Simplified Analysis Method for Vibration of Fusion Reactor Components with Magnetic Damping

Yoshikazu Tanaka, Tomoyoshi Horie and Tomoya Niho

Department of Mechanical System Engineering,
Kyushu Institute of Technology
680–4 Kawazu, Iizuka, Fukuoka 820–8502, Japan

Corresponding to :
Yoshikazu Tanaka,
Department of Mechanical System Engineering,
Kyushu Institute of Technology,
680–4 Kawazu, Iizuka, Fukuoka 820–8502, Japan
Tel. (+81)948-29-7757
Fax. (+81)948-29-7751
e-mail : yoshi@solid.mse.kyutech.ac.jp

abstract

This paper describes two simplified analysis methods for the magnetically damped vibration. One is the method modifying the result of finite element uncoupled analysis using the coupling intensity parameter, and the other is the method using the solution and coupled eigenvalues of the single-degree-of-freedom coupled model. To verify these methods, numerical analyses of a plate and a thin cylinder are performed. The comparison between the results of the former method and the finite element tightly coupled analysis show almost satisfactory agreement. The results of the latter method agree very well with the finite element tightly coupled results because of the coupled eigenvalues. Since the vibration with magnetic damping can be evaluated using these methods without finite element coupled analysis, these approximate methods will be practical and useful for the wide range of design analyses taking account of the magnetic damping effect.

1 Introduction

A large Lorentz force generated by the interaction of eddy current due to plasma disruption and the toroidal magnetic field is applied to conductive thin shell structures of fusion reactor components. When the structures deform in the magnetic field, the electromotive force, which is produced by the deformation velocity and the magnetic field, reduces the eddy current [1]. Therefore, it is important to evaluate the electromagnetic and structural coupled effect or magnetic damping effect for the design of fusion reactor components. The evaluation of magnetic damping effect may avoid too conservative structural design of the reactor components. In the design work, since the finite element coupled analyses of the components take much time to prepare and execute the analyses, they will be performed to confirm the integrity of the design. Simplified analysis methods will be useful for so many analysis cases with various design parameters especially in deciding on a design concept.

Over the past few years, several attempts have been made on parameters related to the magnetic damping effect. The critical magnetic viscous damping ratio, which can be used for the approximate solution method(MMD method) for electromagnetic mechanical coupling, has been proposed and its dependence on material properties was examined by Takagi et al. [2], [3]. The magnetic damping parameters, which were obtained by non-dimensionalizing a set of equations describing the coupled problem and can be used for reduced experimental model, have been proposed by Yoshida et al. [4]. The authors have defined the coupling intensity parameter and examined its dependence on magnetic field, plate thickness and material properties [5]. The advantages of the coupling intensity parameter are that it can represent the characteristics of the magnetically damped vibration [6] and that the parameter can be used to determine the conditions for the reduced experimental model [7].

In this paper, two simplified analysis methods are proposed and described for the vibration with magnetic damping. In one method, the result of finite element uncoupled analysis is modified using the coupling intensity parameter. The other method utilizes the single-degree-of-freedom coupled model taking into account the change of angular frequency caused by the coupling effect. To verify the validity of these methods, numerical analyses are performed for a cantilevered plate in an electromagnetic field. A problem of a cylinder is also analyzed to confirm the use for practical application.

2 Coupling intensity parameter

2.1 Definition of coupling intensity parameter

The symmetric matrix equation of magnetic damping problem [8] is expressed using displacement u, the normal component of current vector potential T and velocity v by

$$\begin{bmatrix} -\mathbf{K} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{U} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{v}} \\ \dot{\mathbf{T}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{K} & \mathbf{0} \\ \mathbf{K} & \mathbf{0} & \mathbf{C}_s \\ \mathbf{0} & -\mathbf{C}_e & -\mathbf{R} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{T} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{F}^{ex} \\ -\dot{\mathbf{B}}^{ex} \end{Bmatrix}, \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ the stiffness matrix, $\mathbf{C}_s$ the coupling sub–matrix by the electromagnetic force, $\mathbf{C}_e$ the coupling sub–matrix by the electromotive force, $\mathbf{U}$ the inductance matrix, and $\mathbf{R}$ the resistance matrix; and vectors $\mathbf{F}^{ex}$, $\dot{\mathbf{B}}^{ex}$ are the external mechanical force and the change of the external magnetic field respectively.

