EFFECT OF VARYING THE DELAY DISTRIBUTION IN DIFFERENT CLASSES OF NETWORKS: RANDOM, SCALE-FREE, AND SMALL-WORLD

A Thesis

by

BUM SOON JANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2007

Major Subject: Computer Science

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Approved by:

Chair of Committee,	Yoonsuck Choe
Committee Members,	Eun Jung Kim
	Alexander Sprintson
Head of Department,	Valerie E. Taylor

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ABSTRACT

Effect of Varying the Delay Distribution in Different
Classes of Networks:
Random, Scale-free, and Small-world. (August 2007)

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Chair of Advisory Committee: Dr. Yoonsuck Choe

Networks, and associative properties, prevalent in natural and artificial systems have been investigated extensively. A common method for network analysis is based on graph theory because graphs naturally represent the relationship between objects in a network. In this context, three classes of networks are frequently investigated: random, scale-free, and small-world network. The three classes of networks have been studied extensively, to find properties and to analyze the structure of each network type using various measurements. Despite that all real networks have time delays, researchers relying on graph theory commonly disregarded delay or considered them only as being homogeneous. Delay cannot be ignored because delay has a critical role in many types of networks, such as the internet, business networks, and biological networks. The role and effect of delay, however, are still not clearly understood in the context of graph-based analysis. Furthermore, graph-based analysis of networks containing delay has not been attempted so far. In this thesis, I compared multiple network structures with delay in a graph context. I incorporated delay information into the network topology by a simple technique called temporal augmentation. Also, I investigated the effect of varying the delay distribution in these different network classes with added delay.

In this thesis, several experiments were conducted based on two network construction methods (naive, and modified conventional method) and three types of delay distributions (peaked, uniform, and unimodal), with different network parameters. From the experiments, I found that the effect of the number of hubs in scale-free network was negligible, while the role of neighborhood size in small-world networks was significant. Also, neighborhood size affect smallworldness of networks.

Effect of delay was expressed differently based on different patterns of delay distribution and network structures. Networks with uniformly randomly distributed delay had the best robustness in dealing with delay. Unimodal cases had larger increases in shortest path sum than uniform case. Peaked cases showed the worst increase in shortest path sum. Also, sparse networks with high smallworldness was less affected by delay while dense networks with high smallworldness more affected by delay. These results extended understanding of the relationship between network structures and delay.

To my God and my parents, Ranwoo Jang and Myoungsook Oh

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CHAPTER I

INTRODUCTION

We call a system of interacting components a 'Network'. By this definition, there are various types of networks we encounter in our lives. Friendships, ecosystems, food web, WWW, and even the brain are all networks we can find in our everyday life, with different scales, components, and connecting links. Thus, it is hard to imagine our life without networks. In spite of the importance and prevalence of networks around our environment, physical or functional structures and properties of networks are only partially understood.

The analysis of the brain, a kind of a complex network, has not been fully conducted. The interaction between neurons in the brain gives rise to complicated functions such as perception and cognition. No single neuron controls where or how information is processed. That is, neurons themselves are trivial units. But their connectivity has a nontrivial structure and is capable of producing complex functions. Therefore, to understand how neural networks function, we have to investigate the relationship between neurons.

There are three main classes of networks: random, scale-free, and small-world. The random model suggested by Erdos and Renyi consists of randomly connected nodes [7]. When their model was proposed, many researchers believed that the random model could explain all types of networks. However, new classes of networks were discovered that were accountable by the random model. It turns out that there are some components in the network that are not randomly connected [8]. Barabasi and Albert [9] proposed a new model called a scale-free network that is constructed

This thesis follows the style of *IEEE Transactions on Neural Networks*.

based on ‘growth’ and ‘preferential attachment’ inspired by mechanisms in the evolution of real networks. Another model proposed by Watts and Strogatz [10] was the small-world network. This model imitated social networks that showed a unique property: Pairs of people connected by a path of six hops on average fall under this type of network. Subsequently, more networks with the small-world property were discovered in real world networks and most scientists have come to regard the brain as a small-world network [4, 11, 5, 12, 13, 14].

The three classes of networks above have been studied extensively, in order to find out their properties and to analyze the structure of each network type using various measurements [15]. However, this analysis did not take into account delay. In idealized models it is easy to forget that real world networks are subject to connective delays. Delay is caused by the finite speed at which signals are propagated throughout the network. If every neural connection has the same latency, the effects are trivial. Neural networks must deal with delay which may vary for each connection. As we know, delay cannot be ignored as delay has a critical role in networks such as the internet, business networks, biological networks, etc. So far, only few studies have included time delay in dynamic network analysis [3, 16, 17, 18, 19, 20]. These studies used delay differential equations. These equations represent a set of action potentials of neurons over time offering detailed information about how a network reacts to a specific stimulus. Therefore, specific information about the network structure is required to construct such models. The analysis based on delay differential equations rely on other information (topology of networks, function of nodes, and weight of edges) as well and did not compare different network classes. Alternative approaches are required to compare multiple network classes with delay.

A graph consists of a set of vertices and a set of edges which connect one vertex to another. Graphs provide a significantly higher level of abstraction compared to delay differential equations and have been used to elegantly represent and analyze networks. Therefore, a network can be analyzed as a graph at a higher level. Also, delay can be simulated in graphs by a temporal augmentation method [21].

In this thesis, I will show that delay influences networks and different types of network respond differently to such added delay. In addition, I will demonstrate which type of network topology can deal with delay more effectively.

A. Problem overview

For a better understanding of the problem addressed in this thesis, a brief review of the importance of network structure is first provided. Next, the role of delay based on current studies and the approach taken in this thesis are introduced.

1. Importance of network structure

Networks can be classified into three broad groups: regular networks (lattice), random networks, and complex networks, including scale-free and small-world networks. Complex networks have a topology in between those of random and regular networks. We generally focus on the complex network because the simple networks, such as random or regular network (lattice), show trivial functions or properties.

In the past few years, the progress in the research on complex networks has been made regarding the effect of network topology on the network's dynamical behavior. Moreover, it was found that the topology of a network often plays a crucial role in the synchronization of chaotic dynamical systems [22]. For example, Sporns et al. investigated the relationship between the anatomical connectivity (synaptic connection linking neurons) and the functional connectivity (a pattern of temporal correlation between neuronal elements). Functional connectivity is obtained from the network's dynamics (Fig. 1). They proposed that the dynamical functional complexity (functional connectivity) of the brain derives from its small-world attributes: integration and segregation in anatomical structure [1, 20, 23]. Vragovic et al. showed different efficiency of information transmission in complex networks [24] and Simard et al. found that a neural network with small-world structure reduced learning error and training time compared to those based on different type of connectivity [25].

From the above studies, we can see that the network structure or type plays a vital role in the network function. In this thesis, we will investigate the effect of delay on networks with different topology.

2. Effect of distributed delay

The effect of delay is one key issue that has drawn attention in networking research because there exists a non-zero communication delay between components in all real networks. Excluding a few studies, however, most studies on network connectivity have assumed that each connection has the same delay. The effect of uniform delay is trivial; on the contrary, real networks show that they have distance-dependent delay or randomly distributed delay. Only a few studies in the context of delay differential equation illustrated the effect of distributed delay [3, 16, 17, 18, 19, 26]. We have partial knowledge about distributed delay in networks represented as graphs. Therefore, in this thesis, I will measure the effect of different delay distributions on multiple network structures in the context of graph-based analysis.

B. Approach

Existing studies are based on coupled chaotic map and phase oscillators using delay differential model. These models need specific information about network structure.

