

# TRANSIENT ANALYSES OF SYNCHRONOUS MOTOR TRAINS

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## ABSTRACT

This paper discusses the philosophy of optimum location of torsional critical speeds which will be excited by the transient startup, as well as methods of adjusting the resonant frequencies. Pulsating torque, average torque, and acceleration rate during startup are considered in choosing the best frequency range for the resonances. Both steady-state and transient torsional excitation are discussed along with the applicable stress criterion. A method to determine the allowable number of startups is presented involving cumulative fatigue considerations which apply to the transient torsional stresses. The severity of backlash in gears and geared couplings is discussed and methods for calculating the response of the system to instantaneous negative torque are presented. The proper choice of couplings can change mode shapes, reduce backlash severity, and add damping which can reduce the detrimental effects to the torsional system. The proper specifications of couplings and gears in the design stage and the proper definition of service conditions greatly aid the gear and coupling manufacturers and can result in a system which will operate reliably, even under backlash conditions.

## INTRODUCTION

The philosophy of locating the torsional critical speeds with respect to the transient excitation of a synchronous motor driven system is the most important consideration in designing a reliable system. Several items must be examined: When does the dynamic torque reach its maximum during the startup? How does the average torque vary? Should the critical speed be located early in the startup or later when the acceleration of startup is at a maximum? Answers to these questions must be obtained to develop the philosophy for optimum location of critical speeds for transient startup.

The excitation from a synchronous motor during startup is caused by the electrical "slipping" of the rotor with respect to the line frequency. The slipping effect produces a dynamic exciting torque that initially begins at 120 Hz when the motor starts and reduces to 0 Hz when the motor reaches its syn-

chronous speed. During the startup, all of the torsional natural frequencies below 120 Hz can be excited. Obviously, if the system can be designed so that no critical speeds occur below 120 Hz, then the transient startup provides no problem. However, in large systems with massive motors, compressors, pumps, or generators, several critical speeds generally occur below 120 Hz.

The location of each natural frequency, the shape of the torque-speed curves, and the acceleration of the system will determine the amount of torsional stress which occurs in the shafts during startup. The stresses in the system should be compared to applicable endurance limits to determine the fatigue life and reliability of the system to both transient and continuous operation.

Sources of continuous excitation can also cause torsional failures and must be considered as seriously as the transient excitation. Sufficient margins must be maintained between the torsional resonances and sources of continuous excitation such as operating speed, its multiples, or other energy at discrete frequencies in the system. In many cases, couplings can be chosen to adjust the system torsional response and minimize torsional vibrations and stresses to acceptable levels.

### STEADY STATE TORSIONAL ANALYSIS

The steady state torsional vibration response of rotating machinery must be determined when designing an equipment train. Accurate response prediction requires analysis techniques which include consideration of all forcing functions in the system in addition to the mass-elastic properties of the shafts and couplings. The first step in the solution is to calculate the torsional natural frequencies. Matrix methods, utilizing an eigenvalue solution in conjunction with normalized coordinates, are the most practical and accurate method for calculating both the torsional resonant frequencies and the forced vibration response. [1] With this type of analysis, stress levels can be calculated and compared to an applicable stress criterion.

An advantage of an eigenvector-eigenvalue matrix solution is that all the possible modes of vibration are calculated. To calculate the torsional resonant frequencies of a system, a mathematical model is synthesized which should respond in the same manner as the actual system. All of the elastic, mass and damping properties of the system are required to assemble this mathematical model. The required elastic properties and the mass inertia can be calculated, measured or obtained from the manufacturer of the specific elements.

A steady state torsional response includes the dynamic torques and the interaction of the system while at steady operating conditions. The exciting torques which must be considered are at frequencies which include shaft operating speeds, line frequency and their multiples. One of the primary functions of the torsional analysis is to adjust the natural frequencies so that they are not within a 20 percent margin of a possible excitation frequency. Since the torsional natural frequencies are a function of the mass and stiffness, adjustments in the stiffness or mass of an element can usually shift the resonant frequencies as required.

For best reliability all torsional natural frequencies should have a sufficient margin between any exciting energy source and a torsional natural frequency to minimize the response. If the natural frequency is determined by actual measurement, 10 percent should be adequate, however, if the natural frequencies are determined by calculation an additional percentage margin should be added to allow for calculation inaccuracies which can easily arise in the determination of shaft

stiffness, mass inertia ( $WR^2$ ), degree of coupling-to-shaft fit, etc. A 20 percent margin is recommended when the torsional natural frequencies are determined solely by an analytical method. If a 20 percent margin cannot be attained, then a forced vibration analysis of the condition should be made assuming a conservative forcing function at the resonant frequency.

