ON THE SELF-SYNCHRONIZATION OF MECHANICAL VIBRATORS OF THE RESONANCE TYPE VIBRATING MACHINERY WITH MULTIDEGREE-OF-FREEDOM.

by

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SYNOPSIS

This paper deals with the self-synchronization of mechanical vibrators in the multidegree-of-freedom system, when the frequencies of the unbalanced rotors are near a natural frequency ω_r of the system.

By means of a transformation to the principal coordinate, the equation of motion of the masses of the vibrating system can be represented as a series of elementary oscillators. And to study single frequency resonance conditions, we can, to a first approximation consider only one of the n equations of the vibrating system.

The generating phase angle of the unbalanced rotors and the steady state vibration of the system are derived by the method of averaging. The stability of the steady state solution are analyzed by use of Routh-Hurwitz criterion.

1. INTRODUCTION

The phenomenon of synchronization occurs in dynamical system as well as in electric ciruits and automatic control systems.

In the previous papers [1, 2, 3], the authors have discussed some problems, i. e. the rotation of an unbalanced rotor dependent on oscillation of its axis, the rolling mechanism due to small oscillation, the gearless low head screen or the vibro-motor which has, on a rigid body two unbalanced rotors without the coupling to each, and the automatic balancer.

I.I. Blekhman [4] has studied the synchronization of mechanical vibrators on a rigid body which can accomplish a translational oscillations with one degree-of-freedom, by method of Poincare-Liapnov.

In this paper, we investigate the synchronization of unbalanced rotors on some rigid bodies of a n degree-of-freedom vibrating system by the method of averaging and derive the periodic mode of the resonant vibrating conveyer with multi-vibrators.

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NOMENCLATURE

O_0 : center of mass of the system at rest
O : center of mass of the system in operation
O_i : center of revolution of the i-th unbalanced rotor
O_0-xy : fixed rectangular coordinate, axis O_0-y is parallel to the direction of motion of vibrator body
O-uv : moving coordinate fixed to conveyer trough
φ : angle of rotation of conveyer
M : total mass of the system
M_c : mass of conveyer trough
I_c : moment of inertia of conveyer trough about center of mass
R_c : distance between mass center of conveyer and that of system
M_{io} : mass of i-th vibrator body
I_{io} : moment of inertia of vibrator body about O_i
R_i, δ_i : position of shaft of the i-th rotor, distance and angle from mass center, at rest
δ_i : angle which makes O_0u makes with O_0O_i
v_i : relative displacement of i-th vibrator body with respect to conveyer trough
k_i, c_i : spring constant and damping coefficient of the spring which connects i-th vibrator body to conveyer trough
m_i : unbalanced mass of the i-th rotor
$m_i r_i$: unbalanced mass moment of the i-th rotor
J_i : moment of inertia of the i-th unbalanced rotor about center of shaft
φ_i : angle of rotation of the i-th unbalanced rotor
λ_i : viscous damping of the i-th unbalanced rotor
L_i : torque of motor driving the i-th rotor
s : number of vibrators
ω_r : one of natural frequencies of the system
z_r : principal coordinate
Ω : frequency of stationary rotation of vibrator

2. EQUATION OF MOTION

Fig.1 shows the model of the system and coordinates.

Vibrator body M_i is constrained to move on a line v_i which is parallel to the axis v.

The kinetic energy, the potential energy and Rayleigh's dissipation function, neglecting the energies of the force of gravity and the damping force of the support, can be written :

