

**Superconducting transition temperature in heterogeneous ferromagnet-superconductor systems**Valery L. Pokrovsky<sup>1,2</sup> and Hongduo Wei<sup>1</sup><sup>1</sup>*Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA*<sup>2</sup>*Landau Institute for Theoretical Physics, Chernogolovka, Moscow 142432, Russia*

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We study the superconducting phase transition in two systems: ferromagnet-superconductor bilayer (FSB) and a thin superconducting film with a periodic array of magnetic dots (SFMD) upon it. We show that this transition is of the first order in FSB and of the second order in SFMD. The shift of the transition temperature  $\Delta T_c$  due to the presence of a ferromagnetic layer may be positive or negative in the FSB and is always negative in the SFMD. The dependence of  $\Delta T_c$  on geometrical factors and external magnetic field is found. Theory is extended to multilayers.

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**I. INTRODUCTION**

Heterogeneous ferromagnetic-superconducting (FM-SC) systems have attracted much attention recently<sup>1-14</sup> If the proximity effect is suppressed by the oxide layer between the FM and SC components, they interact via magnetic field. Any inhomogeneous magnetization produces magnetic field penetrating into the superconductor and inducing supercurrents. The supercurrents in turn generate magnetic field acting on the magnetization. Systems in which both, FM and SC parts are thin films represent a special interest for the experiment and can be analyzed theoretically. In these systems, spontaneous vortices appear due to the magnetic interaction.<sup>15</sup> Erdin *et al.*<sup>16</sup> have developed a method to calculate the arrangement of the magnetization in the FM film and supercurrents including vortices in the SC film in the London's approximation. The London's approximation is justified for these mesoscopic systems because characteristic length scales for magnetic field (the effective penetration depth and the period of textures) are much larger than the coherence length  $\xi$  of the superconductor. This method was applied recently<sup>17</sup> to study topological textures in the ferromagnet-superconductor bilayer (FSB). It was shown that the homogeneous state of the FSB with the magnetization perpendicular to the layer is unstable with respect to the formation of vortices. The ground state of the FSB represents a periodic array of stripe domains in which the direction of the magnetization in the FM film and the vorticity of vortices in the SC film alternate together.

In this paper we study the SC transition in heterogeneous FM-SC systems including the FSB, multilayers, and superconducting film with a periodic array of magnetic dots (SFMD). For this purpose we extend the theory of spontaneous SC-FM structures developed in the work<sup>17</sup> to the case of multilayers. We demonstrate that in the FSB the transition proceeds discontinuously (the first-order phase transition) as a result of competition between the stripe domain structure in a FM layer at suppressed superconductivity and the combined vortex-domain structure in the FSB. Spontaneous vortex-domain structures in the FSB tend to increase the transition temperature, whereas the effect of the FM self-interaction decreases it. The final shift of transition temperature  $\Delta T_c$  depends on several parameters characterizing the

SC and FM films and varies typically between  $-0.03T_c$  and  $0.03T_c$ .

In the SFMD the superconductivity appears continuously (the second-order phase transition). The shift of the transition temperature is always negative in this system.

Though the influence of the textures on the transition temperature is akin to the influence of the homogeneous magnetic field, there are important differences between these two phenomena: first, the average magnetic field may be zero for magnetic textures; second, the reciprocal action of the magnetic field generated by vortices onto magnetization is substantial.

The plan of this paper is as follows. In the following section we consider the change of the transition temperature due to spontaneous stripe structures in the FSB. In Sec. III we analyze how this stripe structure and the transition temperature change in the presence of an external magnetic field. In Sec. IV we study the shift of the transition temperature in the SFMD. Sec. V is devoted to theory of spontaneous textures in a multilayer FM-SC structure and to the shift of the transition temperature in it. Our conclusions are given in Sec. VI.

**II. TRANSITION TEMPERATURE IN THE SPONTANEOUS STRIPE STRUCTURE OF FSB**

As it was shown in Ref. 17, the homogeneous state of the FSB with the magnetization perpendicular to the layer is unstable with respect to the formation of a stripe domain structure, in which both the direction of the magnetization in the FM film and the circulation of the vortices in the SC film alternate together. Let the stripe width be  $L_s$ . The magnetization can be written as  $\mathbf{m} = ms(x)\hat{z}$ , where the coordinate  $x$  is along the direction perpendicular to the domain walls,  $\hat{z}$  denotes the unit vector perpendicular to the layers, and  $s(x)$  is a periodic step function with period  $2L_s$ :

$$s(x) = \begin{cases} +1 & 0 < x < L_s \\ -1 & L_s < x < 2L_s \end{cases}$$

The energy of the stripe structure per unit area  $U$  and the equilibrium stripe width  $L_s$  were calculated in Ref. 17. Here we correct a calculational mistake of that work:<sup>18,19</sup>

$$U = \frac{-16\tilde{m}^2}{\lambda_{eff}} \exp\left(\frac{-\tilde{\epsilon}_{dw}}{4\tilde{m}^2} + C - 1\right), \quad (1)$$

$$L_s = \frac{\lambda_{eff}}{4} \exp\left(\frac{\tilde{\epsilon}_{dw}}{4\tilde{m}^2} - C + 1\right). \quad (2)$$

The notations in Eqs. (1) and (2) are as follows:  $\lambda_{eff} = \lambda^2/d_s$  is the effective penetration depth of the SC film whose thickness is denoted  $d_s$ ,  $\lambda$  is the London penetration depth,  $\tilde{\epsilon}_{dw}$  is the renormalized linear tension of the domain wall,  $\epsilon_v = (\phi_0^2/16\pi^2\lambda_{eff})\ln(\lambda_{eff}/\xi)$  is the single vortex energy in the absence of the FM film;  $m$  is the magnetization per unit area of the FM film,  $\tilde{m} = m - \epsilon_v/\phi_0$  is the renormalized magnetization (due to the screening effect of vortices), and  $C \approx 0.57721$  is the Euler constant. To find the transition temperature, we combine the energy given by Eq. (1) with the Ginzburg-Landau free energy. The total free energy per unit area reads

$$F = U + F_{GL} = \frac{-16\tilde{m}^2}{\lambda_{eff}} \exp\left(\frac{-\tilde{\epsilon}_{dw}}{4\tilde{m}^2} + C - 1\right) + n_s d_s \left[ \alpha(T - T_c) + \frac{\beta}{2} n_s \right]. \quad (3)$$

Here  $\alpha$  and  $\beta$  are the Ginzburg-Landau parameters. We have omitted the gradient term in the Ginzburg-Landau equation since the gradient of the phase is included in the energy (1), whereas the gradient of the SC electron density can be neglected everywhere beyond the vortex cores. Recalling that  $\lambda^2 = m_s c^2 / 4\pi n_s e^2$  and plugging it into Eq. (3), we find the free energy as function of  $n_s$ ,  $T - T_c$ , and  $m$ . Note that

$$\tilde{m} = m + \frac{\phi_0 e^2 d_s n_s}{4\pi m_s c^2} \ln \frac{4\pi e^2 d_s n_s \xi}{m_s c^2}. \quad (4)$$