Since the coupled eigenvalue α of Eq.(1) includes the magnetic damping effect, the coupling intensity parameter C_{es} is defined [5] by

$$C_{es} = \frac{\mathrm{Re}\ \alpha}{|\alpha|} \tag{2}$$

to evaluate the magnetic damping effect.

2.2 Dependence of the coupling intensity parameter

Dependence of the coupling intensity parameter has been obtained from the single-degree-of-freedom coupled model [5]. The equation of motion for the coupled problem is expressed using u and T as

$$m\ddot{u} + ku + C_s T = f^{ex} \ , \tag{3}$$

where F^{ex} and $C_s T$ mean the external force and the coupling term by the electromagnetic force respectively. The equation of the eddy current becomes

$$U\dot{T} + C_e\dot{u} + RT = \dot{B}^{ex} \ , \tag{4}$$

where $U, R, \dot{B}^{ex}$, and $C_e\dot{u}$ mean the inductance, the resistance, the change of external magnetic field, and the coupling term by the electromotive force respectively. Elimination of T from Eqs.(3) and (4) yields the third order differential equation

$$mU\,\dddot{u} + mR\,\ddot{u} + (kU - C_s C_e)\,\dot{u} + Rku = -C_s\dot{B}^{ex} + \dot{f}^{ex} + Rf^{ex} \ . \tag{5}$$

Upon solving the characteristic equation of Eq.(5), the coupled eigenvalues

$$\gamma_e = -\frac{R}{3U} + \left\{b - \sqrt{b^2 + a^3}\right\}^{\frac{1}{3}} + \left\{b + \sqrt{b^2 + a^3}\right\}^{\frac{1}{3}} \tag{6}$$

$$\begin{aligned}\gamma_s \pm \omega_s i = &-\frac{R}{3U} - \frac{\left\{b - \sqrt{b^2 + a^3}\right\}^{\frac{1}{3}} + \left\{b + \sqrt{b^2 + a^3}\right\}^{\frac{1}{3}}}{2} \\ &\pm \frac{-\left\{b - \sqrt{b^2 + a^3}\right\}^{\frac{1}{3}} + \left\{b + \sqrt{b^2 + a^3}\right\}^{\frac{1}{3}}}{2}\sqrt{3}\ i\end{aligned} \tag{7}$$

have been obtained, where a and b are the functions of Young's modulus E, density ρ, electric conductivity κ, and thickness h. Substituting these coupled eigenvalues $\gamma_s \pm \omega_s i$ into Eq.(2), the coupling intensity parameter is expressed [5] by

$$C_{es} = \frac{\mathrm{Re}\ \alpha}{|\alpha|} = C_{es}(B, h, E, \rho, \kappa, P_s, P_e, P_c) \ , \tag{8}$$

where P_s, P_e, and P_c are the proportional constants which can be determined by the finite element eigenvalue analysis.

3 Simplified analysis methods for magnetically damped vibration

Two simplified analysis methods are presented for a magnetically damped vibration problem with an initial deflection. Although the problem is simple, these methods are also applicable to the magnetically damped vibration with dynamic force.

3.1 Modification of the finite element uncoupled result using the coupling intensity parameter (method 1)

The magnetically damped vibration response can be approximately obtained by modifying the finite element uncoupled results to include the coupling effect using the coupling intensity parameter, which is based on the single-degree-of-freedom coupled model. Since the structural and eddy current equations are solved alternately by neglecting the electromotive force in the finite element uncoupled analysis, the computation time can be much saved compared to the finite element tightly coupled analysis where the equations are solved simultaneously.