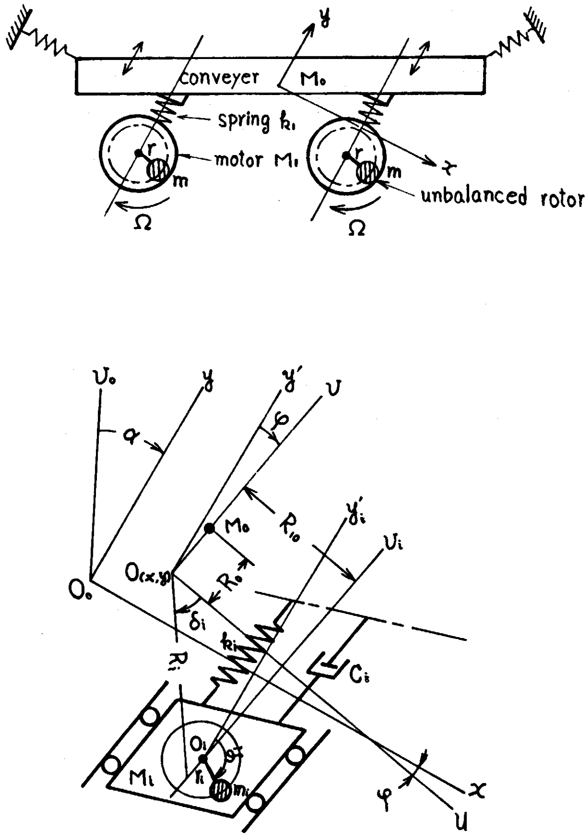

Fig. 1

$$T=\frac{1}{2}M(\dot{x}^2+\dot{y}^2)+\frac{1}{2}I\dot{\varphi}^2+M_0R_0\dot{x}\dot{\varphi}+\frac{1}{2}\sum_{i=1}^{s}M_i(\dot{v}_i^2+2\dot{v}_i\dot{y})$$

$$-\sum_{i=1}^{s}M_iR_i\dot{\varphi}(\dot{x}\sin\delta_i+\dot{y}\cos\delta_i+\dot{v}_i\cos\delta_i)$$

$$+\frac{1}{2}\sum_{i=1}^{s}[J_i\dot{\varphi}_i^2+2m_ir_i\dot{\varphi}_i\{\dot{x}\cos\varphi_i-\dot{y}\sin\varphi_i$$

$$-R_i\dot{\varphi}\sin(\delta_i-\varphi_i)-\dot{v}_i\sin\varphi_i\}] \qquad (1)$$

$$V=\frac{1}{2}\sum_{i=1}^{s}k_iv_i^2+\frac{1}{2}(k_xx^2+k_yy^2+k_\varphi\varphi^2)$$

$$F=\frac{1}{2}\sum_{i=1}^{s}c_i\dot{v}_i^2+\frac{1}{2}\sum_{i=1}^{s}\lambda_i(\dot{\varphi}_i-\dot{\varphi})^2$$

$$Q_{\varphi i}=L_i$$

where

$$M = M_C + \sum_{i=1}^{s}(M_{i0} + m_i), \quad M_i = M_{i0} + m_i$$

$$I = I_C + M_C R_C^2 + \sum_{i=1}^{s}(I_{i0} + M_i R_i^2)$$

and the directions of rotations of all unbalanced rotors are assumed clockwise.

Applying Lagrange's equation, we obtain the differential equations of motion :

$$J_i \ddot{\varphi}_i + \lambda_i \dot{\varphi}_i = m_i r_i \{ \ddot{y} \sin \varphi_i - \ddot{x} \cos \varphi_i + R_i \ddot{\varphi} \sin(\delta_i - \varphi_i) + \ddot{v}_i \sin \varphi_i \} + L_i \quad (i = 1, 2, \cdots, s)$$

$$M_i \ddot{v}_i + c_i \dot{v}_i + k_i v_i = M_i (R_i \ddot{\varphi} \cos \delta_i - \ddot{y}) + m_i r_i (\dot{\varphi}_i^2 \cos \varphi_i + \ddot{\varphi}_i \sin \varphi_i) \quad (i = 1, 2, \cdots, s)$$

$$M \ddot{x} + k_x x = \sum_{j=1}^{s} m_j r_j (\dot{\varphi}_j^2 \sin \varphi_j - \ddot{\varphi}_j \cos \varphi_j)$$

$$M \ddot{y} + k_y y = \sum_{j=1}^{s} m_j r_j (\dot{\varphi}_j^2 \cos \varphi_j + \ddot{\varphi}_j \sin \varphi_j) - \sum_{j=1}^{s} M_j \ddot{v}_j$$

$$I \ddot{\varphi} + k_\varphi \varphi = \sum_{j=1}^{s} m_j r_j R_j \{ \ddot{\varphi}_j \sin(\delta_j - \varphi_j) - \dot{\varphi}_j^2 \cos(\delta_j - \varphi_j) + \sum_{j=1}^{s} M_j R_j \cos \delta_j \cdot \ddot{v}_j \quad (2)$$

Here we shall assume that each unbalanced rotor runs at high speed in comparison with the frequencies ω_x, ω_y and ω_φ, then the second terms on the left hand side in last three equations of Eq.(2) may be neglected.