We expect that  $n_s$  is small near the transition point  $T_c$  and, therefore, retain only the linear in  $n_s$  part in the first term in Eq. (3). This term can be included in the Ginzburg-Landau free energy and resulting in a shift of the Ginzburg-Landau transition temperature:

$$F = n_s d_s \left[ \alpha(T - T_r) + \frac{\beta}{2} n_s \right], \quad (5)$$

where

$$T_r = T_c + \frac{64\pi m^2 e^2}{\alpha m_s c^2} \exp\left(\frac{-\tilde{\epsilon}_{dw}}{4m^2} + C - 1\right). \quad (6)$$

Minimizing the total free energy over  $n_s$ , we find the equilibrium value of  $n_s$  (for  $T < T_r$ ):  $n_s = -(\alpha/\beta)(T - T_r)$ . Substituting it back to Eq. (5), we find the equilibrium free energy

$$F = -\frac{\alpha^2(T - T_r)^2}{2\beta} d_s. \quad (7)$$

The SC phase is stable if its free energy (7) is less than the free energy of a single FM film with the stripe domain structure, which has the following form:<sup>20,21</sup>  $F_{fm} = -4m^2/L_f$ , where  $L_f$  is the stripe width of the single FM film. Near the SC transition point the temperature dependence of the variation of this magnetic energy is negligible. Hence, when  $T$  increases, the SC film transforms into a normal state at some temperature  $T_c^*$  below  $T_r$ . This is the first-order phase transition. At transition point both energies equal to each other:

$$\frac{\alpha^2(T_c^* - T_r)^2}{2\beta} d_s = \frac{4m^2}{L_f}. \quad (8)$$

Thus, the shift of the transition temperature is determined by a following equation:

$$T_c^* - T_c = \frac{64\pi m^2 e^2}{\alpha m_s c^2} \exp\left(\frac{-\tilde{\epsilon}_{dw}}{4m^2} + C - 1\right) - \sqrt{\frac{8\beta m^2}{\alpha^2 d_s L_f}}. \quad (9)$$

Two terms in Eq. (9) play opposite roles. The first one is due to the appearance of spontaneous vortices which lowers the free energy of the system and tends to increase the transition temperature. The second term is the contribution of the purely magnetic energy, which tends to decrease the transition temperature. The values of parameters entering Eq. (9) can be estimated as follows. The dimensionless Ginzburg-Landau parameter is  $\alpha = 7.04T_c/\epsilon_F$ , where  $\epsilon_F$  is the Fermi energy. A typical value of  $\alpha$  is about  $10^{-3}$  for low-temperature superconductors. The second Ginzburg-Landau parameter is  $\beta = \alpha T_c/n_e$ , where  $n_e$  is the electron density. For estimates we take  $T_c \sim 3$  K,  $n_e \sim 10^{23}$  cm<sup>3</sup>. The magnetization per unit area  $m$  is the product of the magnetization per unit volume  $M$  and the thickness of the FM film  $d_m$ . We accept a typical value of  $M \sim 10^2$  Oe and  $d_m \sim 10^2$  Å. Then  $m = 10^{-4}$  Gs/cm<sup>2</sup>. In an ultrathin thin magnetic film the observed values of  $L_f$  vary in the range 1 to 100 μm.<sup>22,23</sup> If  $L_f \sim 1$  μm,  $d_s = d_m = 10^2$  Å, and  $\exp(-\tilde{\epsilon}_{dw}/4m^2 + C - 1) \approx 10^{-3}$ , we obtain  $\Delta T_c/T_c \sim -0.03$ . For  $L_f = 100$  μm,  $d_s = 5 \times 10^2$  Å, and  $\exp(-\tilde{\epsilon}_{dw}/4m^2 + C - 1) \approx 10^{-2}$ , we find that  $\Delta T_c/T_c \sim 0.02$ .

### III. SPONTANEOUS STRIPE STRUCTURE IN AN EXTERNAL MAGNETIC FIELD

In this section we study the spontaneous stripe system in the FM-SC bilayer in the presence of an external perpendicular magnetic field  $\mathbf{B}$  (along the  $\hat{z}$  direction). Since the external magnetic field tends to align the magnetization parallel to itself, we anticipate that the width  $L_1$  of stripes with the magnetization parallel to the external magnetic field increases, whereas the width  $L_2$  of the stripes with the antiparallel magnetization decreases. Let us define a step function with the period  $L = L_1 + L_2$  as follows:

$$s(x) = \begin{cases} +1 & (0 < x < L_1) \\ -1 & (L_1 < x < L) \end{cases}.$$

The Fourier-transform of  $s(x)$  is

$$s_G = \begin{cases} 2i(1 - e^{iGL_1})/(LG) & (G \neq 0) \\ (L_1 - L_2)/L & (G = 0). \end{cases} \quad (10)$$

Here  $G = 2\pi r/L$  and  $r = 0, \pm 1, \pm 2, \dots$ . For the sake of brevity, we denote  $t = L_1 - L_2$ . At large distance from the bilayer the magnetic field asymptotically becomes equal to the external magnetic field. The total magnetic flux is the same in any cross section of the space. Thus, the average magnetic field through the SC layer is

$$\frac{\phi_0}{L} \int_0^L n(x) dx = B_{ext}, \quad (11)$$

where  $n(x)$  is the density of vortices. The general expression for the free energy of a periodic stripe system of magnetization and vortices is given by Eq. (10) of the work.<sup>17</sup> Employing this equation and the Fourier expansion for the step function  $s(x)$  [see Eq. (10)] and denoting  $n_G$  the Fourier transform of the vortex density  $n(x)$ , we obtain:

$$U_v = \sum_G \tilde{\epsilon}_v s_G n_{-G} + \frac{1}{2} \sum_{G \neq 0} V_G n_G n_{-G}, \quad (12)$$

where  $\tilde{\epsilon}_v = \epsilon_0 - m\phi_0$  is the renormalized energy of a vortex.  $V_G = \phi_0^2/(2\pi|G|)$  is the Fourier transform of the vortex interaction energy. An infinitely large term  $V_{G=0}n_{G=0}^2$  has been omitted since it corresponds to the energy of the external magnetic field. From Eq. (12) we readily find that the constraint condition implies

$$n_{G=0} = \frac{B_{ext}}{\phi_0}. \quad (13)$$

This equation confirms that  $V_{G=0}n_{G=0}^2$  is the energy of the uniform external field. Minimization of the total vortex energy  $U_v$  over the vortex density  $n_G$  results in equation:

$$\tilde{\epsilon}_v s_G + V_G n_G = 0 \quad (G \neq 0). \quad (14)$$

Plugging the solutions  $n_G$  from Eqs. (13) and (14) into Eq. (12) and adding the energy of domain walls, we arrive at the following expression for the total energy per unit area:

$$\begin{aligned} \tilde{U} = & \frac{-8\tilde{m}^2}{L} \left[ C + \ln \frac{L}{\lambda_{eff}} + \frac{1}{2} \ln \left( 2 + 2 \cos \frac{\pi t}{L} \right) \right] \\ & - \frac{\tilde{m} B_{ext} t}{L} + \frac{2\epsilon_{dw}}{L}. \end{aligned} \quad (15)$$