In general free vibration problem with damping

$$m\ddot{u} + c\dot{u} + ku = 0 \ , \tag{9}$$

with the initial conditions $u(0) = U_0$ and $\dot{u}(0) = 0$, the solution characterized by the damping ratio ζ and the natural angular frequency ω_0 is expressed as

$$u = \sqrt{\frac{1}{1-\zeta^2}} U_0 \exp\left(-\zeta\omega_0 t\right) \cos\left(\omega_0\sqrt{1-\zeta^2}t - \beta\right) \ , \tag{10}$$

where

$$\beta = \tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}} \ . \tag{11}$$

Since ζ is the ratio of the damping coefficient c and the critical damping coefficient $2\sqrt{mk}$, it can also be written as

$$\zeta = \frac{\mathrm{Re}\ \alpha'}{|\alpha'|} \ , \tag{12}$$

where α' is the eigenvalue of Eq.(9). While ζ is defined based on the second order differential equation (9), C_{es} is based on the third order differential equation (5). According to the similarity, however , in the definitions of C_{es} in Eq.(8) and ζ in Eq.(12), we assume

$$C_{es} \simeq \zeta \ . \tag{13}$$

Then the magnetically damped vibration response is approximately written by

$$u_c = \sqrt{\frac{1}{1 - {C_{es}}^2}} U_0 \exp\left(-C_{es}\omega_0 t\right) \cos\left(\sqrt{1 - C_{es}^2}\omega_0 t - \beta\right) \ . \tag{14}$$

By substituting $\zeta = 0$ into Eq.(10), the uncoupled vibration response without damping is written as

$$u_g = U_0 \cos \omega_0 t \ . \tag{15}$$

Comparing the magnetically damped vibration of Eq.(14) with the uncoupled vibration of Eq.(15), the amplitude is decreased by $\sqrt{\frac{1}{1-C_{es}^2}} \exp\left(-C_{es}\omega_0 t\right)$ times as large as that of uncoupled vibration, the angular frequency is multiplied by $\sqrt{1 - C_{es}^2}$, and the phase angle is delayed for β. Therefore, the magnetically damped vibration response is obtained from the result of the finite element uncoupled analysis using this analogy(Fig. 1). The value of C_{es} is calculated from Eq.(8) based on the single-degree-of-freedom coupled model.

This method may be useful for the parametric survey in the design taking account of the magnetically damped vibration response, because the coupling intensity parameter is expressed as the function of the magnetic field, material properties and thickness. This method can be applied to the response with more than one structural mode. After evaluating each mode using this method, the response is obtained by superposing these modes.

3.2 The method using the solution and coupled eigenvalues of the single-degree-of-freedom coupled model (method 2)

The analytical solution and coupled eigenvalues γ_s , $\gamma_s \pm \omega_s i$ of the single-degree-of-freedom coupled model are used to approximately evaluate the magnetic damped vibration response. The geometry of the structure is taken into account by the proportional constants P_s, P_e and P_c of the single-degree-of-freedom coupled model.

Using the coupled eigenvalues γ_s in Eq.(6) and $\gamma_s \pm \omega_s i$ in Eq.(7), the general solution of Eq.(5) is given by

$$u = D_1 \mathrm{e}^{\gamma_e t} + \mathrm{e}^{\gamma_s t} \left\{ D_2 \cos \omega_s t + D_3 \sin \omega_s t \right\} \ , \tag{16}$$

where D_1, D_2 and D_3 are the constants determined from initial conditions. For the problem with the initial conditions

$$u(0) = U_0, \quad \dot{u}(0) = 0, \quad \ddot{u}(0) = -\frac{k}{m} U_0 \ , \tag{17}$$

the constants are

$$D_1 = U_0(1 - A) \tag{18}$$

$$D_2 = A U_0 \tag{19}$$

$$D_3 = -\frac{U_0}{\omega_s} \left\{ \gamma_e - (\gamma_e - \gamma_s) A \right\} \ , \tag{20}$$

where

$$\omega_0^2 = \frac{k}{m}, \quad A = \frac{\omega_0^2 + \gamma_e^2 - 2\gamma_e \gamma_s}{\gamma_e^2 - 2\gamma_e \gamma_s + \gamma_s^2 + \omega_s^2} \ . \tag{21}$$

The magnetically damped vibration response is obtained from Eq.(16) by substituting D_1, D_2 and D_3, which are determined from Eqs.(18), (19) and (20) using the coupled eigenvalues γ_e and $\gamma_s \pm \omega_s i$ of Eqs.(6) and (7) (Fig. 2).