The vibrators are considered alike, i.e.

$M_1 = M_2 = \cdots$, $k_1 = k_2 = \cdots = k_s$

$m_1 = m_2 = \cdots = m_s = m$, $r_1 = r_2 = \cdots r_s = r$

$J_1 = J_2 = \cdots = J_s = J$, $\lambda_1 = \lambda_2 = \cdots = \lambda_s = \lambda$, $L_1 = \cdots L_s = L$

Eliminating x, y and φ from Eq.(2) and neglecting smaller terms, we have the following equations.

$$\boldsymbol{M} \ddot{\boldsymbol{v}} + \boldsymbol{K} \boldsymbol{v} = \varepsilon \boldsymbol{F}$$

$$\ddot{\varphi}_i + \varepsilon \rho \dot{\varphi}_i = \varepsilon q_2 [\ddot{v}_i \sin \varphi_i + \sum_{j=1}^{s} \{ -b \sin \varphi_i + h_j' \sin(\delta_i - \varphi_i) \cdot R_i \} \ddot{v}_j] + \varepsilon N \quad (3)$$

$$(i = 1, 2, \cdots, s)$$

where

$$\boldsymbol{v} = \begin{Bmatrix} \boldsymbol{v}_1 \\ \boldsymbol{v}_2 \\ \vdots \\ \boldsymbol{v}_3 \end{Bmatrix}, \quad \boldsymbol{M} = [m_{ij}], \quad m_{ij} = \begin{cases} 1 - (b + h_{ij}) & (i = j) \\ -(b + h_{ij}) & (i \neq j) \end{cases}$$

$$K=\omega_n^2[\delta_{ij}],\quad \delta_{ij}=\begin{cases}1 & (i=j)\\ 0 & (i\neq j)\end{cases}$$

$$\varepsilon F=\varepsilon\begin{bmatrix}F_1\\ F_2\\ \vdots\\ F_s\end{bmatrix},\qquad \varepsilon F_i=\varepsilon q_1(\dot{\varphi}_i^2\cos\varphi_i+\ddot{\varphi}_i\sin\varphi_i)-\varepsilon\zeta\dot{v}_i \quad (i=1,2,\cdots,s)$$

$$b=\frac{M_1}{M},\qquad \omega_n^2=\frac{k_1}{M_1},\qquad \varepsilon q_1=\frac{mr}{M_1}$$

$$h_{ij}=\frac{M_1R_iR_j}{I}\cos\delta_i\cos\delta_j,\qquad \varepsilon q_2=\frac{mr}{J}$$

$$h'_j=\frac{M_1R_i}{I}\cos\delta_i\cos\delta_j,\qquad \varepsilon\zeta=\frac{c_1}{M_1}$$

$$\varepsilon\rho=\frac{\lambda}{J},\qquad \varepsilon N=\frac{L}{J}$$

Putting $\varepsilon\mathbf{F}=\{0\}$ in the fist equation of Eq.(3), we can get the natural frequencie $\omega_1,\cdots,\omega_s$ and modal column matrix $[\mu_{ij}]$ of the system.

The coordinates $\boldsymbol{v}$ can be related to the principal coordinates $\mathbf{Z}$ by the relatio:

$$\boldsymbol{v}=[\mu_{ij}]\,\mathbf{Z} \tag{4}$$

By means of transformation to the principal coordinates of the undamped system, th initial system (3) can be represented as follows:

$$\ddot{Z}+\Lambda Z=\varepsilon\Phi$$

$$\ddot{\varphi}_i=\varepsilon q_2[[\mu_i]\ddot{Z}\sin\varphi_i+\sum_{j=1}^{s}\{-b\sin\varphi_i$$

$$+h'_jR_i\sin(\delta_i-\varphi_i)\}[\mu_i]\ddot{Z}]+\varepsilon(N-\rho\dot{\varphi}_i) \tag{5}$$

where $(i=1,2,\cdots,s)$

$$\Lambda=\begin{bmatrix}\omega_1^2 & 0 & \cdots & 0\\ 0 & \omega_2^2 & & \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & \omega_s^2\end{bmatrix},\qquad \varepsilon\Phi=[a_{ij}]^{-1}[\mu_{ij}]^T\varepsilon F,\quad [a_{ij}]=[\mu_{ij}]^TM[\mu_{ij}]$$

$[\mu_i]$ and $[\mu_j]$ are ith and jth row vectors of the modal column matrix $[\mu_{ij}]$ respectively.

