FAN LAWS, THE USE AND LIMITS IN PREDICTING CENTRIFUGAL COMPRESSOR OFF DESIGN PERFORMANCE

by

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ABSTRACT

The fan laws, particularly what the limits are in their use is described. The derivation is covered very briefly. To help understand some of the fan law uses, centrifugal impeller characteristics are covered, particularly some of the things controlling the shape of the curve. A discussion of the fan law deviations in an individual impeller is covered. Multistage centrifugals are covered using two, four, and six stage configurations. A set of stage curves are made up for use of the study. These curves reflect two basic impeller designs. The designs result in a high (15 percent) and a low (five percent) head rise. Also included is a study where the effect of molecular weight is presented. Speed variations are covered and compared to the fan law prediction with the deviations being shown.

INTRODUCTION

The inspiration for this paper came from a question raised at one of the Performance Testing Discussion Groups during the 1990 Turbomachinery Symposium. While discussing the subject of doing a field performance test on an alternative gas, the fan law question was raised. Specifically, the question was "What are the limits of use of the fan laws?" There was silence, then the question was dismissed with a "it depends" kind of answer. After the session, it occurred to the author that this question deserved a better answer. The following will attempt to do just that.

Before proceeding, it seems in order to issue a disclaimer or two. To give some idea of magnitude, a few numerical examples will be used. For comparison to the fan law, centrifugal stage curves will be used. Any resemblance of these curves to machines currently in service or being designed is purely coincidental. The author believes these to be typical of real curves, but these curves are the result of imagineering on the part of the author and some of his colleagues. The purpose here is to illustrate a point or two, not present a design dissertation. Having said all of this, it is time to address the subject.

FAN LAWS

The fan laws are derived from a dimensional analysis, using the Pi-theorem as a formal procedure. The reader is directed to Shepherd [1] for a complete derivation of the π-terms. The following three π-terms are used for the simplified fan law approach.

$$\Pi_1 = \frac{Q}{ND^3} \tag{1}$$

$$\Pi_2 = \frac{H}{N^2D^2} \tag{2}$$

$$\Pi_3 = \frac{P}{\rho N^3D^5} \tag{3}$$

Where:

- Q = volumetric flow, acfm
- N = rotational speed, rpm
- D = diameter, ft
- H = head, ft-lb/lb
- P = power, ft-lb/min
- ρ = density, lb/ft3

If the effect of compressibility is ignored, then density becomes a constant. The constants of density and geometry (diameter) can be disregarded and the relationships commonly referred to as the fan laws result.

$$Q \propto N \tag{4}$$

$$H \propto N^2 \tag{5}$$

$$P \propto N^3 \tag{6}$$

These relationships provide a useful tool for taking a point on a compressor curve at one speed and deriving a comparable point at a different speed, for similar conditions, of pressure, temperature, molecular weight and compression exponent.

Shape of the Curve

The shape of the stage curve has an influence on the predictability of off design performance by the fan laws. A review of the factors that influence curve shape would appear in order. Impeller cross sections with simplified velocity triangles are shown in Figure 1. For this discussion, reference will be made to the enlarged impeller vector tip triangle in Figure 2. The figure represents an ideal vector tip triangle, ignoring the effect of slip (the gas is assumed to follow the blade angle without deviation). When an impeller is operating within its design head flow envelope, the flow is dictated by an area made up of the impeller outside diameter multiplied by the tip width, minus the blockage caused by the impeller vanes. The velocity vector V_{r2} (90 degree) results from the division of the impeller discharge volumetric flow by the discharge area for a 90 degree blade angle. The vector V_{r2} then is proportional to the impeller discharge flow. When there is a backward lean to the blade, the relative flow is calculated using the blade angle and the radial component of the relative velocity, but the proportionality, V_{r2} and flow, is still correct, particularly for the assumption made earlier about zero slip (gas follows the blade angle without deviation).

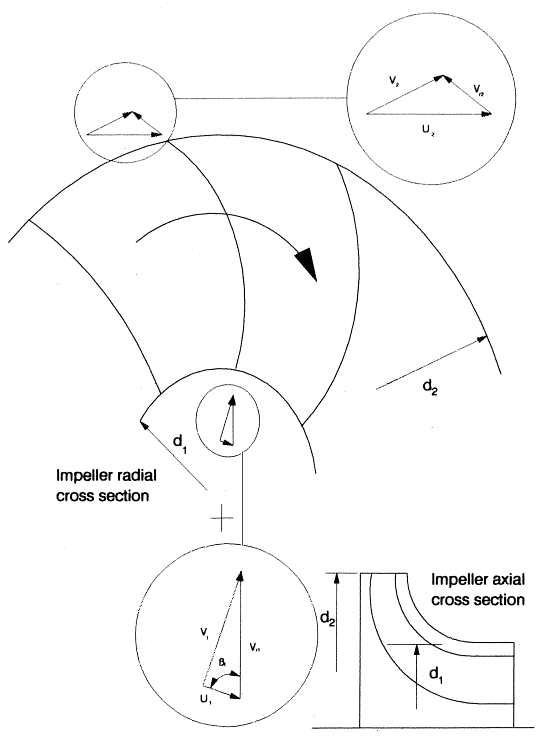


Figure 1. Impeller Inlet and Outlet Vector Triangle.

