Shell model for rotating turbulence

著者	Hattori Yuji, Rubinstein R, Ishizawa A
journal or publication title	Physical Review E
volume	70
number	046311
page range	046311-1-046311-6
year	2004-10
URL	http://hdl.handle.net/10228/566

doi: 10.1103/PhysRevE.70.046311

PHYSICAL REVIEW E **70**, 046311 (2004)

Shell model for rotating turbulence

Y. Hattori
Division of Computer Aided Science, Kyushu Institute of Technology, Tobata, Kitakyushu 804-8550, Japan

R. Rubinstein
NASA Langley Research Center, Hampton, Virginia 23681, USA

A. Ishizawa
Theory and Computer Simulation Center, National Institute for Fusion Science, Toki, Gifu 509-5292, Japan
(Received 24 March 2004; published 29 October 2004)

A modified shell model for rotating turbulence is proposed. The effect of rotation is introduced by a randomized linear term. Randomization is shown to be important in correctly modeling the rotation effect. Numerical simulation shows that the exponent of the energy spectrum in the inertial range changes from $-5/3$ to -2 as rotation rate increases. The mechanism behind this change is explained by weak turbulence theory and supported by numerical results.

DOI: 10.1103/PhysRevE.70.046311 PACS number(s): 47.27.Eq, 47.32.−y

I. INTRODUCTION

Statistical properties of rotating turbulence are less understood than those of homogeneous isotropic turbulence, although rotating turbulence is important in fluid motion both in the atmosphere and oceans and in turbomachinery. The equations of motion are the Navier-Stokes equations for an incompressible fluid in a rotating frame,

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u}, \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{\Omega}$ is the angular velocity of system rotation. When the rotation is strong, or in other words, the Rossby number $\mathrm{Ro} = U/\Omega L \sim |\boldsymbol{u} \cdot \nabla \boldsymbol{u}|/|\boldsymbol{\Omega} \times \boldsymbol{u}|$ is much smaller than 1 and the Reynolds number is sufficiently large, the flow is considered to approach an essentially two-dimensional (quasi-2D) state as expected from the Taylor-Proudman theorem after a (possibly long) transient process. The small-scale turbulence is important not only in the transient process but also in the quasi-2D state where some kind of forcing injects energy so that the turbulent component is maintained. In the quasi-2D state, stretching and folding of vorticity lines in the direction of the rotation axis are suppressed; energy transfer is also suppressed. It should be noted that the quasi-2D state is usually different from the two-dimensional turbulence as discussed below.

There have been different predictions for the power law of the energy spectrum in the inertial range: the same argument used for nonrotating homogeneous turbulence gives $E(k) \propto k^{-5/3}$; two-dimensionalization due to strong rotation can lead to $E(k) \propto k^{-3}$ for two-dimensional homogeneous turbulence; assuming that the energy dissipation is proportional to the rotating frequency, $E(k) \propto k^{-2}$ is obtained by dimensional analysis. The exponent is most likely to depend on some additional parameters, the most important one being the Rossby number. The direct numerical simulation by Smith and Waleffe [1] found the exponent -3. Recently the experiment by Baroud *et al.* [2] found the exponent -2 for strong rotation. They also studied the scaling of the probability distribution functions of the velocity difference and the structure function.

In this paper, we propose a modified shell model for rotating turbulence and use it to study statistical properties of the rotating turbulence. Shell models have been successfully used to study statistical properties of turbulence by many authors (see Biferale [3] for a review). Most of them dealt with homogeneous isotropic turbulence. Hattori and Ishizawa [4] studied magnetohydrodynamic (MHD) turbulence using a shell model. They were concerned mostly with the two-dimensional case, where direct numerical simulations show that large-scale coherent magnetic structures are formed and control the dynamics; for example, the energy spectrum scales as $k^{-3/2}$, which is explained by an argument similar to weak turbulence theory. A linear term which represents the effect of the coherent structures is introduced to the shell model for MHD turbulence. The modified model successfully predicts $k^{-3/2}$ spectrum when B_C, which is the strength of the coherent structures, is large and randomized. Note that randomization is necessary in this model as the energy spectrum obeys different scaling when B_C is constant. The same idea can be applied to the rotational turbulence as there is strong similarity between the MHD turbulence and the rotating turbulence as recognized by Zhou [5].