3. ANALYSIS

To study single frequency resonance conditions of forced oscillation in the systen we can, to a first approximation consider the equations describing the rotation motions of unbalanced vibrators and one of n equations of the oscillating systerr

If the forced oscillations near the frequency ω_r are to be determined then th equation in (5) referring to ω_r is chosen. Thus we consider the equations.

$$\ddot{Z}_r + \omega_r^2 Z_r = \varepsilon \Phi_r$$
$$\ddot{\varphi}_i = \varepsilon f(\varphi_i, \dot{\varphi}_i, \ddot{Z}_r) \quad (i=1,2,\cdots,s) \tag{6}$$

The remaining (n—l) coordinates of the system are assumed to be far from resonance; their oscillations will then be small in comparison with the resonance oscillations of the coordinate Zr, and they will be neglected in the first approximation.

By use of the method of averaging, we can get the conditions of stationary synchronous rotations of unbalanced vibrators as following sections.

3.1 Conveyer with 2 Resonant Vibrators.

For the system with two resonant vibrators, we obtain the natural frequencies and the modal column matrix in the form

$$\omega_1^2 = \frac{\omega_n^2}{1-2h}, \qquad \omega_2^2 = \frac{\omega_n^2}{1-2b}$$
$$[\mu_{ij}] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \tag{7}$$

where

$$h = h_{12} = h_{21} = M_1 R_{10}^2 / I$$
$$R_{10} = R_1 \cos\delta_1 = -R_2 \cos\delta_2$$

3.1.1 In The Case of $\Omega = \omega_2$

Let us consider the problem in the useful case that the angular frequencies of two unbalanced rotors are near the natural frequency ω_2. Neglecting the coordinate Z_1, Eq. (6) can be written in the form.

$$\ddot{Z}_2 + \omega_2^2 Z_2 = \frac{\varepsilon q_1}{2(1-2b)}(\dot{\varphi}_1^2 \cos\varphi_1 + \ddot{\varphi}_1 \sin\varphi_1 + \dot{\varphi}_2^2 \cos\varphi_2$$
$$+ \ddot{\varphi}_2 \sin\varphi_2) - \frac{\varepsilon\zeta}{1-2b}\dot{Z}_1$$
$$\ddot{\varphi}_1 = \varepsilon q_2 (1-2b) \ddot{Z}_2 \sin\varphi_1 + \varepsilon N - \varepsilon\rho\dot{\varphi}_1 \tag{8}$$
$$\ddot{\varphi}_2 = \varepsilon q_2^2 (1-2b) \ddot{Z}_2 \sin\varphi_2 + \varepsilon N - \varepsilon\rho\dot{\varphi}_2$$

For $\varepsilon = 0$ Eq.(8) describes a harmonic oscillation and rotations with constant frequencies. Therefore for $\varepsilon \neq 0$ it is natural to expect that the oscillations are approximately harmonic and frequencies φ_1, φ_2 are approximately constant, i.e. they vary slowly.

It is convenient to introduce the substitutions

$$\varphi_1 = \Omega t + \theta_1$$
$$\varphi_2 = \Omega t + \theta_2 \tag{9}$$

$$Z_2 = A_2 cos\ (\Omega t + \Xi_2)$$
$$Z_2 = -A_2 \omega_2 sin\ (\Omega t + \Xi_2)$$

The new variables $\theta_1, \theta_2, A_2, \Xi_2$ will be slowly varying functions of time. They represent the essential parameters of motion, i.e. θ_1, θ_2 are the generating phase angle of unbalanced rotors, A_2 is the amplitude and Ξ_2 is the angular phase of oscillations.

