal-down basis* mass matrix [16], respectively. They are obtained performing a WB transformation on the general quark mass matrix bases [17, 18].

C. A negative quark mass eigenvalue

Based on the completeness of WB transformations [16, 18], it permits us to use the diagonal-up basis (2.8a) (or the diagonal-down basis (2.8b)) as the starting matrices to generate any one of quark mass matrix pictures. If there are zero textures in quark mass matrices, this transformation is able to find them. Since some zero textures must be located on the diagonal entries of hermitian up- and down-quark mass matrices, it implies that at least one and at most two of their eigenvalues be negative [1]. Furthermore, for the case of two negative eigenvalues, these mass matrices can be reduced to have only one, by adding a minus sign to the mass matrix basis (2.8a) (or (2.8b)) as follows:

$$M_u = -(-M_u) \quad \text{or/and} \quad M_d = -(-M_d),$$

and implementing the WB transformations for the terms in parenthesis. Thus, zero textures in models can be deduced by assuming that each quark mass matrix M_u and M_d contains exactly one negative eigenvalue [16], i.e.,

$$\lambda_{iq} < 0 \ (i = 1, 2 \text{ or } 3) \text{ and } \lambda_{jq} > 0 \text{ for } j \neq i. \quad (2.11)$$

Thus, we can assume without loss of generality that each quark mass matrix contains only one negative eigenvalue. And the minus sign in masses can be removed later by redefining the right-handed field singlets.

III. NUMERICAL FIVE-ZERO TEXTURES

It is important to say that *realistic* quark mass matrix may contain at most three-zero textures in their matrix elements. Furthermore, we have only two realistic type of patterns depending how the three-zero textures are distributed in the mass matrix entries. In one case we have a matrix with two zeros on its diagonal (*two-zero diagonal pattern*) and the other one is a matrix with one zero on its diagonal (*one-zero diagonal pattern*), as they are indicated in each column of Table I. From which you can observe that making WB transformations using the permutation matrices p, we obtain all the possible viable cases for each pattern. It is important to emphasize that no more viable configurations with three zeros are possible to obtain. Hence, Table I summarizes all the viable three-zero texture for up- and down-quark mass matrices. These patterns are general and the phases are not necessary to be included, as will be shown below when the initial bases (2.8) be used. Let us study each pattern.

Permutation matrices	two-zero diagonal pattern $(p_i \, M_q \, p_i^T)$	one-zero diagonal pattern $(p_i \, M_q \, p_i^T)$
$p_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & 0 \\ \|\xi_q\| & 0 & \|\beta_q\| \\ 0 & \|\beta_q\| & \alpha_q \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & 0 \\ \|\xi_q\| & \gamma_q & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_2 = \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \|\xi_q\| \\ 0 & \alpha_q & \|\beta_q\| \\ \|\xi_q\| & \|\beta_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & \|\xi_q\| \\ 0 & \alpha_q & 0 \\ \|\xi_q\| & 0 & \gamma_q \end{pmatrix}$
$p_3 = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & \|\beta_q\| & 0 \\ \|\beta_q\| & 0 & \|\xi_q\| \\ 0 & \|\xi_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & \gamma_q & \|\xi_q\| \\ 0 & \|\xi_q\| & 0 \end{pmatrix}$
$p_4 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \|\xi_q\| & \|\beta_q\| \\ \|\xi_q\| & 0 & 0 \\ \|\beta_q\| & 0 & \alpha_q \end{pmatrix}$	$\begin{pmatrix} \|\gamma_q\| & \|\xi_q\| & 0 \\ \|\xi_q\| & 0 & 0 \\ 0 & 0 & \alpha_q \end{pmatrix}$
$p_5 = \begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & \|\beta_q\| \\ 0 & 0 & \|\xi_q\| \\ \|\beta_q\| & \|\xi_q\| & 0 \end{pmatrix}$	$\begin{pmatrix} \alpha_q & 0 & 0 \\ 0 & 0 & \|\xi_q\| \\ 0 & \|\xi_q\| & \gamma_q \end{pmatrix}$
$p_6 = \begin{pmatrix} & 1 & \\ & & 1 \\ 1 & & \end{pmatrix}$	$\begin{pmatrix} 0 & \|\beta_q\| & \|\xi_q\| \\ \|\beta_q\| & \alpha_q & 0 \\ \|\xi_q\| & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \gamma_q & 0 & \|\xi_q\| \\ 0 & \alpha_q & 0 \\ \|\xi_q\| & 0 & 0 \end{pmatrix}$

TABLE I. One- and two-zero diagonal pattern. The phases are not necessary to be included.