The paper is organized as follows. The shell model for rotating turbulence is introduced and its properties are studied using weak turbulence theory in Sec. II. Then we use the model to study statistical properties of the rotating turbulence in Sec. III. Concluding remarks are given in Sec. IV.

II. MODIFIED SHELL MODEL

A. Equation

Let us consider the following shell model for the rotating turbulence:

$$\frac{dZ_n}{dt} = i\alpha k_{n+1}Z_{n+2}Z^*_{n+1} + i\beta k_n Z_{n+1}Z^*_{n-1} - i\gamma k_{n-1}Z_{n-1}Z_{n-2} - \nu k_n^2 Z_n + i\Omega_n(t)Z_n + f_n, \tag{3}$$

where Z_n is a representative mode of $\boldsymbol{u}$ corresponding to the wave number $k_n = k_0\lambda^n$ and $\alpha+\beta+\gamma=0$. When $\Omega_n(t)=0$, it is an improved shell model by L'vov *et al.* [7] for nonrotating turbulence. The additional term $i\Omega_n(t)Z_n$ models the effect of rotation. The rotation rate $\Omega_n(t)$ is defined by

$$\Omega_n = \Omega_c + \Omega'_n, \quad \frac{d\Omega'_n}{dt} = -\frac{\Omega'_n}{T} + \frac{\tilde{\Omega}_n(t)}{T}, \tag{4}$$

$$\frac{d\tilde{\Omega}_n}{dt} = -\frac{\tilde{\Omega}_n}{\tau} + \frac{g_n(t)}{\tau}, \tag{5}$$

where $g_n(t)$ is Gaussian white noise with

$$\langle g_n(t)g_n(s)\rangle = \sigma_n \delta(t-s). \tag{6}$$

Thus Ω_n is a sum of the mean value Ω_c and the fluctuating part Ω'_n, which has the correlation time τ. In fact Ω'_n is a filtered noise of $\tilde{\Omega}_n$, T^{-1} being the low-pass cutoff frequency; $\tilde{\Omega}_n$ is correlated as $\langle\tilde{\Omega}_n(t)\tilde{\Omega}_n(s)\rangle \propto \exp(-|t-s|/\tau)$. Note that the fluctuating part has a significant role in correctly taking account of the rotational effect as shown later.

We may interpret the randomized linear term introduced above as follows. The inertial waves

$$\boldsymbol{u} = \boldsymbol{U}\exp[i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)],$$

$$\boldsymbol{k}\cdot\boldsymbol{U} = 0, \quad \omega = 2\boldsymbol{\Omega}\cdot\frac{\boldsymbol{k}}{k} = 2\Omega\cos\theta, \tag{7}$$

are solutions of Eqs. (1) and (2) if we neglect the nonlinear and viscous terms. Here θ is the angle between $\boldsymbol{\Omega}$ and $\boldsymbol{k}$. Since Z_n represents Fourier modes whose wave vectors satisfy $k_0\lambda^n \leqslant k < k_0\lambda^{n+1}$, θ should vary as Z_n moves among the Fourier modes. In other words, Z_n stochastically represents the corresponding set of Fourier modes. The corresponding variation of the angular frequency ω is taken account into Ω_n. The time scale of variation is τ; in the numerical computation we set $\tau=\Omega_c^{-1}$ as it would be reasonable that these two time scales are in the same order of magnitude. The low-pass cutoff is introduced so that Ω_n is smooth; we set $T=0.1\Omega_c^{-1}$. The results below are insensitive to τ and T as far as they are in reasonable ranges of values, while choosing too large τ is essentially same with constant Ω.

The shell model equation (3) has two invariants,

$$E = \sum_n |Z_n|^2, \quad H = \sum_n \left(\frac{\alpha}{\gamma}\right)^n |Z_n|^2, \tag{8}$$

when $\alpha+\beta+\gamma=0, \nu=0, f_n=0$. In the following, the parameters are set to $N=26, \lambda=k_{n+1}/k_n=2, \alpha=1, \beta=\gamma=-0.5$; we have confirmed that the results below are essentially unchanged by varying these parameters as long as the model stays in a chaotic regime. Forcing is given at $n=1$: $f_1=0.5+0.5i, f_n=0(n\geqslant 2)$.

