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# Agricultural monopolistic competitor and the Pigovian tax

A monopolistically competitive agricultural market structure has some features of competition and some features of monopoly. Monopolistic competition has the following attributes: (a) many sellers; (b) product differentiation; and (c) free entry. In the long-run equilibrium, price equals average total cost, and the agricultural firm earns zero economic profit. The aim of this paper is to construct a relatively simple chaotic long-run monopolistic competitor's agricultural output growth model that is capable of generating stable equilibria, cycles or chaos. A key hypothesis of this work is based on the idea that the coefficient $\pi = \frac{d+m}{(\alpha-1)b\left(1+\frac{1}{e}\right)}$ plays a crucial role in explaining local stability of the monopolistic competitor's agricultural output, where d is the coefficient of the marginal cost function of the agricultural monopolistic competitor; b is the coefficient of the inverse demand function; a is the coefficient of average cost growth; m is the Pigovian tax rate; and e is the coefficient of the price elasticity of demand.

**Keywords:** monopolistic competition, agriculture, long-run, equilibrium conditions, the Pigovian tax rate, chaos

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## Introduction

Monopolistic competition has characteristics of both competition and monopoly. Similar to competition, monopolistic competition has many firms, and free exit and entry. Similar to monopoly, the products are differentiated and each company faces a downward sloping demand curve. Monopolistic competition refers to a market situation with a relatively large number of sellers offering similar and differentiated products.

Food products are increasingly heterogeneous as firms are able to create and market branded products. As agricultural firms turn to new branded product development to defend market share, Boland *et al*. (2012) suggest that many of these industries arguably resemble monopolistically competitive industries. The subject of their study, prunes in the United States, is an example that is consistent with firms operating under monopolistic competition. There are several firms operating in the marketplace, there are no barriers to entry, prunes are sold as a successful brand, and demand curves are downward sloping.

According to Adam Smith (1776), the 'invisible hand' of the marketplace leads self-interested buyers and sellers in a market to maximise the total benefit that society can derive from a market. But market failures can still happen. When a transaction between a buyer and a seller directly affects a third party, the effect is called an externality. Namely, an externality refers to the uncompensated impact of one person's actions on the well-being of a bystander. It is a direct effect of the actions of one person or firm on the welfare of another person or firm, in a way that is not transmitted by market prices.

Externalities cause markets to be inefficient, and thus fail to maximise total surplus. In other words, negative externalities cause the socially optimal quantity in a market to be less than the equilibrium quantity. On the other hand, positive externalities cause the socially optimal quantity in a market to be greater than the equilibrium quantity.

In this theoretical framework, we can say that the quantity produced and consumed in the agricultural market equilibrium is efficient in the sense that it maximises the sum of producer and consumer surplus. However, if an agricultural firm contributes to air, land or water pollution (a negative externality), then the cost to society of producing agricultural products is larger than the cost to the producer. For each unit of agricultural output produced, the social cost includes the private costs of the producers plus the cost to those bystanders adversely affected by the pollution.

The government can internalise an externality by imposing a tax on the agricultural producer to reduce the equilibrium quantity to the socially desirable quantity. Internalising an externality involves altering incentives so that people take account of the external effects of their actions. When externalities are significant and private solutions are not found, government may attempt to solve the problem through: (a) command and control policies (these usually take the form of regulations that forbid or require certain behaviours); and (b) market-based policies (government uses Pigovian taxes and subsidies to correct the effects of a negative externality). In other words, public policies for externalities are: (a) regulation; (b) taxes and subsidies; (c) assign property rights; and (iv) pollution permits.