Once the system has been modeled and the natural frequencies have been determined, then the forcing functions should be applied. The forcing function represents a dynamic torque applied at a location in the system which is likely to generate torque variations. Commonly, a dynamic torque of approximately 1 percent zero-peak of the transmitted torque is assumed for a geared system. This torque is applied at the motor, gear box, and the driven equipment to determine which location will cause the largest deflections. Since the relative deflections between the masses produce a twist in the shaft resulting in stresses, the peak-to-peak stresses are calculated for the system using this applied dynamic torque.

The dynamic torques should then be compared to an applicable endurance criterion. Mil Standard 167 is commonly used which defines a dynamic endurance stress as a function of the ultimate tensile strength.

$$\text{allowable torsional stress} = \frac{\text{ultimate tensile strength}}{25}$$

Stress concentration factors occur at keyways, splines, shrink fits, etc., and should be used in calculating the expected dynamic stress levels. [2]

### TRANSIENT TORSIONAL ANALYSES

After the steady state analysis has been made and the system torsional resonances are adjusted, the transient analysis can be made to evaluate the stresses for the transient startup. The transient analysis evaluates the conditions on startup which are continually changing because of the time varying torques from the motor and load and acceleration of the system. When a synchronous motor starts, an excitation which varies from 120 Hz to 0 Hz is imposed upon the torsional system. During this period of time, the torsional system will be excited at several of its resonant frequencies. The response amplitude and shaft stresses depend upon the resonant frequency, the average and dynamic torque when the system passes through each resonant frequency, the damping in the system, and the system load torques. The transient response is also affected by the acceleration time of the motor. For slow motor startups, the system will stay at a resonant frequency for a longer period of time allowing stresses to be amplified. Rapid acceleration through the resonances will minimize the amplification at resonant frequencies.

Synchronous motors develop a strong oscillating torque during starting because of slippage between the rotor and stator fields. The frequency of the dynamic torque (pulsating torque) is equal to two times the slip frequency which varies from twice line frequency initially, to zero when the motor is synchronized.

$$f(\text{cps}) = 120 \left( 1 - \frac{\text{motor rpm}}{\text{synchronous rpm}} \right)$$

Thus, the excitation frequency varies from 120 cps at standstill to zero at synchronous speed. Any torsional natural frequencies of the system within this frequency range (0 to 120 Hz) will be excited during the starting cycle as the pulsating torque passes through the critical frequency range.

A common analysis technique used for the transient analysis is a numerical integration of the equation of motion in the form of a time-step solution. Beginning with the initial conditions of the motor at rest, the torque as determined from the startup torque curves should be applied. The inertia of the system then responds to this unbalanced torque and the system begins rotating. At each time step, the instantaneous values of pulsating torque, average torque, rotational speed, inertia, and damping are recalculated and the motor startup is simulated mathematically.

The time step for the numerical integration must be chosen to simulate the frequencies of interest. The basic equation of motion at time  $t_{n+1}$  for the torsional system is:

$$[J] \{\ddot{\Theta}_{n+1}\} + [C] \{\dot{\Theta}_{n+1}\} + [K] \{\Theta_{n+1}\} = \{T_{n+1}\} \quad (1)$$

where:  $J$  = mass inertia matrix

$C$  = damping matrix

$K$  = stiffness matrix

$\ddot{\Theta}$  = angular acceleration matrix (column)

$\dot{\Theta}$  = angular velocity matrix (column)

$\Theta$  = angular displacement matrix (column)

$T$  = torque matrix (column)

$n$  = counter for time step

$t_{n+1} = t_n + \Delta t$  let  $\Delta t = h$

The numerical integration may be accomplished with the average acceleration method (at each time interval) by using the following relations for angular velocity and displacement:

$$\{\Theta_{n+1}\} = \{\Theta_n\} + h \{\dot{\Theta}_n\} + \frac{h^2}{4} \{\ddot{\Theta}_n\} + \frac{h^2}{4} \{\ddot{\Theta}_{n+1}\} \quad (2)$$

$$\{\dot{\Theta}_{n+1}\} = \{\dot{\Theta}_n\} + \frac{h}{2} \{\ddot{\Theta}_n\} + \frac{h}{2} \{\ddot{\Theta}_{n+1}\} \quad (3)$$