B. Theoretical approach: Weak turbulence theory

Here we apply a weak turbulence approximation to the present shell model in order to obtain scaling properties of the energy spectrum analytically. Weak turbulence theory originates in the observation of Benney and Saffman [10] that the infinite hierarchy of moment equations generated by the statistical theory of a nonlinear dispersive wave equation admits an "intrinsic closure" provided that the wave amplitudes decorrelate in time due to linear phase scrambling rather than due to nonlinear interactions. The closure is possible because by assumption, this theory contains a small parameter: the ratio of the typical nonlinear decorrelation time to the linear decorrelation time. The condition that this ratio is small proves to limit the wave amplitudes, hence the term "weak" turbulence. More details are available in the recent comprehensive treatise by Zakharov *et al.* [9].

For modes for which both the forcing and viscous damping can be neglected, Eq. (3) becomes

$$\frac{dZ_n}{dt} = i\Omega_n(t)Z_n + i\alpha k_{n+1}Z_{n+2}Z^*_{n+1} + i\beta k_n Z_{n+1}Z^*_{n-1} - i\gamma k_{n-1}Z_{n-1}Z_{n-2}. \tag{9}$$

Define

$$\langle\Omega'_n(t)\Omega'_n(s)\rangle = R_n(t,s). \tag{10}$$

For the model defined by Eq. (4)

$$R_n(t,s) = \sigma_n^2\left\{\frac{\tau}{\tau-T}e^{-|t-s|/\tau} - \frac{T}{\tau-T}e^{-|t-s|/T}\right\}. \tag{11}$$

If $T=0$ so that $\Omega'_n=\tilde{\Omega}_n$,

$$R_n(t,s) = \langle\tilde{\Omega}_n(t)\tilde{\Omega}_n(s)\rangle = \sigma_n^2 e^{-|t-s|/\tau}, \tag{12}$$

but for nonzero T, when $|t-s|$ is small,

$$R_n(t,s) \approx \sigma_n^2 + O(|t-s|^2). \tag{13}$$

The linear part of the problem,

$$\frac{dZ_n}{dt} = i\Omega_n(t)Z_n, \tag{14}$$

has the obvious solution

$$\frac{dZ_n}{dt} = \exp\left[i\int_s^t d\tau\Omega_n(\tau)\right]Z_n(s), \tag{15}$$

so that if the response function is defined by the property

$$\langle Z_n(t)\rangle = G_n(t,s)\langle Z_n(s)\rangle, \tag{16}$$

we have [11]

$$G_n(t,s) = \left\langle \exp\left[i\int_0^t d\tau\Omega_n(\tau)\right]\right\rangle = \exp[-C_n(t,s)/2]H(t-s), \tag{17}$$

where

$$C_n(t,s) = \int_s^t d\tau \int_s^t d\sigma R_n(\tau,\sigma). \tag{18}$$

In particular, if $\Omega_n(t)$ were simply white noise so that $\tau = T = 0$ in Eqs. (4) and (5), then

$$R_n(t,s) = \sigma_n \delta(t-s), \tag{19}$$

and

$$G_n(t,s) = e^{-\sigma_n(t-s)/2} H(t-s). \tag{20}$$

If instead R_n is given by Eq. (12) , then

$$G_n(t,s) = e^{-\sigma_n^2\{(t-s)\tau - \tau^2 + \tau^2 \exp[-(t-s)/\tau]\}} H(t-s). \tag{21}$$