We may transform Eq.(8) to the form

$$\dot{A}_2 = -\frac{\varepsilon q_1 \Omega^2}{2(1-2b)\omega_2}\{(1+2\frac{\dot{\theta}_1}{\Omega})\cos(\Omega t + \theta_2)\}\sin(\Omega t + \Xi_2)$$
$$+(1+2\frac{\dot{\theta}_2}{\Omega})\cos(\Omega t + \theta_2)\}\sin(\Omega t + \Xi_2)$$
$$-\frac{\varepsilon\zeta A_2}{1-2b}\sin^2(\Omega t + \Xi_2)$$
$$\Xi_2 = -\frac{\varepsilon q_1 \Omega^2}{2(1-2b)\omega_2 A_2}\{(1+2\frac{\dot{\theta}_1}{\Omega})\cos(\Omega t + \theta_1)$$
$$+(1+2\frac{\dot{\theta}_2}{\Omega})\cos(\Omega t + \theta_2)\}\cos(\Omega t + \Xi_2)$$
$$-\frac{\varepsilon\zeta}{2(1-2b)}\sin 2(\Omega t + \Xi_2) + (\omega_2 - \Omega) \quad (10)$$
$$\ddot{\theta}_1 = -\varepsilon q_2(1-2b)A_2\omega_2\Omega\sin(\Omega t + \theta_1)\cos(\Omega t + \Xi_2)$$
$$+\ \varepsilon\{N - \rho(\Omega + \dot{\theta}_1)\}$$
$$\ddot{\theta}_2 = -\varepsilon q_2(1-2b)A_2\omega_2\Omega\sin(\Omega t + \theta_2)\cos(\Omega t + \Xi_2)$$
$$+\varepsilon\{N - \rho(\Omega + \dot{\theta}_2)\}$$

Since it is assumed that the variables are slowly varying function, Eq.(10) may be averaged over one cycle as follows

$$\dot{A}_2 = \frac{\varepsilon q_1 \Omega^2}{4(1-2b)\omega_2}\{(1+2\frac{\dot{\theta}_1}{\Omega})\sin(\theta_1 - \Xi_2)$$
$$+(1+2\frac{\dot{\theta}_2}{\Omega})\sin(\theta_2 - \Xi_2)\} - \frac{\varepsilon\zeta A_2}{2(1-2b)}$$
$$\dot{\Xi}_2 = -\frac{\varepsilon q_1 \Omega^2}{4(1-2b)\omega_2 A_2}\{(1+2\frac{\dot{\theta}_1}{\Omega})\cos(\theta_1 - \Xi_2) \quad (11)$$
$$+(1+2\frac{\dot{\theta}_2}{\Omega})\cos(\theta_2 - \Xi_2)\} + (\omega_2 - \Omega)$$
$$\ddot{\theta}_1 = -\frac{1}{2}\varepsilon q_2(1-2b)A_2\omega_2\Omega\sin(\theta_1 - \Xi_2) + \varepsilon\{N - \rho(\Omega + \dot{\theta}_1)\}$$
$$\ddot{\theta}_2 = -\frac{1}{2}\varepsilon q_2(1-2b)A_2\omega_2\Omega\sin(\theta_2 - \Xi_2) + \varepsilon\{N - \rho(\Omega + \dot{\theta}_2)\}$$

The conditions for existence of a stationary motion are

$$A_2=a_2,\qquad \Xi_2=\xi_2,\qquad \theta_1=\theta_{10},\qquad \theta_2=\theta_{20}$$
$$\dot{a}_2=\dot{\xi}_2=\dot{\theta}_{10}=\dot{\theta}_{20}=0 \tag{12}$$

From Eq. (11) and (12) we have

$$\begin{aligned}
&\varepsilon q_1\Omega^2\{\sin(\theta_{10}-\xi_2)+\sin(\theta_{20}-\xi_2)\}-2\varepsilon\zeta\omega_2 a_2=0\\
&\varepsilon q_1\Omega^2\{\cos(\theta_{10}-\xi_2)+\cos(\theta_{20}-\xi_2)\}-4(1-2b)\omega_2 a_2(\omega_2-\Omega)=0\\
&-\frac{1}{2}\varepsilon q_2(1-2b)a_2\omega_2\Omega\sin(\theta_{10}-\xi_2)+\varepsilon(N-\rho\Omega)=0\\
&-\frac{1}{2}\varepsilon q_2(1-2b)a_2\omega_2\Omega\sin(\theta_{20}-\xi_2)+\varepsilon(N-\rho\Omega)=0
\end{aligned} \tag{13}$$

For stationary conditions of motion Eq. (13) gives the following expression for the generating phase relation between the rotations of two rotors and the amplitude of vibration

$$\begin{aligned}
&\theta_{20}-\theta_{10}=0\\
&a_2=\frac{mr\Omega^2}{M_1}\ \frac{1}{\sqrt{4\omega_2^2(1-2b)^2(\omega_2-\Omega)^2+\lambda^2\Omega^2/M_1^2}}
\end{aligned} \tag{14}$$