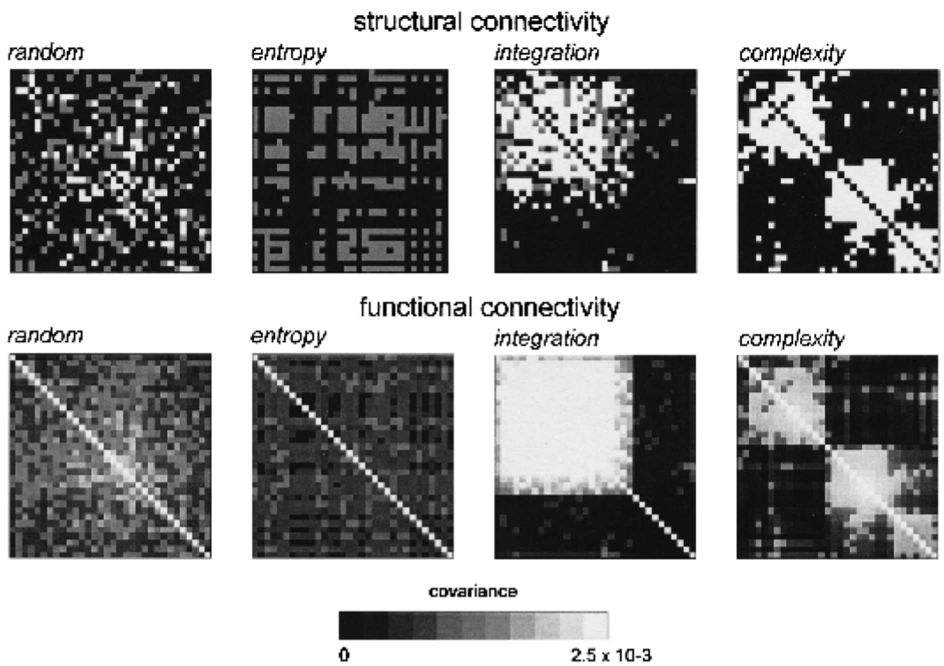


Fig. 1. Structural connectivity and functional connectivity. Comparison of structural and functional connectivity is shown. Connection matrices (top row) show unidirectional connection in gray and bidirectional connection in white. Covariance matrices (bottom row) represent the functional connectivity. Adapted from [1].

We cannot compare networks with diverse structures using these models. Therefore, we need a new frame work, which can compare multiple network structures and at the same time deal with delay.

Using graphs, we can represent very large scale networks such as the brain. Graphs have been used to represent neural networks [2, 5, 20, 27]. In an adjacency matrix, the rows and columns of a two-dimensional array represent source and destination vertices in the graph: each element indicates whether an edge exists between the vertices (1) or (0). Therefore, the adjacency matrix is a representation of a graph for computation. However, insertion of delay in an abstract graph context was not applied in previous studies. In this thesis, delay is added to a network by a temporal augmentation method [21]. Fig. 2 schematically shows temporal augmentation.

By using graphs and temporal augmentation, I will demonstrate not only how delay can influence network function, but also how different network classes respond to added delay.

C. Outline of the thesis

In chapter II, we will review necessary background for comprehending the main classes of networks, importance of delay, and graph theory. Chapter III provides methods and measurements for experiments: Here I will show network modeling methods for network classes and measurements to compare network properties. Chapter IV presents experiments and results. The following chapter V, discussion, will include the interpretation and analysis of the results. Finally, chapter VI will conclude this thesis.

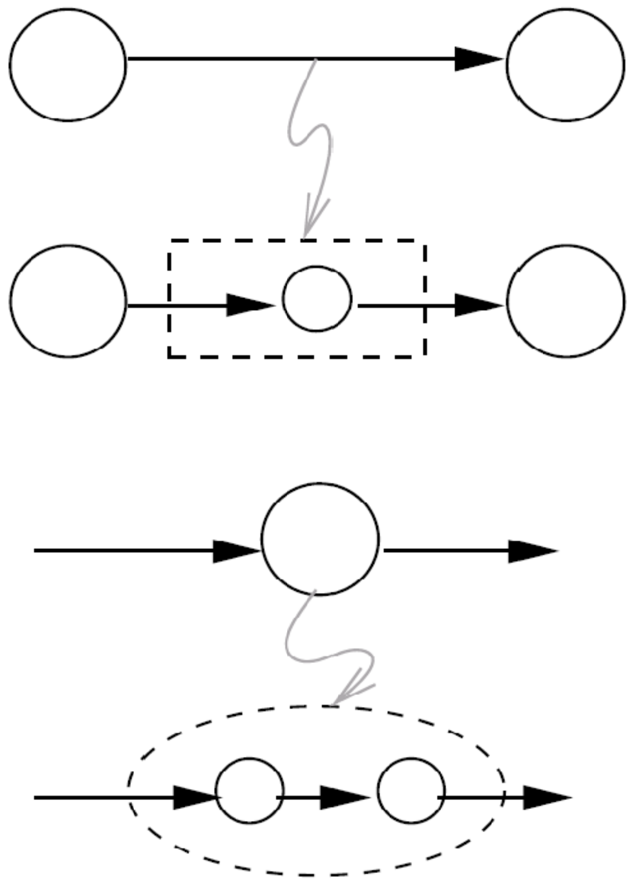


Fig. 2. Simple idea of temporal augmentation. Circles represent nodes and arrows represent edges. The top figure shows a concept of splitting an edge and adding a new node. The bottom figure shows a concept of splitting a node and adding a new edge.

CHAPTER II

BACKGROUND

Random, scale-free, and small-world are three main types of network [8] we will investigate. Many researchers have studied the effects of these structural connectivities on network function. There are many modeling methods for building these networks and understanding network classes. On the other hand, most network connectivity research assumed that delay is homogeneous or zero even though delay is distributed and nonzero in real networks. In this chapter, network structure and delay will be reviewed as main issues in network analysis.

In order to investigate these issues, simulation of network structure and delay is required. Sporns et al. suggested a graph theoretical method for effectively analyzing neural networks [27]. This method is useful for representing and comparing multiple network structures. On the other hand, temporal augmentation proposed by Choe et al. [21] can simulate the effect of delay in a graph-based analysis. In the last part of this chapter, graph theory for network structure representation and temporal augmentation for delay representation are illustrated.

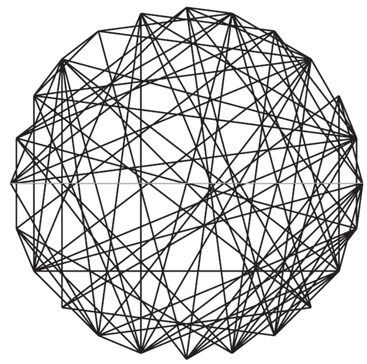
A. Classes of networks

Three classes of networks are frequently investigated: (1) random networks, (2) scale-free networks, and (3) small-world networks. A random network is one in which vertices are connected with random edges as in Fig. 3(a): An arbitrary pair of vertices have an equal probability of being connected by an edge. Although there are few examples of real world networks which fall under this category, they have been thoroughly studied: Random network was the best guess at the time it was proposed because data on the detailed connectivity of large-scale networks were not available

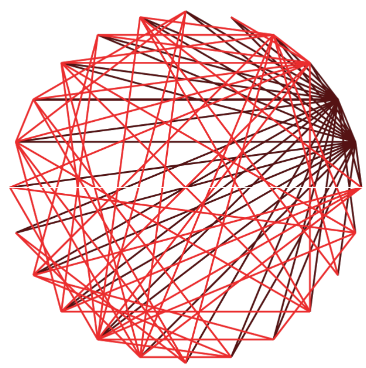
[7] (random networks were used when knowledge of connectivity was missing). In this thesis, random networks are considered as a reference case to compare with other network classes. In scale-free networks, a small number of vertices has a higher number of incoming and outgoing edges than the average in the network. The number of edges going into a vertex is called its in-degree, while the number of edges leaving a vertex is called its out-degree. A vertex with high in/out-degree is called a hub (the number of hubs in Fig. 3(b) is four). Examples of scale-free networks include client-server relationships such as the internet or cable television network. One criticism of scale-free networks is that they can be made extremely vulnerable with the loss of only a small number of highly connected hubs [9]. Small-world networks are characterized by small, highly clustered, interconnected groups referred to as neighborhoods [10] as shown in Fig. 3(c). A common example is that of a social network. Even though small-world networks have highly interconnected groups, they have surprisingly short average shortest path length between vertices [2].