The applicability of weak turbulence theory to Eq. (9) is linked to the linear decorrelation mechanism provided by the random process $\Omega_n'(t)$. First recall the crucial diagonal property of correlations in the shell model of L'vov *et al.* [7],

$$\langle Z_n(t) Z_m^*(s)\rangle \propto \delta_{nm}. \tag{22}$$

Define the correlations

$$U_n(t,s) = \langle Z_n(t) Z_n^*(s)\rangle,$$

$$U_n^\dagger(t,s) = \langle Z_n(t) Z_n(s)\rangle. \tag{23}$$

In the weak turbulence approximation, the dominance of linear decorrelation implies that the two-time and single-time correlations in shell n are related by

$$U_n(t,s) = \left\langle \exp\left[i \int_s^t d\tau\, \Omega_n(\tau)\right] Z_n(s) Z_n^*(s) \right\rangle = G_n(t,s) U_n(s), \tag{24}$$

$$U_n^\dagger(t,s) = \left\langle \exp\left[i \int_s^t d\tau\, \Omega_n(\tau)\right] Z_n(s) Z_n^*(s) \right\rangle = G_n(t,s) U_n^\dagger(s), \tag{25}$$

if $t \geqslant s$, where $U_n(s)$ denotes the single-time correlation $U_n(s) = U_n(s,s)$. Since $U_n^\dagger(t,s) = 0$ if $U_n^\dagger(s) = 0$, this correlation will be ignored in what follows.

The perturbation theory described in detail in Zakharov *et al.* [9] or Benny and Saffman [10] leads to the governing equation for the single-time correlations $U_n(t) = U_n(t,t)$,

$$\begin{aligned}\frac{dU_n}{dt} &= \alpha k_{n+1}^2 \Theta_{n+2,n+1,n}[\gamma U_{n+1}U_n + \beta U_{n+2}U_n + \alpha U_{n+2}U_{n+1}] \\ &\quad + \beta k_n^2 \Theta_{n+1,n,n-1}[\gamma U_n U_{n-1} + \beta U_{n+1}U_{n-1} + \alpha U_{n+1}U_n] \\ &\quad + \gamma k_{n-1}^2 \Theta_{n,n-1,n-2}[\gamma U_{n-1}U_{n-2} + \beta U_n U_{n-2} + \alpha U_n U_{n-1}],\end{aligned} \tag{26}$$

where

$$\Theta_{m+1,m,m-1} = \int_0^\infty d\tau G_{m+1}(\tau) G_m(\tau) G_{m-1}(\tau). \tag{27}$$

Time stationarity has been assumed, so that $G(t,s) = G(t-s)$ is a function of time difference only.

Defining

$$\Xi_p = \Theta_{p+1,p,p-1} k_p^2 [\alpha U_{p+1}U_p + \beta U_{p+1}U_{p-1} + \gamma U_p U_{p-1}], \tag{28}$$

Eq. (26) takes the form

$$\begin{aligned}\frac{dU_n}{dt} &= \alpha \Xi_{n+1} + \beta \Xi_n + \gamma \Xi_{n-1} = [\alpha \Xi_{n+1} - \gamma \Xi_n] \\ &\quad - [\alpha \Xi_n - \gamma \Xi_{n-1}].\end{aligned} \tag{29}$$

A constant flux solution of Eq. (29) is defined by

$$\varepsilon = \alpha \Xi_{n+1} - \gamma \Xi_n, \tag{30}$$

where ε denotes the energy flux. For the linear response function defined by Eq. (20),

$$\Theta_{m+1,m,m-1} = \frac{1}{\sigma_{m+1} + \sigma_m + \sigma_{m-1}}. \tag{31}$$

Equation (31) is also a good approximation for large shell indices m for the more general response function Eq. (21).

Power counting in Eq. (30) shows that if

$$\sigma_m \sim k_m^\mu, \tag{32}$$

and if, as usual, the shells are in a geometric progression, $k_n = k_0 \lambda^n$, then Eq. (26) admits the formal solution

$$U_m \sim k_m^{-1+\mu/2}. \tag{33}$$

Standard arguments (compare L'vov *et al.* [9]) show that a solution of Eq. (26) exists with the scaling Eq. (33) in the "region of transparency" in which the unforced, inviscid equation Eq. (9) is valid.

Equation (33) corresponds to an energy spectrum scaling as

$$E_m \sim k_m^{-2+\mu/2}. \tag{34}$$

In particular, for the case studied by Hattori and Ishizawa [4] in which $\mu = 1$, $E_m \sim k^{-3/2}$, in agreement with their numerical simulations. In the present case with $\mu = 0$, we have the theoretical prediction $E_m \sim k_m^{-2}$.