Substituting the recursion equations 2 and 3 into equation 1 and solving for  $\ddot{\Theta}_{n+1}$  obtains:

$$\{\ddot{\Theta}_{n+1}\} = [D]^{-1} \left\{ \{T_{n+1}\} - [C] \{\dot{\Theta}_n\} - \frac{h}{2} [C] \{\dot{\Theta}_n\} - [K] \{\Theta_n\} - h [K] \{\dot{\Theta}_n\} - \frac{h^2}{4} [K] \{\ddot{\Theta}_n\} \right\} \quad (4)$$

where  $[D] = [J] + \frac{h}{2} [C] + \frac{h^2}{4} [K]$

Equations 2, 3 and 4 define the values of angular displacement, velocity and acceleration for the time  $t_{n+1}$ . The torque  $T_{n+1}$  is obtained from the motor startup torque characteristics [3], which should be available from the motor manufacturer.

The transient torsional analysis program developed and used by SwRI is based on the technique of time-step numerical integration. [4] As is shown in Figure 1, it provides a very convenient and practical graphic presentation of predicted transient torque response of the system. Response to the natural frequencies, initial shock of startup, and motor synchronization are noted. Of particular interest for the gear and geared couplings is the possible negative torque which indicates that the gear teeth can be momentarily unloaded. The negative torque means that for an instant, the system is oscillating such that the dynamic torque at the gear, because of inertia loading, exceeds the average transmitted torque and the teeth actually separate and then recontact with a shock loading. This shock can damage the gear teeth or couplings depending upon the amount of torque reversal and their design characteristics.

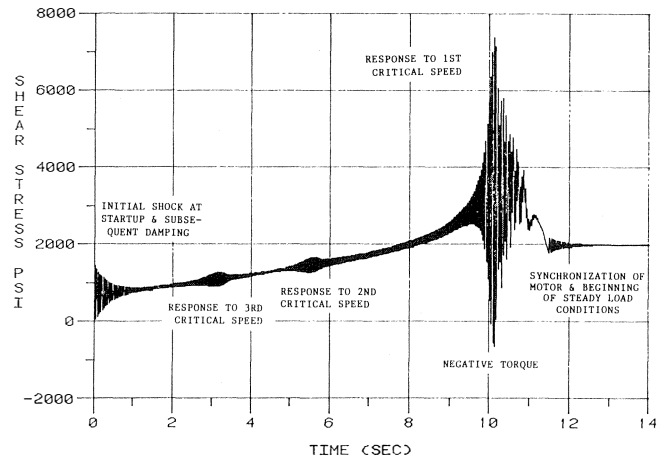


Figure 1. Shaft Transient Response.

### PHILOSOPHY OF RESONANT RESPONSE LOCATION

Since all of the torsional natural frequencies below 120 Hz (two times line frequency) can be excited during a transient startup of a synchronous motor, the question arises, if there is an optimum location for these resonances above the normal consideration of maintaining a sufficient margin from continuous operating frequencies for steady state torsional stress requirements. This question cannot be answered in a general sense; the specifics of each case must be evaluated.

The factors which must be considered in evaluating the optimum location of the transient resonances include the shape of the torque-speed curve, the initial starting torque of the motor, the acceleration rate at the torsional natural frequency and the motor startup time. Although the startup time is closely related to the acceleration, the acceleration rate is more important since it varies throughout the complete startup cycle.

The initial shock to the torsional system is produced by the motor starting torque. The sudden application of the startup torque shocks the shafting system and can cause many of its modes to be excited. The modes with lowest dynamic stiffness will be most likely to respond. A resonance close to 120 Hz (two times line frequency) would be most undesirable since just at the initial shock the slip frequency is equal to twice line frequency. The initial shock could excite the critical speed at 120 Hz and the slip frequency would continue to excite it for a few additional revolutions, making that mode quite responsive.

The voltage curve is directly related to the torque which includes both a steady state torque component and a pulsating or dynamic torque component, as can be seen in Figure 2. The dynamic torque component is of primary importance since it produces the energy which would cause the vibrational response of the torsional natural frequency. The torsional natural frequencies should be located near frequencies having low dynamic torque components. Locating a torsional natural frequency in a region of high average torque during startup can minimize the possibility of a response causing negative torque or backlash. Negative torque is particularly detrimental in geared systems.