Minimizing the total energy  $U$  over  $L$  and  $t$ , we find the equilibrium values of  $L$  and  $t$ :

$$L = \frac{2L_s}{\sqrt{1 - \left( \frac{L_s B_{ext}}{2\pi\tilde{m}} \right)^2}}, \quad (16)$$

$$t = \frac{2L}{\pi} \arctan \frac{L B_{ext}}{4\pi\tilde{m}}, \quad (17)$$

where  $L_s$  is given by Eq. (2). The results of Eqs. (16) and (17) are similar to those for a purely FM stripe structure in a single FM film.<sup>24</sup> The critical external field  $B_{ext}^c$  at which the domain structure vanishes is

$$B_{ext}^c = 2\pi\tilde{m}/L_s. \quad (18)$$

It varies in the range of 1–10 Oe.

In the end of this section, we consider how the SC transition temperature of the bilayer changes in the presence of external magnetic field. Since at a field  $B_{ext}^c \sim 1-10$  Oe the stripe structure vanishes, the SC transition proceeds in the homogeneous state of FM film excluding very small vicinity of  $T_c$ . Therefore, it is determined by the same nucleation process as in the case of a single SC film. The nucleation in a thin film for the field perpendicular to it was considered by Tinkham.<sup>25</sup> Though the geometry is different from the bulk geometry considered by Abrikosov,<sup>26</sup> his solution can be applied directly. The order parameter coincides with the Landau wave function for the first Landau level. In the case of the bilayer the energy of the nucleus reads

$$U = \int \left[ \frac{1}{2m} \left| \left( \frac{\hbar}{i} \nabla - \frac{2e}{c} \mathbf{A}_0 \right) \psi \right|^2 + a|\psi|^2 \right] d^2x + \Delta U. \quad (19)$$

Here  $\mathbf{A}_0$  is the vector potential produced by the critical field  $\mathbf{H}_{c2}$ . The nucleus energy (19) differs from that in the absence of magnetic field by the value  $\Delta U = -m \int B_z^{(n)} d^2x$ , where  $B_z^{(n)}$  is the magnetic field generated by the nucleus at the FM film. We will prove that this additional term is equal to zero. Indeed, the magnetic field generated by the nucleus reads

$$\mathbf{B}^{(n)}(\mathbf{x}) = \frac{1}{c} \int \nabla \frac{1}{|\mathbf{x} - \mathbf{x}'|} \times \mathbf{j}_n(\mathbf{x}') d^3x', \quad (20)$$

where  $\mathbf{x}'$  is a point inside SC film whose thickness will be put zero in the end,  $\mathbf{x}$  denotes a point in the FM film. We assume that the current flows in the  $x$ - $y$  plane. Since it has zero divergence, it can be represented as  $\mathbf{j}_n = \tilde{z} \times \nabla' f(x', y')$ , where  $f(x', y')$  is a function localized in a finite part of the SC film. The flux of the induced field is

$$\int B_z^{(n)} d^2x = \frac{1}{c} \int \left( \hat{z} \times \nabla \frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) [\hat{z} \times \nabla' f(\mathbf{x}')] d^2x d^3x'. \quad (21)$$

A simple transformation turns this integral into a following form:

$$\int B_z^{(n)} d^2x = \frac{1}{c} \int f(\mathbf{x}') \nabla^2 \frac{1}{|\mathbf{x} - \mathbf{x}'|} d^2x d^3x'. \quad (22)$$

This integral is equal to zero if  $\mathbf{x}$  and  $\mathbf{x}'$  belong to different films. Thus, the interaction between the SC nucleus and the homogeneously magnetized film is zero independently on the wave function of the localized nucleus. Therefore, the transition temperature is the same as that in the absence of the FM film.

Let the external magnetic field  $B_{ext}$  equal to the value, at which the stripe structure in a single FM film vanishes  $B_c = 2\pi m/L_f$ .<sup>24</sup> In the interval of magnetic field  $B_c < B < H_{c2}$  the shift of the transition temperature is the same as in the absence of the FM layer  $\Delta T_c/T_c = B/H_{c2}$ . The typical value of  $B_c$  is  $\sim 1-10$  Oe. On the other hand, the second critical field for the SC film at  $T=T_c^*$  can be estimated as  $H_{c2}(T_c^*) = H_{c2}(T=0)|T_c - T_c^*|/T_c \sim 100$  Oe. Hence  $B_c < H_{c2}(T_c^*)$ . It confirms our assumption that the FM film remains homogeneous at the SC transition. From the formulas  $T_c^* = T_c(1 - B_{ext}/H_{c2})$  and  $B_c = 2\pi m/L_f$  we find the shift of the transition temperature due to  $B_c$  is  $|\Delta T_c|/T_c = B_c/H_{c2}(0) \sim 10^{-3} - 10^{-2}$ . For large  $L_f$  the sensitivity of the shift of the transition temperature to the magnetic field can be rather strong.

#### IV. TRANSITION TEMPERATURE IN A SC FILM WITH A SQUARE ARRAY OF FM DOTS

Recently Erdin considered theoretically the vortex-antivortex textures in SFMD.<sup>27</sup> For the case that only one vortex and one antivortex appear per a magnetic dot, he predicted a symmetry violation in the lowest energy state in a range of parameters. For simplicity we choose another range of parameters in which no symmetry violation proceeds: the vortex centers are located precisely under the centers of the magnetic dots, whereas the antivortex centers are located between them in the centers of elementary cells. Let us assume each dot to be a circular thin disk with a radius  $R$  and a constant magnetization  $m$  per unit area with a direction perpendicular to the plane (along the  $z$  axis). Let  $a$  denote the dot lattice constant. The total energy per unit area of the system is<sup>27</sup>

$$U = u_{vv} + u_{mv} + u_{mm}. \quad (23)$$

The three terms in the right-hand side of the above equation have the following forms:

$$u_{vv} = \frac{\phi_0^2}{4\pi a^4} \sum_{\mathbf{G}} \frac{|F_{\mathbf{G}}|^2}{G(1+2\lambda_{eff}G)}, \quad (24)$$