B. Research on delay

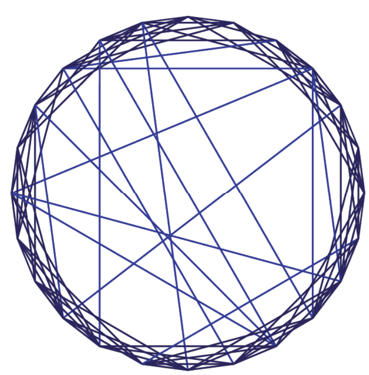
All real networks have time delay due to various factors such as the number of steps between components, physical characteristics of the link, computation time at each component, and so on. However, researchers relying on graph theory commonly disregarded delay or considered them only as being homogeneous. Even for those using dynamical equations, only few have considered non-uniform delay. Eurich et al. showed that dynamical systems converge to a stabilizing state and a simpler dynamical pattern as in Fig. 4 with more distributed delay [17]. In their study, the dynamical pattern of the food-webs with distributed delay was similar to a non-delayed system. Several other studies showed similar results that distributed delay yields stabilizing



(*a*) Random Network



(*b*) Scale-Free Network



(*c*) Small-World Network

Fig. 3. Complex networks. Structure of (*a*) random, (*b*) scale-free, and (*c*) small-world networks are shown. All networks have 24 nodes and 86 edges, with nodes arranged on the circle. Adapted from [2].

state and simpler dynamical pattern [3, 19, 26]. Furthermore, networks with random delay were shown steady-state synchronization of coupled chaotic map whereas those with fixed delay were shown chaotic synchronization [18]. Therefore, broad distribution of time delay is not just noise, but plays an important functional role in networks.

C. Graph theory

A graph is an abstract data type consisting of a set of nodes (or vertices) and a set of edges that establish connectivity between the nodes. In directed graphs edges only connect two vertices in one direction while in ordinary graphs edges connect two vertices in both directions. Directed graphs have been used more commonly in modeling neural networks. A directed graph can be completely specified by an adjacency matrix. An entry (at row i, column j) $\alpha_{ij} = 1$ indicates the existence of an edge from vertex j to i, while $\alpha_{ij} = 0$ indicates that no such edge exists. There exists a unique adjacency matrix for each graph. The adjacency matrix provides a convenient way to represent a graph for computation. We can represent and easily analyze large scale networks by using their adjacency matrices. Also, we can get the distance matrix, which represents the shortest path length between vertices by using powers of an adjacency matrix [27]. Measures of graph properties are detailed in the next chapter.

D. Temporal augmentation

Temporal augmentation [21] can represent delay in networks represented in graphs, such as response time in WWW, membrane time constant in neurons, and geographical distance. This method augments a single edge by adding a number of nodes

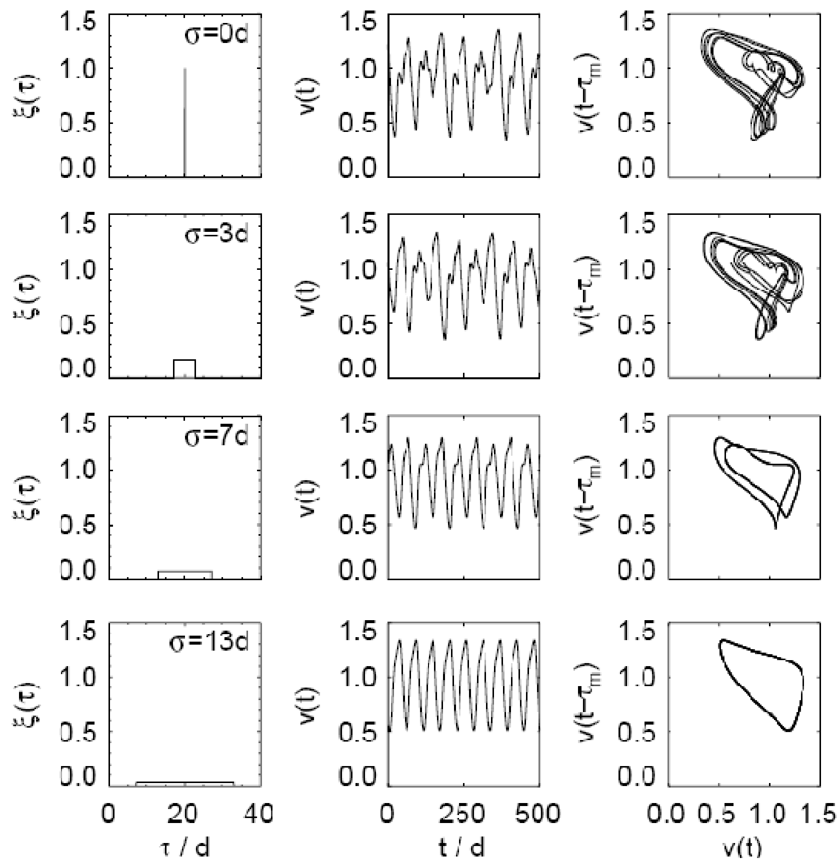
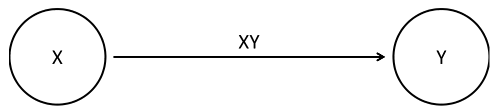
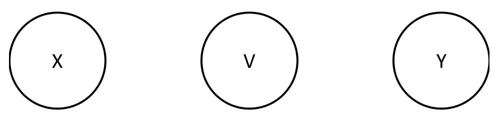


Fig. 4. Delay distribution and dynamic pattern. Broader delay distribution yields simpler dynamics. Delay distribution function (left column) used to compute the blood cell concentration level $V(i)$ (center column) during simulations of Mackey-Glass equation with distributed delays is shown. Also shown are phase plane portraits $V(t-\tau_m)$ vs. $V(i)$ (right column) generated by two dimensional embedding using delay coordinates. σ=0d (days) denotes the singular delay case. In all cases τ_m=20d. Transients are omitted. Adapted from [3].

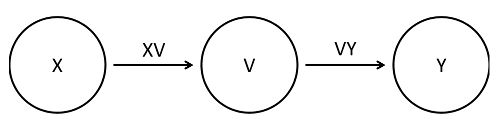
on the edge, proportional to the delay. Fig. 5 shows the process of temporal augmentation and Fig. 6 illustrates an adjacency matrix before and after augmentation. This way we can represent delay as an increasing number of hops. The impact of an increasingly longer path is unlikely to pass short-term causal influence to the target node, because the length of a path represents the time it takes for a signal to travel the path.



(a) Two vertices X and Y are connected by an edge XY.



(b) The edge XY is removed and a new node V is added.



(c) The edges XV and VY are added doubling the path length of the previous connection.

Fig. 5. Temporal augmentation.