Linear analysis used in the theory of economic growth presumes an orderly periodicity that rarely occurs in an economy. In this sense, it is important to construct deterministic, nonlinear economic dynamic models that elucidate irregular, unpredictable economic behaviour. Chaos theory is used to prove that erratic and chaotic fluctuations can arise in completely deterministic models. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions can produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change. Thus chaos embodies three important principles: (a) extreme sensitivity to initial conditions; (b) cause and effect are not proportional; and (c) nonlinearity.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1994), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993) and Jablanovic (2011, 2012a, 2012b, 2012c), among many others. A number of nonlinear business cycle models use chaos theory to explain the complex motion of the economy.

The agricultural economics literature does not have any examples of the externalities analysis in an industry typified by monopolistic competition. The aim of this paper is to develop a theoretical framework of how externalities can influence long-run agricultural monopolistic competitor equilibrium. This is done by constructing a relatively simple chaotic long-run monopolistic competitor's agricultural output growth model that is capable of generating stable equilibria, cycles or chaos.

## The model

In the model of the monopolistically competitive agricultural firm take the inverse demand function:

$$P_t = a - bQ_t \tag{1}$$

where P is the monopolistic competitor's agricultural price; Q is the monopolistic competitor's agricultural output; and a and b are coefficients of the inverse demand function.

Marginal revenue (line MR in Figure 1) is:

$$MR_t = P_t\left[1 + \left(\frac{1}{e}\right)\right] \tag{2}$$

where MR is marginal revenue; P is the monopolistic competitor's agricultural price; and e is the coefficient of the price elasticity of demand.

Further, suppose the quadratic marginal cost function for the monopolistically competitive agricultural firm is:

$$MC_t = c + dQ_t + fQ_t^2 \tag{3}$$

where MC (curve MC in Figure 1) is marginal cost; Q is the monopolistic competitor's agricultural output; and c, d and f are coefficients of the quadratic marginal cost function.

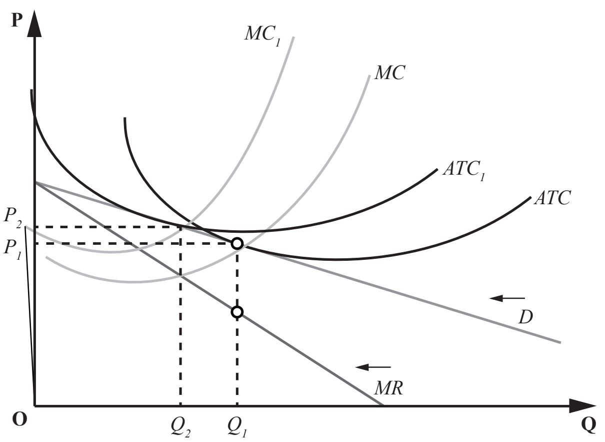


**Figure 1:** Long-run profit maximisation and new long-run equilibrium of a monopolistically competitive agricultural firm and the new marginal cost curve which includes the Pigovian tax.

Because of the externality, the cost to society of producing an agricultural product is larger than the cost to the agricultural producer. For each unit of agricultural output produced, the social cost includes the private costs of the agricultural producer plus the costs to those bystanders adversely affected by the water, land or air pollution. The marginal social costs take into account the external costs imposed on society by the producer. An agricultural producer would take the costs of pollution into account when deciding how much agricultural product to supply because the Pigovian tax now makes him/her pay for these external costs.

It is supposed that the Pigovian tax is:

$$T_t = mQ_t \tag{4}$$

where T is the Pigovian tax; Q is the agricultural output; and m is the Pigovian tax rate. In this sense, the marginal cost function for the agricultural monopolistic competitor is:

$$MC_t = c + (d + m)Q_t + fQ_t^2 \tag{5}$$

where MC (curve $MC_1$ in Figure 1) is the marginal cost; Q is the agricultural output; c, d and f are coefficients of the quadratic marginal private cost function; and m is the Pigovian tax rate.