A. Two-zero diagonal pattern

In what follows, we work the *diagonal-up* and the *diagonal-down* cases simultaneously, both patterns are

indicated indistinctly with q, where $q = u$ or $q = d$ depending if we are working either the up-quark mass matrix or the down-quark mass matrix, respectively. The standard representation for the two-zero diagonal pattern, in Table I, is

$$M_q = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & 0 & |\beta_q| \\ 0 & |\beta_q| & \alpha_q \end{pmatrix}, \tag{3.1}$$

where its diagonalizing matrix U_q satisfies the following relation

$$U_q^\dagger M_q U_q = D_q = \begin{pmatrix} \lambda_{1q} & & \\ & \lambda_{2q} & \\ & & \lambda_{3q} \end{pmatrix}, \tag{3.2}$$

with the diagonal quark mass matrix D_q containing the quark mass eigenvalues $|\lambda_{iq}|$. The invariant matrix operators "det" and "trace" applied on (3.2), gives us

$$\alpha_q = \lambda_{1q} + \lambda_{2q} + \lambda_{3q}, \tag{3.3}$$

$$|\beta_q| = \sqrt{-\frac{(\lambda_{1q} + \lambda_{2q})(\lambda_{1q} + \lambda_{3q})(\lambda_{2q} + \lambda_{3q})}{\alpha_q}}, \tag{3.4}$$

$$|\xi_q| = \sqrt{\frac{-\lambda_{1q}\lambda_{2q}\lambda_{3q}}{\alpha_q}}. \tag{3.5}$$

The expression (3.5) must be a real number, and because only one λ_{iq} eigenvalue is assumed negative (Eq. (2.11)), we have that

$$\alpha_q > 0, \tag{3.6}$$

that together with (3.4) and the hierarchy (2.10) we find that only a possibility is allowed

$$\lambda_{1q}, \lambda_{3q} > 0 \quad \text{and} \quad \lambda_{2q} < 0. \tag{3.7}$$

According to (A9), and assuming $\gamma_q = 0$ in (A1), we have the diagonalizing matrix for (3.1)

$$U_q = \begin{pmatrix} e^{ix}\sqrt{\frac{\lambda_{2q}\lambda_{3q}(\alpha_q - \lambda_{1q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\sqrt{\frac{\lambda_{1q}\lambda_{3q}(\lambda_{2q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{1q}\lambda_{2q}(\alpha_q - \lambda_{3q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ e^{ix}\sqrt{\frac{\lambda_{1q}(\lambda_{1q}-\alpha_q)}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q - \lambda_{2q})}{(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\lambda_{3q}-\alpha_q)}{(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \\ -e^{ix}\sqrt{\frac{\lambda_{1q}(\alpha_q - \lambda_{2q})(\alpha_q - \lambda_{3q})}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{1q})}} & -e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q - \lambda_{1q})(\lambda_{3q}-\alpha_q)}{\alpha_q(\lambda_{2q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\alpha_q - \lambda_{1q})(\alpha_q - \lambda_{2q})}{\alpha_q(\lambda_{3q}-\lambda_{1q})(\lambda_{3q}-\lambda_{2q})}} \end{pmatrix}, \tag{3.8}$$

where α_q is given in (3.3).