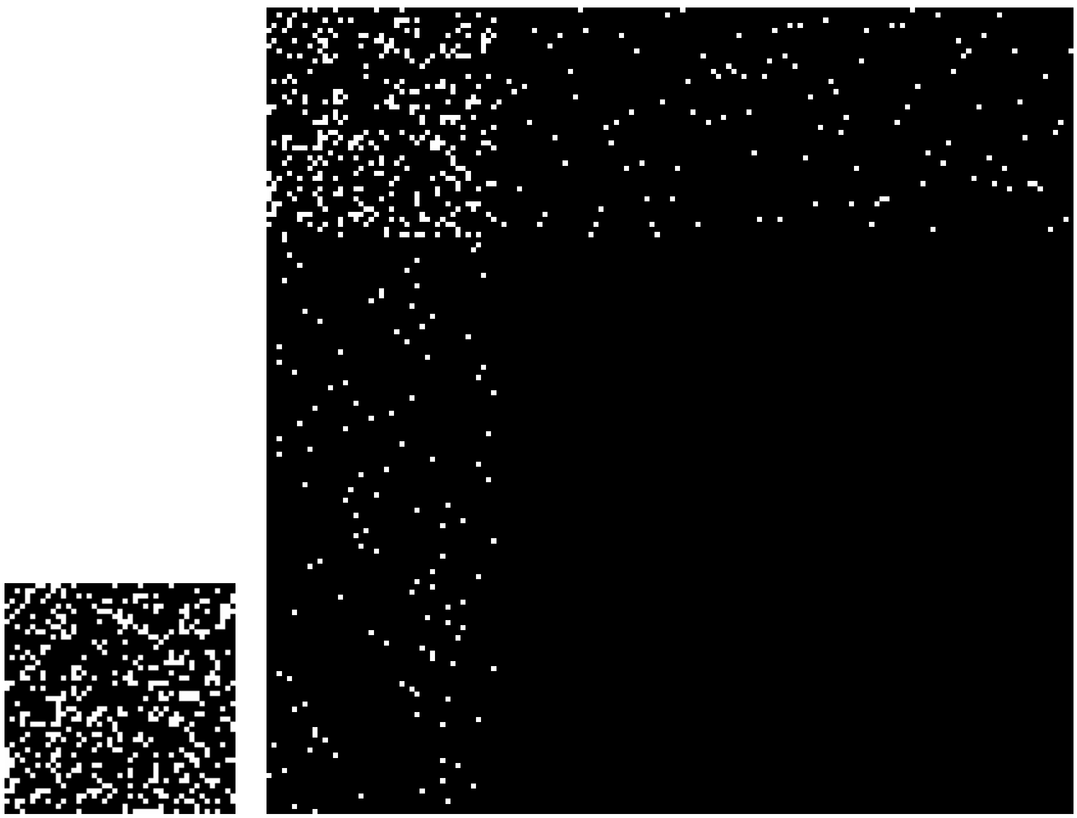


Fig. 6. Augmented adjacency matrix. A random network's connectivity matrix is shown (left). White represents 1 (connection) and black represents 0 (no connection). The same network with $\frac{1}{4}$ of all edges temporally augmented is shown (right).

CHAPTER III

METHODS

This chapter shows how to construct and analyze different network classes. Here, I propose two network modeling methods: naive and MC (modified conventional) method. The two methods form the basis of analyzing and comparing the network classes. These two modeling methods fulfill existing definitions of network classes. The naive method builds a network by simply using a minimal number of essential parameters, while MC (modified conventional) builds a network using more robust modeling methods.

As measurements for comparing the resulting networks, average shortest path length, clustering coefficient, and smallworldness were used. Detailed explanation about these measurements will be provided in the following sections.

A. Network Model

The networks in this thesis are assumed to be connected and do not allow connections from a neuron (or vertex) to itself. These constraints are similar to those imposed in [27]. The networks are represented by directed graphs because vertices are assumed to represent individual neurons which have synapses which are directed. This serves as another constraint of network modeling in this thesis.

Random, scale-free, and small-world are the three general classes of networks that form the basis of experiments. I considered these three networks constructed by two modeling (or construction) methods: Naive and modified conventional.

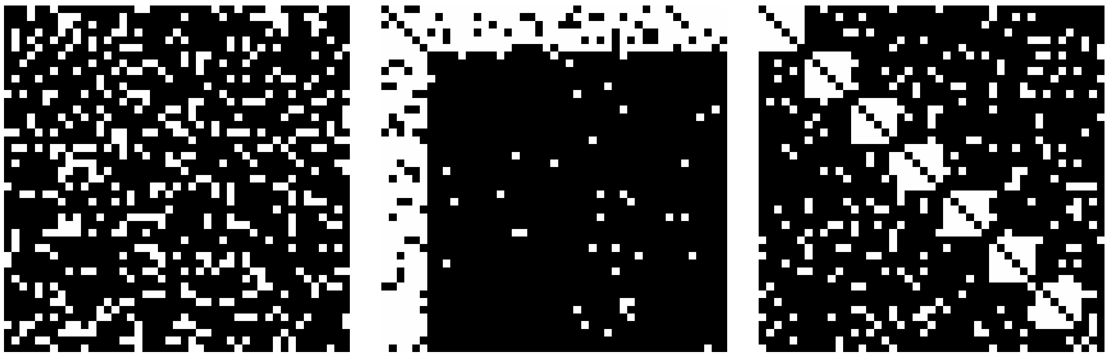
1. Naive method of network construction

The naive method of network construction is based on the definition of each network class and requires a minimum number of parameters. Under this construction method, RN (Random Network) is generated simply by specifying the number of vertices and edges. The number of vertices, edges, and hubs are specified for SFN (Scale-Free Network). SWN (Small-World Network) is designated a neighborhood size as well as the number of vertices and edges. By specifying networks with the same number of nodes and edges, the only variation between the networks is their network class. Given these parameters, the graph of networks is represented by an adjacency matrix for further computation as in Fig. 7.

In Fig. 7(a), RN is built as an empty adjacency matrix with a fixed number of nodes. A fixed number of edges is assigned randomly to the entries of the matrix except for the diagonal entries. In Fig. 7(b), SFN is built as an empty adjacency matrix with the same number of nodes, while the given number of hubs is connected to a large number of other nodes. The remaining edges are assigned randomly. In Fig. 7(c), SWN is built as an empty adjacency matrix with the same number of nodes, while the given number of neighborhood is connected locally and the rest randomly assigned.

2. Modified conventional (MC) method of network construction

The naive method explained above is simple and efficient, but we cannot ensure that this modeling method can exactly generate characteristics of each network class. Therefore, other construction methods, which have been confirmed by many researchers, are needed to overcome the limitation of the naive model [7, 9, 10]. There are well-known methods in network connectivity research that can serve this purpose.



(a) RN (b) SFN (c) SWN

Fig. 7. Naive method. Adjacency matrices for (a) Random network, (b) Scale-free network, and (c) Small-world network are shown. Networks are generated with 45 nodes and 450 edges. SFN has 6 hubs and SWN has a neighborhood size of 5.

On the other hand, these conventional methods assume the graphs to be undirected. I changed the conventional methods based on undirected graphs to construct directed graphs. I call this modeling method MC (Modified Conventional) model. The conventional methods include ER (Erdos and Renyi) for random, BA (Barabasi and Albert) for scale-free, and WS (Watts and Strogatz) for small-world networks. Fig. 8 shows example adjacency matrices for the MC model.

The definition of ER model is the same as the naive model. There is no difference between the random networks in Fig. 7(a) and Fig. 8(a). The BA model does not need a parameter (the number of hubs) unlike the naive method. The number of hubs are determined naturally because BA modeling has two mechanisms: 'growth' and 'preferential attachment'. However, in BA modeling, we need to specify the ratio of in-degree/out-degree, because we are using directed graphs and in/out directions are determined by this ratio. As a result, the hubs do not have bi-directional connections to other vertices, as in Fig. 8(b). Building a BA graph takes more time than the naive method because of the 'growth' mechanism. WS modeling uses two steps: building

(a) ER (b) BA (c) WS

Fig. 8. MC (Modified Conventional) method. Networks are generated with 45 nodes and 450 edges. BA has 0.6 a in-degree/out-degree ratio and WS has 8 as neighborhood size and 0.01 as rewiring ratio.

a regular network (lattice) and rewiring the edges. First, we have to build a regular network (lattice) using a given number of vertices, edges, and neighborhood size. Second, given rewiring rate as a parameter, randomly chosen edges are reassigned. Fig. 8(c) shows the final state after these two steps. The construction methods explained above are subset of MC (Modified Conventional) method for directional graphs. The pseudo codes of the above are provided in Fig. 9, Fig. 10, and Fig. 11.

