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Effect of Material Combinations and Relative Crack Size on the Stress Intensity Factors of Edge Interface Cracks

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Abstract

In this paper, the stress intensity factors (SIFs) of a single edge interface crack in the bi-material bonded strip subjected to in-plane tension and bending moment are investigated systematically. The SIFs are computed for arbitrary material combinations with varying the relative crack size *a/W*. Specifically, some necessary skills as refined mesh and extrapolations of the stress intensity factors are used to improve the accuracy of the calculation. For the edge interface crack, it is found that the dimensionless SIFs are not always finite for the edge interface cracks in the bonded semi-infinite plate depending on Dundurs' material composite parameters.
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Keywords: Stress intensity factors, edge interface crack, bending moment, material combination,bonded strip

1. Introduction

There are many cases in engineering design to employ the bonded structures or multiple layers in industries. The presence of cracks negatively affects a structure's performance and may result in the damage. Therefore, a lot of research has been pursued on the analysis of the edge interface crack where stresses diverge and oscillate. However, according to the author's best knowledge, most published literatures are about the tensile loading case. There are only few solutions concerning the in-plane bending case of the edge-cracked bonded strip problems.

In this research, the zero element method is used to compute the stress intensity factors (SIFs) for the edge-cracked bonded strip subjected to in-plane tensile and bending loading conditions. In the zero element method, very refined meshes and exact analytical solutions of the reference are utilized for extract

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SIFs. Furthermore, the use of post-processing technique of extrapolation reduces the computational cost and improves the accuracy significantly. New results for the edge-cracked bonded strip under bending moment are computed for various material combinations and relative crack sizes. Then, the SIFs are compared systematically for the bonded strip under tensile and bending loading conditions for the whole range of material combinations and crack sizes. Furthermore, an empirical relation for the factor K for any material combinations within the zone of dominance of the free-edge singularity will also be presented in this paper.
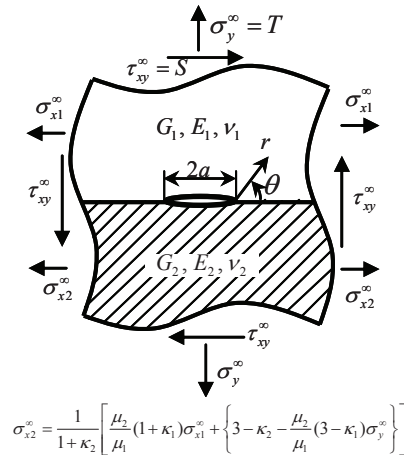

$$\sigma_{x2}^{\infty} = \frac{1}{1+\kappa_2}\left[\frac{\mu_2}{\mu_1}(1+\kappa_1)\sigma_{x1}^{\infty} + \left\{3-\kappa_2-\frac{\mu_2}{\mu_1}(3-\kappa_1)\sigma_y^{\infty}\right\}\right]$$

Fig. 1. The reference problem

2. Numerical methods for the determination of the stress intensity factors

Recently, an effective method was proposed for calculating the stress intensity factors in homogenous plates. Then, the method is successfully extended to the interface crack problems [1]. Both of those methods utilize the stress values at the crack tip computed by FEM. For a given bi-material bonded structure, the stress intensity factors are defined as shown in Eq. (1).

$$\sigma_y + i\tau_{xy} = \frac{K_1 + iK_2}{\sqrt{2\pi r}}\left(\frac{r}{2a}\right)^{i\varepsilon} \quad , \quad r \to 0 \tag{1}$$

$$\varepsilon = \frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{G_1}+\frac{1}{G_2}\right)\Big/\left(\frac{\kappa_2}{G_2}+\frac{1}{G_1}\right)\right] \tag{2}$$

$$\kappa_m = \frac{3-\nu_m}{1+\nu_m}(plane \quad stress) \quad (m=1,2) \tag{3}$$

$$\kappa_m = 3-4\nu_m \ (plane \quad strain) \quad (m=1,2) \tag{4}$$

Here, σ_y, τ_{xy} denote the stress components along the interface. From Eq. (1), the stress intensity factors may be separated as

$$K_1 = \lim_{r \to 0} \sqrt{2\pi r}\, \sigma_y \left(\cos Q + \frac{\tau_{xy}}{\sigma_y} \sin Q \right), K_2 = \lim_{r \to 0} \sqrt{2\pi r}\, \tau_{xy} \left(\cos Q + \frac{\sigma_y}{\tau_{xy}} \sin Q \right) \tag{5}$$

$$Q = \varepsilon \ln(\frac{r}{2a}) \tag{6}$$

Where, r and Q can be chosen as constant values since the reference and given unknown problems have the same FEM mesh patterns and material combinations in the process of analysis. Therefore, expression (9) may be derived from Eq. (5) and Eq. (6) if only Eq. (7) is satisfied. Here, the subscript * denotes the value of the reference problem.