Impeller Blade Angle

Reference is made to Figure 2 at the 90 degree V_{r2}, relative velocity vector. If the flow is reduced, the magnitude of vector V_{r2} is reduced, but there is no influence on the tangential component, V_{u2} of the absolute velocity, V. The ratio of V_{u2}/U_2 equals 1.0. The vector U_2 represents the impeller tip velocity. For the radial impeller the theoretical curve is flat, with a theoretical work input

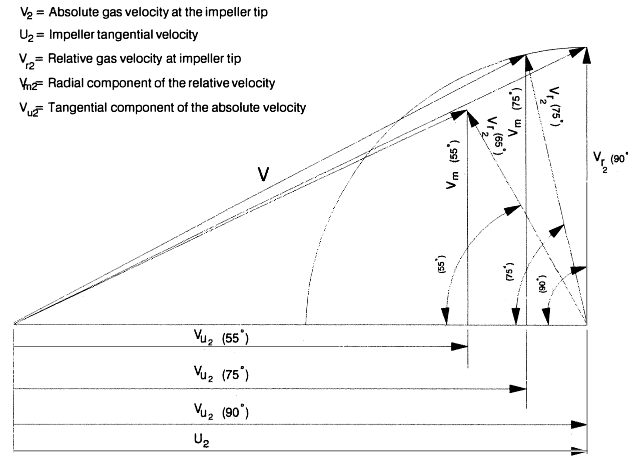


Figure 2. Impeller Tip Vector Triangle.

coefficient equal to 1.0. It should be pointed out that in actual fact this does not occur due to slip.

Reference now made to the 75 degree relative velocity vector. Note that when the magnitude of V_{r2} is reduced because of reduced flow, V_{u2} will be increased. The theoretical work input coefficient changes from a value of less than 1.0 to the value 1.0 as a limit. This change results in a slope to the curve. If the 55 degree relative velocity vector is examined and compared to the 75 degree relative velocity vector, a similar change in slope can be deduced. Further examination would indicate that the vector V_{u2} changes at a faster rate for the 55 degree relative velocity vector. This being true, then leads to the deduction that the more the backward lean of a impeller blade, the steeper the slope of the curve. The relative slope changes are shown in Figure 3.

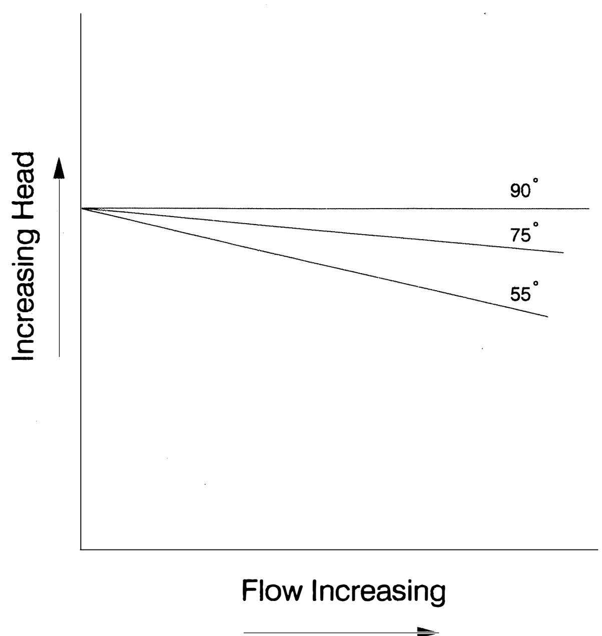


Figure 3. Compressor Ideal Stage Head Input Performance Curves.

Molecular Weight

Gas moving through an impeller is constantly changing in volume due to compressibility. If an impeller of a given geometry is operated on a light gas and then operated on a heavy gas, the curve will be steeper for the light gas. The cause for this is the higher volume ratio of the heavy gas. To explore this further, consider the basic equation for head, H.

$$H = Z_{avg} R T_1 \frac{n}{n-1} \left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right) \qquad (7)$$

Where:

Z_{avg} = compressibility

R = 1545/molecular weight

T_1 = absolute temperature, R

n = polytropic exponent

P_1 = inlet pressure, psia

P_2 = discharge pressure, psia

The relationship for volume ratio is:

$$\text{Volume Ratio} = \frac{Q_1}{Q_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} \qquad (8)$$

Where:

Q_1 = volumetric flow, inlet, acfm

Q_2 = volumetric flow, discharge, acfm

If for a given geometry, speed, and inlet flow rate, a head value is established, it can be assumed that the head stays reasonably constant as the molecular weight is changed. This being true will indicate a larger value for pressure ratio from Equation (7) with an increase in molecular weight. Equation (8) then can be used to indicate an increase in volume ratio. The volume ratio then is increasing with an increase in molecular weight. An increase in volume ratio represents a decrease in impeller exit volume. This in turn shortens the vector V_{r2}. From the foregoing discussion, it was shown that with a decrease in relative velocity, the vector V_{u2} became longer. The effect is the same as having a more radial blade which then results in a flatter curve.

It should also be pointed out that since an increase in the vector V_{u2} results in a higher work input, there is actually a head increase with the higher molecular weight gas. The relationship for work input is given in Equation (9). Earlier, when giving the assumptions used in the development of the fan laws, it was stated that density was constant. This would give cause to think that the performance of an impeller is independent of molecular weight changes. The two phenomenon just cited indicate this is not true.

Speed

According to the fan laws, a speed increase of 10 percent would indicate a 10 percent flow increase, and the head should increase by 21 percent. If the inlet volume was proportional to the impeller exit velocity, as the fan law assumptions state, then this would be true. If a speed increase is viewed in the same light as the increase in molecular weight, where it was shown that the impeller tip geometry vector triangle does not stay proportional; it can be concluded that an increase greater than 21 percent can be expected.

The following example will aid in illustrating the effect of a speed change on a single impeller. This example still uses the blade angle without slip. The values chosen, while believed to be tpical, are postulated rather than taken from any particular suppliers design.

A 20 in impeller operating at 8000 rpm was chosen for the example. For convenience, the nominal flow was chosen to be an actual volumetric flow (acfm) of 8000. The tip speed is a nice round value of 700 fps. The 55 degree vane angle was chosen. The relative velocity is 220 fps. This produces a radial component, V_m, of the relative velocity of 180 fps. The difficult part is to obtain an actual flow at the tip of the impeller.

The characteristics of an impeller are very dependent on the conditions at the outlet of the impeller. Conditions at this point are not easily determined. The performance a user needs is based on inlet and discharge flange conditions, so impeller tip values are quite abstract to the user. Because they are difficult to measure, they are normally not taken for a users use, but are only obtained for use by the design engineer.