A high acceleration rate through a torsional natural frequency is desirable since it will reduce the response magnification of the shafting system. Maximum response would be obtained if the system were running constantly at the torsional

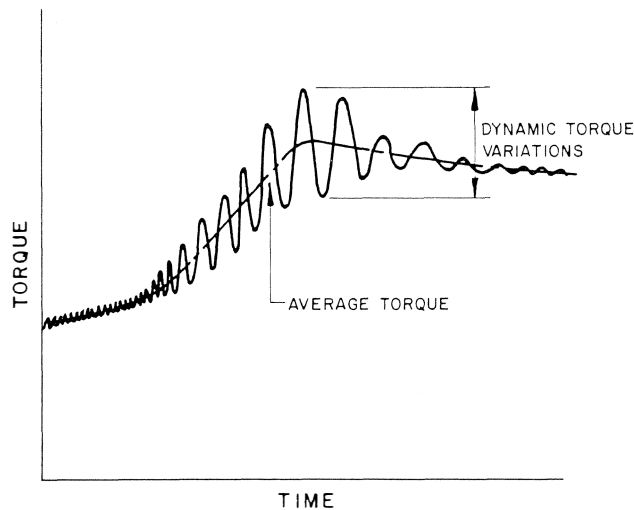


Figure 2. Motor Startup Torque.

natural frequency; the faster the motor passes through that frequency, the less opportunity the shafting system has to respond. [5] The proper phase angle ( $90^\circ$ ) must occur for the system to respond to its maximum. High acceleration rates minimize the time that this optimum phase angle occurs.

Obviously, if high acceleration rates are desirable, this would indicate a short startup time is also desirable. All of these factors must be considered together to obtain the optimum location of the torsional natural frequencies for transient conditions. Fast startup time and high accelerations generally require high initial torques which would introduce shocks into the system and be undesirable. Obviously a balance between fast acceleration rates and initial torque must be obtained to find the optimum combination which will not severely limit the number of allowable startups.

### STRESS CRITERION

A valid stress-strain curve for the particular metal used in the shafts must be available to determine the endurance limit of the shaft material. One good source of this is the Metals Handbook. [6] Since most of the available data on various steels has been obtained from bending reversal tests the bending endurance limit must be reduced by two to obtain the torsional endurance limit, based upon the maximum shear failure theory. Stress concentration factors for keyways and other stress risers must be considered.

High transient stresses (above the endurance limit) cause premature failure of the shafts and consequently limit the allowed number of startups before a failure would occur. For this reason the transient stresses must be calculated and compared to the endurance limit stress. It is not necessary that the transient stresses be less than the endurance stress because the stress levels do not occur continuously. The transient stresses, however, must be sufficiently low to allow an acceptable number of starts. If the transient stresses exceed the endurance limit, then the cumulative fatigue concept should be applied to sum the number of cycles in which the stresses exceed the endurance limit to determine how many starts can be allowed for the system.

The cumulative fatigue theory basically defines how many allowable cycles of a certain level of stress can be tolerated before a failure would occur. This is based upon a plot of stress

versus number of cycles (S-N curve) which defines the stress at which a failure should occur. The S-N curve should be based upon actual tests of specimens of a particular type of metal and it defines the stress levels at which failures have occurred in these test specimens. These S-N curves are available for most types of shafting materials. Using this curve, the allowed number of cycles for a particular stress can be determined, as shown in Figure 3. If a fraction of these cycles has been used in the transient startup, then the total startups of the system before a shaft failure would be expected can be defined. Since the stress levels vary both in amplitude and frequency, a more complex calculation must be made to determine the fraction of the total fatigue which has occurred. The stress levels must be analyzed for each cycle to determine the percentage of cumulative fatigue and then the allowable number of startups can be determined, as shown in Figure 3.