1. The diagonal-down basis

Performing a WB transformation for the diagonal-down basis (2.8b), using this time the unitary matrix given in (3.8) with $q = d$, i.e., U_d, we have

$$M_d' = U_d(D_d)U_d^\dagger = \begin{pmatrix} 0 & |\xi_d| & 0 \\ |\xi_d| & 0 & |\beta_d| \\ 0 & |\beta_d| & \alpha_d \end{pmatrix}, \tag{3.9}$$

$$M_u' = U_d\left(V^\dagger D_u V\right)U_d^\dagger, \tag{3.10}$$

where (3.2) was used. According to (3.6) and (3.7), in this case we have

$$\lambda_{1d}, \ \lambda_{3d} > 0, \ \lambda_{2d} < 0, \ \alpha_d = \lambda_{1d} + \lambda_{2d} + \lambda_{3d} > 0. \tag{3.11}$$

The computation is facilitated defining the following new variables for the phases introduced in (3.8)

$$\begin{aligned} e^{ix} &= x_1 + ix_2, \quad \text{with} \quad x_1^2 + x_2^2 = 1, \\ e^{iy} &= y_1 + iy_2, \quad \text{with} \quad y_1^2 + y_2^2 = 1. \end{aligned} \tag{3.12}$$

Hence, the variabbles satisfies

$$|x_1|, |x_2| \leq 1, \quad \text{and} \quad |y_1|, |y_2| \leq 1. \tag{3.13}$$

With the former definitions and using data from the CKM mixing matrix V in (2.5), the matrix entries of M_u' in (3.10), become *hypersurfaces* for the set of points (x_1, x_2, y_1, y_2) in $\mathbb{R}^4$ for each case to be considered: $\lambda_{1u} = -m_u$ or $\lambda_{2u} = -m_c$ or $\lambda_{3u} = -m_t$. The analysis of these hypersurfaces, by considering conditions (3.13), shows that only the entries (1,2) and (1,3) of M_u' could have possible zero (texture) solutions.

a. $\lambda_{1u} = -m_u$: In this case the best results are obtained by considering the following quark masses (in MeV units): $m_u = 1.71604$, $m_d = 2.9042$, $m_s = 65$, $m_c = 567$, $m_b = 2860$, $m_t = 172100$ which are close to the central values and are within the allowable range permitted by (2.1). The variables (x_1, x_2, y_1, y_2) satisfy the implicit equations

whose solutions are

$\text{Re}[M'_u(1,2)] = -138.321 + 201.317x_1 - 80.6426x_2 + 204.719y_1$
$\qquad - 164.358x_1y_1 + 21.6323x_2y_1 + 3.85055y_2 - 21.6323x_1y_2$
$\qquad - 164.358x_2y_2 = 0,$

$\text{Im}[M'_u(1,2)] = 80.479x_1 + 200.909x_2 - 4.02965y_1 - 23.6558x_1y_1$
$\qquad - 179.732x_2y_1 + 214.241y_2 + 179.732x_1y_2$
$\qquad - 23.6558x_2y_2 = 0,$

$x_1 = 0.684994, \quad y_1 = -0.500433,$
$x_2 = 0.728548, \quad y_2 = -0.865775,$

such that $M'_u(1,1) = 0$. The corresponding numerical five-zero texture quark mass matrices obtained are

$$M'_u = \begin{pmatrix} 0 & 0 & -79.32299208381 + 154.7195315i \\ 0 & 5539.23021 & 28125.945500584217 + 6112.7938593i \\ -79.32299208381 - 154.7195315i & 28125.9455 - 6112.7938593i & 167126.0537497 \end{pmatrix} \text{ MeV}, \qquad (3.14a)$$

$$M'_d = \begin{pmatrix} 0 & 13.891097 & 0 \\ 13.891097 & 0 & 421.41405 \\ 0 & 421.41405 & 2797.9042 \end{pmatrix} \text{ MeV}, \qquad (3.14b)$$

and their diagonalizing matrices are respectively:

$$U_u = \begin{pmatrix} 0.6762634914995 + 0.734812367i & -0.050244372843746 + 0.01389198632i & -0.0004494994264332 + 0.00088826384734i \\ 0.027504657705 - 0.0431835830349i & -0.4970428606 - 0.849312285i & 0.1665852443687 + 0.03528548775i \\ -0.00340858221659 + 0.00924818235635i & 0.1150663143 + 0.12513711i & 0.98538127634 - 0.00519998920457i \end{pmatrix}, \qquad (3.15a)$$

