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Modern Nuclear Force Predictions for the α Particle

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We present new calculations of the α particle which are based on the most modern nucleon-nucleon interactions alone and combined with the Tucson-Melbourne or the Urbana IX three-nucleon interaction. Results for the binding energies and some properties of the wave function are given. On that phenomenological level little room is left for the action of a possible four-nucleon force.

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Few nucleon bound states have received increasing attention in recent years. The possibility of solving increasingly complex systems allows one to probe the underlying dynamics directly (for a detailed review, see [1]). Several methods have been developed and applied to the $4N$ system and realistic forces, the GFMC [2], the CHH [3], the SV [4], the CRCGV [5], and the FY [6,7] methods. Very recently a new no-core shell model calculation appeared [8]. The $4N$ system is an important test ground for both the NN and the $3N$ nuclear interactions because of its strong binding. In this article we will address the question of whether at least the ground state energy of the α particle can be described by the most modern nuclear Hamiltonians.

In recent years, NN forces have been tuned very well to the rich NN data base, which led to a new generation of so-called realistic NN forces: Nijm I, II [9], AV18 [10], and CD-Bonn [11]. While Nijm II and AV18 are purely local, Nijm I has a weak nonlocality in the form of a ∇^2 dependence, and CD-Bonn is quite nonlocal (keeping the underlying Dirac structure of the one-boson exchange without p/m expansions). All potentials describe the NN data base with χ^2/data very close to 1. But they also have a large phenomenological character with a typical number of 40 fit parameters. The AV18 and CD-Bonn forces distinguish nn, pp, and np interactions and thus include charge-symmetry breaking (CSB) and charge-independence breaking (CIB). The interaction AV18 has, in addition, a whole set of electromagnetic corrections built in.

It is well known that the two $3N$ bound states, ^{3}H and ^{3}He, are theoretically underbound using only NN forces [12–15]. We compare our latest theoretical results, based on fully converged Faddeev calculations, to the experimental binding energies in Table I. The two nonlocal potentials lead to less underbinding than the local ones. The $3N$ calculations take the full CSB and CIB of the NN force into account including the isospin $T = 3/2$ admixtures. In the case of ^{3}He and ^{4}He the Coulomb interaction is included and, in addition, we use also the electromagnetic corrections in the case of AV18. Because of an implementation error, the results for Nijm I, II changed with respect to [13]. Table I also includes the strongly model-dependent kinetic energies which are correlated to the NN correlations. The local potentials have a somewhat harder core [13] leading to a higher kinetic energy.

There are two additional dynamical ingredients, which should cure that underbinding, relativistic effects and three-nucleon forces (3NF). Although one can, in principle, always find a unitary (but very complex) transformation to a Hamiltonian without any 3NF [16], we keep the given $2N$ interactions for practical reasons and because of their generally accepted physical origin. In relation to those forces, one or both of the two mentioned dynamical ingredients are needed. There are still controversies about the role of relativistic effects [17,18] and, as we think, also open conceptual questions. Thus we consider here only a strictly nonrelativistic framework, keeping in mind, however, that relativistic corrections should finally be added.

The topic of 3NF's is as old as nuclear physics [19] and, based on meson exchanges, various processes have been proposed in the past (for a review, see [20]). Among them, the Fujita-Miyazawa force [21] with an intermediate Δ generated by the exchange of two pions is most

TABLE I. $3N$ and $4N$ binding energies for various NN potentials together with expectation values T of the kinetic energy.

	^{3}H		^{3}He		^{4}He	
Potential	E_B [MeV]	T [MeV]	E_B [MeV]	T [MeV]	E_B [MeV]	T [MeV]
CD-Bonn	−8.012	37.42	−7.272	36.55	−26.26	77.15
AV18	−7.623	46.73	−6.924	45.68	−24.28	97.83
Nijm I	−7.736	40.73	−7.085	39.97	−24.98	84.19
Nijm II	−7.654	47.51	−7.012	46.62	−24.56	100.31
Exp.	−8.48	–	−7.72	–	−28.30	–

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obvious and is implemented in all modern 3NF models. Here we mention the rather popular 2π-exchange Tucson-Melbourne (TM) model [22], the Brazilian version thereof [23], and the Urbana 3NF [2]. There are also extensions to π-ρ and ρ-ρ exchanges [24]. The 2π-exchange model has been critically reviewed recently and a modified version, TM $'$, has been proposed in [25,26] which satisfies at least chiral symmetry. In this article we use the TM, TM $'$, and the Urbana IX 3NF's.