The long-run equilibrium of agricultural monopolistically competitive industry generates two equilibrium conditions. Firstly, a monopolistic competitor maximises profit by producing the quantity at which marginal revenue equals marginal cost. Thus the profit-maximising condition is that:

$$MR_t = MC_t \tag{6}$$

In an agricultural monopolistically competitive market, price exceeds marginal cost because profit maximisation requires marginal revenue to equal marginal cost and because the downward-sloping demand curve makes marginal revenue less than the price. Equivalently, equation (7) expresses price directly as a mark-up over marginal cost, i.e.:

$$P_t = \frac{MC_t}{\left(1 + \frac{1}{e}\right)} \tag{7}$$

The second condition, price (P) equal to average cost (ATC):

$$P_t = ATC_t \tag{8}$$

means that each agricultural firm in the industry is earning only a normal profit. Economic profit is zero and there is no economic loss.

In accordance with (7) and (8) we obtain (curve ATC in Figure 1):

$$ATC_t = \frac{MC_t}{\left(1 + \frac{1}{e}\right)} \tag{9}$$

Further:

$$ATC_{t+1} = ATC_t + \Delta ATC \tag{10}$$

i.e.:

$$(1-\alpha)ATC_{t+1} = ATC_t \tag{11}$$

Substituting (7) and (8) in (11) gives (curve $ATC_1$ in Figure 1):

$$(1-\alpha)P_{t+1} = \frac{MC_t}{\left(1+\frac{1}{e}\right)} \tag{12}$$

Substituting (1) in (12) gives:

$$(1-\alpha)(a-bQ_{t+1}) = \frac{MC_t}{\left(1+\frac{1}{e}\right)} \tag{13}$$

Firstly, it is supposed that a = 0 and c =0. By substitution one derives:

$$Q_{t+1} = \frac{d+m}{b(\alpha-1)\left(1+\frac{1}{e}\right)}Q_t - \frac{f}{b(1-\alpha)\left(1+\frac{1}{e}\right)}Q_t^2 \tag{14}$$

Further, it is assumed that the long-run agricultural monopolistic competitor's output is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the long-run agricultural monopolistic competitor's output growth rate depends on the current size of the long-run monopolistic competitor's output, Q, relative to its maximal size in its time series $Q^m$. We introduce q as $q = Q/Q^m$. Thus q ranges between 0 and 1. Again we index q by t, i.e. write $q_t$ to refer to the size at time steps t = 0, 1, 2, 3, ... Now the growth rate of the long-run agricultural monopolistic competitor's output is measured as:

$$q_{t+1} = \frac{d+m}{b(\alpha-1)\left(1+\frac{1}{e}\right)}q_t - \frac{f}{b(1-\alpha)\left(1+\frac{1}{e}\right)}q_t^2 \tag{15}$$

This model given by equation (15) is called the logistic model. For most choices of α, b, d, f, m and e there is no explicit solution for (15). Namely, knowing α, b, d, f, m and e and measuring $q_0$ would not suffice to predict $q_t$ for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (15) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behaviour of $q_t$. This difference equation (15) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point $q_0$ the solution is highly sensitive to variations of the parameters α, b, d, f, m and e; secondly, given the parameters α, b, d, f, m and e the solution is highly sensitive to variations of the initial point $q_0$. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

## The logistic equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamic equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process (Figure 2) for the logistic equation:

$$z_{t+1} = \pi z_t(1-z_t), \pi \in [0,4], z_t \in [0,1] \tag{16}$$

is equivalent to the iteration of growth model (15) when we use the following identification:

$$z_t = \frac{(\alpha-1)f}{(1-\alpha)(d+m)}q_t \qquad \pi = \frac{d+m}{(\alpha-1)b\left(1+\frac{1}{e}\right)} \tag{17}$$