$$\tau_{xy}^{*} \Big/ \sigma_{xy}^{*} = \tau_{xy} \Big/ \sigma_y \tag{7}$$

$$K_1^{*}/\sigma_y^{*} = K_1/\sigma_y, \quad K_2^{*}/\tau_{xy}^{*} = K_2/\tau_{xy} \tag{8}$$

The stress intensity factors of the given unknown problem can be obtained by :

$$K_1 = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}^{*}} K_1^{*}, K_2 = \frac{\tau_{xy0,FEM}}{\tau_{xy0,FEM}^{*}} K_2^{*} \tag{9}$$

Here $\sigma_{y0,FEM}^{*}, \tau_{xy0,FEM}^{*}$ are the stress components at the crack tip (the zero element) of the reference problem calculated by FEM, and $\sigma_{y0,FEM}, \tau_{xy0,FEM}$ are those of the given unknown problem. In this study, stress intensity factors of the reference problem are given by the exact theoretical solution of the single central interface crack in an infinite dissimilar plate (Fig.1) subjected to tension T and shear S .

$$K_I^{*} + iK_{II}^{*} = (T + iS)\sqrt{\pi a}(1 + 2i\varepsilon) \tag{10}$$

Regarding the reference problem in Fig.3, $\sigma_{y0,FEM}^{T=1,S=0}{}^{*}$ and $\tau_{xy0,FEM}^{T=1,S=0}{}^{*}$ are values of stresses for $(T,S)=(1,0)$, and $\sigma_{y0,FEM}^{T=1,S=0}{}^{*} \tau_{xy0,FEM}^{T=0,S=1}{}^{*}$ are those for $(T,S)=(0,1)$ The stress components of the reference problem can be express by the form of superposing stresses at the crack tip as

$$\sigma_{y0,FEM}^{*} = \sigma_{y0,FEM}^{T=1,S=0}{}^{*} \times T + \sigma_{y0,FEM}^{T=0,S=1}{}^{*} \times S \tag{11}$$

$$\tau_{xy0,FEM}^{*} = \tau_{xy0,FEM}^{T=1,S=0}{}^{*} \times T + \tau_{xy0,FEM}^{T=0,S=1}{}^{*} \times S \tag{12}$$

By assuming $T = 1$ the value of S can be obtained from Eq. (11) and Eq. (12) as:

$$S = \frac{\sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=1,S=0}{}^{*} - \tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=1,S=0}{}^{*}}{\tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=0,S=1}{}^{*} - \sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=0,S=1}{}^{*}} \tag{13}$$

The condition shown in Eq. (7) can be satisfied by applying $(T=1, S)$ to the reference problem. Then, the zero element method can be extended to the interfacial crack problems, and the stress intensity factors can be calculated using Eq.(9). For more details about the zero element method, see Oda K et al.,2000[1].
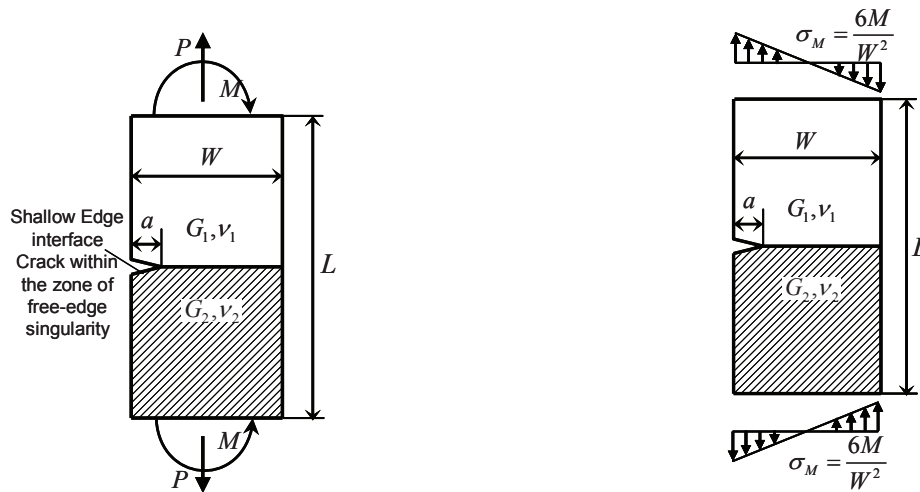
Fig. 2. (a) a bonded strip subjected to tensile and bending loads; (b) loading boundaries to counter the bending moment

3. Numerical results and discussion

3.1. Formulation of the problem

The geometric configuration for the bonded strip is shown in Fig.2(a). It is composed of two elastic isotropic and homogenous strips that are perfectly bonded along the interface. It is supposed that an interface crack with a length of a has initiated at the free edge corner. The strip is subjected to in-plane tensile and bending loading conditions. The material above the interface is termed material 1, and the material below is termed material 2. The length L is assumed to be much greater than the width W ($L \geq 2W$). The counter-moments can be modeled using the tensile stresses applied at the top and the bottom boundaries of the strip shown in Fig.2(b).