$$U_d = \begin{pmatrix} 0.67017852 + 0.71279034i & 0.10351850 + 0.17909241i & 0.00070804245 \\ 0.14011366 + 0.14902248i & -0.48438959 - 0.83801926i & 0.14577693 \\ -0.021125534 - 0.022468754i & 0.071301225 + 0.12335484i & 0.98931723 \end{pmatrix}, \qquad (3.15b)$$

which gives the exact CKM mixing matrix (2.5) $U_u^\dagger U_d = V$ and the CP violating phase, in agreement with the measured values up to 1σ [25, 29]; although a large number of significant digits were required.

$\text{Re}[M'_u(1,2)] = 69.8779 + 212.583x_1 - 84.5059x_2 + 203.455y_1$
$\qquad + 66.7775x_1y_1 + 21.9698x_2y_1 + 3.80114y_2$
$\qquad - 21.9698x_1y_2 + 66.7775x_2y_2 = 0,$

$\text{Im}[M'_u(1,2)] = 84.3394x_1 + 212.165x_2 - 3.99422y_1 - 23.7916x_1y_1$
$\qquad + 72.3149x_2y_1 + 213.789y_2 - 72.3149x_1y_2$
$\qquad - 23.7916x_2y_2 = 0$

b. $\lambda_{2u} = -m_c$: For this case the quark masses that best fit results are (in MeV units): $m_u = 1.38$, $m_d = 2.82$, $m_s = 70.8356$, $m_c = 592.3$, $m_b = 2860$, $m_t = 172100$, where the implicit equations

have the solutions $x_1 = -0.99609$, $x_2 = -0.0883441$, $y_1 = 0.949769$, $y_2 = 0.312952$, such that $M'_u(1,1) = 0$. The corresponding numerical five-zero texture quark mass matrices obtained are

$$M'_u = \begin{pmatrix} 0 & 0 & 101.16321 - 273.96222i \\ 0 & 1649.3286 & 19419.195 - 2159.2334i \\ 101.16321 + 273.96222i & 19419.195 + 2159.2334i & 169859.75. \end{pmatrix} \text{ MeV}, \qquad (3.16a)$$

$$M'_d = \begin{pmatrix} 0 & 14.304633 & 0 \\ 14.304633 & 0 & 441.0438 \\ 0 & 441.0438 & 2791.9844 \end{pmatrix} \text{ MeV}, \qquad (3.16b)$$

and their respective diagonalizing matrices are

$$U_u = \begin{pmatrix} -0.99358138 - 0.098207896i & 0.039092042 - 0.040251962i & 0.0005870283 - 0.0015803942i \\ 0.020016032 + 0.052233615i & 0.94574516 + 0.29911097i & 0.11321111 - 0.012367896i \\ -0.0011910042 - 0.0045650693i & -0.10404534 - 0.046096344i & 0.99349071 + 0.0019085337i \end{pmatrix}, \qquad (3.17a)$$

$$U_d = \begin{pmatrix} -0.97682387 - 0.086635334i & -0.18589492 - 0.061253052i & 0.0007623121 \\ -0.19257000 - 0.017079196i & 0.92053883 + 0.30332089i & 0.15241321 \\ 0.030450627 + 0.0027006918i & -0.14181749 - 0.046729377i & 0.98831656 \end{pmatrix}, \qquad (3.17b)$$

which gives the exact CKM matrix (2.5): $U_u^\dagger U_d = V$. A complete analysis for this last case was already made in paper [16].

We must say that we have taken into account all possibilities, but at most five-zero textures were found. As a result, textures with six zeros are not realistic models. Additionally, for the diagonal-up basis (2.8a) *no* five-zero textures were found. Similarly, for the one-zero diagonal pattern case (Table I), no solutions with five-zeros were found.