Compared to the method in section 3.1(method 1), the response with high accuracy may be obtained because this method is based on the analytical damped solution of the third order differential equation. Since the dynamic finite element analysis is not required in this method, the computation time may be shorter. Like method 1, the parametric study for design analysis and the application to the response with more than one mode are also available using this method.

4 Magnetic damping problem of a cantilevered plate

4.1 Analysis model

To verify the validity of these simplified analysis methods, a cantilevered copper plate in an electromagnetic field shown in Fig. 3 is solved. While the plate is vibrating by the initial deflection (10 mm), the deformation velocity $\dot{u}_z$ and the magnetic field B_x generate an electromotive force, which generates the magnetic damping force to the plate.

4.2 Results with different magnetic fields

Analyses of dynamic behavior of the plate are performed and compared with the finite element coupled solution for the cases of $B_x = 0.2, 0.5, 0.7$ T.

Figure 4 shows the deflection at the free end of the plate. The result of method 1 agrees well with that of the finite element tightly coupled analysis when B_x is 0.2 T(Fig. 4(a)). For higher magnetic field, the results of method 1 are delayed in time compared to those of the finite element coupled analysis because the angular frequency difference between $\sqrt{1-C_{es}^2}\,\omega_0$ and ω_s for each magnetic field (5.03 % for $B_x = 0.5$ T and 14.3 % for 0.7 T according to Table 1) is larger than that for 0.2 T. Although small delay in time is observed for large C_{es} values, the results obtained from method 1 satisfactory predict the response of magnetically damped vibration. According to Fig. 4, the results of method 2 for all cases are in good agreement with the finite element coupled results because the coupled eigenvalues used include the coupling effect between the electromagnetic and mechanical systems.

4.3 Results for different materials

These methods are applied to the plate with different materials such as aluminum and Type 316 stainless steel with a magnetic field $B_x = 0.5$ T. Material properties used for the analyses are summarized in Table 2.

Figure 5 shows the deflection at the free end of the plate. According to Table 3, the angular frequency difference of Type 316 stainless steel is 2.96×10^{-3} %, which is so small that the result of method 1 agrees with the finite element tightly coupled result as shown in Fig. 5(a). As for aluminum, the result of method 1 is slightly delayed in time as shown in Fig. 5(b) because the angular frequency difference of 6.17 % is not so small. This property attributes to the good electric conductivity of aluminum compared to Type 316 stainless steel. Then, the magnetic damping characteristics of aluminum is similar to those of copper of Fig. 4(b). Method 1 is valid for the materials with different properties and can be used for simplified magnetic damping analysis. Figure 5 also shows the results of method 2, which agree very well with those of the finite element tightly coupled analysis. Since the coupled eigenvalues are used in this method, the response with good accuracy is obtained even for the material with large magnetic damping effect or good electric conductivity.

4.4 Usability of these simplified analysis methods

Since many cases with combinations of structural geometry, thickness, magnetic field and material properties should be analyzed for design with the effect of magnetic damping, finite element coupled analysis takes much computation time. In both simplified analysis methods, a finite element eigenvalue analysis is needed once for each geometry of the structure to determine the proportional constants P_s, P_e and P_c, which lead to the coupling intensity parameter and the coupled eigenvalues using the single-degree-of-freedom coupled model. Although a finite element uncoupled analysis is required for each combination of material properties and thickness for method 1, the magnetically damped vibration is approximately obtained by modifying the result using the coupling intensity parameter for cases with different intensity of magnetic field. In method 2, the vibrations for all analysis cases are approximately obtained by using the dependence of the coupled eigenvalues on magnetic field, material properties and thickness. Therefore, the computation cost for the design analyses will be much saved using these simplified analysis methods.

5 Magnetic damping of a thin cylinder

These simplified analysis methods are applied to the magnetically damped vibration of a cylinder to verify the applicability of these methods.

5.1 Analysis model

A thin cylinder made of Type 316 stainless steel of radius 1 m and length 2 m shown in Fig. 6 is analyzed. It is supported rigidly along its bottom line and vibrated by the initial deflection of 1.3 mm along point A to B. Since the region of this problem is multiply connected, a lid with much lower Young's modulus, mass density and electric conductivity is attached on one end of the cylinder.

