LOCATION ANALYSIS - POSSIBILITIES OF USE IN PUBLIC ADMINISTRATION

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Abstrakt: *V článku jsou stručně popsána teoretická východiska lokační teorie a možnosti využití v oblasti veřejné správy jako je navrhování sítí a lokace různých zařízení v geografickém prostoru regionů.*

Abstract: *The paper under consideration describes key theoretical issues of continuous/discrete location theory and possibilities of applications in the area of public administration activities such as networks design and location of different facilities in geographical area of regions.*

Key words: *facility location, continuous/discrete location, public administration, location analysis*

1 Introduction

Location analysis is relatively new branch of science belonging into the group of operational research disciplines like Theory of Graphs, Mathematical Programming, Theory of Games, etc. For successful application of theoretical issues in practice the team of research workers requires effective cooperation with other specialists like economists, geographers, informatics, etc. There is a plenty of practical possibilities of location analysis application "in real life". Ones of the most amazing and useful are applications in public administration. We could list long inventory of illustrative cases from the different branches of public affaires like determination of number and location of hospitals or first aid stations, locations of offices of regional/state level, location of power stations, stocks, warehouses, logistics centres, etc. All of practical illustrative cases differ as to objective function, conditions, constraints and other relevant features. At the same time we can find a lot of common features that allows do classification of location tasks. In general the location analysis problem can be formulated as follows:

Let us have determined geographical area (population centre, micro region, region, etc.) There is a need in this area to locate k devices (markets, warehouses, hospitals, offices, automated teller machines, etc.) so that the objective function would be minimizes/maximized according to the objective function construction. In next paragraph we mention some of the historical roots and issues of location analysis.

2 Historical issues of optimal location

The roots of location analysis extend deep to the history. One of the former scientists that can be assumed to be the inventor of location task was the great French mathematician Pierre Fermat. He solved following task: In the plane of any triangle ABC (see *Fig. 1*) find the point (in the picture labeled F) minimizing the sum of line segments $AF+BF+CF$.

The problem was proposed by Fermat to Torricelli who solved it geometrically. Let $A'BC$ is the outside equilateral triangle on side BC, $AB'C$ is the outside equilateral triangle on side CA and ABC' is the outside equilateral triangle on side AB. The lines AA', BB', CC' meet in the point that is nowadays known as Fermat point. This point is said to be the first triangle center discovered after ancient Greek times. Torricelli proved that the Fermat point is the solution if each angle of triangle ABC is less than 120°. Sometimes, F is called the Fermat-Torricelli point.

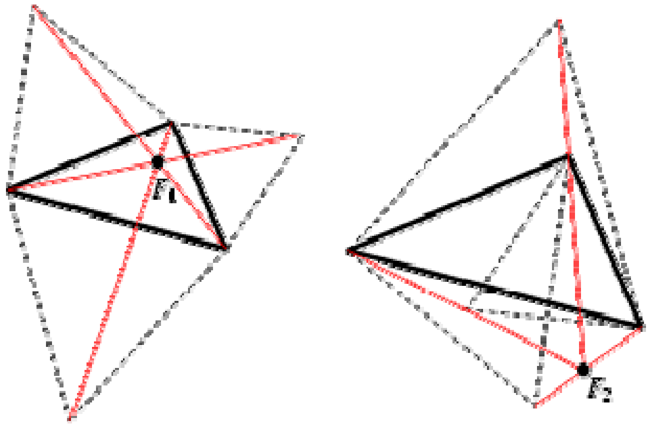

Figure 1

The Fermat point is also known as the 1st isogonic centre, the Greek word *iso* means *equal* and *gon* meaning *angle*. This is because the angles *BFC*, *CFA*, *AFB* are all equal (120°). There are two isogonic centres, the 2nd isogonic centre is obtained using the other three equilateral triangles on the sides of triangle *ABC* pointing inwards. Torricelli´s solution was published by his pupil Viviani in 1659. Torricelli also proved that if all of the triangle angles are less than 120° ($2\pi/3$), then the first Fermat point is in the triangle interior point from which each side subtends an angle of 120°. If one angle equals or is greater than 120°, then Fermat point coincides with the offending angle.