IV. ANALYTICAL FIVE-ZERO TEXTURE NON-FRITZSCH-LIKE QUARK MASS MATRIX

It is well known that certain relationship exists between the quark mass matrices and the CKM mixing angles and CP violating phase, so an analytical study unraveling the deeper aspects of flavor physics must be done. For our case, the numerical five-zero textures for quark mass matrices given in Eqs. (3.14) and (3.16) are viable models according with the latest low energy data. Thus, let us consider the following analytical five-zero textures model

$$
\begin{aligned}
M_u &= P^\dagger \begin{pmatrix} 0 & 0 & |\xi_u| \\ 0 & \alpha_u & |\beta_u| \\ |\xi_u| & |\beta_u| & \gamma_u \end{pmatrix} P, \\
M_d &= \begin{pmatrix} 0 & |\xi_d| & 0 \\ |\xi_d| & 0 & |\beta_d| \\ 0 & |\beta_d| & \alpha_d \end{pmatrix},
\end{aligned} \tag{4.1}
$$

where all the phases are reduced to those included in $P = \text{diag}(e^{-i\phi_{\xi_u}}, e^{-i\phi_{\beta_u}}, 1)$ (with $\phi_{\beta_u} \equiv \arg(\beta_u)$ and $\phi_{\xi_u} \equiv \arg(\xi_u)$) which was achieved by making a WB transformation, in such a way that the phases for M_d are

not necessary to be considered. So we have nine free parameters, to reproduce ten physical quantities: 6 quark masses and 3 mixing angles and 1 phase from the CKM matrix, which implies physical relationships between the quark masses and/or mixings. The five-zero texture (4.1) is a non-Fritzsch-like due to their zero patterns do not fit with those zero textures proposed by Fritzsch in (1.4).

Making the permutation $P_2 = [(1, 0, 0), (0, 0, 1), (0, 1, 0)]$, the matrix M_u in (4.1), can be written as

$$
M_u = P^\dagger P_2 \begin{pmatrix} 0 & |\xi_u| & 0 \\ |\xi_u| & \gamma_u & |\beta_u| \\ 0 & |\beta_u| & \alpha_u \end{pmatrix} P_2 P, \tag{4.2}
$$

such that the permuted matrix has the same structure of (A1) with $q = u$. Thus, considering the case

$$
\lambda_{1u} = -m_u, \quad \lambda_{2u} = m_c, \quad \text{and} \quad \lambda_{3u} = m_t,
$$

we have from (A3) through (A5) that

$$
\begin{aligned}
\gamma_u &= m_t + m_c - m_u - \alpha_u, \\
|\beta_u| &= \frac{\sqrt{\alpha_u - m_c}\sqrt{m_t - \alpha_u}\sqrt{\alpha_u + m_u}}{\sqrt{\alpha_u}}, \\
|\xi_u| &= \frac{\sqrt{m_c}\sqrt{m_t}\sqrt{m_u}}{\sqrt{\alpha_u}},
\end{aligned} \tag{4.3}
$$

and using (A6), we obtain

$$
m_c < \alpha_u < m_t. \tag{4.4}
$$

According to (4.2) and (A9), an approach to diagonalizing M_u is the unitary matrix U_u ($\to P^\dagger P_2 U_u$), giving by

$$
U_u \approx \begin{pmatrix} e^{i(\phi_{\xi_u})} & \frac{\sqrt{\alpha_u - m_c}\sqrt{m_u}e^{i(\phi_{\xi_u})}}{\sqrt{\alpha_u}\sqrt{m_c}} & \frac{\sqrt{m_c}\sqrt{m_t - \alpha_u}\sqrt{m_u}e^{i(\phi_{\xi_u})}}{\sqrt{\alpha_u}\,m_t} \\ \frac{\sqrt{\alpha_u - m_c}\sqrt{m_t - \alpha_u}\sqrt{m_u}\,e^{i(\phi_{\beta_u})}}{\sqrt{\alpha_u}\sqrt{m_c}\sqrt{m_t}} & -\frac{\sqrt{m_t - \alpha_u}\,e^{i(\phi_{\beta_u})}}{\sqrt{m_t}} & \frac{\sqrt{\alpha_u - m_c}\,e^{i(\phi_{\beta_u})}}{\sqrt{m_t}} \\ -\frac{\sqrt{\alpha_u}\sqrt{m_u}}{\sqrt{m_c}\sqrt{m_t}} & \frac{\sqrt{\alpha_u - m_c}}{\sqrt{m_t}} & \frac{\sqrt{m_t - \alpha_u}}{\sqrt{m_t}} \end{pmatrix}, \tag{4.5}
$$

where the hierarchy established in (2.10) and Eq. (4.4) were considered. The phases x and y are not included because we are only interested in magnitudes.