To study the stability of the stationary motions, let η_1, η_2, α_2 and α_3 be small perturbations and put

$$\begin{aligned}
&A_2=a_2+\alpha_2, &&\Xi_2=\xi_2+\alpha_3\\
&\theta_1=\theta_{10}+\eta_1, &&\theta_2=\theta_{20}+\eta_2
\end{aligned} \tag{15}$$

If we substitute Eq. (15) into (11) and use the Eq. (13), (14), the corresponding characteristic equation becomes

$$\begin{aligned}
&p\{p^2+\varepsilon\rho p+\frac{2q_2\omega_2^2}{q_1}(1-2b)^2a_2^2(\omega_2-\Omega)\}\times\\
&\quad\times[p^3+(\frac{\varepsilon\zeta}{1-2b}+\varepsilon\rho)p^2+(\omega_2-\Omega)^2\{2+\frac{8q_2\omega_2^2a_2^2}{q_1\Omega^2}(1-2b)^2\}p\\
&\quad+(\omega_2-\Omega)^2\{2\varepsilon\rho+\frac{8q_2\omega_2^2a_2^2}{q_1\omega^2}\varepsilon\zeta(1-2b)\}]=0
\end{aligned} \tag{16}$$

Applying the Routh-Hurwitz criterion to Eq. (16), the condition for stability is finally obtained as follows.

$$\omega_2-\Omega>0 \tag{17}$$

When the angular frequencies of the vibrators are just under the natural frequency ω_2, the motions of the conveyer trough are represented in the form

$$y=-\frac{a_2}{1-2b}\cos(\Omega t+\xi_2)$$
$$x=\varphi=0 \tag{18}$$

A strong linear vibration is produced in the direction of axis y.

3.1.2 In the Case of $\Omega=\omega_1$

Here let us consider the problem of synchronization in the case that the frequencies of two unbalanced rotors are near the natural frequency ω_1. For this case, the coordinate Z_2 in Eq. (6) will be neglected. Following the same method which was applied in the previous section, we obtain the following equations for stationary conditions of motion

$$\varepsilon q_1\Omega^2\{\sin(\theta_{10}-\xi_1)-\sin(\theta_{20}-\xi_1)\}-2\varepsilon\zeta\omega_1 a_1=0$$
$$\varepsilon q_1\Omega^2\{\cos(\theta_{10}-\xi_1)-\cos(\theta_{20}-\xi_1)\}-4(1-2h)\omega_1 a_1(\omega_1-\Omega)=0$$
$$-\frac{1}{2}\varepsilon q_2\omega_1\Omega a_1(1-2h)\sin(\theta_{10}-\xi_1)+\varepsilon(N-\rho\Omega)=0$$
$$\frac{1}{2}\varepsilon q_2\omega_1\Omega a_1(1-2h)\sin(\theta_{20}-\xi_1)+\varepsilon(N-\rho\Omega)=0 \tag{19}$$

Eq. (19) gives the following connection between the generating phases of two rotors.

$$\theta_{20}-\theta_{10}=\pi \tag{20}$$

By the same method of stability analysis as in the previous section, we obtain the condition for the stability of the solution (20) in the form

$$\omega_1-\Omega>0 \tag{21}$$

From the above results we conclude that the motion of the conveyer trough becomes the rotational motion when the frequencies of the vibrators are just under the frequency ω_1.

3.2 Conveyer with 3 Resonant Vibrators. ($\Omega=\omega_3$)

For the system with three resonant vibrators, letting

$$s=3\ ,\quad R_1\cos\delta_1=-R_3\cos\delta_3=R_{10}$$
$$\cos\delta_2=0\ ,\quad R_2\sin\delta_2=R_2$$

in Eq. (5), we obtain the natural frequencies and the modal column matrix of the vibrating system in the form

$$\omega_1^2=\omega_n^2$$

$$\omega_2^2 = \frac{\omega_n^2}{1-2h}$$
$$\omega_3^2 = \frac{\omega_n^2}{1-3b} \qquad (22)$$

$$[\mu_{ij}] = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

Here we consider the problem of synchronization, when the frequencies of unbalanced rotors are near the natural frequency ω_3 which corresponds to the most useful one of the normal modes of vibrations for the conveyer.