SINGLE IMPELLER

Again, at the risk of insulting professional compressor impeller designers, there have been a few liberties taken. A relative velocity, V_{r2}, of 220 fps was assumed. The radial component, V_m, of the absolute velocity, V, was calculated.

The example could have been done by use of pure ratios, but this seemed to make the entire exercise somewhat abstract. A set of values for head and pressure ratio were developed to help illustrate the example. The following relationships are presented, as they are needed in the analysis.

$$\zeta = \frac{V_{u2}}{u_2} \qquad (9)$$

$$\mu = \eta \, \zeta \qquad (10)$$

$$H = \frac{\mu \, u_2^2}{g} \qquad (11)$$

Where:

ζ = work input coefficient

η = head coefficient

μ = efficiency, polytropic

g = gravitational constant, fps^2

Using the initial conditions, a volume ratio of 1.24 is calculated. For the example, a flow increase of 10 percent is used. The fan law would indicate that if a 10 percent flow increase is desired, a 10 percent increase in speed is called for from the relationship of Equation (4). The speed boost increases the tip speed V_2 by 10 percent to 770 fps. It would appear prudent, that since the blade exit angle is fixed, the relative velocity should be increased by 10 percent. This then keeps the vector tip triangle similar to the original. The efficiency is actually just incidental, since the purpose of the example is to see how well the fan law is predicting the performance. Because the example uses the tip triangle, and performance is based on inlet condition, the volume ratios are compared. The head, based on the triangle is 121 percent, as can be seen from the tip speed squared term from Equation (10). For a more in depth coverage of the relationships, refer to Brown [2].

The volume ratio for the 10 percent increase is 1.285. Earlier it was stated that the volume ratio for the base case was 1.240. With the fan law this value should not have changed. Taking the ratio of the two values indicates a 3.6 percent increase. If the inlet volume were adjusted back to the 10 percent value, the head would increase by the slope of the curve which produces a deviation of 0.76 percent. This is not a serious deviation but does indicate directionally that even in one impeller that there is an inherent error. The steps used to obtain the values given are outlined in the APPENDIX.

One word of caution might be in order, there are limits to the amount of speed increase an impeller can tolerate. The obvious one is the mechanical limits due to stress. As has been stated, the impeller tip generally controls the flow of the compressor. For a given geometry, when the volume ratio is sufficiently increased over design, the inlet vane geometry becomes more of a consideration. This could be a Mach number limit (heavy molecular weight gases), or it could be a high negative incidence angle at the inlet vane. These factors are normally difficult for a user to evaluate and are one reason to consider getting the original equipment manufacturer getting involved with a rating change.

MULTISTAGE COMPRESSORS

For the multistage study, a set of stage curves were developed. These curves were based on two basic slopes. For this, the 15 degree backward lean (75 degree) and 35 degree backward lean (55 degree) blade angles were selected. These provided a five percent and 15 percent rise to surge respectively. The impeller curves are included in the APPENDIX.

The study used combinations of two, four, and six impellers (stages) in series at the same speed. The fan laws tend to be most applicable to a series of impellers without cooling or side stream intervention. Uncooled section curves are generally available from the compressor supplier. Some effects that were not included were the entrance and exit losses, which help distort the calculations. Some of these are built into the stage efficiency but for a precise study they should be evaluated as a separate item.

REYNOLDS NUMBER

The effect of any Reynolds number correction has been neglected. This generally would seem to be recommended in the majority of the applications for multistage compressors. The ASME PTC-10 [3] includes a correction, which was included in the original writing of the code to account for loss of performance at very low Reynolds numbers, typical of high vacuum applications. For pressure service and at Reynolds numbers on the order of 10^6, the correction is too optimistic. When used in practice, only half the value is used. A better approach to Reynold's number performance correction was presented by Wiesner [4]. Generally, however, the conservative approach is to ignore any Reynolds number correction for head or efficiency.

MULTISTAGE COMPRESSOR MOLECULAR WEIGHT STUDY

Because molecular weight is often changed, the first study concerned itself with a molecular weight change. The fan laws would indicate that, all other factors being equal, the head would be constant for a multistage compressor uncooled section. A base case using a molecular weight of 26 was used. This is a medium value, close to air and probably one of the most common molecular weights encountered in practice. The compressor two, four, and six stage configurations were then analyzed on an impeller by impeller basis. The conditions used in the base case are outlined in Table 1. As would be expected there is a deviation. The higher than design molecular weight tended to be overly optimistic for both the high head rise case as well as for the low head rise case. See Tables 2 and 3 for a tabulation of the results for low and high backward lean impellers respectively. As would be expected, the more stages, the more the deviation. Also the high head rise case indicated more deviation than the low head rise case. These deviations are the result of compressibility changes between the stages, causing a stack-up error. The internal impeller effects were not included, to keep the stage curves simple. A deviation plot of the high and low head rise case are shown in Figure 4 and 5

Table 1. Fan Law Cases.

basis of design	low backward	lean	(5% head rise)
no of stages	2	4	6
design MW	26	26	26
design K	1.4	1.4	1.4
design Z	1	1	1
design cfm	8000	8000	8000
design speed	8000	8000	8000
design head	18179	36285	54366
design eff	0.79	0.79	0.79
design ghp	369	737	1105

basis of design	high backward	lean	(15% head rise)
no of stages	2	4	6
design MW	26	26	26
design K	1.4	1.4	1.4
design Z	1	1	1
design CFM	8000	8000	8000
design speed	8000	8000	8000
design Head	16400	32720	48988
design eff	0.79	0.79	0.79
design GHP	333	664	995

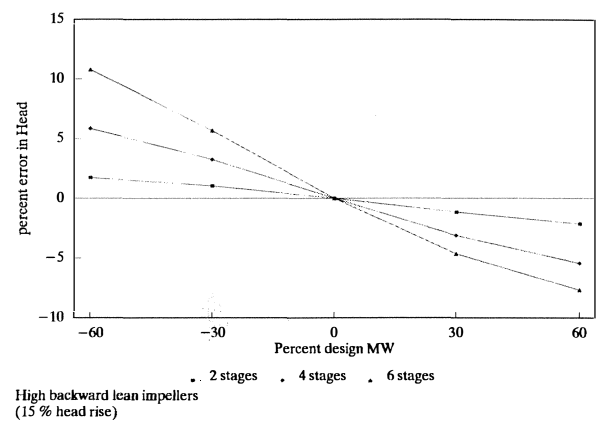


Figure 4. Fan Law Head Prediction Deviation for Molecular Weight Change, High Head Rise Impeller Case.

