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## SEVERAL SOLUTIONS OF INTENSITY OF SINGULAR STRESS USEFUL FOR EVALUATING BONDED STRENGTH

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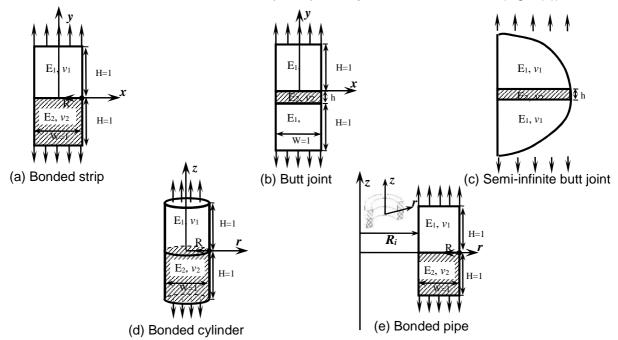
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**Abstract:** Adhesive joints are widely used in many industrial sectors since the numerous advantages over other traditional joints. However the mismatch of different materials may cause debonding of the joints. Since few studies are available for evaluating the bonded strength, this study will focuses on the solutions for the intensity of the singular stress for several typical adhesive joints, such as bonded plate and butt joint in plane stress state, bonded cylinder/pipe under axial-symmetric state. In this research, an effective numerical method combined with FEM results is used to obtain the SIFs at corner of adhesive joints in arbitrary material combinations. Here the simple bonded plate, which has already been solved accurately in our previous study, is used as the reference problem. We will discuss the effect of the adhesive thickness in a butt joint, and defined a new parameter for semi-infinite structure; and for the bonded joints in axial-symmetric, the non-singular terms are derived so that the SIFs can be obtained, and it is found that the axial-symmetric problem are quite different since the SIFs cannot be dominated by the Dunders' parameters. And at last, the SIFs for several problems are discussed and compared with each other.

Keywords: Adhesion, Stress Intensity Factor, Fracture Mechanics, Interface, Finite Element Method

#### 1 INTRODUCTION

The adhesive joints have been widely used in automobile, space and aviation engineering recent years since it has number of advantages over the traditional joints, such as no welding residual stress, lightweight, lower costs and easy to process. There are all kinds of adhesive joints in different application area, such as pipe joint (Fig. 1(f)) in couplings, and butt joint (Fig. 1(b)) in microelectronic packaging. Among these adhesive joints, the most fundamental one is a bonded strip composed by two different materials (Fig. 1(a)).

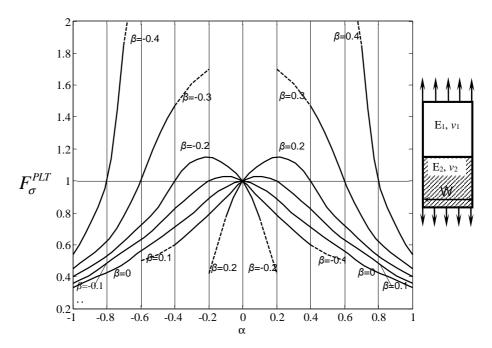


**Fig. 1** Several bonded structures having singularity at interface corner in the form  $\sigma_{ii} \propto K_\sigma/r^{1-\lambda}$ 

It is known that different material properties cause singular stress at the end of interface which leads to the failure of these structures. All these problems need to consider the intensity of singular stress.

So far, many studies have been done for these kinds of bonded structures. The fundamental problem of bonded plate in Fig. 1(a) with the expression of intensity of singular stress as Eq.(1) has the has been analysed by Chen-Nisitani[1] and Noda et.al[2]. Fig.2 shows the stress intensity factor of this problem.

$$K_{\sigma} = \lim_{r \to 0} \left[ r^{1-\lambda} \times \sigma_{\theta \mid \theta = \frac{\pi}{2}}(r) \right], \quad F_{\sigma} = \frac{K_{\sigma}}{\sigma^{\infty} W^{1-\lambda}}, \tag{1}$$



**Fig. 2**  $F_{\sigma}$  for a boned strip in Fig. 2(a)

Also many researchers studied the adhesive thickness effect on the adhesive join shown in Fig. 1(b) [3,4,5]; and this kind of adhesive specimen was also studied under bending tests [6]. In Ref. [7], the effect of adhesive thickness was considered with varying material properties for a bonded cylinder specimen. Zhang and Noda et.al [8] have analysed the effect of adhesive thickness for the butt joint under tension by investigating all material combinations systematically. And in reference [9], it's found that the critical values of the stress intensity factors are almost constant (see Fig.3)