The stress intensity factors for the aforementioned problems in plane strain or plane stress are only determined on the two elastic mismatch parameters α and β (Dundurs,1969). And the Dundurs' material composite parameters are defined as

$$\alpha = \frac{G_1(\kappa_2 + 1) - G_2(\kappa_1 + 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)}, \beta = \frac{G_1(\kappa_2 - 1) - G_2(\kappa_1 - 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)} \tag{14}$$

where, the subscripts denote material 1 or 2, G_m, E_m and v_m denote shear modulus, Young's modulus and Poisson's ratio for material m, respectively. In this research, only the stress intensity factors for $\beta \geq 0$ in $\alpha - \beta$ space has been investigated since switching material 1 and 2 ($mat1 \Leftrightarrow mat2$) will only reverse the signs of α and β ($(\alpha, \beta) \Leftrightarrow (-\alpha, -\beta)$). As a result, all the data are only given for the right part of the $\alpha - \beta$ space ($\alpha > 0$).

3.2. Comparison of the SIFs for the bonded strip subjected to tensile and bending loads

The dimensionless stress intensity factors at the crack tip of the edge interface crack in bi-material bonded strip are systematically investigated by varying the relative crack size a/W, as well as material

parameters α and β. In this paper, we restrict our discussions to material combinations with $\beta = 0.3$, and the same phenomenon can be found from others material combinations. The double logarithmic distributions of the dimensionless stress intensity factors F_1 and F_2 at the crack tip are shown in Fig. 3(a) and Fig. 3(b) respectively. In Fig.3, F_1, F_2 for the bonded plate subjected to remote tension are plotted in solid curves and those for bending loading condition are plotted in dashed ones. From this figure, it can be found that the double logarithmic distributions behave linearity when $a/W < 0.01$ and differ within about 10% at $a/W < 0.05$. Furthermore, the slopes correspond to the oscillatory stress singularities λ of the bonded strip. It means that the stress intensity factors at the crack tip in a bonded semi-infinite plate are significantly determined by the free-edge singularity. For an arbitrary bonded strip, the values of F_1 and F_2 behave good linearity within the zone of dominance of the free-edge singularity. It can also be found from Fig.3 that the sign of slope for each curve varies with the changing of $\alpha(\alpha - 2\beta)$. Specifically, the slope for each line is positive when $\alpha(\alpha - 2\beta) < 0$, zero when $\alpha(\alpha - 2\beta) = 0$ and is negative when $\alpha(\alpha - 2\beta) > 0$. Thus, it can also be deduced for the limiting case ($a/W \to 0$), see, the bonded semi-infinite plate ($a/W \to 0$), the following relations can be deduced as

$$F_1 \to 0, F_2 \to 0 \ \ when \ \ \alpha(\alpha - 2\beta) < 0,$$

$$F_1, F_2 \ \ are \ finite \ when \ \ \alpha(\alpha - 2\beta) = 0,$$

$$F_1 \to \infty, F_2 \to \infty \ \ when \ \ \alpha(\alpha - 2\beta) > 0.$$

Here, all the results and phenomenon are in agreement with those for the bonded strip subjected to remote tension[2]. Moreover, F_1, F_2 are in good agreement with the two loading conditions within the whole range of zone of dominance of free-edge singularity when $\alpha(\alpha - 2\beta) = 0$. The values of F_1, F_2 for a joint under remote tensile loading condition are bigger than those for bending loading condition within the zone of free-edge singularity when $\alpha(\alpha - 2\beta) > 0$. However, a reverse conclusion can be given for the singular zone when $\alpha(\alpha - 2\beta) < 0$.