The problem seems to be very easy and it can be expressed in general form for more than three points as follow:
Let there are given n distinct points P_i in the plane with coordinates (x_i, y_i) for $1 \le i \le n$ and a point P with coordinates (x,y) . We have to minimize the objective function:

$$f(x,y) = \sum_{i=1}^{n} \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

The task can be also expressed in another way like: Given n points, find the line segments connecting all points with point P with the shortest possible total length of segments[1]. For four points, P is the intersection of the two diagonals, but the required minimum segments are not necessarily these diagonals. The problem is often called Steiner´s Segment Problem. A modified version of the problem is, given two points, to find the segments with the shortest total length connecting the points such that each branch point may be connected to only three segments. Even if the problem looks simple, in general there is no explicit solution to this version of the problem [3].

3 Steiner and Fermat-Weber problems

The above mentioned problems can be find more attractive and practical when considering the objective function for example as the cost of oil or natural gas pipelines (per unit) which have to connect the stock point/points with places of consume. There already exist solutions of variety different types of location problems under a range of metrics and variable cost. An important problem for transportation networks is that generalizes Steiner and Fermat – Weber problem. It consists in finding a point $P(x,y)$ lying intermediate to the

[1] Segments need not necessarily be straight from one point to another.

fixed vertices/geographical points $A_1, A_2, ..., A_i, ..., A_n$ such that the total network length is minimized, i.e. such that the function:

$$f(x,y) = \sum_{i=1}^{n} \left(\sqrt{(x-x_i)^2 + (y-y_i)^2} \right) \cdot w_i = \sum_{i=1}^{n} w_i \cdot d_i$$

is minimised, where (x_i, y_i) are the coordinates of the vertices/points A_i, w_i is a positive weight associated with the j^{th} vertex [6]. The point *P*, satisfying this equation for *n=3* is known as Steiner point. If all the weights are equal the Steiner point lies inside or on the triangle A_1, A_2, A_3. If all the vertices weight are equal and one of the triangle angles equals 120° (or more), the Steiner point lies at appropriate vertex. If all the weights of vertices are equal the Steiner point coincides with the Fermat-Torricelli point.

4 Mathematical model of location problem

In transportation practise the location problems can be denoted by simple mathematical model belonging to the area of mathematical programming. The most important models are those called UFLP – Uncapacitaded Facility Location Problem and CFLP – Capacitated Facility Location Problem. Models include three obligatory parts that are: criterion function, constraint conditions and non-negativity conditions:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} f_i y_i$$

$$x_{ij} \geq 0 \quad i = 1, \dots, m; \ j = 1, \dots, n$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j = 1, \dots, n$$

$$x_{ij} \leq y_i \quad i = 1, \dots, m; \ j = 1, \dots, n$$

$$y_i \in \{0, 1\} \quad i = 1, \dots, m$$

$$(condition \ \ for \ \ CFLP)$$

$$\sum_{j=1}^{n} d_j x_{ij} \leq s_i y_i \quad i = 1, \dots, m$$

Where

$J = \{1,2,...,n\}; \ j = 1,...,n$ is a set of requirements for service,

d_j = total amount of demand of j^{th} customer $(d_j \geq 0)$,

$I = \{1,2,...,m\}; \ i = 1,...,m$ is a set of service centres locations,

$f_i : (f_i \geq 0)$ are the fixed costs for installation of service centre in vertex v_i,

$c_{ij} =$ cost of satisfying demand $\underline{d_j}$ of customer *j* by device installed in v_i,

$x_{ij} =$ part of demand of customer $j - d_j$ satisfied by device in v_i,

$s_i =$ capacity of device located in v_i,

$y_i =$ binary variable denoting installation of device in v_i ($y_i = 1$), in opposite case ($y_i = 0$)

5 Applications

In this chapter we mention three applications of location analysis in practice.