$$u_{mv} = -\frac{\phi_0}{a^2} \sum_{\mathbf{G}} \frac{m_z \mathbf{G} F_{-\mathbf{G}}}{1+2\lambda_{eff}G}, \quad (25)$$

$$u_{mm} = -2\pi\lambda_{eff} \sum_{\mathbf{G}} \frac{G^2 |\mathbf{m}_z \mathbf{G}|^2}{1+2\lambda_{eff}G}. \quad (26)$$

where  $\mathbf{G} = (2\pi/a)(r, s)$  ( $r, s$  are integers) are the reciprocal lattice vectors;  $F_{\mathbf{G}} = \sum_i n_i e^{i\mathbf{G} \cdot \mathbf{r}_i}$  is the structure factor of the vortex lattice;  $n_i$  and  $\mathbf{r}_i$  indicate the vorticity and the position of the  $i$ th vortex in the elementary cell, respectively. In the purely magnetic term  $u_{mm}$  it is necessary to perform a regularization since only the difference between energies of the SC and normal-state matters:

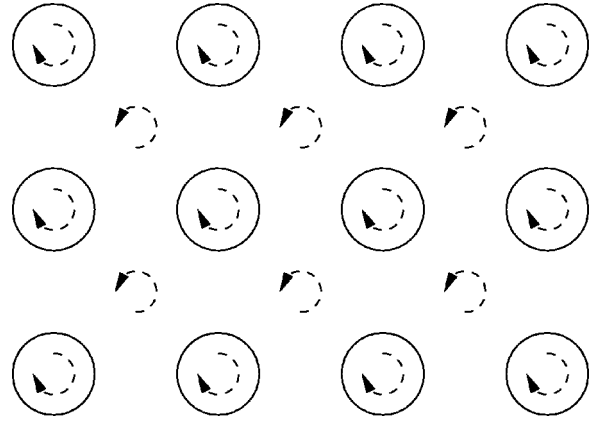


FIG. 1. Schematics representation of FM dots with spontaneous vortices and antivortices. The circles drawn by solid line represent FM dots. The dash half circles with clockwise and counterclockwise arrows indicate vortices and antivortices, respectively.

$$u_{mm} \rightarrow \tilde{u}_{mm} = u_{mm}(\lambda_{eff}) - u_{mm}(\lambda_{eff} = \infty). \quad (27)$$

The last term in the right-hand side of Eq. (27) is the dipolar energy of the FM dots above the SC transition (Fig. 1). At temperature below the SC transition the magnetic field generated by the dots penetrates into the SC film and creates vortices and antivortices if the magnetization and the size of the dots are large enough.<sup>16</sup> Keeping in mind that  $\lambda_{eff} \gg a$  near the new transition temperature  $T_c^*$ , we can rewrite the total energy Eq. (23) as follows:

$$u = \frac{\phi_0^2 e^2 d_s n_s}{2\pi m_s c^2 a^2} \ln \frac{a}{\xi} - \frac{\phi_0^2 e^4 d_s^2 n_s^2}{4\pi^2 m_s^2 c^4 a} I_0 - \frac{\phi_0^2 e^2 d_s n_s}{4\pi^2 m_s c^2 a^2} \times \left( I_1 + \frac{4\pi^2 m R}{\phi_0} I_2 \right) + \frac{2\pi^2 m^2 e^2 d_s n_s R^2}{m_s c^2 a^2} I_3, \quad (28)$$

where  $\Sigma'$  means that the term  $r=s=0$  is omitted.  $I_1$ ,  $I_2$ , and  $I_3$  are defined as series:

$$I_0 = \sum_{n,s=-\infty}^{+\infty'} \frac{1}{(n^2 + s^2)^{3/2}},$$

$$I_1 = \sum_{n,s=-\infty}^{+\infty'} \frac{(-1)^n + (-1)^s}{n^2 + s^2},$$

$$I_2 = \sum_{n,s=-\infty}^{+\infty'} \frac{J_1\left(\frac{2\pi R}{a} \sqrt{n^2 + s^2}\right) [1 - (-1)^{n+s}]}{n^2 + s^2},$$

$$I_3 = \sum_{n,s=-\infty}^{+\infty'} \frac{J_1^2\left(\frac{2\pi R}{a} \sqrt{n^2 + s^2}\right)}{n^2 + s^2}. \quad (29)$$

We combine this energy with the Ginzburg-Landau free energy for the SC film as it was done in Sec. II:

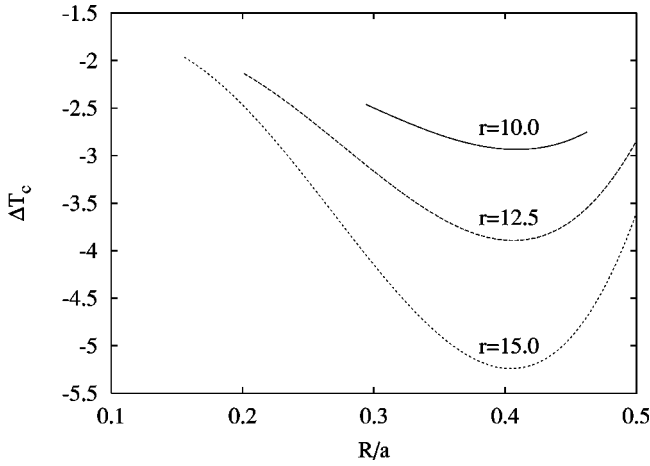


FIG. 2.  $\Delta T_c$  vs  $R$  for  $\xi=0.21a$ , respectively, for  $r=10.0$ ,  $12.5$ , and  $15.0$ , here  $r=4\pi^2 ma/\phi_0$ .  $\Delta T_c$  is in the unit  $\hbar^2/4\alpha m_s a^2$ , which is about  $0.02$  K for  $\alpha=10^3$  and  $a=3$   $\mu\text{m}$ .

$$\begin{aligned}
 F = & \frac{\phi_0^2 e^2 d_s n_s}{2\pi m_s c^2 a^2} \ln \frac{a}{\xi} - \frac{\phi_0^2 e^4 d_s^2 n_s^2}{4\pi^2 m_s^2 c^4 a} I_0 - \frac{\phi_0^2 e^2 d_s n_s}{4\pi^2 a^2 m_s c^2} \\
 & \times \left( I_1 + \frac{4\pi^2 m R}{\phi_0} I_2 \right) + \frac{2\pi^2 m^2 e^2 d_s n_s R^2}{m_s c^2 a^2} I_3 \\
 & + \left[ \alpha(T - T_c) + \frac{\beta}{2} n_s \right] n_s d_s. \quad (30)
 \end{aligned}$$