5.2 Numerical results

The simplified analysis methods are applied separately to vibration modes 3 and 6 shown in Fig. 7, then superposed. Table 4 summarizes C_{es} and the angular frequency difference of the dominant modes for each magnetic field. Figure 8 shows the deflection at point A of the cylinder. When B_x is 0.5 T, the result of method 1 agrees well with that of the finite element tightly coupled analysis with direct time integration method. When B_x is higher such as 1.5 T, the result of method 1 is slightly delayed in time as with the result of the plate because of the angular frequency difference summarized in Table 4. The results of method 2 are fitted to the finite element tightly coupled results as shown in Fig. 8. These simplified methods are applicable to the magnetic damping effect of the cylinder as well as that of the plate.

6 Conclusion

Two simplified analysis methods to approximately obtain dynamic response with the effect of magnetic damping have been proposed and applied to a plate and a thin cylinder in magnetic fields.

As for the method modifying the finite element uncoupled result using the coupling intensity parameter, although the results are slightly delayed in time compared to those of the finite element tightly coupled analysis as the magnetic damping effect increases, the results obtained from this method satisfactory predict the response of magnetically damped vibration. The results of the method using the solution and coupled eigenvalues of the single-degree-of-freedom coupled model agree very well with the finite element tightly coupled results because of the coupled eigenvalues. Since the vibration with magnetic damping can be evaluated using these methods without finite element coupled analysis, these simplified analysis methods will be practical and useful for the wide range of design analyses taking account of the magnetic damping effect.

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Table 1 Coupling intensity parameter and angular frequencies for different magnetic field

Magnetic field B_x [T]	C_{es}	$\sqrt{1-C_{es}^2}\,\omega_0$ [rad/s]	ω_s [rad/s]	Difference between $\sqrt{1-C_{es}^2}\,\omega_0$ and ω_s [%]
0.2	0.0374	67.0	67.4	0.704
0.5	0.241	65.0	68.7	5.03
0.7	0.498	58.1	67.8	14.3

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Table 2 Material properties for the analysis

Material	Young's modulus [Pa]	Density [kg/m^3]	Electric conductivity [S/m]
Type 316 SS	1.80×10^{11}	7.90×10^{3}	1.38×10^{6}
Al	6.90×10^{10}	2.71×10^{3}	3.30×10^{7}

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Table 3 Coupling intensity parameter and angular frequencies for different materials

Material	C_{es}	$\sqrt{1-C_{es}^2}\,\omega_0$ [rad/s]	ω_s [rad/s]	Difference between $\sqrt{1-C_{es}^2}\,\omega_0$ and ω_s [%]
Type 316 SS	0.00474	91.1	91.1	0.003
Al	0.320	91.3	97.3	6.17

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Table 4 Coupling intensity parameter and angular frequencies of each structural mode

Magnetic field B_x [T]		C_{es}	$\sqrt{1-C_{es}^2}\,\omega_0$ [rad/s]	ω_s [rad/s]	Difference between $\sqrt{1-C_{es}^2}\,\omega_0$ and ω_s [%]
Mode 3	0.5	0.0443	23.6	23.7	0.399
	1.5	0.413	21.5	22.8	5.76
Mode 6	0.5	0.0451	85.4	86.4	1.22
	1.5	0.446	76.5	94.0	18.6