5.1 Q-REC

In the frame of international research program COPERNICUS named Q-REC (Quality Recycling) 4 involved countries (Czech Republic, Germany, Ireland, Poland) tried to solve problem of end of life/crash cars.

5.1.1 Background

Worn-out and unusable trash automobiles, trucks, and miscellaneous other powered vehicles, henceforth called „Junk Cars“, have been collecting throughout the Czech Republic for some years, becoming both a public nuisance and an eyesore. It is appropriate to consider organized methods to control, if not outright eliminate, this situation.

5.1.2 Solution

An appealing possibility is the establishment, at suitable locations, of facilities devoted to the dismantling of Junk Cars and preparation of their salvageable components and materials for recycling; this could include salable used parts as well as raw materials such as steel, copper, glass, and rubber. It might be expected that in due course of time a profit/centered industry would develop independently to address these ends. Even so, a case can be advanced for the formal study of this problem, with the objective of outlining a system with less overall grief and total expense ... than might be expected if the job is left to market forces alone.

5.1.3 Elements

The overall Junk Car problem is perceived to consist of the following elements.

a. Identify the location for potential recycling facilities, henceforth called Recycling Centers. This will involve economic, engineering, and environmental impact studies of the possible locations, as well as the differing setup costs which may be expected to be involved.
b. Asses the quantity of Junk Cars in inventory at all Junk Car accumulation sites ... both large and small ... throughout the country, as well as econometric assessment of the accumulation-rate of Junk Cars into the future. In addition, a determination should be made of the unit cost of transporting Junk Cars from the various sites to each of the proposed recycling centres. This determination (in naive form) could involve no more than a table of the distances involved; compounding such a table could range from a practical ’’common sense“ approach to the solution of a formal mathematical problem.
c. Determination of the ’’production horizon “ for the recycling centres. This will necessitate the specification of a target date when the recycling process should reach statistical equilibrium, and will (of necessity) include a schedule for the processing of Junk Cars accumulated to date. (Mathematical models developed for epidemiological purposes may be helpful to this latter and.)
d. Determine which of the potential sites for the construction of recycling centres should (in fact) be established, and develop a schedule for the construction sequence. The first problem is solvable via Mathematical Programming Techniques, while the second could involve similar methodology, together with addressing the financial and legalistic issues which attend all construction tasks.

5.1.4 Issues

The project from the point of view the location analysis issued into the methodology of determination of number recycling centres, its location and allocation task solution (that means assignment for every source point of crash/junk car the recycling centre where it will be dismantled/ recycled). The methodology was supported by automated decision making support tool in form of programme in C++. Second issue was the solution of logistics of spare parts, recycled materials and not reusable materials distribution to the destination treatment plants/waste dumps.

5.2 ATM-Automated Teller Machines

In 2002 the location/allocation problem of the Firm MUZO, a.s. was proposed to the Department of Logistics and Technology Processes of the Transport Faculty of Czech Technical University in Prague for solution. The problem was raised by enormous increase of ATM in Czech Republic. The firm MUZO, a.s, being one of the two firms dealing in the field of installation and servicing ATM, had problems with deficiency of service operators´ capacity. On the other side the performance of service vehicles measured in kilometres, began increase rapidly. So the management of the firm decided to enlarge the number of service centres. The project issued into the programme that enables accordingly to the demand of customers of ATM determine the appropriate number of centres and to assign all of the ATM location to appropriate centre so that the performance would be optimal and the time of response would be in determine limits.

6 Conclusion

Large interest of practise for location problems solutions by means of information techniques gives raise to develop new effective methods and algorithms for different problems in all branches of human activities like transportation, urban planning, public administration, defence, social affaires, environment, etc. Great attractiveness of the problematique for scientists, research workers, pedagogical workers and students as well caused that at the Jan Perner Faculty of Transport at the Department of Informatics, sub department of Quantitative methods in Transport was created a group that follows the need of practice of location problems solutions and develops friendly use software tools enabling incorporate into solution all the newest methods, techniques and technologies like telematics, logistics, information systems a applied mathematics.

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