It was noted earlier that weak turbulence theory applies only when linear phase decorrelation dominates nonlinear phase decorrelation, so that

$$\sigma_m \geqslant \sqrt{k_m^3 E_m}. \tag{35}$$

If weak turbulence generates an energy spectrum for which $k_m^3 E_m$ increases with m, the inequality in Eq. (35) can be satisfied for small m but violated for sufficiently large m. For large k_m, linear decorrelation is a perturbation of nonlinear decorrelation, and Kolmogorov scaling $E_m \sim k_m^{-5/3}$ is recovered.

The phenomenological theory proposed by Zhou [5] and the closure theory of Canuto and Dubovikov [6] both predict the same picture of the spectral scaling in rotating turbulence, with $E(k) \sim k^{-2}$ at large scales, Kolmogorov scaling at small scales, and a transition between these two regimes when the rotation rate Ω satisfies $\Omega \approx \epsilon^{1/3} k^{2/3}$. These arguments perhaps apply better to the present problem, which is

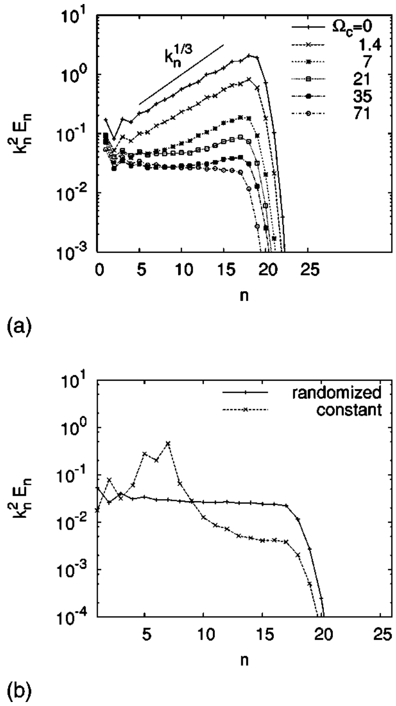

FIG. 1. Energy spectrum. (a) $\Omega_c = 0, 1.4, 7, 21, 35$, and 71. (b) Comparison of constant and random large-scale effects. $\Omega_c = 71$. Note that $k_n^{1/3}$ in (a) corresponds to the Kolmogorov 5/3 law.

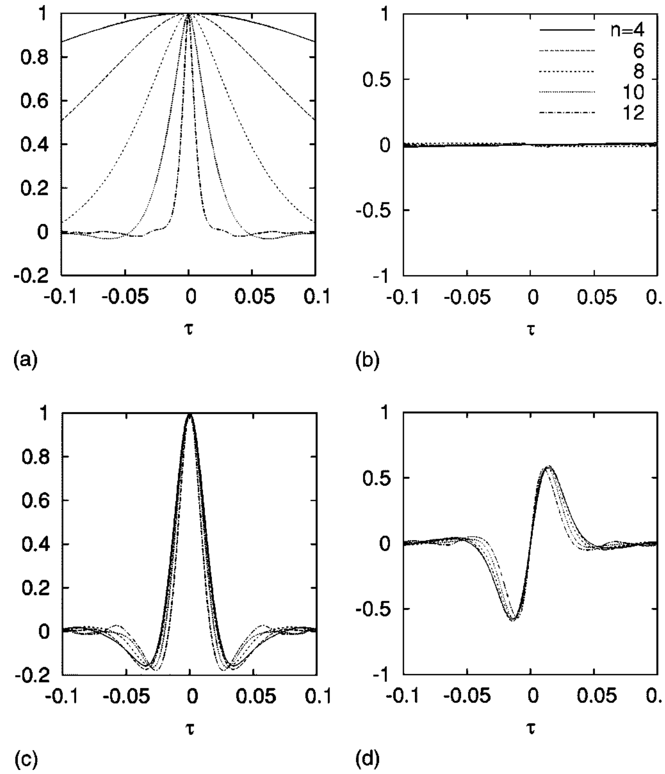

FIG. 2. Correlation (see text for the definition). (a) $\Omega_c = 0$, real part, (b) $\Omega_c = 0$, imaginary part, (c) $\Omega_c = 0$, real part, (d) $\Omega_c = 71$, imaginary part.

effectively "isotropic," than to rotating turbulence, in which anisotropy has an important role. The role of anisotropy in the spectral scaling of rotating turbulence remains an open question; "anisotropic" shell models in which the complex amplitude Z_n is replaced by a higher dimensional geometric quantity may have a role in deciding this question.