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List of Figures

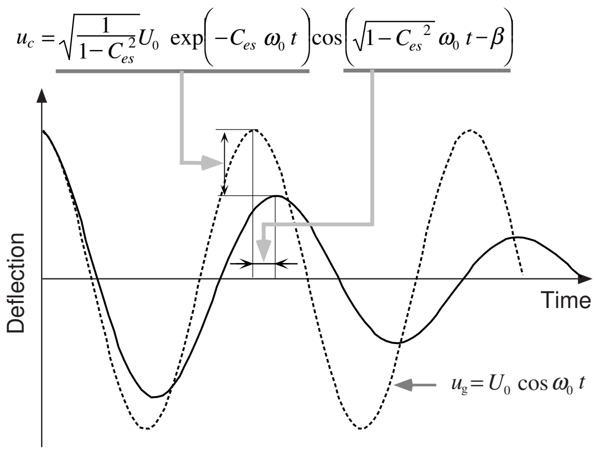

Figure 1

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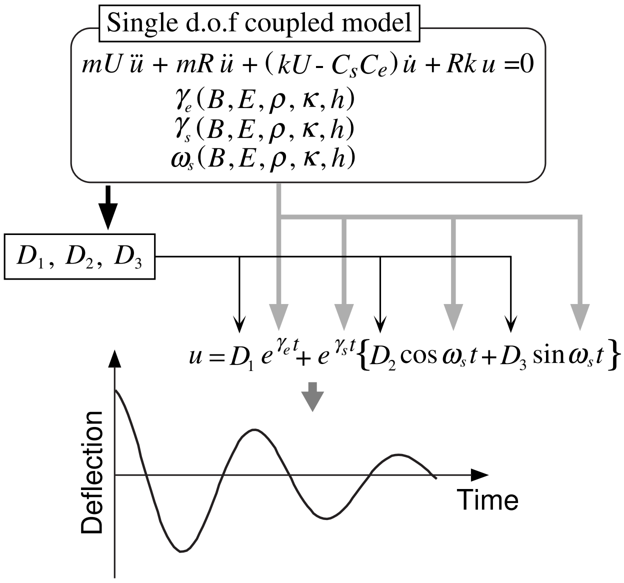

Figure 2

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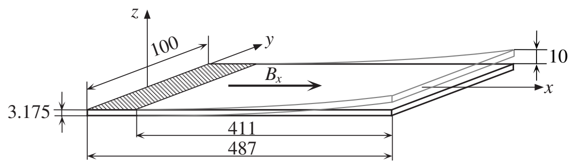

Young's modulus	:	1.10×10^{11}Pa
Density	:	8.91×10^{3}kg/m^3
Electric conductivity	:	5.81×10^{7}S/m

Figure 3

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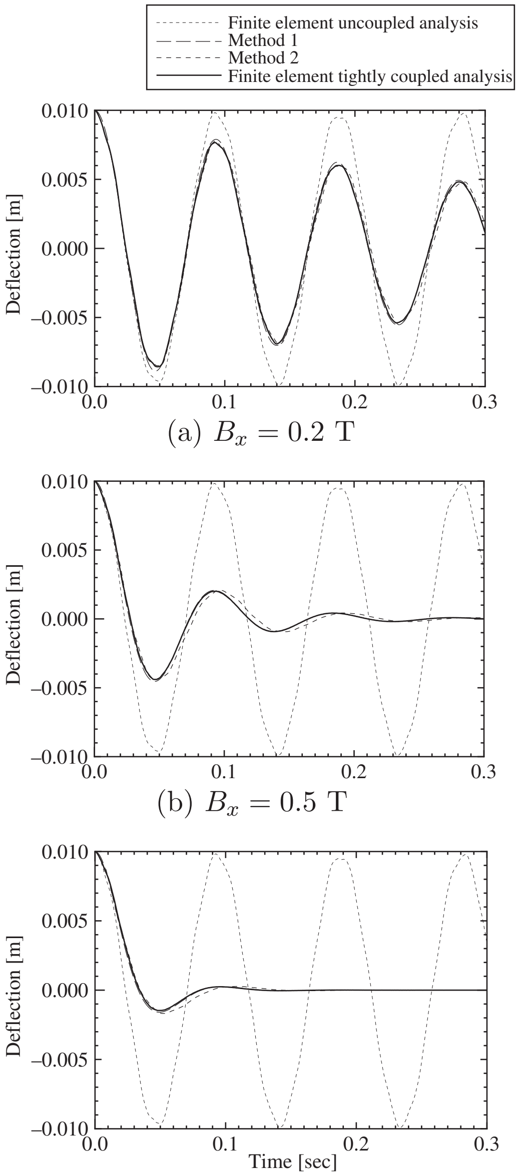