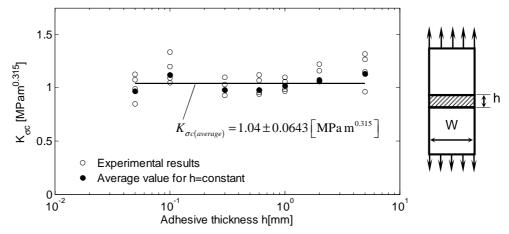


Fig. 3 Relationship between K<sub>σc</sub> and h

This study will mainly focus on the corner stress intensity factors (SIFs) for several typical adhesive joints as shown in Fig. 1, and aims to give the solutions of SIFs for the future study and application in engineering. In this research, an effective numerical method combined with FEM results is used to obtain the SIFs at corner

of adhesive joints in arbitrary material combinations.

#### 2. ANALYSIS ON BUTT JOINT

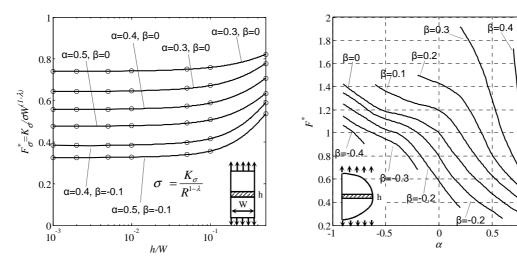
This section deals with a butt joint as shown in Fig. 1(b), (c). Then, the effect of adhesive thickness on the corner stress intensity factor is discussed. This analytical study focuses on the FEM stress at the first node of interface considering together with the exact solution in Fig. 2 by applying the same mesh division [8, 10]. For the adhesive joint in a bonded finite plate as shown in Fig.1 (b), the dimensionless stress intensity factor  $F_{\sigma}$  defined in Equation (2) is suitable because  $F_{\sigma}$  has the same value if the adhesive thickness  $h \ge W$  [2].

$$F_{\sigma} = K_{\sigma} / \sigma W^{1-\lambda} \tag{2}$$

Here,  $\sigma$  denotes the remote tensile stress. On the other hand, if the adhesive thickness h is much smaller than the adhesive width W ( $h/W \to 0$ ), the results correspond to the solution for bonded semi-infinite plate in Fig. 1 (c). In this case, the dimensionless stress intensity factor  $F_{\sigma}^{\ *}$  defined in Equation (3) is suitable because  $F_{\sigma}^{\ *}$  has the same value if h is small enough.

$$F_{\sigma}^{*} = K_{\sigma} / \sigma h^{1-\lambda} \tag{3}$$

Fig.4 (a) shows an example of the relation between the adhesive thickness h and the corner stress intensity factor. It is seen that  $F_{\sigma}^*$  becomes constant when adhesive layer is thin enough. In other words, it is found that the adhesive butt joint in Fig.1 (b) can be regarded as the bonded semi-infinite plate in Fig.1 (c) when the ratio  $h/W \le 0.005$ . The solution of bonded semi-infinite plate is obtained as shown in Fig.4 (a). Fig. 4 (b) shows the dimensionless stress intensity factor  $F_{\sigma}^*$  in Fig.1 (c) under arbitrary material combination.



**Fig. 4(a)**  $F_{\sigma}^*$  vs. adhesive thickness in Fig. 1 (b)

**Fig. 4(b)**  $F_{\sigma}^*$  with varying material combination

#### 3. ANALYSIS ON BONDED CYLINDER AND BONDED PIPE

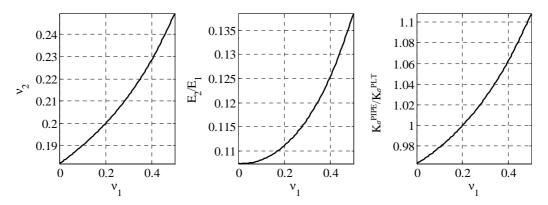
Here, two extreme solutions are discussed; one is a bonded pipe with  $W/R_i \rightarrow 0$  in Fig. 1(e), the other is a bonded cylinder equivalent to Fig. 1(d) when  $W/R_i \rightarrow \infty$ . If these two extreme solutions are available under arbitrary material combinations, the authors think general solutions may be conjectured without difficulty.

The singular stresses of the unknown problem in Fig. 1(d) and (e) include the non-singular term  $\mathcal{E}_{j}^{AXIL}$ ,  $\mathcal{E}_{j}^{AXIL}$  and can be expressed as:

$$\sigma_{j}^{AXIL} = \frac{K_{\sigma_{j}}^{AXIL}}{R^{1-\lambda}} + \partial_{j}^{AXIL} \left(j = r, z, \theta\right); \quad \tau_{rz}^{AXIL} = \frac{K_{\tau_{xy}}^{AXIL}}{R^{1-\lambda}} + \partial_{jz}^{AXIL} \tag{4}$$

In our previous paper, the corner stress intensity factor for the bonded strip was analysed accurately, and based on this result, the stress intensity shown in Eq. (4) can be obtained by using the zero element method.