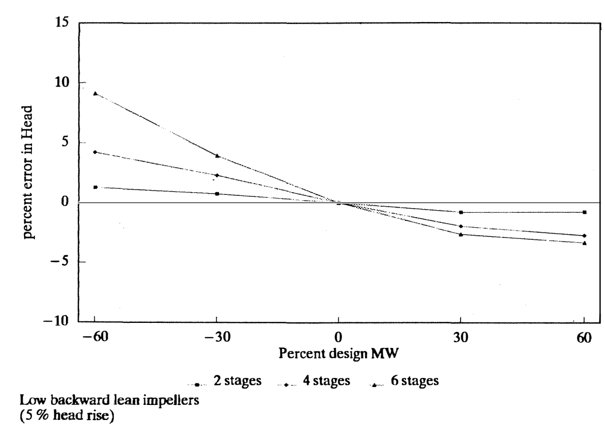


Figure 5. Fan Law Head Prediction Deviation for Molecular Weight Change, Low Head Rise Impeller Case.

respectively. The gas horsepower deviations which tend to track the head fairly well in deviation direction. Magnitude is somewhat less. These are shown in Figures 6 and 7.

The deviation curves seem to indicate that stages operating at a nominal tip speed (700 fps), performance can be reasonably well predicted using overall correction methods for a change in molecular weight up to 30 percent in either direction.

Table 2. Low Backward Lean - MW Variation.

MW + 30%				(MW=33.8)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	18179	36285	54366	fanlaw
eff	0.79	0.79	0.79	
ghp	479.7	958.1	1436.5	
head	18324	37006	55836	actual
eff	0.792	0.793	0.794	
ghp	483	973	1468	
head	–0.79%	–1.95%	–2.63%	error
eff	–0.25%	–0.38%	–0.50%	
ghp	–0.68%	–1.53%	–2.15%	
MW – 30%				(MW=20.0)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	18179	36285	54366	fanlaw
eff	0.79	0.79	0.79	
ghp	283.8	566.9	850	
head	18045	35469	52309	actual
eff	0.789	0.783	0.778	
ghp	282	559	831	
head	0.74%	2.30%	3.93%	error
eff	0.13%	0.89%	1.54%	
ghp	0.65%	1.42%	2.29%	
MW + 60%				(MW=41.6)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	18179	36285	54366	fanlaw
eff	0.79	0.79	0.79	
ghp	590.4	1179.2	1768	
head	18431	37314	56250	actual
eff	0.793	0.793	0.793	
ghp	597	1208	1822	
head	–1.37%	–2.76%	–3.35%	error
eff	–0.38%	–0.38%	–0.38%	
ghp	–1.11%	–2.38%	–2.96%	
MW – 60%				(MW=16.25)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	18179	36285	54366	fanlaw
eff	0.79	0.79	0.79	
ghp	230.6	460.6	690.6	
head	17952	34818	49831	actual
eff	0.787	0.777	0.764	
ghp	229	450	655	
head	1.26%	4.21%	9.10%	error
eff	0.38%	1.67%	3.40%	
ghp	0.71%	2.36%	5.44%	

Table 3. High Backward Lean - MW Variation.

MW + 30%				(MW=33.8)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	16400	32720	48988	fanlaw
eff	0.79	0.79	0.79	
ghp	432.9	863.2	1293.5	
head	16592	33774	51376	actual
eff	0.792	0.793	0.794	
ghp	437	888	1351	
head	–1.16%	–3.12%	–4.65%	error
eff	–0.25%	–0.38%	–0.50%	
ghp	–0.94%	–2.79%	–4.26%	
MW – 30%				(MW=20.0)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	16400	32720	48988	fanlaw
eff	0.79	0.79	0.79	
ghp	256.2	510.8	765.4	
head	16232	31683	46354	actual
eff	0.789	0.785	0.779	
ghp	254	499	735	
head	1.03%	3.27%	5.68%	error
eff	0.13%	0.64%	1.41%	
ghp	0.85%	2.36%	4.13%	
MW + 60%				(MW=41.6)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	16400	32720	48988	fanlaw
eff	0.79	0.79	0.79	
ghp	532.8	1062.4	1592	
head	16761	34600	53069	actual
eff	0.793	0.794	0.793	
ghp	543	1102	1719	
head	–2.15%	–5.43%	–7.69%	error
eff	–0.38%	–0.50%	–0.38%	
ghp	–1.88%	–5.14%	–7.39%	
MW – 60%				(MW=16.25)
cfm	8000	8000	8000	
speed	8000	8000	8000	
head	16400	32720	48988	fanlaw
eff	0.79	0.79	0.79	
ghp	208.1	415.0	621.9	
head	16118	30913	44226	actual
eff	0.788	0.779	0.766	
ghp	205	398	579	
head	1.75%	5.85%	10.77%	error
eff	0.25%	1.41%	3.13%	
ghp	1.52%	4.27%	7.41%	

MULTISTAGE COMPRESSOR SPEED STUDY

The speed variation study used the same impeller curves as the molecular weight change study. All conditions used to set up the base case are outlined in Table 1. This is the case to which all speed variations are compared. Tables 4 and 5 outline the values for a speed range of +10 percent through –20 percent. Again, both the low head and high head rise designs were evaluated and the results compared. The results are tabulated in Table 6.