The condition of minimum over  $n_s$  from the free energy, Eq. (30), reads:

$$\begin{aligned}
 & \frac{\phi_0^2 e^2 d_s}{2\pi m_s c^2 a^2} \ln \frac{a}{\xi} - \frac{\phi_0^2 e^4 d_s^2 n_s}{2\pi^2 m_s^2 c^4 a} I_0 - \frac{\phi_0^2 e^2 d_s}{4\pi^2 a^2 m_s c^2} \\
 & \times \left( I_1 + \frac{4\pi^2 m R}{\phi_0} I_2 \right) + \frac{2\pi^2 m^2 e^2 d_s R^2}{m_s c^2 a^2} I_3 \\
 & + \alpha(T - T_c) d_s + \beta n_s d_s = 0. \quad (31)
 \end{aligned}$$

At a new critical temperature  $T_c^*$  the density of SC carriers must be zero. Plugging  $n_s(T_c^*)=0$  into Eq. (31), we obtain the shift of the critical temperature:

$$\Delta T_c = \frac{\hbar^2}{4\alpha m_s a^2} \left( \frac{4\pi^2 m R}{\phi_0} I_2 + I_1 - 2\pi \ln \frac{a}{\xi} - \frac{8\pi^4 m^2 R^2}{\phi_0^2} I_3 \right). \quad (32)$$

Figure 2 shows the relation between  $\Delta T_c$  and  $R$  for  $\xi=0.21a$ . To ensure spontaneous occurrence of the vortices the inequality  $u_{mv} + u_{vv} < 0$  must be satisfied. It is equivalent to the following relation:

$$\frac{4\pi^2 m R}{\phi_0} I_2 + I_1 - 2\pi \ln \frac{a}{\xi} < 0. \quad (33)$$

The London's approximation is valid if  $\xi \ll a$ . This condition is violated in a close vicinity of the transition temperature. For  $a \sim 3$   $\mu\text{m}$  and  $\xi(T=0) = 0.1$   $\mu\text{m}$  this vicinity is of the

order of  $0.001T_c$  and further we neglect it. Figure 2 shows that the shift of transition temperature is a rather complicated function of the dots radius  $R$  and the ratio  $r=4\pi^2 ma/\phi_0$ . For each value  $r$ , there exists a threshold radius  $R_0$ , at which the vortices first appear. The shift of the transition temperature grows by absolute value with  $R$  increasing, reaches a maximum at  $R/a \approx 0.4$  and then decreases. It remains negative at any  $R$  in the interval between  $R_0$  and  $a/2$ . At a fixed  $R > R_0$  the absolute value of  $\Delta T_c$  increases with the ratio  $r$  and is negative.

## V. FM TEXTURES IN THE MULTILAYERS

We consider a FM-SC multilayer system consisting of  $N$  bilayers with a distance  $d$  between two neighboring ones. Let us start with the limit  $Nd \gg R_s$ , where  $R_s$  is the lateral linear size of a layer. If the magnetic films are magnetized perpendicularly to layers, the average induction inside the multilayer is  $B=4\pi m/d$  and its direction is perpendicular to the layers. The situation is the same as in a layered superconductor placed into an external magnetic field.<sup>28</sup> Therefore, pancake vortices in each SC layer may appear. Together they form the Abrikosov linear vortices if a condition  $m\phi_0/d > \epsilon_a$  is satisfied, which guarantees that the vortex line is energy favorable. Here  $\epsilon_a = \epsilon_0 \ln \lambda / \xi$  is the vortex line energy per unit length,<sup>29</sup> and  $\epsilon_0 = \phi_0^2 / (4\pi \lambda)^2$ . There is no need to consider the Josephson coupling effect in this case since the phase difference between SC layers is zero if the vortex lines are perpendicular to the layers. On the other hand, the Josephson vortices appear along the layers if the magnetization  $\mathbf{m}$  is parallel to the layers and satisfy a condition  $m\phi_0/d > \epsilon_J$ , where  $\epsilon_J = \gamma \epsilon_0 \ln \lambda / d$  is the Josephson vortex line energy and  $\gamma$  is the anisotropy parameter for the layered superconductor.<sup>28</sup> These ideas were applied by M. Houzet *et al.*<sup>30</sup> to explain properties of the magnetic superconductor  $\text{RuSr}_2\text{GdCu}_2\text{O}_8$ . We will focus on a FM-SC multilayer in the opposite limit  $Nd \ll \Lambda \ll R_s$ , where  $\Lambda = \lambda^2/d$  is the effective penetration depth for layered superconductors. In such a multilayer one should expect spontaneous vortices and antivortices combined with the domains in the FM films for the same reason as in the case of a single FM-SC bilayer.<sup>17</sup>

We first analyze a multilayer superconductor without any FM texture. Pancake vortices in a finite stack of layers were discussed by Mints *et al.*<sup>31</sup> We reproduce here some of their results and derive new ones, substantial for our purposes by applying a modified approach proposed by Efetov<sup>32</sup> (see also Ref. 33) (they considered a layered superconductor with infinite number of layers). To simplify the calculation, we assume that layers are infinitely thin and located at the planes  $z_n = nd$  ( $n$  is an integer). The vector potential  $\mathbf{A}_v$  due to the pancake vortices at SC layers satisfies a following equation:

$$\begin{aligned}
 & -\Delta \mathbf{A}_v(\boldsymbol{\rho}, z) + \frac{1}{\Lambda} \sum_n \delta(z - z_n) \mathbf{A}_v(\boldsymbol{\rho}, z) \\
 & = \frac{\phi_0}{2\pi \Lambda} \sum_n \delta(z - z_n) \sum_{n_p} \delta_{n_p} \nabla^{(2)} \varphi_n(\boldsymbol{\rho} - \boldsymbol{\rho}_{n_p}). \quad (34)
 \end{aligned}$$

The vector potential in Eq. (34) is induced by pancake vortices with the vorticity  $\delta_{np} = \pm 1$  placed at the position  $\boldsymbol{\rho}_{np}$ , where  $p$  enumerates vortices in the  $n$ th plane. The Coulomb gauge  $\nabla \cdot \mathbf{A}_v = 0$  is used. In addition,  $A_{vz} = 0$  because the direction of  $\nabla^{(2)} \varphi_n$  is along the layers. It is useful to introduce an auxiliary potential  $\tilde{\mathbf{A}}_v(\boldsymbol{\rho}, z) = \sum_n \delta(z - z_n) \mathbf{A}_v(\boldsymbol{\rho}, z)$  confined to the layers, the ‘‘London vector’’<sup>33</sup>  $\boldsymbol{\phi}_n(\boldsymbol{\rho}) = \sum_{n,p} \delta_{np} (\phi_0/2\pi) \nabla^{(2)} \varphi_n(\boldsymbol{\rho} - \boldsymbol{\rho}_{np})$ , and the corresponding auxiliary vector  $\tilde{\boldsymbol{\phi}}_n(\boldsymbol{\rho}, z) = \sum_n \boldsymbol{\phi}_n(\boldsymbol{\rho}) \delta(z - z_n)$ . In terms of these variables Eq. (34) can be rewritten as follows:

$$-\Delta \mathbf{A}_v(\boldsymbol{\rho}, z) + \frac{1}{\Lambda} \tilde{\mathbf{A}}_v(\boldsymbol{\rho}, z) = \frac{1}{\Lambda} \tilde{\boldsymbol{\phi}}_n(\boldsymbol{\rho}, z). \quad (35)$$