B. Measurements

Networks have been studied extensively using various measurements [15]. Among these measurements, average shortest path length, clustering coefficient, and small-worldness are usually used to measure the network's structural properties. A network with short average shortest path length means that the network is more integrated between vertices. High clustering coefficient means that the network is more segregated. That is, there exists dense local connections and many local clusters in the network. Smallworldness is based on the two above measurements: average shortest

```
RN(n,e)
1. make an empty n × n matrix A
2. while (e ≠ 0)
3.   i=random(1,n)
4.   j=random(1,n)
5.   if i ≠ j and A(i,j) ≠1
6.     A(i,j)=1
7.     e=e-1
8.   end
9. end
```

Fig. 9. Pseudo code for RN.

path length and clustering coefficient. Therefore, smallworldness measures the degree of small-world properties [14]. In this thesis, shortest path sum (average shortest path length) is used to measure delayed network's function. Smallworldness is used to compare the naive and modified conventional method. When a network is delayed, we can see the effect of delay due to smallworldness of the non-delayed network.

1. Average shortest path length

Path length is defined as the number of distinct directed edges that are linked between a source vertex j and a target vertex i. Fig. 12 shows an example. The average shortest path length L is an average of the shortest path length among all possible paths linking all pairs of vertices. Note that when calculating the average, loops are ignored. A loop is a path that has its beginning and ending in the same vertex. Therefore, self to self paths are not counted. Using an adjacency matrix, we can easily

```
BA(n,e,r)
make an empty n×n matrix A
randomly pick two initial nodes
 A(i,j)=1,A(k,l)=1
e=e-2
while (R≠0)
ratio=rand()
 if ratio ≤ r
  choose a node X among existing nodes based on its indegree
  randomly choose a node Y≠X
  if A(X,Y)≠1
   A(X,Y)=1
    e=e-1
   end
  else
   choose a node Y among existing nodes based on its outdegree
   randomly choose a node X≠Y
   if A(X,Y)≠1
    A(X,Y)=1
    e=e-1
   end
  end
 end
```

Fig. 10. Pseudo code for BA.

```
WS(n,e,h,r)
make a regular n×n matrix A with neighborhood h
R=e×r
while (R≠0)
  i=random(1,n), r−i=random(1,n)
  j=random(1,n), r−j=random(1,n)
  if i≠j and A(i,j)=1 and A(r−i,r−j)≠1
    A(i,j)=0
    A(r−i,r−j)=1
  end
end
```

Fig. 11. Pseudo code for WS.

calculate the average shortest path length of a network. A distance matrix is derived from powers of the adjacency matrix. Powers of the adjacency matrix indicate the existence of paths of a particular length in graphs. If graph G has adjacency matrix A, then its kth power has one at the ith row and jth column if and only if there is a single path of length exactly k from node j to i. The elements of a distance matrix are the shortest path length between vertices. Thus, L of a graph is an average of all entries of the distance matrix:

$$L(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} d_{ij}, \tag{3.1}$$

where N is the number of nodes of graph G, and d_{ij} is the distance from node j to i where $i \neq j$.

It is known that the average shortest path length of a random network is typically small. We can also use the shortest path sum of a network to compare differences between a network, with delay and without delay.

2. Clustering coefficient

Clustering coefficient C_i is defined as the ratio between (1) the number of connections among a node i's immediate neighbors and (2) the maximum possible number of connections among these neighbors:

$$C_i = \frac{\text{number of existing edges in } G_i}{\text{maximum possible number of edges in } G_i}, \tag{3.2}$$

where G_i is a subgraph of neighbors connecting to/from node i. Examples are shown in Fig. 13. That is, clustering coefficient means how tightly the neighborhood of a particular node is connected. C of a graph is an average of clustering coefficients of all vertices and measures the locality of a network:

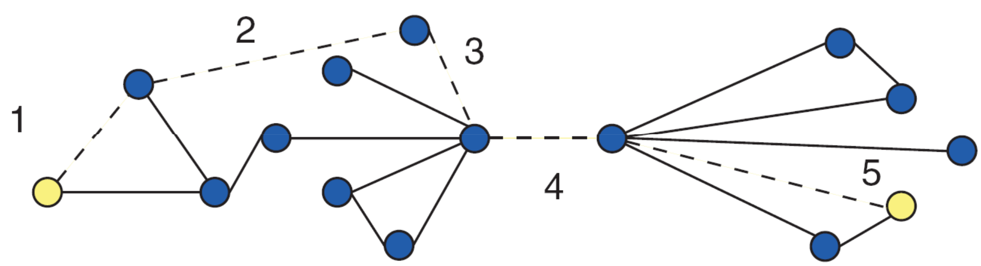


Fig. 12. Shortest path length. The path length between nodes is defined as the possible number of edges that need to be traversed to get from one to the other. The shortest path between the yellow nodes is marked by the dotted lines which has a length of five. Adapted from [4].

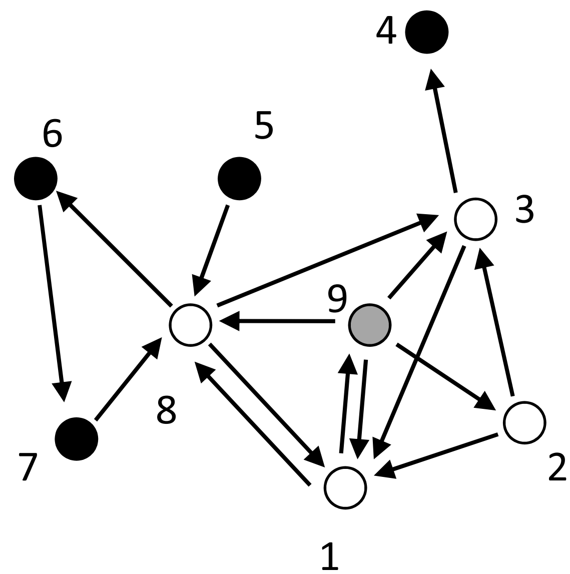


Fig. 13. Clustering coefficient. Clustering coefficients of directed graphs for the shaded node 9 are shown. This node's neighbors are 1, 2, 3, and 8, which maintain 6 connections among them out of 12 possible (4^2-4). This results in a clustering coefficient of $\frac{6}{12}$=0.5. Adapted from [5].

$$C(G) = \frac{1}{N}\sum_{i \in G} C_i, \tag{3.3}$$

where N is the number of nodes in graph G. In random networks, C_{rand} is generally small.

3. Smallworldness

Smallworldness is a simple scalar measure based on L and C of a graph. L of small-world networks is similar to L of random networks and C of small-world networks is greater than C of random networks [14]. Therefore, we can expect a network with a ratio $\lambda = L/L_{\text{rand}}$ approximately 1 and $\gamma = C/C_{\text{rand}}$ greater than 1 to be a small-world network. Based on these given criteria, smallworldness is defined as $\sigma = \gamma/\lambda$, where $\sigma > 1$ indicates a small-world property. We can use this parameter as an indicator for smallworldness of a network. I assumed that the smallworldness of an original network prior to adding delay will affect the properties of the network after adding delay through temporal augmentation. In spite of certain networks belonging to the same network class, smallworldness can be different based on the methods of construction. The random network can be considered a reference case because L_{rand} and C_{rand} are used in λ and γ respectively as a baseline.