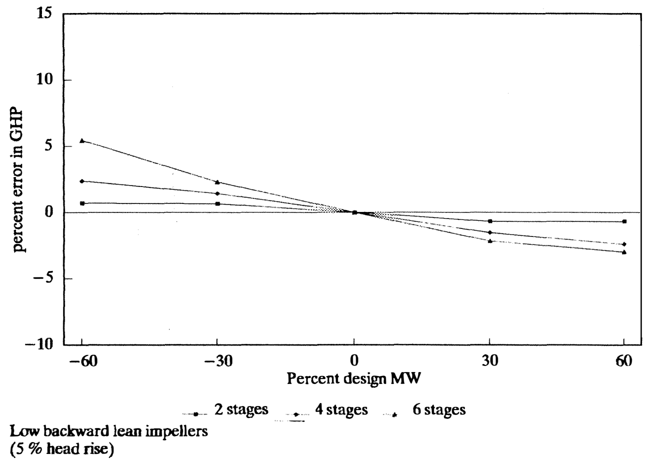


Figure 6. Fan Law Gas Horsepower Deviation for a Molecular Weight Change, Low Head Rise Impeller Case.

Table 4. Low Backward Lean - Speed Variation.

no of stages	2	4	6	
cfm	8800	8800	8800	
speed + 10%	8800	8800	8800	
head	21997	43905	65783	fanlaw
eff	0.79	0.79	0.79	
ghp	491.1	980.9	1470.8	
head	22124	44572	67215	actual
eff	0.792	0.793	0.794	
ghp	493	993	1495	
head	–0.58%	–1.50%	–2.13%	error
eff	–0.25%	–0.38%	–0.50%	
ghp	–0.38%	–1.21%	–1.62%	
cfm	7200	7200	7200	
speed – 10%	7200	7200	7200	
head	14725	29391	44036	fanlaw
eff	0.79	0.79	0.79	
ghp	269.0	537.3	805.5	
head	14637	28859	42721	actual
eff	0.789	0.781	0.781	
ghp	268	531	790	
head	0.60%	1.84%	3.08%	error
eff	0.13%	0.00%	1.15%	
ghp	0.37%	1.18%	1.97%	
cfm	6400	6400	6400	
speed – 20%	6400	6400	6400	
head	11635	23222	34794	fanlaw
eff	0.79	0.79	0.79	
ghp	188.9	377.3	565.8	
head	11496	22333	32141	actual
eff	0.788	0.777	0.765	
ghp	187	369	539	
head	1.21%	3.98%	8.26%	error
eff	0.25%	1.67%	3.27%	
ghp	1.03%	2.26%	4.96%	

Table 5. High Backward Lean - Speed Variation.

no of stages	2	4	6	
cfm	8800	8800	8800	
speed + 10%	8800	8800	8800	
head	19844	39591	59275	fanlaw
eff	0.79	0.79	0.79	
ghp	443.2	883.8	1324.3	
head	20010	40518	61408	actual
eff	0.792	0.793	0.793	
ghp	446	902	1367	
head	–0.83%	–2.29%	–3.47%	error
eff	–0.25%	–0.38%	–0.38%	
ghp	–0.62%	–2.02%	–3.12%	
cfm	7200	7200	7200	
speed – 10%	7200	7200	7200	
head	13284	26503	39680	fanlaw
eff	0.79	0.79	0.79	
ghp	242.8	484.1	725.4	
head	13173	25825	37973	actual
eff	0.79	0.786	0.782	
ghp	241	475	702	
head	0.84%	2.63%	4.50%	error
eff	0.00%	0.51%	1.02%	
ghp	0.73%	1.91%	3.33%	
cfm	6400	6400	6400	
speed – 20%	6400	6400	6400	
head	10496	20941	31352	fanlaw
eff	0.79	0.79	0.79	
ghp	170.5	340.0	509.4	
head	10323	19387	28454	actual
eff	0.788	0.78	0.767	
ghp	168	327	476	
head	1.68%	8.01%	10.19%	error
eff	0.25%	1.28%	3.00%	
ghp	1.49%	3.97%	7.03%	

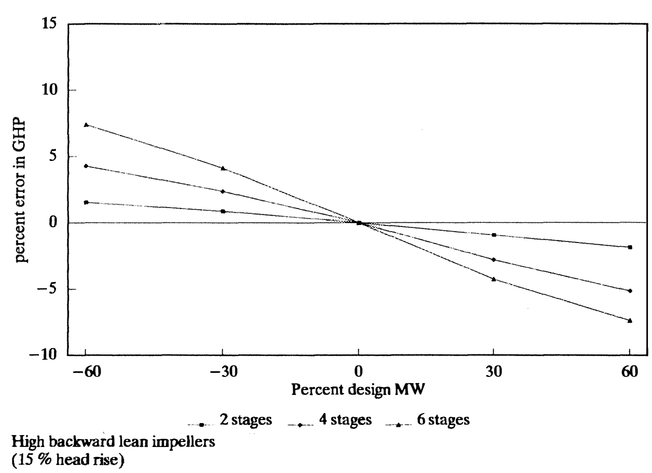


Figure 7. Fan Law Gas Horsepower Deviation for a Molecular Weight Change, High Head Rise Impeller Case.

Table 6. Low Backward Lean.