mass matrix M_d case ($q = d$) in (4.1), we have that

$$
\begin{aligned}
\alpha_d &= m_d + m_b - m_s, \\
|\beta_d| &= \frac{\sqrt{m_d + m_b}\sqrt{m_b - m_s}\sqrt{m_s - m_d}}{\sqrt{m_d + m_b - m_s}}, \\
|\xi_d| &= \frac{\sqrt{m_b}\sqrt{m_d}\sqrt{m_s}}{\sqrt{m_d + m_b - m_s}}.
\end{aligned} \tag{4.6}
$$

Taking into account (3.3) through (3.8) for the down

And the unitary matrix U_d, which diagonalizes M_d, ac-

cording to (3.7) and (3.8) for $q = d$ is given by the approach

$$U_d \approx \begin{pmatrix} 1 & -\frac{\sqrt{m_d}}{\sqrt{m_s}} & \frac{\sqrt{m_d}\,m_s}{(m_b)^{3/2}} \\ \frac{\sqrt{m_d}}{\sqrt{m_s}} & 1 & \frac{\sqrt{m_s}}{\sqrt{m_b}} \\ -\frac{\sqrt{m_d}}{\sqrt{m_b}} & -\frac{\sqrt{m_s}}{\sqrt{m_b}} & 1 \end{pmatrix}, \qquad (4.7)$$

where the hierarchy (2.10) was considered and the phases x and y be omitted.

Now, we can easily find the CKM matrix $V = U_u^\dagger U_d$. In particular, using the matrix form (4.5) and (4.7) for U_u, U_d respectively, can survive current experimental tests. To leading order, we obtain.

$$|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1, \qquad (4.8a)$$

$$|V_{us}| \approx \left| \sqrt{\frac{\alpha_u - m_c}{\alpha_u}} \sqrt{\frac{m_u}{m_c}} - e^{i(\phi_{\beta_u} - \phi_{\xi_u})} \sqrt{\frac{m_d}{m_s}} \right|, \quad (4.8b)$$

$$|V_{cd}| \approx \left| \sqrt{\frac{\alpha_u - m_c}{\alpha_u}} \sqrt{\frac{m_u}{m_c}} - e^{i(\phi_{\xi_u} - \phi_{\beta_u})} \sqrt{\frac{m_d}{m_s}} \right|, \quad (4.8c)$$

$$|V_{cb}| \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{i\phi_{\beta_u}} \sqrt{\frac{\alpha_u - m_c}{m_t}} \right|, \qquad (4.8d)$$

$$|V_{ts}| \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_{\beta_u}} \sqrt{\frac{\alpha_u - m_c}{m_t}} \right|, \qquad (4.8e)$$

$$\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}} \left| \frac{\sqrt{\frac{\alpha_u}{m_t}} - e^{-i\phi_{\beta_u}} \sqrt{\frac{\alpha_u - m_c}{\alpha_u}} \sqrt{\frac{m_s}{m_b}}}{\sqrt{\frac{\alpha_u - m_c}{m_t}} - e^{-i\phi_{\beta_u}} \sqrt{\frac{m_s}{m_b}}} \right|, \quad (4.8f)$$

$$\frac{|V_{td}|}{|V_{ts}|} \approx \sqrt{\frac{m_d}{m_s}}, \qquad (4.8g)$$

where $\alpha_u \ll m_t$ was assumed. Hence, α_u is an apparently free parameter that must be adjusted to give physical quantities. Note that if $\alpha_u \gg m_c$ then $\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}}$, but we shall consider $\alpha_u \gtrsim m_c$ in order to fit experimental data. Also, as you can observe the famous Gatto-Sartori-Tonin (GST) relation $\sin\theta_{12} \approx m_d/m_s$ is maintained [14].