(a) $B_x = 0.2$ T

(b) $B_x = 0.5$ T

(c) $B_x = 0.7$ T

Figure 4

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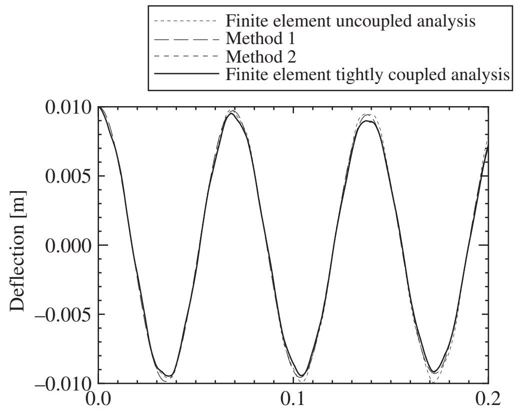
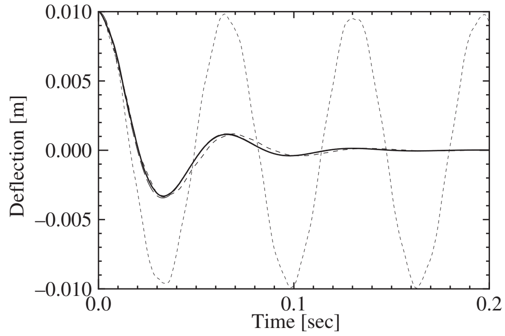


(a) Type 316 SS ($B_x = 0.5$ T)



(b) Al ($B_x = 0.5$ T)

Figure 5

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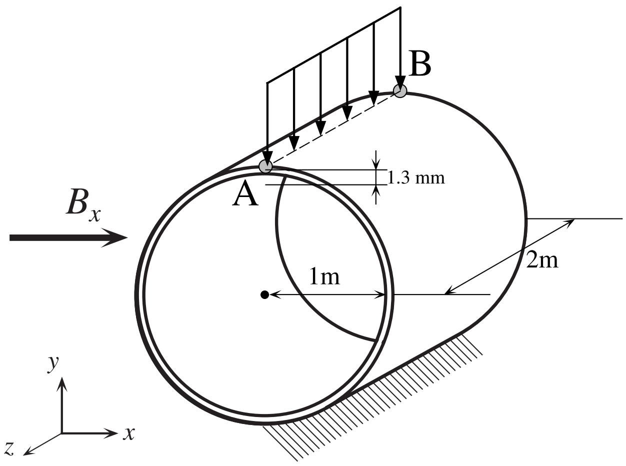

Figure 6

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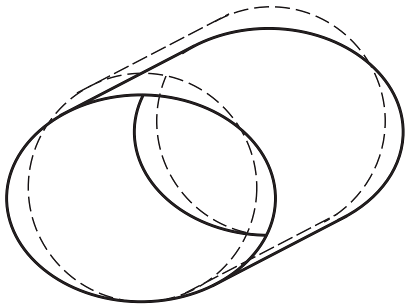
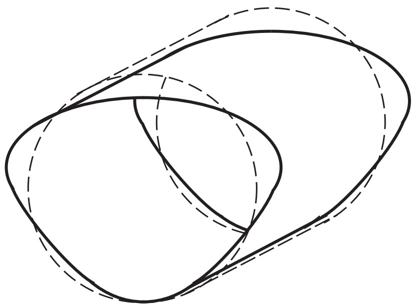


Vibration mode 3



Vibration mode 6

Figure 7

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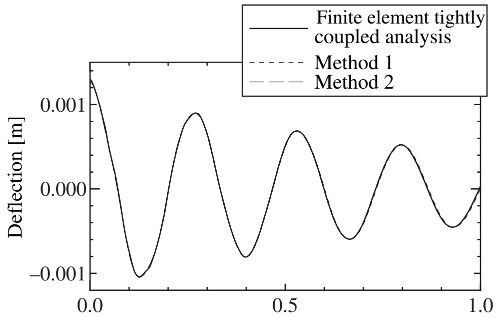
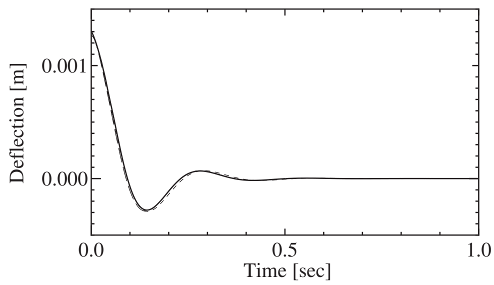


(a) $B_x = 0.5$ T



(b) $B_x = 1.5$ T

Figure 8

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