Put $Z_1 = Z_2 = 0$ in Eq. (5).

Then Eq. (6) can be written in the form

$$\ddot{Z}_3 + \omega_3 Z_3 = \frac{\varepsilon q_1}{3(1-3b)} \sum_{j=1}^{3} (\dot{\varphi}_j^2 \cos\varphi_j + \ddot{\varphi}_j \sin\varphi_j) - \frac{\varepsilon\zeta}{1-3b} \dot{Z}_3$$

$$\ddot{\varphi}_1 = \varepsilon q_2(1-3b)\ddot{Z}_3 \sin\varphi_1 + \varepsilon N - \varepsilon\rho\dot{\varphi}_1 \qquad (23)$$
$$\ddot{\varphi}_2 = \varepsilon q_2(1-3b)\ddot{Z}_3 \sin\varphi_2 + \varepsilon N - \varepsilon\rho\dot{\varphi}_2$$
$$\ddot{\varphi}_3 = \varepsilon q_2(1-3b)\ddot{Z}_3 \sin\varphi_3 + \varepsilon N - \varepsilon\rho\dot{\varphi}_3$$

Introduce the substitutions of the new variables θ_1, θ_2, θ_3, A_3, Ξ_3 for the variables φ_1, φ_2, φ_3, Z_3, $\dot{Z}_3$, according to the expressions

$$\varphi_i = \Omega t + \theta_i \qquad (i = 1, 2, 3)$$
$$Z_3 = A_3 \cos(\Omega t + \Xi_3) \qquad (24)$$
$$\dot{Z}_3 = -A_3\omega_3 \sin(\Omega t + \Xi_3)$$

Following the same method which was applied in the previous sections, we obtain the following equations for stationary conditions of motion

$$\varepsilon q_1 \Omega^2 \{\sin(\theta_{10} - \xi_3) + \sin(\theta_{20} - \xi_3) + \sin(\theta_{30} - \xi_3)\} - 3\dot{\varepsilon}\zeta\omega_3 a_3 = 0$$
$$\varepsilon q_1 \Omega^2 \{\cos(\theta_{10} - \xi_3) + \cos(\theta_{20} - \xi_3) + \cos(\theta_{30} - \xi_3)\}$$
$$-6(1-3b)\omega_3 a_3(\omega_3 - \Omega) = 0$$
$$-\frac{1}{2}\varepsilon q_2(1-3b) a_3\omega_3\Omega \sin(\theta_{10} - \xi_3) + \varepsilon N - \varepsilon\rho\Omega = 0 \qquad (25)$$
$$-\frac{1}{2}\varepsilon q_2(1-3b) a_3\omega_3\Omega \sin(\theta_{20} - \xi_3) + \varepsilon N - \varepsilon\rho\Omega = 0$$

$$-\frac{1}{2}\varepsilon q_2(1-3b)a_3\omega_3\Omega\sin(\theta_{30}-\xi_3)+\varepsilon N-\varepsilon\rho\Omega=0$$

From Eq. (25) we obtain the solutions for stationary motion, i.e. phase relations and amplitude of vibration as follows

$$\theta_{20}-\theta_{10}=0$$
$$\theta_{30}-\theta_{10}=0 \qquad (26)$$
$$a_3=\frac{\varepsilon q_1\Omega^2}{\sqrt{4(1-3b)^2\omega_3^2(\omega_3-\Omega)^2+(\varepsilon\zeta\omega_3)^2}}$$

The characteristic equation of the system corresponding to the above solutions becomes

$$p\{p^2+\varepsilon\rho p+\frac{(1-3b)^2\omega_3^2a_3^2}{\varepsilon q_1\Omega}(\omega_3-\Omega)\}[p^3+(\varepsilon\rho+\frac{\varepsilon\zeta}{1-3b})p^2$$
$$+\{\frac{(\varepsilon\zeta)^2}{4(1-3b)^2}+(\omega_3-\Omega)^2+\frac{\varepsilon\rho\varepsilon\zeta}{1-3b}+\frac{(1-3b)^2\omega_3^2a_3^2}{\varepsilon q_1\Omega}+\frac{\varepsilon q_1\varepsilon q_2\Omega^2}{2}\}p \qquad (27)$$
$$+\frac{\varepsilon\zeta(1-3b)\omega_3^2a_3^2}{\varepsilon q_1\Omega}(\omega_3-\Omega)-\varepsilon\rho\{\frac{(\varepsilon\zeta)^2}{4(1-3b)^2}+(\omega_3-\Omega)^2\}+\frac{\varepsilon\zeta\varepsilon q_1\varepsilon q_2\Omega^2}{4(1-3b)}]=0$$

Applying Routh-Hurwitz criterion to Eq.(27) the condition for stability is obtained, as follows

$$\omega_3-\Omega>0 \qquad (28)$$

4. EXPERIMENTAL RESULTS

To verify the results of the above theory, the test apparatuses were set up as shown in photo. 1 and 3.