It is obvious that Eqs. (4.8a), (4.8b), (4.8c) and (4.8g) are consistent with the previous results [12, 15, 33]. A good fit of Eqs. (4.8) and the CKM to the experimental data suggests

$$\alpha_u = 5539.2302 \text{ MeV}, \ \phi_{\beta_u} = 0.214008, \ \phi_{\xi_u} = 2.044543, \quad (4.9)$$

which does not differ from the values given in [12, 15, 33] for $\phi_1 \approx -\pi/2 \sim (\phi_{\beta_u} - \phi_{\xi_u})$, which it is an important contribution term for CP-violation in the context of present mass matrices, and $\phi_2 \approx \pi/15 \sim \phi_{\beta_u} \to 0$.

V. CONCLUSIONS

We have made a complete survey of zero textures in general quark mass matrices based on the completeness of the WB transformation [16, 18]. We are being able to reproduce all possible zero textures, by starting from specific quark mass matrices bases, like (2.8a) or (2.8b). In that way, we just find only two different numerical five-zero texture patterns: they are the matrices bases (3.14)

and (3.16), which gives precise quark masses, CKM mixing angles, and the Jarlskog quantity, at 1σ confidence level (C.L.). We must stress that only these zero patterns are possible, including their permutations, in the two-zero diagonal pattern, something indicated in Table I. A standard five-zero texture basis for this viable model is giving in (4.1). It has nine free parameters to reproduce ten physical quantities: 6 quark masses, 3 mixing angles and 1 phase from the CKM matrix, which implies physical relationships between the quark masses and/or mixings, summarized in the approach relations (4.8), which have not been studied before; though, their structures are not so simple as presented by Fritzsch according to (1.4); and they depend from a parameter α_u. However, the GST relation is maintained, and an important contribution for CP violation is still presented in the context of the model. Additionally, our five-zero texture model is a non-Fritzsch like quark mass matrices, because it does not have the same zero texture distribution as given in (1.4).

A recent aspect being claimed by some authors is about the viability of the five-zero texture models. According to these authors the five-zero texture models have a limited viability into the Fritzsch-like model context, and as a result they recommend working only four-zero texture models [8, 19, 22, 26–28]. However, there have been many five-zero textures proposed by other authors [6, 8, 22, 23, 25], although most of them are like-Fritzsch zero textures, so they do not coincide with that configuration obtained by me in Eq. (4.1). In that sense we have found new five-zero texture non-Fritzsch like quark mass matrices which are viable models reproducing experimental data. Even further, our models are the unique viable models that can accommodate data at 1σ C.L.

Appendix A: Four-zero textures

Let us start by implementing a method that we shall apply later to special cases. Let us consider the following structure for the up- $(q = u)$ and/or down- $(q = d)$ quark mass matrix [1]

$$M_q = \begin{pmatrix} 0 & |\xi_q| & 0 \\ |\xi_q| & \gamma_q & |\beta_q| \\ 0 & |\beta_q| & \alpha_q \end{pmatrix}, \qquad (A1)$$

----

[1] The phases in parameters can be included later by means of a WB transformation.

where γ_q and α_q are real numbers because of the hemiticity of M_q. The mass matrix M_q can be diagonalized by using the transformation

$$U_q^\dagger M_q U_q = \begin{pmatrix} \lambda_{1q} & & \\ & \lambda_{2q} & \\ & & \lambda_{3q} \end{pmatrix}, \quad (A2)$$

where λ_{iq} $(i = 1, 2, 3)$ are defined in (2.9). Note that γ_q, $|\beta_q|$ and $|\xi_q|$ can be expressed in terms of λ_{iq} and α_q, by using the invariant matrix functions, $\mathrm{tr} M_q$, $\mathrm{tr} M_q^2$ and $\det M_q$ as follows

$$\gamma_q = \lambda_{1q} + \lambda_{2q} + \lambda_{3q} - \alpha_q, \quad (A3)$$

$$|\beta_q| = \sqrt{\frac{(\alpha_q - \lambda_{1q})(\alpha_q - \lambda_{2q})(\lambda_{3q} - \alpha_q)}{\alpha_q}}, \quad (A4)$$

$$|\xi_q| = \sqrt{\frac{-\lambda_{1q}\lambda_{2q}\lambda_{3q}}{\alpha_q}}. \quad (A5)$$