The transient levels are a function of the location of the critical speeds during the transient startup which reflect a dependence upon the acceleration rate, startup time, initial starting torque, etc. If the number of allowable startups is insufficient for the proposed application of the system then it may be necessary to readjust the location of the torsional natural frequencies, considering the factors discussed under the philosophy of locating the criticals, to obtain a reduction in the transient stresses.

$$\sigma_i = \text{STRESS AT } i^{\text{th}} \text{ CYCLE}$$

$$\sigma_e = \text{ENDURANCE STRESS}$$

$$N_i = \text{NO. OF CYCLES TO FAILURE AT } \sigma_i$$

$$n_i = \text{ACTUAL NO. OF CYCLES AT } \sigma_i$$

$$S = \text{NO. OF CYCLES TO SYNCHRONOUS SPEED}$$

$$q = \sum_{i=1}^S \frac{n_i}{N_i}$$

$$\frac{1}{q} = \text{ALLOWABLE NUMBER OF START UPS}$$

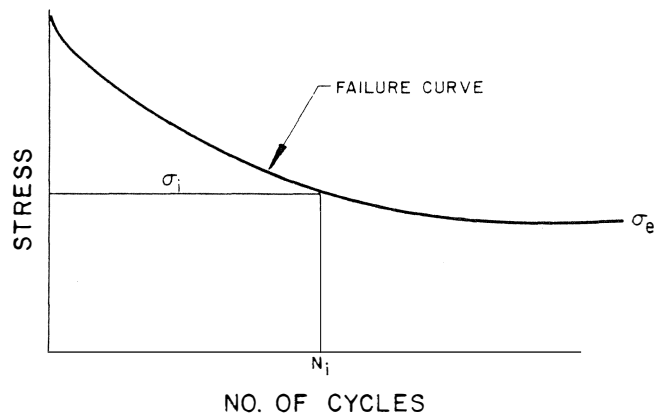


Figure 3. Cumulative Fatigue Method for Allowable Number of Startups.

**TORQUE REVERSALS**

If the dynamic torques exceed the average torque then a negative torque will occur for an instant which will unload the teeth in a gear or geared coupling. This backlash condition (tooth unloading) can result in the teeth separating and then recontacting with a shock. It is this mechanism which causes tooth surface damage (spalling) or early tooth failure. This condition should be avoided if at all possible; however, it can be tolerated to a minimal degree if the proper specifications regarding accuracy of tooth profile and hardening techniques are included in the gear design. Informing the manufacturer of the conditions expected during the transient startup as early as possible is important if torque reversals are expected.

It is not uncommon for torque reversals to occur during synchronous motor startups. Most gear manufacturers view this as a probable occurrence and control the backlash allowance, as shown in Figure 4a, and set limits upon the amount of allowable torque reversal. The coupling manufacturers apply a torque overload factor and specify special manufacturing procedures to build couplings for a synchronous motor driven system to allow for the torque reversals.

**ANALYSIS OF TORQUE REVERSALS**

The evaluation of tooth loading for the occurrence of torque reversals is a nonlinear effect because the torsional stiffness goes to zero upon tooth separation. An analysis of this dynamic mechanism must include careful evaluation of the inertial effects of the complete system to determine the time interval between tooth separation and recontact (Figure 4b). The negative torque reacting against the inertia of the part of the system attached to the drive gear will determine whether the tooth will move through the complete backlash allowance and recontact the unloaded side of the adjacent tooth or whether it will stop in the mid range of the backlash allowance zone. The amount of gap between the loaded teeth, the rate of applied positive torque, and the system inertia will help determine the differential tooth velocities upon recontact and will aid in determining the tooth loading force. Accurate determination of the tooth loading force on contact, however, also depends upon the torsional stiffnesses and inertias of the driven portion of the system as well as the bearing stiffness, as can be seen in Figure 4c. The bearing stiffnesses are involved because the shock force of the gear teeth produces a torque which is a result of a side force perpendicular to the shaft centerline which is affected by the bearing oil film flexibility and the amount of motion that occurs. This is the mechanism which couples torsional and lateral shaft vibrations and must be considered to obtain an accurate evaluation of the actual tooth forces.

The type of tooth failure and the extent of damage depends upon these recontact tooth forces which must be properly considered in the design stage, if torque reversals are suspected.

**SAMPLE CASE**

To demonstrate the effect of changing torsional natural frequencies upon the transient response, including shear stresses and gear tooth forces, an actual system analysis is presented. A representative diagram of an 1800 rpm synchronous motor driven compressor train is shown in Figure 5. The first step in evaluating the reliability of a torsional system is the calculation of the natural frequencies of the system which are shown in Table I for the example system. The original system was undesirable because a torsional natural frequency occurred at 30 Hz (1800 rpm), which is an insufficient margin between

the torsional critical speed and the operating speed. This problem would normally eliminate the need for a transient analysis of the proposed system since modifications must be made to provide sufficient margin between the torsional critical speed and operating speed. However, for comparison purposes a transient analysis was made of the proposed system to determine the degree of severity of the transient stresses.