In the TM, force enters the strong πNN vertex function parametrized in the form of a monopole form factor with a cutoff parameter Λ. Choosing it around a generally accepted value, one achieves the right order of the lacking binding energy in the $3N$ system [12,14,27,28]. Because there is a strong correlation between the α binding energy, some $3N$ scattering observables [29], and the $3N$ binding energy, we fine-tune Λ to the $3N$ binding energies of ^{3}H and ^{3}He and do this separately for each of the four NN forces. In this step we have not yet included $T = 3/2$ admixtures. We list the fit results in Table II. Thereby the original TM parameters for the constants a, b, c, and d have been used [30]. In case of the Nijmegen interactions we did not adjust Λ to the triton, because they do not include a modern specification of the $^1S_0$ nn force. Since for ^{3}H the lack of binding energy is a bit larger than that for ^{3}He, the Λ values for ^{3}H are slightly larger than for ^{3}He.

Now by having those $3N$ Hamiltonians at our disposal we can study the α particle. For results referring to older forces see, for instance, Refs. [1,6,7]. We rewrite the Schrödinger equation into the Yakubovsky equations (YE) [31] and thus decompose the wave function Ψ into 18 Yakubovsky components (YC). Because of the identity of the nucleons, the YC's are not independent from each other and we can reduce their number to two: ψ_1 and ψ_2. Then the wave function reads

$$\Psi = [1 - (1 + P)P_{34}](1 + P)\psi_1 + (1 + P)(1 + \tilde{P})\psi_2 \quad (1)$$

and is expressed with the help of the permutations $P = P_{12}P_{23} + P_{13}P_{23}$ and $\tilde{P} = P_{13}P_{24}$, where P_{ij} are transpositions of particles i and j. Thus P acts on the three-body subcluster (123) and $\tilde{P}$ interchanges the two two-body subclusters (12) and (34). This decomposition is highly advisable for scattering states since the boundary conditions can most easily be expressed for YC's. In our case of a bound state it would, in principle, be possible to solve directly the Schrödinger equation, but the usage of two YC's introduces in a natural manner two types of Jacobi coordinates which accelerate the convergence of a partial wave decomposition.

For four identical particles the YE's reduce to two coupled integral equations

$$\psi_1 = G_0 t_{12} P[(1 - P_{34})\psi_1 + \psi_2], \quad (2)$$

$$\psi_2 = G_0 t_{12} \tilde{P}[(1 - P_{34})\psi_1 + \psi_2]. \quad (3)$$

Here in addition to the permutations the free $4N$ propagator G_0 and the NN t-operator t_{12} occur. t_{12} is driven by the NN force V_{12} through the Lippmann-Schwinger equation $t_{12} = V_{12} + V_{12}G_0t_{12}$. In case of $4N$ scattering, one has to go one step further and solve the three-body and $2 + 2$ subcluster problems beforehand in order to define the correct cut structure [6]. This is not necessary for bound states, and the form of Eqs. (2) and (3) is easier to handle numerically. One, presumably the most effective manner, which includes 3NF's has been given in [32]. We stick to that. Then only the first of the two YE's is changed into

$$\psi_1 = G_0 t_{12} P[(1 - P_{34})\psi_1 + \psi_2] + (1 + G_0 t_{12}) G_0 V^{(3)}_{123} \Psi . \quad (4)$$

Here $V^{(3)}_{123}$ is that part of the 3NF which is symmetrical under exchange of particles 1 and 2. In case of the 2π-exchange TM or the Urbana 3NF, such a separation into three parts is very natural. We solved the sets Eqs. (2) and (3) or Eqs. (4) and (3) in momentum space and in a partial wave representation. The first YC ψ_1 stands for the "3 + 1" partition and is naturally described by two

TABLE II. Cutoff parameters Λ, adjusted $3N$ binding energies, and resulting α-particle binding energies for various force combinations. Expectation values of the kinetic energy are also shown. Bold-faced results have been adjusted to the experiment.