Steel pipes of diameter 90mm were used as conveyer troughs. 75W variable speed electric motors with unbalance weights were employed as the vibrators and connected to the conveyer trough by the rubber springs.

The main dimensions of apparatuses are shown in Table 1.

Photo. 1 shows the conveyer with two vibrators in operation at the frequency $\Omega=50\,\text{Hz}\leqq\omega_2$. The phase relation of revolution of the unbalalanced rotors can be observed $\theta_{20}-\theta_{10}\approx0°$.

Photo. 2 shows 2 vibrators at the frequency $\Omega=40.7$ Hz$\leqq\omega_1$ and $\theta_{20}-\theta_{10}\approx\pi$.

Photo. 3 shows the conveyer with three vibrators in operation at the frequency $\Omega=49.5$ Hz$\leqq\omega_3$ and the phase relations $\theta_{30}-\theta_{10}\approx0°$, $\theta_{20}-\theta_{10}\approx0°$

These experimental results are in good agreement with the theoretical results.

Table 1

Items	2 vibrators	3 vibrators
Weight of conveyer trough M_c	83 kg	123 kg
Length of conveyer trough	2000 mm	3000 mm
Weight of vibrator body M_1	23 kg(×2)	23 kg(×3)
Unbalanced mass moment mr	4.83 kg-mm	4.83 kg-mm
Spring constant of the spring which connects the Vibrator body to conveyer k_1	148 kg/mm	148 kg/mm
Natural frequencies ω_r	ω_1 = 46 Hz ω_2 = 52 Hz	ω_3=52Hz
Spring constant of the spring which supports the system	3.36 kg/mm	4.48 kg/mm

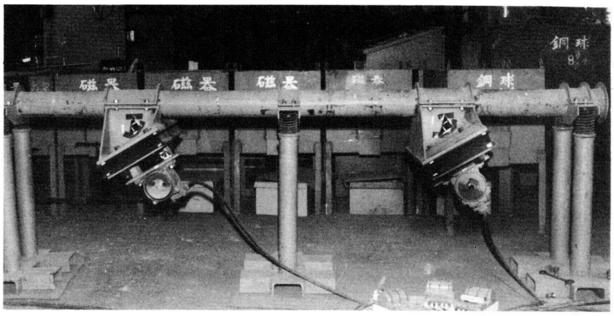

Photo. 1

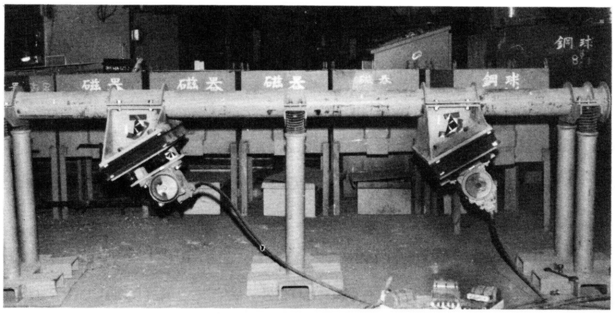

Photo. 2

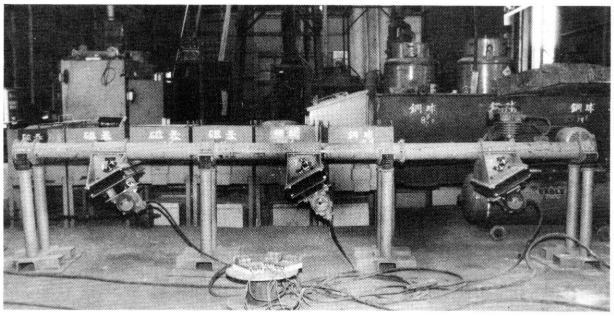

Photo. 3

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