It is noteworthy that weak turbulence scalings can be obtained from a problem in which resonant triads are entirely absent. The essential feature which permits weak turbulence scaling is the existence of the linear decorrelation mechanism provided by the random phase factors $\Omega_n(t)$. But we stress that in a shell model, linear decorrelation only occurs if the phase factors are *random*; the introduction of *deterministic* phase factors cannot disrupt normal Kolmogorov scaling. This conclusion agrees with the numerical simulations of Hattori and Ishizawa [4] in which the scaling exponent depends on λ when, in effect, Ω_n is constant in each shell. The situation is of course entirely different for dispersive waves in two or three dimensions: although the linear decorrelation mechanism is deterministic, the interaction of waves with different wavevectors introduces the necessary phase randomization.

III. NUMERICAL RESULTS

A. Energy spectrum

Figure 1 shows the energy spectrum obtained by the present shell model. The spectrum is multiplied by k^2 so that we can see subtle differences in the scaling exponent. In Fig. 1(a), the scaling law is close to the Kolmogorov law $E_n \propto k_n^{-5/3}$ for $\Omega_c = 0$, the nonrotating case; actually the exponent is slightly smaller than $-5/3$. As Ω_c increases, there appears a region of n for which the scaling exponent is close to -2. For $\Omega_c = 71$, we see $E_n \propto k_n^{-2}$ in the entire inertial range. In Fig. 1(b), we compare the cases with and without the fluctuation Ω_n' in the rotation term. The energy spectrum does not show power law behavior for the case without fluctuation. Therefore the fluctuation is a key factor for the shell model to take account of strong rotation as discussed in the previous section.

B. Correlation

According to weak turbulence theory, the transition from the k^{-2} spectrum to the $K41$ spectrum is caused by the change in the characteristic time scale. We can see this change in Fig. 2, where the correlation defined by

$$\mathrm{Cor}(n;\tau) = \frac{\langle Z_n(t) Z_n^*(t+\tau)\rangle}{\langle |Z_n(t)|^2\rangle} = \frac{U_n(t,t+\tau)}{U_n(t,t)},$$

is shown for various modes. For $\Omega_c = 0$, the time scale is proportional to $k_n^{-1/3}$. On the other hand, for $\Omega_c = 71$, the time scale is the same for all modes. The minima of the real part of the correlation are seen to be around 0.03; this is of the same order as $\pi/\Omega_c \sim 0.044$, which is the phase reversal time. Therefore the phase has the correlation time determined by the rotation effect.

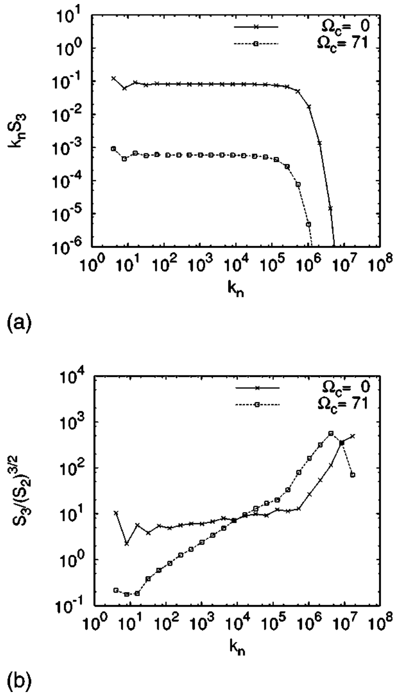

FIG. 3. Third-order structure function. $\Omega_c=0$ and 71. (a) k_nS_3, (b) $S_3/(S_2)^{3/2}$.