Equation (35) can be solved by the Fourier transformation. An intermediate result following directly from Eq. (35) reads

$$\mathbf{A}_v(\mathbf{q}, k) = \sum_n e^{-ikz_n} \frac{\boldsymbol{\phi}_n(\mathbf{q}) - \mathbf{A}_{vn}(\mathbf{q})}{\Lambda(q^2 + k^2)}, \quad (36)$$

where  $\mathbf{A}_v(\mathbf{q}, k)$  is the Fourier transform of the vector potential  $\mathbf{A}_v(\boldsymbol{\rho}, z)$ ,  $\mathbf{A}_{vn}(\mathbf{q})$  is the plane Fourier-transform of the vector-potential  $\mathbf{A}_v(\boldsymbol{\rho}, z_n)$  taken at the  $n$ th SC plane, and  $\boldsymbol{\phi}_n(\mathbf{q})$  is the Fourier-transform of the London vector  $\boldsymbol{\phi}_n(\boldsymbol{\rho})$ . Performing the inverse Fourier transform with respect to the variable  $k$  in both sides of Eq. (36), we find a system of equations for  $\mathbf{A}_{vn}(\mathbf{q})$  at a fixed value of  $\mathbf{q}$  for each  $m$ :

$$\sum_n \left( \frac{1}{2\Lambda q} e^{-q|m-n|d} + \delta_{mn} \right) \mathbf{A}_{vn}(\mathbf{q}) = \frac{1}{2\Lambda q} \sum_n \boldsymbol{\phi}_n(\mathbf{q}) e^{-q|m-n|d}. \quad (37)$$

We apply Eq. (37) to study the simplest case: two SC layers. Let only one pancake vortex be placed in the center of the layer  $z=0$  at  $\boldsymbol{\rho}_1=0$ . The other layer is located at  $z=d$  with-out vortices on it. The solution of Eq. (37) for this situation reads

$$\begin{aligned} \mathbf{A}_{v1}(\mathbf{q}) &= \frac{1 + 2\Lambda q - e^{-2qd}}{1 + 4\Lambda q + 4\Lambda^2 q^2 - e^{-2qd}} \boldsymbol{\phi}_1(\mathbf{q}), \\ \mathbf{A}_{v2}(\mathbf{q}) &= \frac{2\Lambda q e^{-2qd}}{1 + 4\Lambda q + 4\Lambda^2 q^2 - e^{-2qd}} \boldsymbol{\phi}_1(\mathbf{q}). \end{aligned} \quad (38)$$

Here  $\boldsymbol{\phi}_1(\mathbf{q}) = i\phi_0/q \hat{\boldsymbol{\phi}}$  and  $\hat{\boldsymbol{\phi}} = \hat{z} \times \hat{\boldsymbol{q}}$ . In the limit  $qd \ll 1$  the above solution becomes simple:

$$\mathbf{A}_{v1}(\mathbf{q}) = \mathbf{A}_{v2}(\mathbf{q}) = \frac{1}{2 + 2\Lambda q} \boldsymbol{\phi}_1(\mathbf{q}). \quad (39)$$

The current density in each layer is given by

$$\begin{aligned} \mathbf{J}_1(\mathbf{q}) &= \frac{c}{4\pi\Lambda} [\boldsymbol{\phi}_1(\mathbf{q}) - \mathbf{A}_{v1}(\mathbf{q})], \\ \mathbf{J}_2(\mathbf{q}) &= -\frac{c}{4\pi\Lambda} \mathbf{A}_{v2}(\mathbf{q}). \end{aligned} \quad (40)$$

TABLE I. The asymptotic form of the current density in each layer.

	$\rho \gg \Lambda$	$d \ll \rho \ll \Lambda$
$\mathbf{J}_1(\rho)$	$\frac{\phi_0 c}{16\pi^2 \Lambda \rho} \hat{\boldsymbol{\phi}}$	$\frac{\phi_0 c}{8\pi^2 \Lambda \rho} \hat{\boldsymbol{\phi}}$
$\mathbf{J}_2(\rho)$	$-\frac{\phi_0 c}{16\pi^2 \Lambda \rho} \hat{\boldsymbol{\phi}}$	$-\frac{\phi_0 c}{4\Lambda^2} \hat{\boldsymbol{\phi}}$

The asymptotic formulas for the current density in the coordinate representation are shown in the Table I. The force acting between two pancake vortices is  $\mathbf{F} = -\phi_0/c \hat{z} \times \mathbf{J}$ , where  $\mathbf{J}$  is the current produced by one of them at the center of another one. Table I demonstrates that the interaction energy between two pancakes with the same vorticity at the same layer is logarithmic and repulsive at large distance  $R \gg \Lambda$  and at small distance  $d \ll R \ll \Lambda$ , but with different coefficients in front of the logarithm. A peculiarity of the two-layer structure is that the interaction energy of two pancake vortices with the same vorticity located in different layers and separated by the lateral distance  $R \gg \Lambda$ , is logarithmic but attractive. It has the same absolute value as the repulsion of two pancake vortices in the same layer. It can be interpreted as the attraction of two ‘‘half-vortices’’ in the two plane, one carrying the flux  $+\phi_0/2$ , the other carrying the flux  $-\phi_0/2$ . This interaction dramatically differs from the interaction of two vortices in different layers for an infinite number of layers. In the latter case the interaction in different layers is weaker than the interaction in the same layer by a small prefactor  $d/\lambda$ . It can be shown that the logarithmic attraction of two pancakes in different layers with distance  $R \gg \Lambda$  persists at any number of layers  $N$  provided  $Nd \ll \Lambda$ .