CHAPTER IV

EXPERIMENTS AND RESULTS

In order to measure the effect of delay and to investigate the relationship between network structures and delay, several experiments were performed. All experiments were performed based on the two network construction methods described in the previous chapter. Then, a comparison was made between the increase in the ratio of shortest path sum before and after the introduction of distributed delay in the network connections. Increased shortest path sum refers to the increase in the total length of the shortest paths of a network due to temporal augmentation. To calculate the difference in shortest path sum between a network N and its temporally augmented version D, first we let S_N be the distance matrix's sum for N and S_D be the distance matrix's sum for D. The difference in the shortest path sum can then be calculated as $1 - S_N/S_D$.

The first experiment used temporal augmentation by a given percentage, which is the number of augmented edges vs. the total number of edges, in a network using the naive construction method. When delay is added to a network, a fixed percentage of its edges are randomly augmented by one node , then the difference in shortest path sum is calculated. Here, we can consider the augmented number of edges as added delay. For example, if $\frac{1}{4}$ of the edges in a network with 45 vertices and 800 edges is delayed, then 200 edges are temporally augmented. These 200 augmented edges can be considered as delay. The resulting network contains 245 vertices and 1000 edges. In the first experiment, we can investigate the relationship between delay and network structure, the role of the number of hubs in scale-free networks, and the role of the neighborhood size in small-world networks.

In the second experiment, networks are temporally augmented by different delay distributions based on two different construction methods, naive and MC method. In the experiments where delay is introduced into a network, all edges are augmented by delay following a particular distribution (e.g. peaked vs. uniform). If a network with 45 vertices and 800 edges is delayed by a peaked delay distribution with two-hop mean, then 1600 edges are augmented. The resulting network contains 1645 vertices and 2400 edges. This insures that two networks with equal vertices and edges will produce delayed networks with the same number of edges. That means total delay between two networks with two different delay distributions is the same.

A. Networks with a fixed percentage of network connections augmented

The first experiment in this section compares temporally augmented random, scale-free, and small-world networks with a fixed number of nodes and varying number of edges. The second experiment concentrates on how scale-free networks react differently to delay, depending on the number of hubs in the network. The last experiment demonstrates the role of neighborhood size in delayed small-world networks. All these experiments are based on the naive network construction algorithm.

1. Difference in shortest path sum by network class

The three network classes generated using the naive network construction method were compared by the change in the sum of shortest path due to added delay. The number of vertices and the ratio of delay remained constant throughout each trial. The ratio of delay means the ratio of the number of augmented edges vs. total number of edges in a network. Each SFN was assumed to have five hubs and the neighborhood size for SWN was eight. The change in delay of 20 RN, SFN, and SWN was averaged together for each number-of-edge condition. The results for networks with 40, 45,

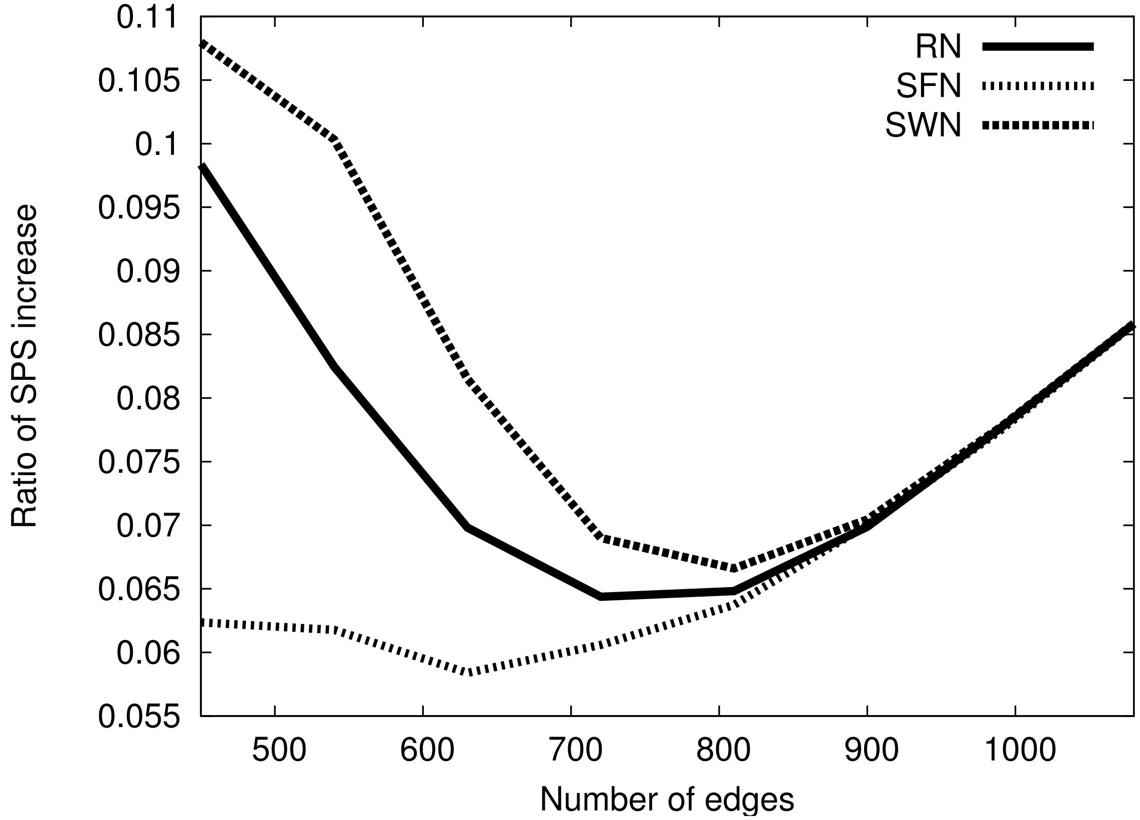


Fig. 14. The ratio of increase in shortest path sum (SPS) of normal vs. delayed networks. The increase ratio in shortest path sum under delay as the initial number of edge increases is shown. Each network contained 45 nodes and $\frac{1}{4}$ of the edges were delayed. SFNs have the smallest increase in SPS, SWNs have the largest increase, and RNs fall in between SFNs and SWNs.

and 50 vertices with $\frac{1}{4}$ of the edges delayed were similar. Here I show a 45-vertex case as in Fig. 14. This plot shows that for higher vertex to edge ratios, SFNs have the smallest increase in shortest path sum, SWNs have the largest increase, and RNs fall in between SFNs and SWNs. Fig. 14 shows that SWNs perform poorly for a small number of edges, but there is a point at which all three networks begin to act almost identically (around 900). This demonstrates that the number of edges has a definite impact on the shortest path sum difference caused by added delay.

2. Difference in shortest path sum dependent on the number of hubs in SFN

SFNs with various numbers of hubs were compared by their increase in the shortest path sum when delay was introduced. For each hub size, 20 non-delayed SFNs were generated and 20 delayed SFNs were derived by temporally augmenting $\frac{1}{4}$ of the edges in the non-delayed SFNs.

Each non-delayed network was compared with its delayed counterpart by their shortest path sum (SPS). Increasing hub count decreases the amount of increase in SPS caused by delay. Fig. 15 clearly shows a downward trend. We can also see that adding more than 13 hubs does not seem to make any further difference. One thing to note is that the scale of the y-axis only accounts for roughly one percent change, so the effect is marginal. The relatively high number of edges may be one reason why there is so little change.