	2 stg pct error			
head	1.205	0.601	0	–0.576
eff	0.254	0.127	0	–0.253
ghp	1.031	0.374	0	–0.377
% speed	–20	–10	0	10
	4 stg pct error			
head	3.982	1.843	0	–1.497
eff	1.673	1.152	0	–0.378
ghp	2.261	1.181	0	–1.214
% speed	–20	–10	0	10
	6 stg pct error			
head	8.255	3.079	0	–2.131
eff	3.268	1.152	0	–0.504
ghp	4.965	1.968	0	–1.622
% speed	–20	–10	0	10
High Backward Lean				
2 stg pct error				
head	1.676	0.843	0	–0.83
eff	0.254	0	0	–0.253
ghp	1.486	0.729	0	–0.623
% speed	–20	–10	0	10
4 stg pct error				
head	8.015	2.626	0	–2.28
eff	1.282	0.509	0	–0.378
ghp	3.966	1.907	0	–2.02
% speed	–20	–10	0	10
6 stg pct error				
head	10.186	4.496	0	–3.473
eff	2.999	1.023	0	–0.378
ghp	7.025	3.327	0	–3.12
% speed	–20	–10	0	10

Deviation plots for head are shown in Figures 8 and 9 for the high and low head rise cases, respectively. As expected, the low head rise case shows a lower deviation than the high head rise case. Just as in the molecular weight variations, the higher head rise is more sensitive to impeller stackup error. The gas horsepower deviations are presented in Figures 10 and 11. Also, as in the molecular weight cases, the deviations are somewhat less. For the two and four stage configurations within ±10 percent speed variation with the low head rise curves, the deviations are relatively modest. While they are, of course, somewhat higher on the high head rise case they are still reasonable. For these cases, the fan law use is quite reasonable. Outside these areas, if it is recognized that the deviation becomes significant, the fan law is feasible for a rough estimate type calculation.

CONCLUSION

In the evaluations, deviations due to effects not considered by the fan laws have been evaluated. Even in a single impeller, there is some deviation. The deviations tend to be more for higher head rise impellers. A tip speed of 700 fps was used, as typical of many industrial centrifugal compressors. Directionally, from the data, it could be concluded that for higher tip speed designs that deviations would be higher. The deviations of two, four, and six stages

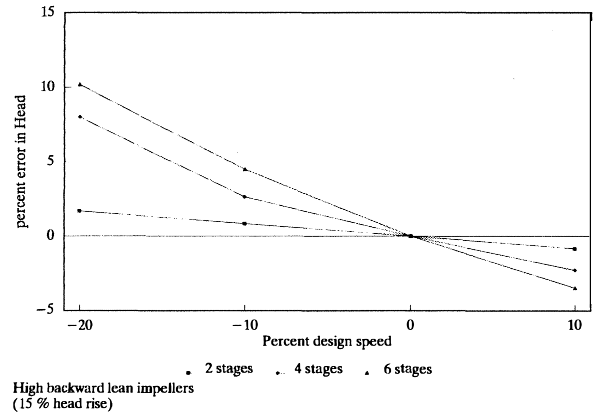


Figure 8. Fan Law Head Prediction Deviation for a Speed Change, High Head Rise Impeller Case.

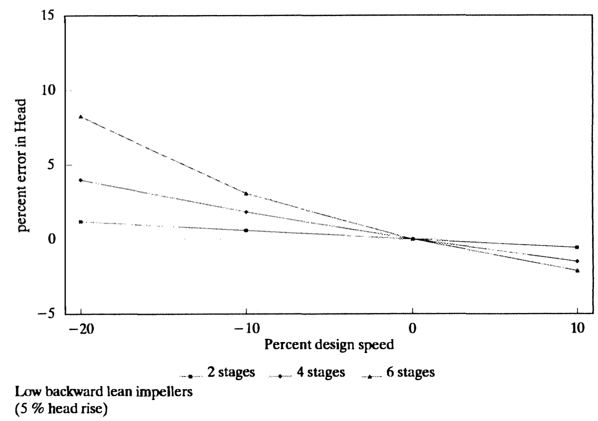


Figure 9. Fan Law Head Prediction Deviation for a Speed Change, Low Head Rise Impeller Case.

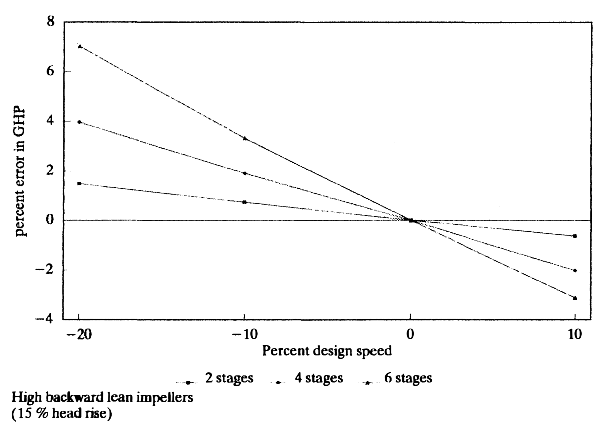


Figure 10. Fan Law Gas Horsepower Prediction Deviation for a Speed Change, High Head Rise Impeller Case.

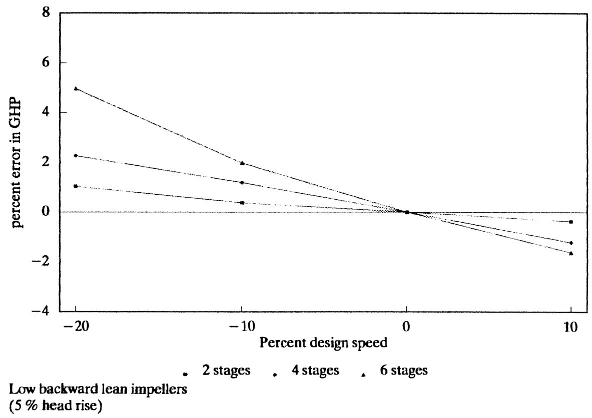


Figure 11. Fan Law Gas Horsepower Prediction Deviation for a Speed Change, Low Head Rise Impeller Case.

impellers were evaluated. Deviations for molecular weight were modest in the ±30 percent region for all combinations. For speed variation cases, the deviations for six stage cases were more significant. For speed variations within the ±10 percent band, the deviations were acceptable for the low head rise cases. For the high head rise cases, the deviation was higher. For considerations beyond these limits, while still useful, caution would be in order.