In the two-layer system the asymptotic for the components of the magnetic field produced by a pancake vortex located in the plane  $z=0$  at its origin directly follow from Eq. (38). In the range  $\rho \gg \Lambda$  they are

$$\begin{aligned} B_z(\rho, z) &= \frac{\phi_0}{8\pi\Lambda} \left[ \frac{1}{\sqrt{\rho^2 + z^2}} - \frac{1}{\sqrt{\rho^2 + (z-d)^2}} \right] \\ &+ \frac{\phi_0}{8\pi} \left[ \frac{|z|}{(z^2 + \rho^2)^{3/2}} + \frac{|z-d|}{[(z-d)^2 + \rho^2]^{3/2}} \right], \end{aligned}$$

$$\begin{aligned} B_\rho(\rho, z) &= \frac{\phi_0}{8\pi\Lambda\rho} \text{sgn}(z) \left( 1 - \frac{|z|}{\sqrt{\rho^2 + z^2}} \right) - \frac{\phi_0}{8\pi\Lambda\rho} \text{sgn}(z-d) \\ &\times \left( 1 - \frac{|z-d|}{\sqrt{\rho^2 + (z-d)^2}} \right) \\ &+ \frac{\phi_0}{8\pi} \left[ \frac{z}{(\rho^2 + z^2)^{3/2}} + \frac{z-d}{[\rho^2 + (z-d)^2]^{3/2}} \right]. \end{aligned}$$

In another region  $d \ll \rho \ll \Lambda$  we find

$$B_z(\rho, z) = \frac{\phi_0}{4\pi\Lambda\sqrt{\rho^2+z^2}},$$

$$B_\rho(\rho, z) = \frac{\phi_0}{4\pi\Lambda\rho} \operatorname{sgn}(z) \left( 1 - \frac{|z|}{\sqrt{\rho^2+z^2}} \right).$$

Due to the strong screening effect exerted by one layer onto another, the magnetic field decays more quickly in the  $z$  direction than in plane (the  $\rho$  direction). The total magnetic flux through the plane  $z=0$  and  $z=d$  are  $\Phi(z=0) = B_z(\mathbf{q}=0, z=0) = (\Lambda + d/2\Lambda + d)\phi_0 \approx \phi_0/2$ , and  $\Phi(z=d) = B_z(\mathbf{q}=0, z=d) = (\Lambda/2\Lambda + d)\phi_0 \approx \phi_0/2$ , respectively. The two fluxes are not exactly equal, and the net flux  $\phi_0 d/(2\Lambda + d)$  escapes through the remote side surface.

The self-energy of a single pancake vortex reads

$$E_{sv} = \frac{1}{8\pi\Lambda} \int \frac{d^2q}{(2\pi)^2} [|\boldsymbol{\phi}_1(\mathbf{q})|^2 - \boldsymbol{\phi}_1(-\mathbf{q}) \cdot \mathbf{A}_{v1}(\mathbf{q})]$$

$$= \frac{1}{8\pi\Lambda} \int \frac{d^2q}{(2\pi)^2} \left[ \frac{\phi_0^2}{q^2} - \frac{\phi_0^2}{2q^2(1+\Lambda q)} \right]$$

$$= \frac{\phi_0^2}{32\pi^2\Lambda} \ln \frac{R_s\Lambda}{\xi^2}, \quad (41)$$

where  $R_s$  is the lateral linear size of the layers as mentioned before. We see that  $E_{sv}$  diverges logarithmically when  $R_s$  goes to infinity. Thus, it is energy unfavorable to produce a single pancake vortex in a layer below the Berezinsky-Kosterlitz-Touless transition. The energy of a pair of pancake vortices located one opposite the other at different planes is

$$E_{lv} = \frac{2}{8\pi\Lambda} \int \frac{d^2q}{(2\pi)^2} \{ |\boldsymbol{\phi}_1(\mathbf{q})|^2 - \boldsymbol{\phi}_1(-\mathbf{q}) \times [\mathbf{A}_{v1}(\mathbf{q}) + \mathbf{A}_{v2}(\mathbf{q})] \}$$

$$= \frac{1}{4\pi\Lambda} \int \frac{d^2q}{(2\pi)^2} \left[ \frac{\phi_0^2}{q^2} - \frac{\phi_0^2}{q^2(1+\Lambda q)} \right]$$

$$= \frac{\phi_0^2}{8\pi^2\Lambda} \ln \frac{\Lambda}{\xi}. \quad (42)$$

The interaction energy of two such pairs separated by a distance  $R \gg d$  is

$$V_{ll}(R) = \frac{2}{8\pi\Lambda} \int \frac{d^2q}{(2\pi)^2} \{ |\tilde{\boldsymbol{\phi}}_1(\mathbf{q})(1 + e^{-i\mathbf{q}\cdot\mathbf{R}})|^2 - \boldsymbol{\phi}_1 \times (-\mathbf{q}) \cdot [\mathbf{A}_{v1}(\mathbf{q}) + \mathbf{A}_{v2}(\mathbf{q})|1 + e^{-i\mathbf{q}\cdot\mathbf{R}}|^2] \} - 2E_{lv}$$

$$= \frac{\phi_0^2}{4\pi^2} \int \frac{J_0(qR)}{1+\Lambda q} dq = \frac{\phi_0^2}{8\pi\Lambda} \left[ \mathbf{H}_0\left(\frac{R}{\Lambda}\right) - N_0\left(\frac{R}{\Lambda}\right) \right]. \quad (43)$$

In the last step we have used the formula:<sup>34</sup>

$$\int_0^\infty \frac{1}{x+z} J_0(cx) dx = \frac{\pi}{2} [\mathbf{H}_0(cz) - N_0(cz)], \quad (44)$$

where  $\mathbf{H}_0(x)$  is the zeroth Struve function and  $N_0(x)$  is the zeroth Neumann function. The asymptotic form of the interaction energy (43) is as follows:

$$V_{ll}(R) = \begin{cases} \frac{\phi_0^2}{4\pi^2\Lambda} \ln \frac{\Lambda}{R} & (d \ll R \ll \Lambda) \\ \frac{\phi_0^2}{4\pi^2 R} & (R \gg \Lambda). \end{cases} \quad (45)$$

Equation (37) can be solved by the same method for any number of layers, though calculations become more cumbersome. However, in the region  $R \gg Nd$  Eq. (37) can be solved quite easily. The vector potential of a pancake vortex, identical at all layers read

$$\mathbf{A}_{v1}(\mathbf{q}) = \dots = \mathbf{A}_{vN}(\mathbf{q}) = \frac{i\phi_0 \hat{z} \times \hat{q}}{q(N+2\Lambda q)}. \quad (46)$$

Equation (46) allows to calculate the magnetic field, the current, and the interaction energy. Specifically, the single linear self-energy and the interaction energy of two linear vortices for an  $N$  multilayer superconductor are

$$E_{lv} = \frac{N\phi_0^2}{16\pi^2\Lambda} \ln \frac{\Lambda}{\xi}, \quad (47)$$

$$V_{ll}(R) = \begin{cases} \frac{N\phi_0^2}{8\pi^2\Lambda} \ln \frac{\Lambda}{R} & (Nd \ll R \ll \Lambda) \\ \frac{\phi_0^2}{4\pi^2 R} & (R \gg \Lambda). \end{cases} \quad (48)$$

We see that the energy of a single linear vortex in a  $N$ -layers SC system is proportional to the number of the layer  $N$ . The interaction energy between two linear vortices is  $N$  times stronger than the corresponding form for two Pearl's vortices at a short distance if we replace  $\Lambda$  by  $\lambda_{eff}$ , but at a long distance, the interaction energy has the same form as that for the Pearl vortices.