3. Difference in shortest path sum dependent on the neighborhood size in SWN

SWNs with varying neighborhood size were compared by their increase in shortest path sum due to added delay. For each neighborhood size, 20 non-delayed SWNs were generated and 20 delayed SWNs were derived by temporally augmenting $\frac{1}{4}$ of the edges in the non-delayed SWNs. Increasing neighborhood size increased the amount of change caused by delay. This is because more connections are used to make redundant pathways between small populations to form neighborhoods. After populating the neighborhoods with local connections, there are fewer edges remaining to connect the distant neighborhoods together. As a result, small-world networks have longer shortest paths when neighborhood size increases. This means that signals traveling across neighborhoods may have high travel costs, which translates to delay. A noticeable fact in Fig. 16 is that the change in neighborhood size affects the network's

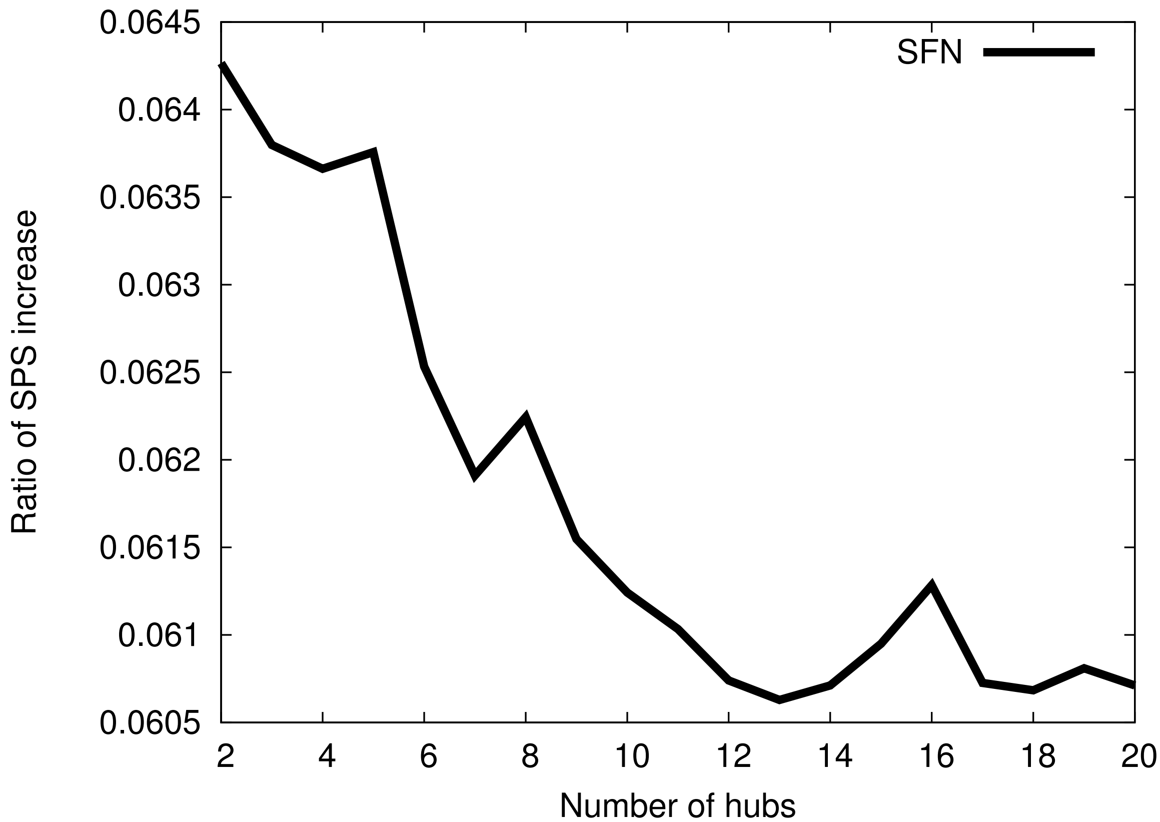


Fig. 15. The delay increase ratio in SFNs with varying number of hubs. The increase ratio in the shortest path sum under delay is shown as the number of hubs is increased. Each network contained 45 nodes and 800 edges, and $\frac{1}{4}$ of the edges delayed. Increasing the hub count decreases the increase in SPS and adding more than 13 hubs does not make any further difference.

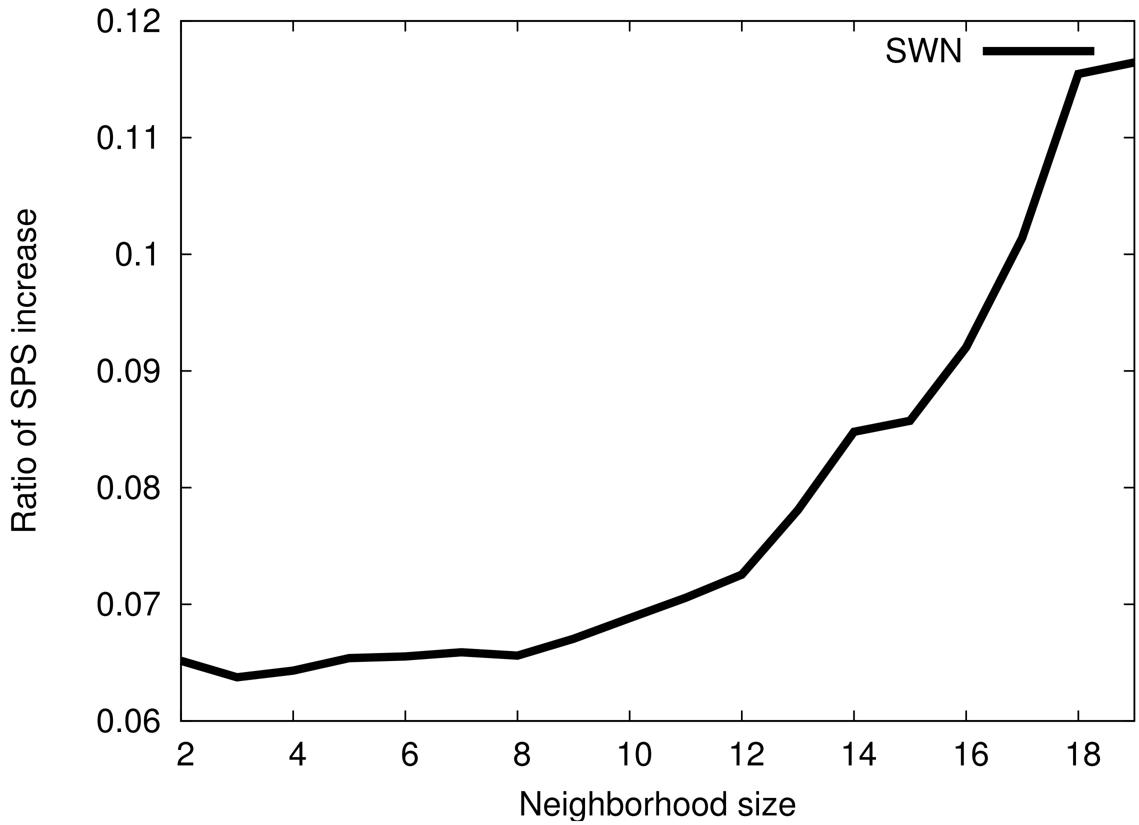


Fig. 16. The delay increase ratio in SWNs with varying neighborhood size. The increase ratio in the shortest path sum before and after the addition of delay is shown as the neighborhood size increases. Each network contained 45 nodes and 800 edges, and $\frac{1}{4}$ of the edges were delayed. Increasing neighborhood size resulted in the increase in the SPS increase ratio.

characteristics more significantly than the number of hubs in scale-free networks.

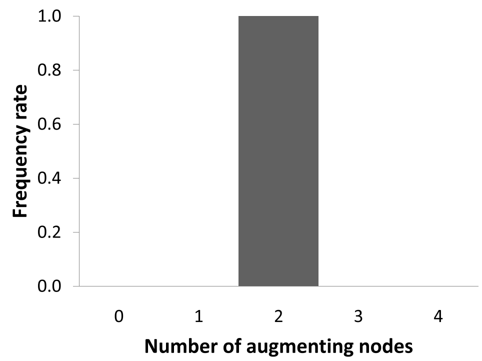
B. Augmentation by different delay distributions

As mentioned in Chapter II, delay is a key issue in network analysis. Especially, distributed delay has received much attention from researchers. Therefore, I will investigate the effect of delay by using three different delay distribution patterns, i.e., peaked, uniform, and unimodal as in Fig. 17. The total number of delayed edges was made to be the same between networks with different delay distribution patterns. That is, I assumed that the integral of the delay distribution is 1 for all distributions.