APPENDIX

Example

Given:
Inlet volume = 8000 acfm
Inlet temperature = 80°F
Impeller diameter = 20 in
Rotative speed = 8000 rpm
Impeller vane angle = 55 degree
n = 1.56
Molecular weight = 29

Calculate tip speed.

$$U_2 = \frac{\pi\, d_2\, N}{720} \tag{A-1}$$

Where:
d_2 = Impeller diameter, in.
N = Rotative speed, rpm

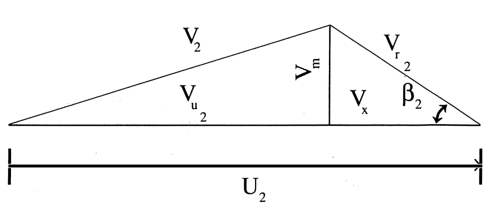
Substituting into Equation (A-1) yields:

$$U_2 = \frac{\pi \times 20 \times 8000}{720}$$

$$U_2 = 698 \text{ fps}$$

Round to: 700 fps

Assume:
Relative velocity V_{r2} = 220 fps



Solve for V_m.

$$V_m = V_{r2} \text{ SIN } \beta_2 = 220 \text{ SIN } 55 \text{ degree} = 180 \text{ fps}$$

Solve for V_x.

$$V_x = V_{r2} \text{ COS } \beta_2 = 220 \text{ COS } 55 \text{ degree} = 126 \text{ fps}$$

Solve for V_{u2}.

$$V_{u2} = U_2 - V_x = 700 - 126 = 574 \text{ fps}$$

Solve for work input, ζ using Equation (9).

$$\zeta = 574/700 = 0.82$$

Assume an efficiency, $\eta = 0.80$

Solve for the head coefficient, μ using Equation (10).

$$\mu = 0.80 \quad 0.82 = 0.656$$

Calculate the head, H, using Equation (11).

= 9983 ft lb/lb

Using Equation (7) and rearranging:

$$\left[\frac{P_2}{P_1}^{\frac{n-1}{n}} - 1 \right] = \frac{H}{R\, T_1 \; \frac{n}{n-1}}$$

$$= \frac{9983}{53.3 \times 540 \times 2.8}$$

$$= 0.124$$

Solve for P_2/P_1

$$\frac{P_2}{P_1} = (1.4124)^{2.8} = 1.39$$

Solve for Volume ratio using Equation (8).

$$\frac{Q_2}{Q_1} = 1.39^{\frac{1}{1.56}} = 1.24 \tag{A-2}$$

Increase the speed by 10 percent.

The predicted inlet volume is 1.10 × 8000 = 8800 acfm.
U_2' = 1.10 × 700 = 770 fps

Volume increases by 10 percent.

Since the outlet area is constant V_m increases.

V_m' = 1.10 X 180 = 198 fps

The relative velocity also increases 10 percent.
Vr_2' = 1.10 220 = 242 fps
V_x' = 242 cos 55 = 139 fps
V_{u2}' = 770 − 139 = 631 fps

Work input coefficient is shown to be the same (for equivalent tip triangle).

$$\zeta' = \frac{631}{770} = 0.82$$

Solving for the head coefficient

$$\mu' = 0.8 \times 0.82 = 0.656$$

Calculate the corresponding head

$$H' = \frac{0.656 \times 770^2}{32.2} = 12{,}079 \text{ ft lb/lb.}$$

$$\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{12{,}079}{53.3 \times 540 \times 2.8} = 0.15$$

$$\frac{P_2'}{P_1} = (1.15)^{2.8} = 1.48$$

The volume ratio for the 10 percent speed increase is, using Equation (8).

$$\frac{Q_2'}{Q_1} = (1.48)^{\frac{1}{1.56}} = 1.285$$

Error from fan law

$$\frac{1.285}{1.24} = 1.036$$

This is 3.6 percent higher than predicted or $110 \times 1.036 = 114$ percent and results in an inlet volume of 9120 acfm rather than the 8800 acfm predicted.

If it is desired to approximate the head generated at an even 10 percent flow increase for the 10 percent speed increase, one can calculate the increased work input for the modified tip triangle.

A close approximation to the tip conditions using a single iteration. Assume the velocity at the impeller tip is reduced by the new volume ratio.

$$V''_{r2} = V'_{r2} \times \frac{\text{Initial Volume Ratio (100\% Speed)}}{\text{New Volume Ratio (110\% Speed)}}$$

$$Vr_2'' = 242 \times 1.24/1.285$$
$$Vr_2'' = 233.5 \text{ fps}$$
$$V_x'' = 233.5 \times \cos 55 = 134 \text{ fps}$$
$$CU_2'' = 770 - 134 = 636 \text{ fps}$$
$$\zeta'' = \frac{636}{770} = 0.826 \text{ (higher work input coefficient)}$$
$$\mu'' = 0.8 \times 0.826 = 0.661$$
$$H'' = \frac{0.661 \times 770^2}{32.2} = 12{,}171$$

Using Equation (A-2)

$$\frac{P_2''}{P_1} = \left[\frac{H}{ZRT\,\frac{n}{n-1}} + 1\right]^{\frac{n}{n-1}} = \left[\frac{12{,}171}{53.3 \times 540 \times 2.8} + 1\right]^{2.8} = 1.4826$$

Check the volume ratio, using Equation (8).