Next, we discuss FM textures in a multilayer system. We assume that the SC and FM layers form very thin bilayers separated by a finite distance  $d$ . The London-Pearl equation for the vector potential  $\mathbf{A}_m$  induced by the magnetic layers and screened by the SC layers is

$$-\Delta \mathbf{A}_m(\boldsymbol{\rho}, z) + \frac{1}{\Lambda} \sum_n \delta(z - z_n) \mathbf{A}_m(\boldsymbol{\rho}, z) = 4\pi \sum_n \nabla \times [\mathbf{m} \delta(z - z_n)]. \quad (49)$$

Comparing it with Eq. (34), we find that they become identical if we replace  $i\phi_0 \hat{z} \times \hat{q}/q$  by  $i4\pi m_q \Lambda \hat{z} \times \mathbf{q}$  after Fourier transform. Therefore, it is straightforward to obtain the result

for the magnetic vector potential from the vector potential induced by vortices. The Fourier transform of the vector potential at each layer produced by an FM texture, identical in each plane, reads

$$\mathbf{A}_{m1}(\mathbf{q}) = \dots = \mathbf{A}_{mN}(\mathbf{q}) = \frac{i4\pi\Lambda m_q \hat{z} \times \mathbf{q}}{N + 2\Lambda q}. \quad (50)$$

Equations (46) and (50) allow to calculate the interaction energy of FM textures and vortex-ferromagnet interaction energy for a given magnetic texture.

Let us consider the spontaneous stripe vortex-domain structure in a  $N$ -layer FM-SC, assuming as before that both the stripe width  $L'_s$  and the distances between vortices are much larger than  $\Lambda$ . As we mentioned before, the interaction energy between two linear vortices has the same form as in a single layer, but the energy of a linear vortex is proportional to  $N$ . The vortex-ferromagnet interaction energy is also proportional to  $N$ . That means that the condition required for spontaneous formation of vortices and antivortices remains the same as for the bilayer

$$m\phi_0 > \epsilon'_v, \quad (51)$$

where  $\epsilon'_v = \phi_0^2/16\pi^2\Lambda \ln\Lambda/\xi$ . A consideration similar to that of Secs. II and III leads to following results. The equilibrium domain width for a  $N$  layer is

$$L'_s = \frac{\Lambda}{4} \exp\left(\frac{\tilde{\epsilon}_{dw}}{4N\tilde{m}_l^2} - C + 1\right), \quad (52)$$

where  $\tilde{m}_l = m - \epsilon'_v/\phi_0$ . The factor  $1/N$  in the exponent (52) significantly reduces the domain width. The total width of parallel and antiparallel domains in an external magnetic field (the period of the domain structure) is

$$L'(B_{ext}) = \frac{2L'_s}{\sqrt{1 - \left(\frac{L'_s B_{ext}}{2N\pi\tilde{m}_l}\right)^2}}. \quad (53)$$

The difference of the widths of parallel and antiparallel domains in an external magnetic field reads

$$t' = \frac{2L'_s}{\pi} \arctan \frac{L'_s B_{ext}}{4N\pi\tilde{m}_l}. \quad (54)$$

The critical field, at which the stripe structure vanishes follows from Eq. (53):

$$B_{ext}^{c'} = \frac{2N\pi\tilde{m}_l}{L'_s}. \quad (55)$$

Note that it increases with the number of layers  $N$ . The shift of the transition temperature  $\Delta T'_c$  in the multilayer case is

$$T_c^* - T_c = \frac{64N\pi m^2 e^2}{\alpha m_s c^2} \exp\left(\frac{-\tilde{\epsilon}_{dw}}{4Nm^2} + C - 1\right) - N \sqrt{\frac{8\beta m^2}{\alpha^2 d L'_f}}. \quad (56)$$

Here  $L'_f$  is the stripe width for the  $N$ -layer consisting only of FM films, i.e. without any SC film. This length is proportional to a modified exponent:  $L'_f \propto \exp(-\epsilon_{dw}/4Nm^2)$ , which can be obtained similarly to Eq. (53). Thus, the second term in Eq. (56) is proportional to  $N \exp(-\epsilon_{dw}/8Nm^2)$ , whereas the first term is proportional to  $N \exp(-\epsilon_{dw}/4Nm^2)$ . Even if the second term in Eq. (56) dominates at small  $N$  and  $\Delta T_c$  is negative, it can change sign at larger  $N$  provided a following inequality is true:  $(2^{9/2}\pi m e^2 \sqrt{dl}/m_s c^2 \sqrt{\beta}) \exp(C-1/2) < 1$ , where  $l$  is the width of the domain wall of FM films.

For the case of a few SC films with square array of FM columnar dots, the shift of the SC transition temperature can be readily obtained from the observation that the distance  $R$  between two vortices satisfies an inequality  $R \ll \Lambda$  near the transition temperature. Then Eq. (47) implies that the vortex line energy in a  $N$  multilayer system is proportional to  $N$ . We see that each term in the Ginzburg-Landau free energy is proportional to  $N$ . Therefore, the shift of the transition temperature is the same as that for a single SC film with FM dots [Eq. (32)].

## VI. CONCLUSIONS

We have studied the characteristics of the SC transition and the shift of the transition temperature in heterogeneous FM-SC systems by using the Ginzburg-Landau equation. The competition between combined vortex-domain structure in the FSB and domain structure in the FM film with the suppressed superconductivity leads to the first-order phase transition. The shift of transition temperature can be positive or negative, depending on parameters of materials used. Typical values of the relative shift  $\Delta T_c/T_c$  range from  $-0.03$  to  $0.02$ . It has been demonstrated that the stripe structure must vanish at a very small external magnetic field about 1–10 Oersted. Simultaneously the transition temperature may change by the value  $\Delta T_c/T_c \sim -0.03-0.02$ .

In the multilayers case, the critical magnetic field at which the stripe disappears increases with the number of layers  $N$ . The shift of the transition temperature can change sign from negative to positive with  $N$  increasing. The reduction of the transition temperature in the SFMD may be of the same order of magnitude as in the stripe structure at reasonable values of parameters. In the FM-SC multilayer, this magnitude is the same as that in a single isolated FM-SC bilayer.

The stripes are expected to appear in the multilayer samples whose total thickness is much smaller than their lateral size. No stripes will exist in the opposite limiting case. This implies that there must exist a critical value of ratio of the thickness to the transverse size, at which the stripe structure disappear. The accepted approximation does not allow to calculate this ratio and the corresponding critical behavior.

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