Using (15) and (17) we obtain:

$$z_{t+1} = \frac{(\alpha-1)f}{(1-\alpha)(d+m)}q_{t+1} =$$
$$= \frac{(\alpha-1)f}{(1-\alpha)(d+m)}\left[\frac{d+m}{(\alpha-1)b\left(1+\frac{1}{e}\right)}q_t - \frac{f}{(1-\alpha)b\left(1+\frac{1}{e}\right)}q_t^2\right] =$$
$$= \frac{f}{(1-\alpha)b\left(1+\frac{1}{e}\right)}q_t - \frac{(\alpha-1)f^2}{(1-\alpha)^2 b(d+m)\left(1+\frac{1}{e}\right)}q_t^2$$

On the other hand, using (15) and (16) we obtain:

$$z_{t+1} = \pi z_t(1-z_t) =$$
$$= \left[\frac{d+m}{(\alpha-1)b\left(1+\frac{1}{e}\right)}\right]\left[\frac{(\alpha-1)f}{(1-\alpha)(d+m)}\right]q_t\left\{1-\left[\frac{(\alpha-1)f}{(1-\alpha)(d+m)}\right]q_t\right\} =$$
$$= \frac{f}{(1-\alpha)b\left(1+\frac{1}{e}\right)}q_t - \frac{(\alpha-1)f^2}{(1-\alpha)^2 b(d+m)\left(1+\frac{1}{e}\right)}q_t^2$$

Thus we have that iterating

$$q_{t+1} = \frac{d+m}{b(\alpha-1)\left(1+\frac{1}{e}\right)}q_t - \frac{f}{b(1-\alpha)\left(1+\frac{1}{e}\right)}q_t^2 \text{ is really}$$

the same as iterating $z_{t+1} = \pi z_t(1-z_t)$ using

$$z_t = \frac{(\alpha-1)f}{(1-\alpha)(d+m)}q_t \text{ and } \pi = \frac{d+m}{(\alpha-1)b\left(1+\frac{1}{e}\right)}.$$

It is important because the dynamic properties of the logistic equation (16) have been widely analysed (Li and Yorke, 1975; May, 1976).

It is obtained that:

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
(ii) For $1 < \pi < 3.57$ there exist fixed points the number of which depends on π;
(iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

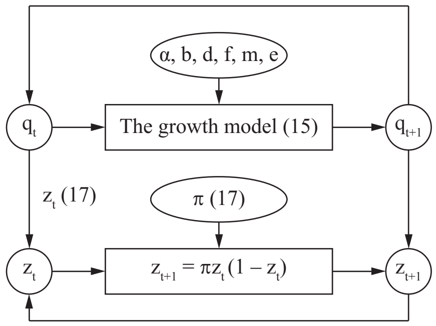


**Figure 2:** Two quadratic iterators running in phase are tightly coupled by the transformations indicated.

(v) For $3 < \pi < 4$ all solutions will continuously fluctuate;

(vi) For $3.57 < \pi < 4$ the solution becomes 'chaotic' which means that there exist a totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of $z_t$ fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

# Conclusion

This paper suggests the use of the simple chaotic model of a profit maximising agricultural monopolistic competitor in predicting the long-run fluctuations of the agricultural monopolistic competitor's output. The model (15) has to rely on specified parameters α, b, d, f, m and e, and an initial value of the long-run monopolistic competitor's output, $q_0$. But even slight deviations from the values of these parameters and the initial value of the long-run agricultural monopolistic competitor's output show the difficulty of predicting a long-term behaviour of the long-run agricultural monopolistic competitor's output, $q_0$. A key hypothesis of this work is based on the idea that the coefficient $\pi = \frac{d+m}{(\alpha-1)b\left(1+\frac{1}{e}\right)}$ plays a crucial role in explaining local stability of the long-run agricultural monopolistic competitor's output where d is the coefficient of the marginal cost function of the agricultural monopolistic competitor; b is the coefficient of the inverse demand function; α is the growth coefficient of the average cost, m is the Pigovian tax rate and e is the coefficient of the price elasticity of demand. The quadratic form of the marginal cost function of the agricultural monopolistic competitor is an important ingredient of the presented chaotic long-run monopolistic competitor's output growth model (15).

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