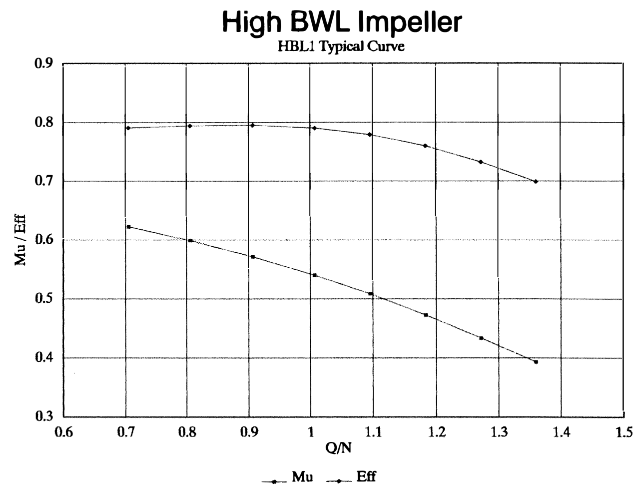
$$\frac{Q''_2}{Q_1} = 1.4826^{\frac{1}{1.56}} = 1.287$$

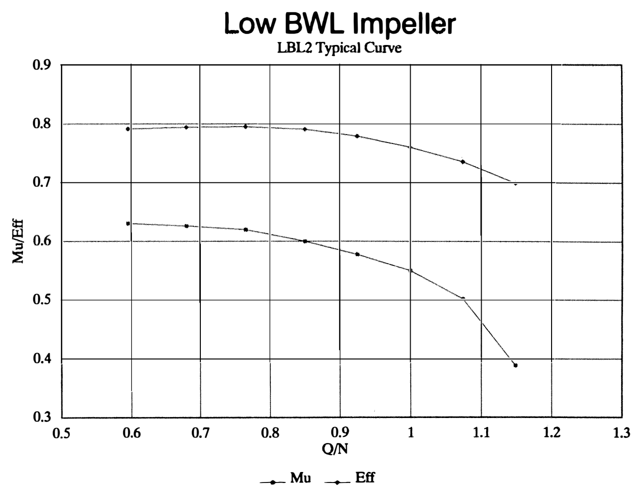
1.287 is very near to 1.285, so this can be considered a close approximation.

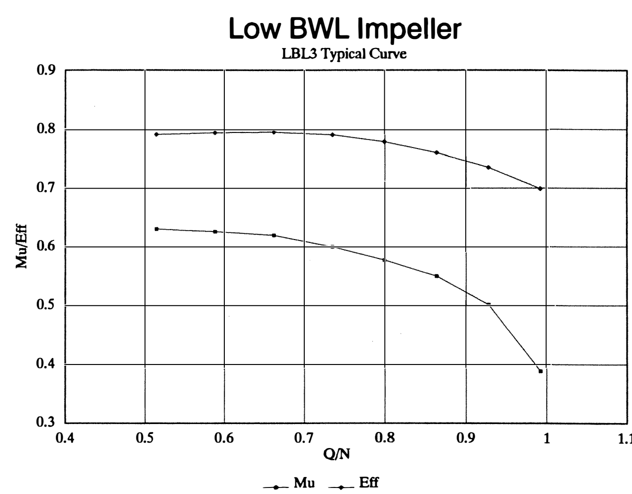
with Q_1 = 8,800 acfm (8000×1.1)

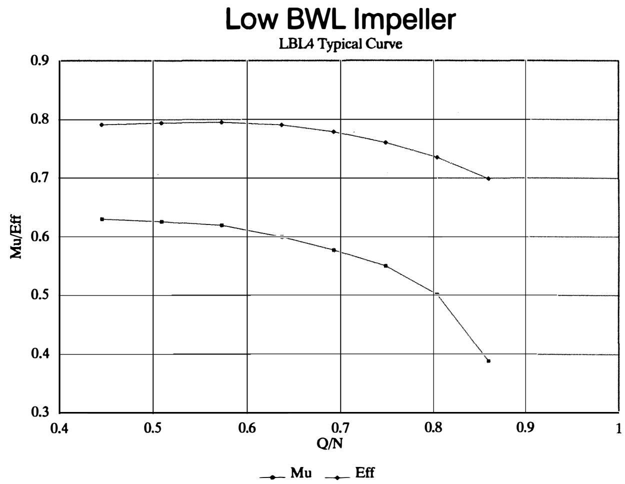
$$\text{Head Deviation} = \frac{0.661}{8.656} = 1.0076$$

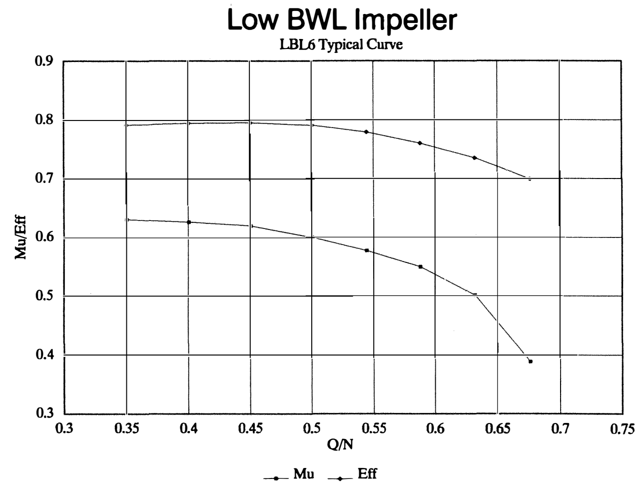
The Head Error is 0.76 percent

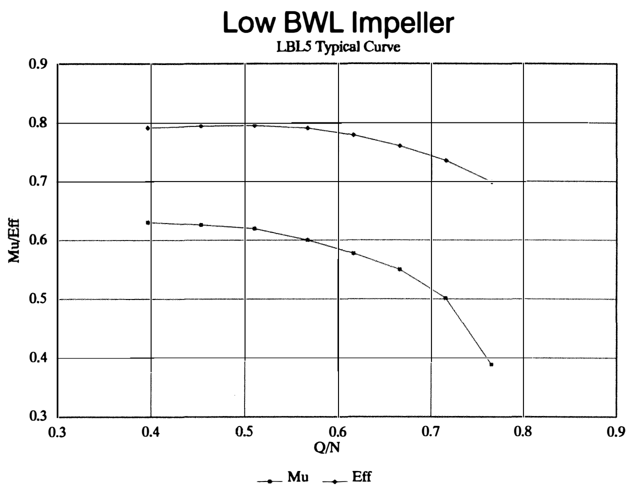












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