The Eqs. (A3) through (A5) are real numbers, because of that, the parameter α_q must lie within an interval. Let

us see the different possibilities.

- If $\lambda_{1q} < 0$, $\lambda_{2q} > 0$ and $\lambda_{3q} > 0$ then

$$|\lambda_{2q}| < \alpha_q < |\lambda_{3q}|. \quad (A6)$$

- If $\lambda_{1q} > 0$, $\lambda_{2q} < 0$ and $\lambda_{3q} > 0$ then

$$|\lambda_{1q}| < \alpha_q < |\lambda_{3q}|. \quad (A7)$$

- If $\lambda_{1q} > 0$, $\lambda_{2q} > 0$ and $\lambda_{3q} < 0$ then

$$|\lambda_{1q}| < \alpha_q < |\lambda_{2q}|. \quad (A8)$$

In the analysis, the hierarchy (2.10) was taken into account, and only a negative eigenvalue was considered according to the justification given in Sect. II C.

The exact analytical result for the diagonalizing matrix U_q given in (A2) is [12, 16, 33]

$$U_q = \begin{pmatrix} e^{ix}\frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{2q}\lambda_{3q}(\alpha_q - \lambda_{1q})}{\alpha_q(\lambda_{2q} - \lambda_{1q})(\lambda_{3q} - \lambda_{1q})}} & e^{iy}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}\lambda_{3q}(\lambda_{2q} - \alpha_q)}{\alpha_q(\lambda_{2q} - \lambda_{1q})(\lambda_{3q} - \lambda_{2q})}} & \sqrt{\frac{\lambda_{1q}\lambda_{2q}(\alpha_q - \lambda_{3q})}{\alpha_q(\lambda_{3q} - \lambda_{1q})(\lambda_{3q} - \lambda_{2q})}} \\ -e^{ix}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}(\lambda_{1q} - \alpha_q)}{(\lambda_{2q} - \lambda_{1q})(\lambda_{3q} - \lambda_{1q})}} & e^{iy}\sqrt{\frac{\lambda_{2q}(\alpha_q - \lambda_{2q})}{(\lambda_{2q} - \lambda_{1q})(\lambda_{3q} - \lambda_{2q})}} & \frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{3q}(\lambda_{3q} - \alpha_q)}{(\lambda_{3q} - \lambda_{1q})(\lambda_{3q} - \lambda_{2q})}} \\ e^{ix}\frac{|\lambda_{2q}|}{\lambda_{2q}}\sqrt{\frac{\lambda_{1q}(\alpha_q - \lambda_{2q})(\alpha_q - \lambda_{3q})}{\alpha_q(\lambda_{2q} - \lambda_{1q})(\lambda_{3q} - \lambda_{1q})}} & -e^{iy}\frac{|\lambda_{3q}|}{\lambda_{3q}}\sqrt{\frac{\lambda_{2q}(\alpha_q - \lambda_{1q})(\lambda_{3q} - \alpha_q)}{\alpha_q(\lambda_{2q} - \lambda_{1q})(\lambda_{3q} - \lambda_{2q})}} & \sqrt{\frac{\lambda_{3q}(\alpha_q - \lambda_{1q})(\alpha_q - \lambda_{2q})}{\alpha_q(\lambda_{3q} - \lambda_{1q})(\lambda_{3q} - \lambda_{2q})}} \end{pmatrix}, \quad (A9)$$

where we have added phases to make compatible the generated CKM matrix with the chosen convention (2.2),

somethig justified in [17]. The phase in third column is not necessary to be included because it can be absorbed into the remaining phases.

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