Figure 4a. Gear Tooth Clearance.

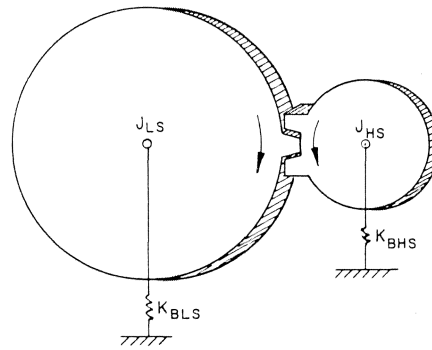


Figure 4b. Gear Tooth Shock Loading.

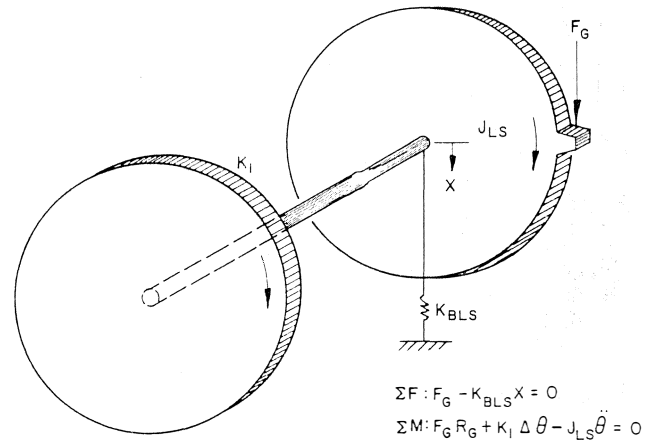


Figure 4c. Gear Response-Equations of Motion.

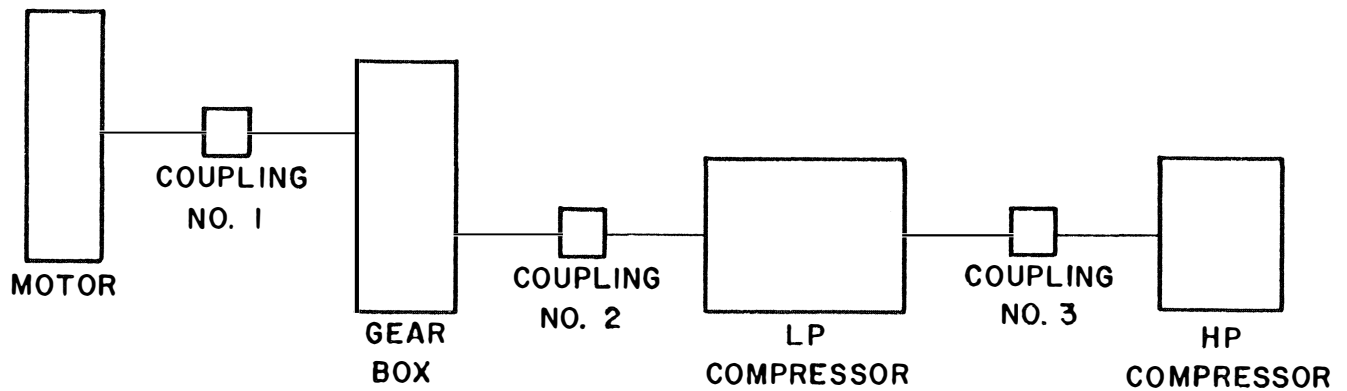


Figure 5. System Schematic.

TABLE 1. TORSIONAL NATURAL FREQUENCIES, HZ

Proposed System		Modified System	
HZ	% Speed	HZ	% Speed
30	75	21	83
53	56	39	68
86	28	69	43

The transient response of a system depends upon the location of the torsional natural frequencies with respect to the motor speed-time curve during startup as shown in Figure 6 for this system. All of mass-elastic properties, the compressor load, and the motor torques (average and dynamic) are required to develop the speed-time curve and the transient response of the system.

Comparisons of the transient response of the proposed and modified systems can be found in Figures 7 and 8. The gear tooth forces are shown in Figure 7 and the compressor shaft stresses are given in Figure 8. A comparison of the a and b portions of the figures indicate the effect of reducing the torsional natural frequencies so that they are excited later in the startup due to the synchronous slip excitation.