For plane strain problems, the Dunder's parameter  $\alpha$  and  $\beta$  can fully control the intensity of singular stress near the end of interface, however, since the bonded pipe and cylinder are axi-symmetric, the corner stress intensity factors cannot be totally dominated by these parameters. Fig.5 indicates an example of the results when  $(\alpha, \beta) = (0.8, 0.3)$ . As shown in Fig.5, the range of SIFs is considered when  $\alpha$  and  $\beta$  are fixed.



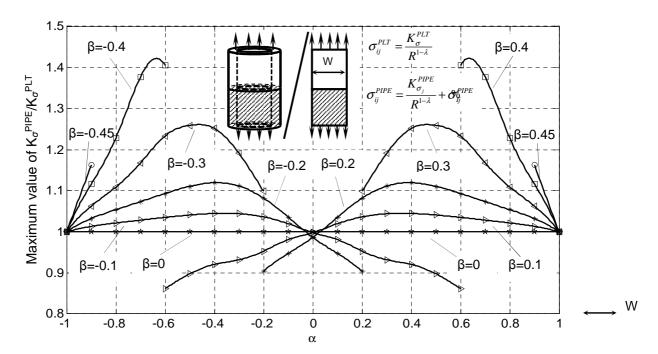
**Fig. 5** Stress ratio and material combination varies when  $(\alpha, \beta) = (0.8, 0.3)$ 

Although under fixed  $(\alpha, \beta)$  the corner stress intensity factor varies slightly, the maximum value can be used in the design of bonded structures. Therefore in this study the maximum value of the SIF ratio is considered. Fig. 6, and 7 show the results of bonded pipe and bonded cylinder respectively, each curve represents the result of fixed  $\beta$  in the  $\alpha$  space.

As shown in the figures, the two problems have similar distributions although the variation of bonded cylinder is larger than that of bonded pipe. And notably, when compared with the result of bonded plate in Fig.2, they all have the similar tendency and distributions, such as the larger value always accompany with large absolute value of  $\beta$ , and all values tend to convergence at approximately 1.

#### 4. CONCLUSIONS

- 1. For the butt joint, the dimensionless intensity of singular stress  $F_{\sigma}^*$ , which is controlled by adhesive thickness h, tends to be a constant with decreasing adhesive thickness when the ratio  $h/W \le 0.005$ . The adhesive butt joint can be regarded in a bonded semi-infinite plate if the adhesive layer is thin enough. For a certain value  $\beta$ , it is found that  $F_{\sigma}^*$  decreases with increasing of  $\alpha$ . The material combinations with large  $\alpha$  should be used to enhance the interface strength.
- 2. From the results in Fig.6 and 7, it is found that the maximum value happens when  $\beta$  is 0.4 or -0.4 and  $\alpha$  is close to 0.6 or -0.6. And both figure are symmetric, since in the  $\alpha$ - $\beta$  space switching material 1 and 2 (mat. 1 $\leftrightarrow$ mat. 2) will only reverse the sings of  $\alpha$  and  $\beta$  (( $\alpha$ ,  $\beta$ ) $\leftrightarrow$ (- $\alpha$ , - $\beta$ )).
- 3. Although the results of axis symmetric problems is different from the one of boned plate and cannot be controlled by the Dunder's parameter  $\alpha$  and  $\beta$ , they all have similar tendency and distribution.



**Fig. 6** Maximum value of  $K_{\sigma}^{PIPE}/K_{\sigma}^{PLT}$  under fixed  $\beta$  when  $R_{i}\!\!\to\!\!\infty$ 

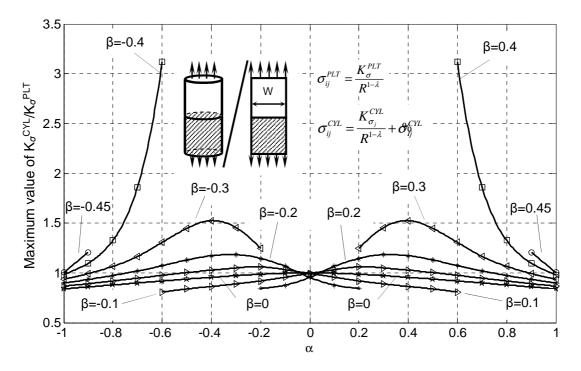


Fig. 7 Maximum value of  $K_{\sigma}^{CYL}/K_{\sigma}^{PLT}$  under fixed  $\beta$ 

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