		^{3}H		^{3}He		^{4}He	
Potential	Λ [m_π]	E_B [MeV]	T [MeV]	E_B [MeV]	T [MeV]	E_B [MeV]	T [MeV]
CD-Bonn + TM	4.784	**−8.480**	39.10	−7.734	38.24	−29.15	83.92
CD-Bonn + TM	4.767	−8.464	39.03	**−7.720**	38.18	−29.06	83.71
AV18 + TM	5.156	**−8.476**	50.76	−7.756	49.69	−28.84	111.84
AV18 + TM	5.109	−8.426	50.51	**−7.709**	49.47	−28.56	110.92
AV18 + TM′	4.756	−8.444	50.55	**−7.728**	49.54	−28.36	110.14
Nijm I + TM	5.035	−8.392	43.35	**−7.720**	42.59	−28.60	93.58
Nijm II + TM	4.975	−8.386	51.02	**−7.720**	50.13	−28.54	113.09
AV18 + Urb IX	–	−8.478	51.28	−7.760	50.23	−28.50	113.21
AV18 + Urb IX (GFMC) [2]	–	−8.47(1)	50.0(8)	–	–	−28.30(2)	112.1(8)
AV18 + Urb IX (CHH) [35]	–	−8.476	51.26				
Exp.		−8.48	–	−7.72	–	−28.30	–

Jacobi momenta for the three-body subcluster and one for the relative motion of the fourth particle to the other three. The second YC ψ_2 stands for the "2 + 2" partition and is naturally described by two relative momenta for the inner motion of the subclusters (12) and (34) and by the relative motion of the two subclusters. There are a lot of orbital and spin angular momenta as well as isospin quantum numbers to be coupled to $J^\pi = 0^+$ and $T = 0$. Both basis sets for the "3 + 1" and "2 + 2" partitions comprise about 1800 different combinations thereof in order to reach a converged description. The various momenta are discretized with roughly 35–45 grid points each. This leads to a huge absolutely full kernel matrix of dimensions $10^8 \times 10^8$. We solve the eigenvalue problem by a Lanczos-type algorithm [33,34] and make intensive use of a massively parallel supercomputer.

We would like to mention that the introduction of the two types of Jacobi momenta in Eqs. (2)–(4) leads to additional coordinate transformations which are hidden in the operator form of the YE's. They are equivalent to permutations. The crucial point in our calculation is the treatment of these permutations and coordinate transformations. The direct interchange of arbitrary particles is unfeasible because of the huge dimension of the problem. It is therefore necessary to interchange particles in two steps in such a way that at least one of the Jacobi momenta is not changed. This guarantees a block diagonal structure for the permutations. Only in this manner does the calculation become feasible. For a detailed description see Ref. [6].

Since we allow for CIB and CSB in the *NN* forces, in principle, the dominant total isospin state $T = 0$ has to be supplemented by $T = 1$ and $T = 2$ admixtures. Our estimations lead to the result that their admixtures will change the binding energy only very slightly (<10 keV) and thus at this stage we neglected them. But CIB and CSB lead to the prescriptions that the *NN* t-operators occur in the form $t = \frac{1}{3}t_{np} + \frac{1}{3}t_{pp} + \frac{1}{3}t_{nn}$ in the *NN* isospin 1 channels. This is different from the 3*N* system, where the corresponding linear combination is $t = \frac{1}{3}t_{np} + \frac{2}{3}t_{pp}$. The pp t-matrix also includes the effect of the Coulomb interaction. Since the bound nucleons are confined to a small space region, we can put to zero the Coulomb interaction outside a radius of 10–20 fm. Then the Fourier transformation of this interaction is nonsingular. The results are cutoff independent and numerically stable.

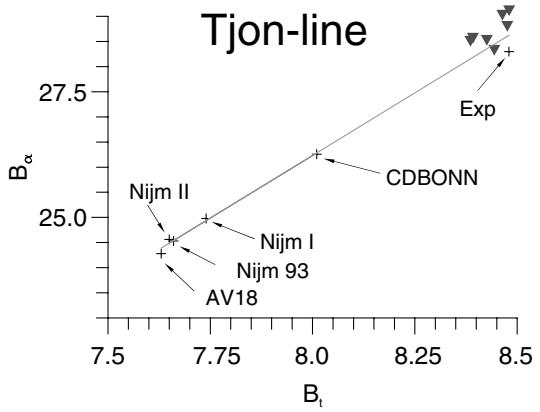

FIG. 1. Correlation of ^{4}He against ^{3}H binding energies in MeV for the different potentials. The triangles mark the predictions with 3NF from Table II.

We show in Table I the α-particle binding energies using *NN* forces only. As expected this theory underbinds the α particle. The CD-Bonn result compares well with [8]. A correlation between the ^{3}H and ^{4}He binding energies found in [36] for simple forces also remains valid for the most modern ones. This is depicted in Fig. 1. The experimental value is close to that straight line correlation, which nourishes the hope that by curing B_t one possibly will also cure B_α.