The shaft stresses and gear tooth forces are lower in the final system because the torsional natural frequencies of the final system occur at a higher percentage of shaft speed which has a greater acceleration rate as can be seen in Figure 6. The higher acceleration rate reduced the dynamic magnification at the torsional natural frequencies, producing a lower response.

The negative gear force shown in Figure 7a indicates a negative torque or backlash would occur for the first torsional mode at the gear which could cause excessive wear or damage to the gear. Modifications to the system were developed which would eliminate the backlash and result in better gear life and reliability. The modifications to eliminate the backlash in the gears required some shaft modifications (diameter and length) as well as coupling changes. The negative torque could not be eliminated at the high pressure compressor shaft since the horsepower transmitted through that shaft has low average torque which is below the dynamic torque.

For practical and economic considerations, shaft modifications were not made on the actual system as installed and only the couplings were modified. As a result a small amount of backlash occurred at the gears for a few cycles during startup. The gear manufacturer was able to adjust backlash clearance and surface hardening procedures to allow the gear to withstand some backlash without detrimental effects. Cou-

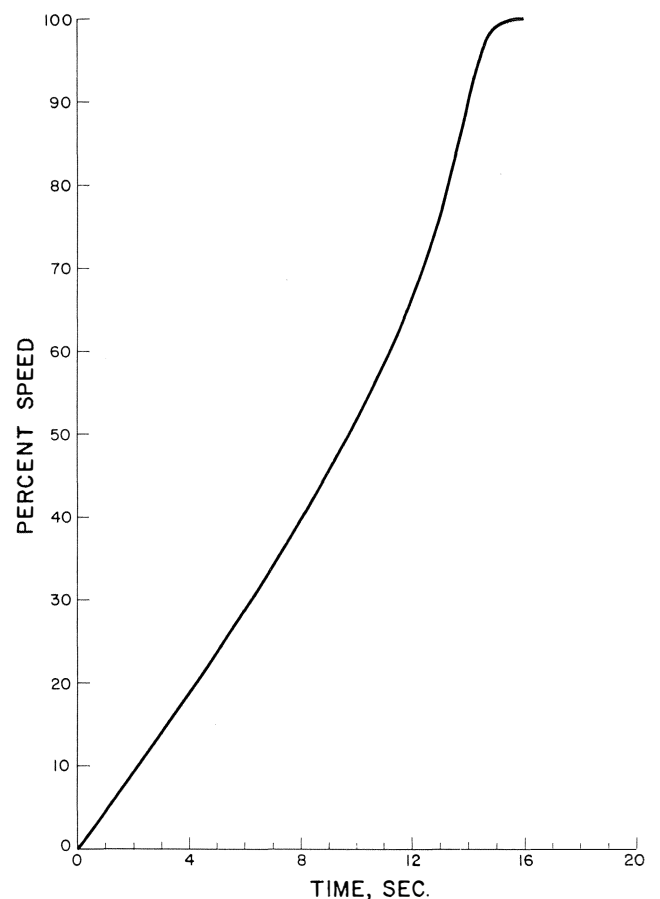


Figure 6. System Speed-Time Curve.

plings were specified which would withstand the negative torques which occurred at the compressor shafts.

The installed system had a shaft torsional stress of 19,000 psi peak-to-peak, compared to 30,000 psi peak-to-peak for the proposed system when the first torsional mode was excited during startup. This reduction in torsional shaft stresses allowed the system to have over 2,000 allowable startups which was sufficient for the application of this system.

For this example, reducing the torsional natural frequencies reduced the transient stresses by taking advantage of

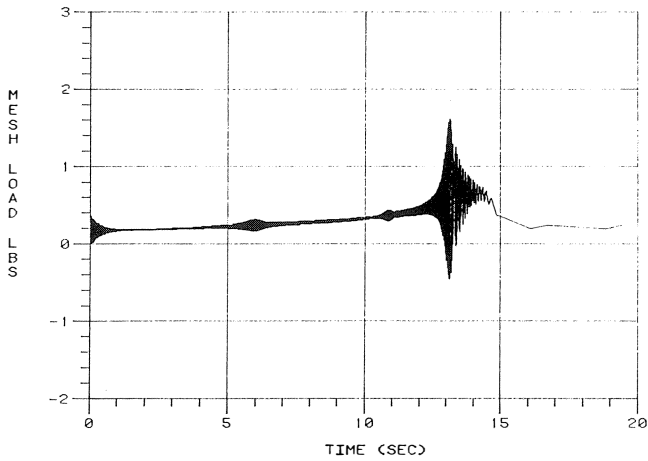


Figure 7a. Proposed System Gear Mesh Load.