C. Third-order structure function

Figure 3 shows the "third-order structure function"

$$S_3(k_n) = \mathrm{Im}\langle Z_{n-1}Z_nZ^*_{n+1}\rangle, \tag{36}$$

introduced by L'vov *et al.* [7]. As in L'vov *et al.* we have

$$\frac{d}{dt}\langle|Z_n|^2\rangle = 2k_n\left[2\alpha S_3(k_{n+1}) + \beta S_3(k_n) + \frac{\gamma}{2}S_3(k_{n-1})\right] + 2\langle f^*_nZ_n\rangle, \tag{37}$$

neglecting viscosity, which in stationary conditions leads to

$$S_3(k_n) = \frac{1}{k_n}\left[A + B\left(\frac{\gamma}{\alpha}\right)^n\right], \tag{38}$$

with some constants A and B. Thus $S_3 \propto k_n^{-1}$ for large n. Figure 3(a) confirms this relation. Note that the magnitude of S_3 is rather small for $\Omega_c=71$. In other words, energy transfer is reduced owing to the rotation effect. This is relatively significant for small wave numbers as shown in Fig. 3(b). For $\Omega_c=0$, $S_3/(S_2)^{3/2}$ (the "second-order structure function" S_2 is $\langle|Z_n|^2\rangle$ as usual) is nearly constant in the inertial range; for $\Omega_c=71$, it increases with k_n. As a result energy accumulates in small-wave-number modes, leading to the steeper energy spectrum $E_n \propto k_n^{-2}$.

D. PDF

The energy spectrum scales as $E_n \propto k_n^{-2}$ when the rotation effect is strong; the same power law is observed in the experiment by Baroud *et al.* [2]. Then it is of interest to see whether or not probability distribution function (PDF) behaves similarly. In order to construct velocity fields from shell variables, we employ the method used in Jensen [8]; that is,

$$\boldsymbol{u}(\boldsymbol{x},t) = \sum_{n=1}^{N} \boldsymbol{c}_n[Z_n(t)\exp(i\boldsymbol{k}_n\cdot\boldsymbol{x}) + \mathrm{c.c.}],$$

where $\boldsymbol{k}_n=k_n\boldsymbol{e}_n$ and $\boldsymbol{e}_n$ and $\boldsymbol{c}_n$ are unit vectors in random directions. We impose

$$\boldsymbol{c}_n\cdot\boldsymbol{e}_n = 0, \quad n = 1,\ldots,N,$$

which is a sufficient condition for incompressibility. We average over a large number of sets $\{(\boldsymbol{c}_n,\boldsymbol{e}_n)\}$ to achieve a good statistical average.

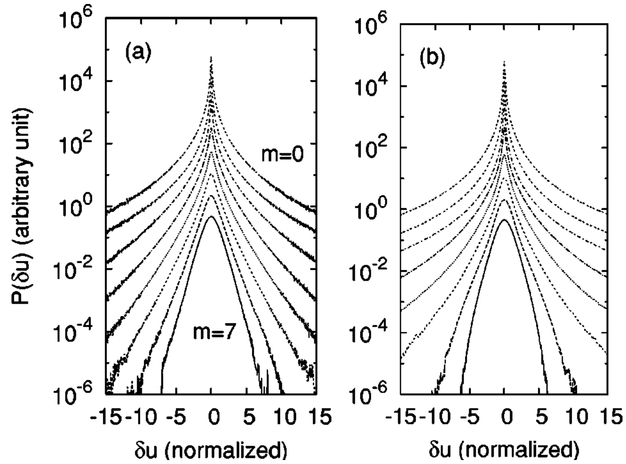

FIG. 4. Normalized PDF. $r=r_02^{1+3m}(m=0,1,\ldots,7), r_0=2\pi/k_N$. (a) $\Omega_c=0$, (b) $\Omega_c=71$.

Figure 4 shows PDF's of the longitudinal velocity difference δu obtained by the method described above. The PDF's are not self-similar either for $\Omega_c=0$ or for $\Omega_c=71$. There is no significant difference between $\Omega_c=0$ and 71. This differs from the experiment which observed self-similar but non-Gaussian PDF's for strong rotation.