In Table II we present our results by adding the TM 3NF adjusted individually to the 3*N* binding energies. We find a slight overbinding of about 300 keV for AV18, Nijm I, and II. In the case of CD-Bonn, the overbinding reaches about 800 keV. Note that adjusting the TM 3NF to ^{3}H leads to a somewhat larger overbinding. Nevertheless, all of these numbers indicate that, with those Hamiltonians, one reaches the α-particle binding energy rather closely and there is little room left for the action of 4*N* forces. On such a phenomenological level, however, it is not possible to decide about the need of 4*N* forces. It is always possible to add another piece of a 3*N* force which in this case should be repulsive to reach the α-particle binding energy more accurately. For instance, the π-ρ exchange 3*N* force would provide at least one more parameter, and both nuclei, ^{3}He and ^{4}He, could be described. In that respect the Urbana 3NF is adjusted in such a manner [1] that, in addition to the 3*N* binding energy, nuclear matter is also

TABLE III. S, S', P, and D state probabilities for α and ^{3}He.

	^{4}He				^{3}He			
Model	S [%]	S' [%]	P [%]	D [%]	S [%]	S' [%]	P [%]	D [%]
AV18	85.45	0.44	0.36	13.74	89.95	1.52	0.06	8.46
AV18 + TM (^{3}He)	85.10	0.30	0.75	13.84	89.86	1.26	0.15	8.72
AV18 + TM′ (^{3}He)	83.27	0.31	0.75	15.68	89.46	1.25	0.13	9.16
AV18 + Urb IX	82.93	0.28	0.75	16.04	89.39	1.23	0.13	9.25
CD-Bonn	88.54	0.50	0.23	10.73	91.45	1.53	0.05	6.98
CD-Bonn + TM (^{3}He)	89.23	0.43	0.45	9.89	91.57	1.40	0.10	6.93

TABLE IV. Expectation values (in MeV) of the four parts of the TM 3NF model with respect to wave functions generated with AV18 + TM and AV18 + TM′ for ^{4}He.

Model	a term	b term	c term	d term
AV18 + TM (^{3}He)	0.003	−4.56	−1.26	−1.72
AV18 + TM′ (^{3}He)	−0.26	−3.90	+3.00	−1.02

taken into account. This then fixes a repulsive piece in that 3NF, with the result that the α-particle binding comes out essentially right. We repeated that calculation for the Urbana 3NF, first performed with the GFMC method [2]. Our result based on YE's is given in Table II together with the previous one. There we also show a triton result by the CHH method [35]. We estimate our numerical accuracy to be about ± 3 keV (± 50 keV) for the $3N$ ($4N$) system. There appear to be small differences between the GFMC results and ours, especially in the kinetic energies.

As in ^{3}H, one can also separate the ^{4}He wave function into S, S', P, and D state probabilities. Here S is spatially symmetric, S' has two-dimensional mixed symmetry, and P and D are the total $L = 1$ and 2 orbital angular momentum parts. We compare the two nuclei in Table III. For ^{4}He, S' is reduced, P is somewhat enhanced, and probabilities are shifted from S to D. It is remarkable that the TM 3NF together with CD-Bonn reduces the D state probability, in contrast to all other cases.

In Ref. [25] it is argued that chiral symmetry requires that the c term in the TM 3NF should be dropped, leading to a TM′ 3NF (one keeps the remaining constants b,d unchanged and a is replaced by $a' = a - 2c$). As seen in Table II the resulting α-particle binding energy after fitting the cutoff to ^{3}He coincides essentially with the α-particle binding energy. It is also interesting to see that the c term has a significant effect on the wave function, as demonstrated in Table IV. In this table we show the expectation values of the four different terms contributing to the TM 3NF evaluated with the wave function, including TM and the a, b, and d expectation values based on the wave function including TM′. In relation to that latter wave function, one can also evaluate the c term. Interestingly it turns out to be repulsive, whereas for the TM wave function it is attractive. Also it is interesting to see that for TM the a term is negligible, whereas it has some importance for TM′. These examples demonstrate again that 3NF's change wave functions and cannot be treated perturbatively, a fact known for a long time [27].

In summary, after adjusting 3NF's to $3N$ binding energies, the α-particle binding energies based on modern nuclear forces are rather close to the experimental value. This indicates that $4N$ forces (at least for $T = 0$) will be unimportant. More details on wave function properties will be published elsewhere. The FY equations are perfectly well under control for realistic NN and 3NF's and will be a perfect tool to study upcoming new force structures given in chiral perturbation theory [37,38].

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