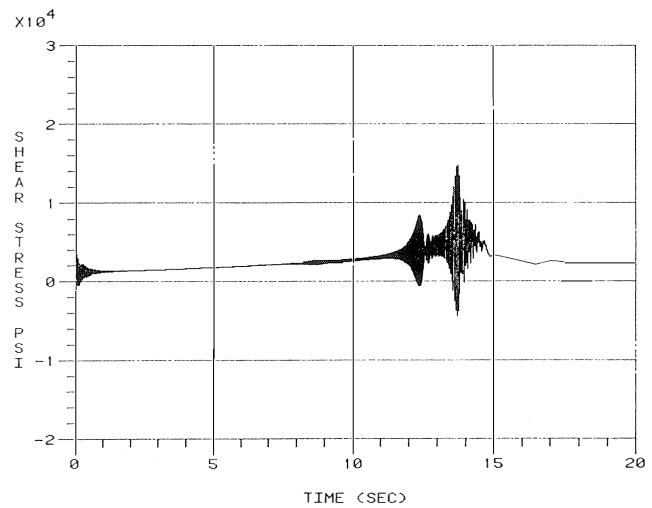


Figure 8b. Modified System High Pressure Compressor - Shaft Stresses.

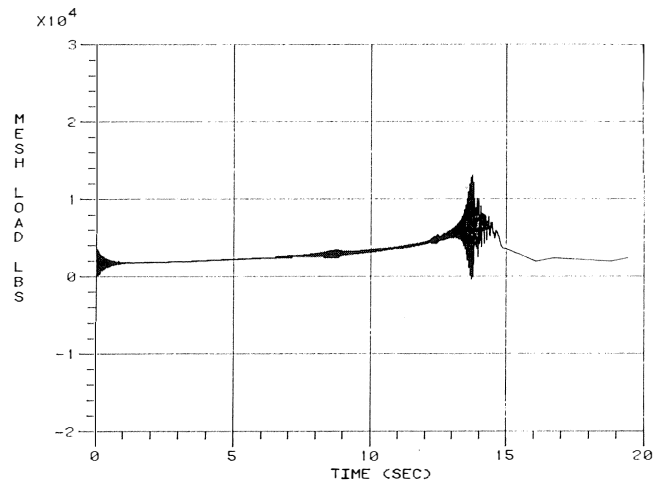


Figure 7b. Modified System Gear Mesh Load.

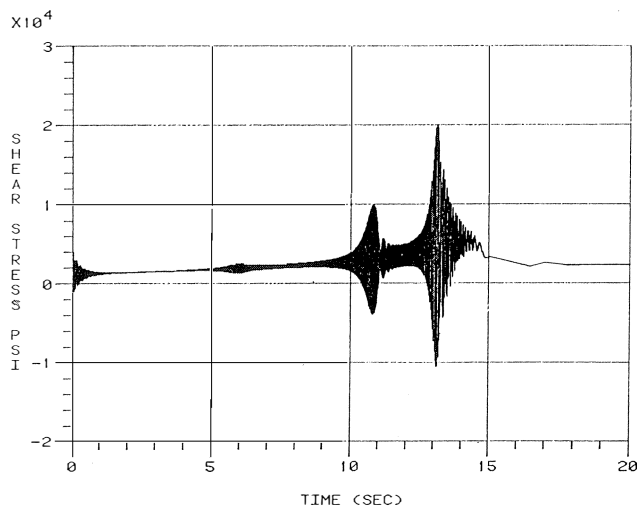


Figure 8a. Proposed System High Pressure Compressor - Shaft Stresses.

higher acceleration rates through the resonant frequencies during startup which resulted in a lower response.

CONCLUSIONS

A transient torsional analysis is necessary to design a reliable synchronous motor system. The system natural frequencies must be properly located for satisfactory continuous operation as well as minimum transient excitation. The level of response to transient excitation depends upon the acceleration rate, average torque, and dynamic torque at the instant of passing through a resonance.

The torsional resonant frequencies can be adjusted by changing stiffness and mass inertia properties of the components and can result in minimizing the transient response.

The allowable number of startups is directly related to stress levels (shaft material, stress concentrations) and depends upon the percentage of fatigue life consumed on each start. A complete torsional analysis should evaluate both the continuous and transient conditions to assess system reliability.

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