These results raise the more general question of anomalous scaling in turbulent systems subject to external agencies. Recent investigations [12,13] of turbulence and shell models driven by forcing with power-law correlation $\langle f(\mathbf{k},t)f(-\mathbf{k},t)\rangle \sim ak^{-\gamma}\delta(t-s)$ (for simplicity, we only write the scalar amplitude of the force correlation; more complete definitions are found in the cited references) suggest that when the forcing dominates nonlinearity, higher order structure functions obey the "normal" scaling obtained by dimensional analysis, and velocity difference PDF's over inertial range separations are self-similar. However, when the forcing decays sufficiently rapidly at large k, nonlinearity becomes dominant, and the non-self-similar velocity difference PDF's of unforced turbulence are recovered.

A heuristic analogy between such forced systems and the present shell model might suggest that when linear decorrelation is dominant over all scales, as when $\Omega_c=71$, self-similar velocity difference PDF's should be observed, with increasingly dominant anomalous effects as Ω_c approaches zero. At this point, we cannot resolve the discrepancy between this expectation and our contrary computational results: perhaps forcing has a more direct impact on shell models than modification of the time scale; alternatively, the linear decorrelation mechanism proposed here may simply be much weaker than the linear decorrelation of dispersive waves. Another possibility is that the subtle interplay between external agencies and nonlinearity is simply beyond the scope of our shell model. In any case, perhaps our results underscore the apparent independence of second order statistical properties like spectral scaling exponents from more refined details like anomalous scaling properties.

IV. CONCLUDING REMARKS

We have proposed a modified shell model for rotating turbulence. The transition from $k^{-5/3}$ spectrum for weak rotation to k^{-2} spectrum for strong rotation is observed as predicted by weak turbulence theory. Although the energy spectrum obeys the same power law as the experiment by Baroud *et al.* [2], the results on PDF's are different. The PDF's obtained by the present model are non-Gaussian as in the experiment. However, unlike the self-similar PDF's observed in the experiment, they are not self-similar. The direction of energy cascade may be responsible for this difference. The present model shows the normal cascade from small wave number to large wave number, while the experiment showed some evidence for the inverse cascade. The present model would be more closely related to the numerical simulation by Smith and Waleffe [1] ; they observed k^{-2} spectrum for $k > k_f$ with k_f being the forcing wave number, which implies that the direction of energy cascade is normal. Simulations with higher Reynolds numbers are expected for more precise evaluation of the present model.

It is shown that introduction of the randomized rotation term leads to k^{-2} spectrum. The same idea can be applied to other cases for which weak turbulence theory works, e.g., turbulence of gravity waves.

[1] L. M. Smith and F. Waleffe, Phys. Fluids **11**, 1906 (1999).
[2] C. N. Baroud, B. B. Plapp, Z.-S. She, and H. L. Swinney, Phys. Rev. Lett. **88**, 114501 (2002); Phys. Fluids **15**, 2091 (2003).
[3] L. Biferale, Annu. Rev. Fluid Mech. **35**, 441 (2003).
[4] Y. Hattori and A. Ishizawa, *Proceedings of IUTAM Symposium on Geometry and Statistics of Turbulence*, edited by T. Kambe, T. Nakano, and T. Miyauchi (Kluwer, Dordrecht, 2001), pp. 89–94.
[5] Y. Zhou, Phys. Fluids **7**, 2092 (1995).
[6] V. M. Canuto and M. S. Dubovikov, Phys. Fluids **9**, 2132 (1997).
[7] V. S. L'vov, E. Podivilov, A. Pomyalov, I. Procaccia, and D. Vandembroucq, Phys. Rev. E **58**, 1811 (1998).
[8] M. H. Jensen, Phys. Rev. Lett. **83**, 76 (1999).
[9] V. E. Zakharov, V. S. L'vov, and G. Falkovich, *Kolmogorov Spectra of Turbulence I, Wave Turbulence* (Springer-Verlag, Berlin, 1992).
[10] D. J. Benney and P. Saffman, Proc. R. Soc. London, Ser. A **289**, 301 (1966).
[11] R. Kubo, J. Math. Phys. **4**, 174 (1961).
[12] B. Mazzi, F. Okkels, and J. C. Vassilicos, Eur. Phys. J. B **28**, 243 (2002).
[13] L. Biferale, M. Cencini, A. S. Lanotte, M. Sbragaglie, and F. Toschi, New J. Phys. **6**, 37 (2004).