3.3. Empirical expression for a bonded strip under bending moment

In the author's previous research, an empirical expression as

$$F_1 \cdot (a/W)^{1-\lambda} = C_1, F_2 \cdot (a/W)^{1-\lambda} = C_2 \tag{15}$$

has been proposed to compute the SIFs at the crack tip for a shallow edge interface crack in a joint subjected to remote tension. Here, C_1, C_2 are constants depending upon the relative elastic properties of materials. In this research, it has been proved that Eq.(15) is also suitable for the case of bending moment except with different coefficients C_1, C_2. Similarly, the coefficients C_1, C_2 are systematically computed against material composite parameters, and are plotted in Fig. 4(a) and Fig. 4(b), respectively. The parameters for a bonded strip subjected to remote tension are plotted in solid curves, and those for the case of bending moment are plotted in dashed ones. It can be easily found that C_1, C_2 are the same for the two loading conditions when $\alpha(\alpha - 2\beta) = 0$, here, the points in well agreement are clearly marked by square frames in Fig.(4). By comparing the coefficients for the two loading conditions, it can be concluded that there are reflection points (marked in box) at $\alpha(\alpha - 2\beta) = 0$ for curves of a given β. The values of $C_1, |C_2|$ for the case of bending loads are always bigger than those for tensile case before this reflection point, and an inverse relationship will be found after this point. The conclusion again confirms the relationship for F_1, F_2 of the two loading conditions deduced in Section 3.2.
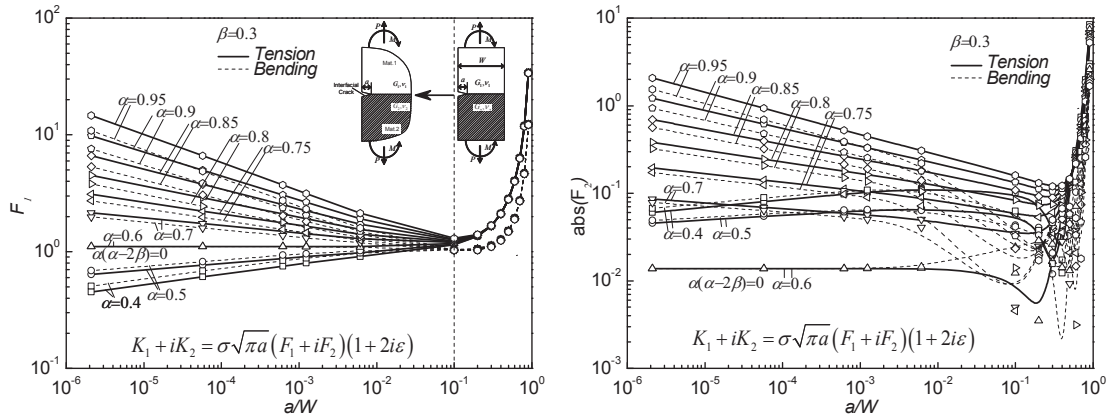
Fig. 3. (a) comparison of F_1 for the joint under tensile and bending loads ; (b) comparison of $\left|F_2\right|$ for the joint under tensile and bending loads
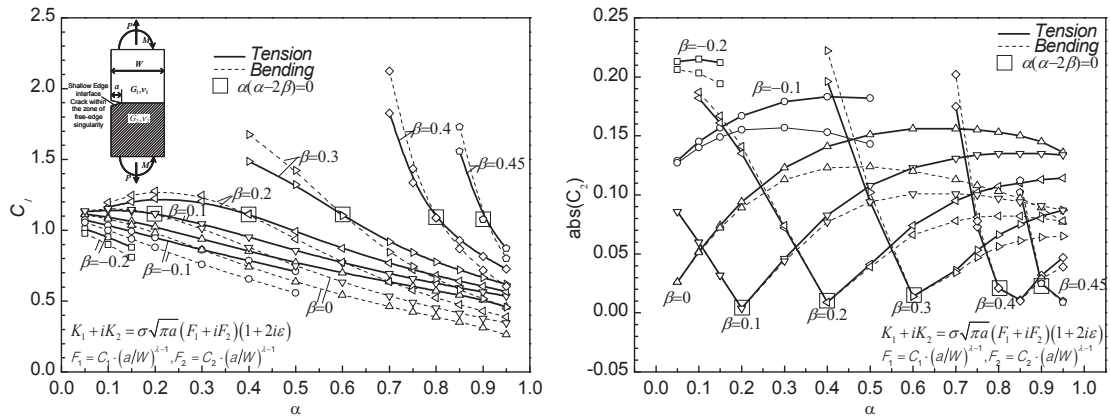

Fig. 4. (a) the values of C_1 for a joint under tensile and bending loads; (b) $\left|C_2\right|$ for a joint under tensile and bending loads

4. Conclusions

The SIFs at the crack tip for a bonded joint under tensile and bending loading conditions are computed and compared for the whole range of material combinations. Within the zone of free-edge singularity, it is certified that the bending loading condition is more dangerous than the tensile case when $\alpha\left(\alpha-2\beta\right)<0$, but is safer when $\alpha\left(\alpha-2\beta\right)>0$. Furthermore, they are totally equivalent for the two loading conditions when $\alpha\left(\alpha-2\beta\right)=0$.

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