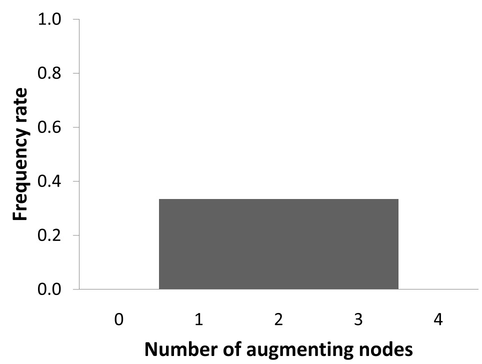
The first experiment compares each shortest path sum of the three network classes constructed by the naive method subject to different delay distributions. Next, to evaluate the naive method of network construction, I will apply the same steps in the first experiment to the MC method. Finally, I will check the smallworldness between scale-free and small-world network from the different construction methods.

1. Effect of delay distribution based on naive construction method

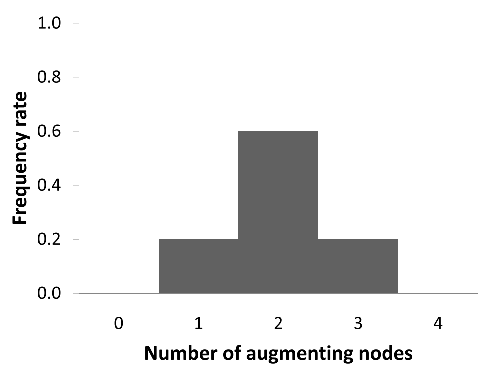
The three network classes constructed with the naive method were compared under different delay distribution conditions using shortest path sum as the measure (Fig. 18). The shortest path sum of the networks with uniformly randomly distributed delay showed the smallest increase in shortest path sum, while peaked delay distribution (homogeneous delay) showed the largest increase. The plots of each type of network respond differently in each delay distribution, but each type of network's response pattern shows similar characteristics across different delay distributions: SFNs have the smallest increase, SWNs have the largest increase, and RNs fall in between SFNs and SWNs. As a result, in spite of the amount of delay being the same, different delay distribution influences a network function because the path lengths of networks are affected by delay distribution and the functional impact depends strongly on the path length. For example, if a longer path between source and target node is present, the impact on the target node may not be strong [22]. From this experiment, we find that networks with uniformly randomly distributed delay show the best robustness in the face of delay. Also, scale-free networks turn out to have the best network structure to deal with delay.



(a) Peaked



(b) Uniform



(c) Unimodal

Fig. 17. Delay distributions. To investigate the effect of delay distribution, three different delay distribution types, (a) Peaked, (b) Uniform, and (c) Unimodal were used. The integral of the delay distribution is 1 for all distribution.

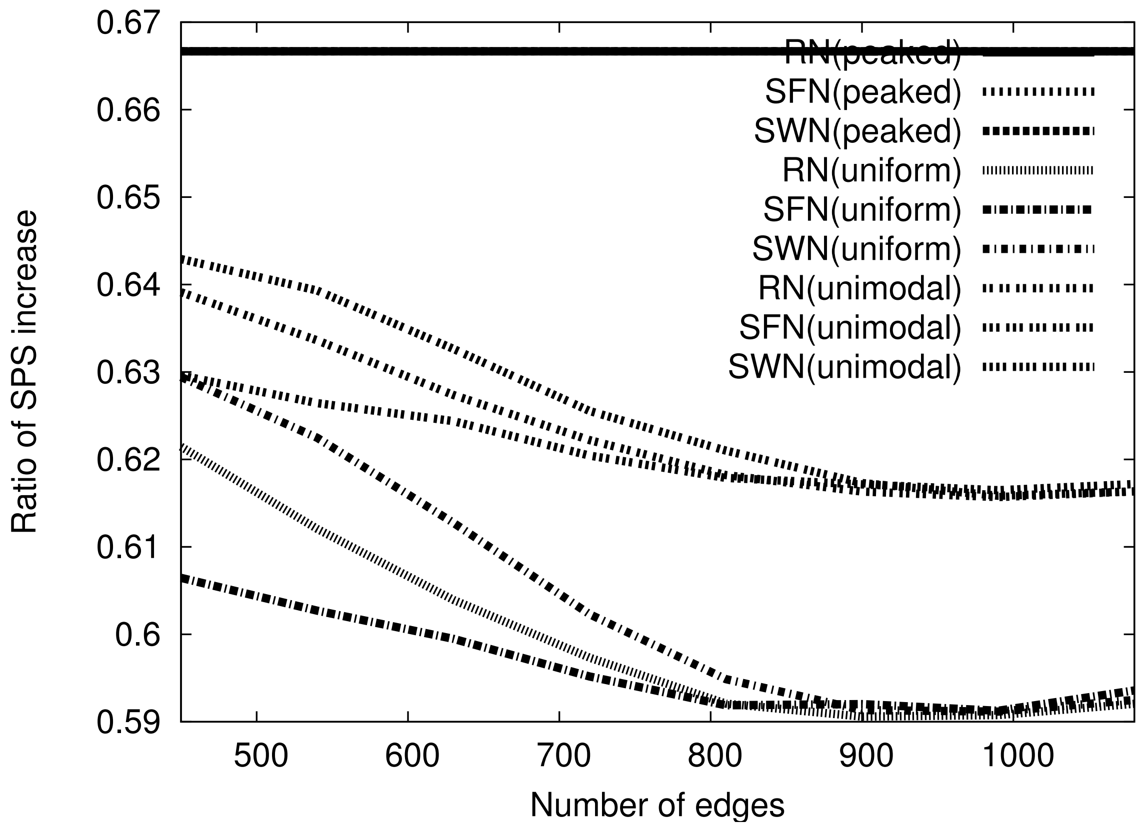


Fig. 18. Increasing shortest path sum in delayed networks with different delay distributions: Naive construction method. The increase ratio in shortest path sum under delay is shown as the number of edges increases. Each network contained 45 nodes. Delay distribution types are indicated in the parentheses. Three networks with uniformly randomly distributed delay showed the smallest increase in the shortest path sum and SFNs have the smallest increase in all delay distribution conditions.

2. Effect of delay distribution based on modified conventional (MC) construction method

To compare previous results based on the naive construction method, the MC construction method was compared under three types of delay distribution conditions. Fig. 19 shows a similar result as the previous experiment: A network with uniformly randomly distributed delay is the least affected by the addition of delay. However, the responses of different network types under a given delay distribution are different from the naive method: BA (a type of scale-free network) has the largest increase, WS (a type of small-world network) has lower increase than BA, and ER (a type of random network) has the smallest increase. Considering ER is the same as RN, we got a contradictory result using a different construction method. The reason is investigated in the following section. In this experiment, uniformly randomly distributed delay continued to show the best robustness in the face of delay.

3. Smallworldness of scale-free and small-world network

In the two preceding sections, two different network construction methods were used, and the resulting networks' responses to different delay distributions were compared. However, the results did not match. How could this be the case? In order to answer this, I ran several analyses on the networks generated by the two construction methods. An unexpected finding was that when the small-world property is quantitatively measured, the results above become consistent. Although a network may be constructed to be a small-world network by a certain construction method, it is possible that the network has less smallworldness than scale-free networks when the smallworldness is quantitatively measured. Fig. 20 and Fig. 21 indicate the smallworldness in scale-free network and small-world network using the two different con-

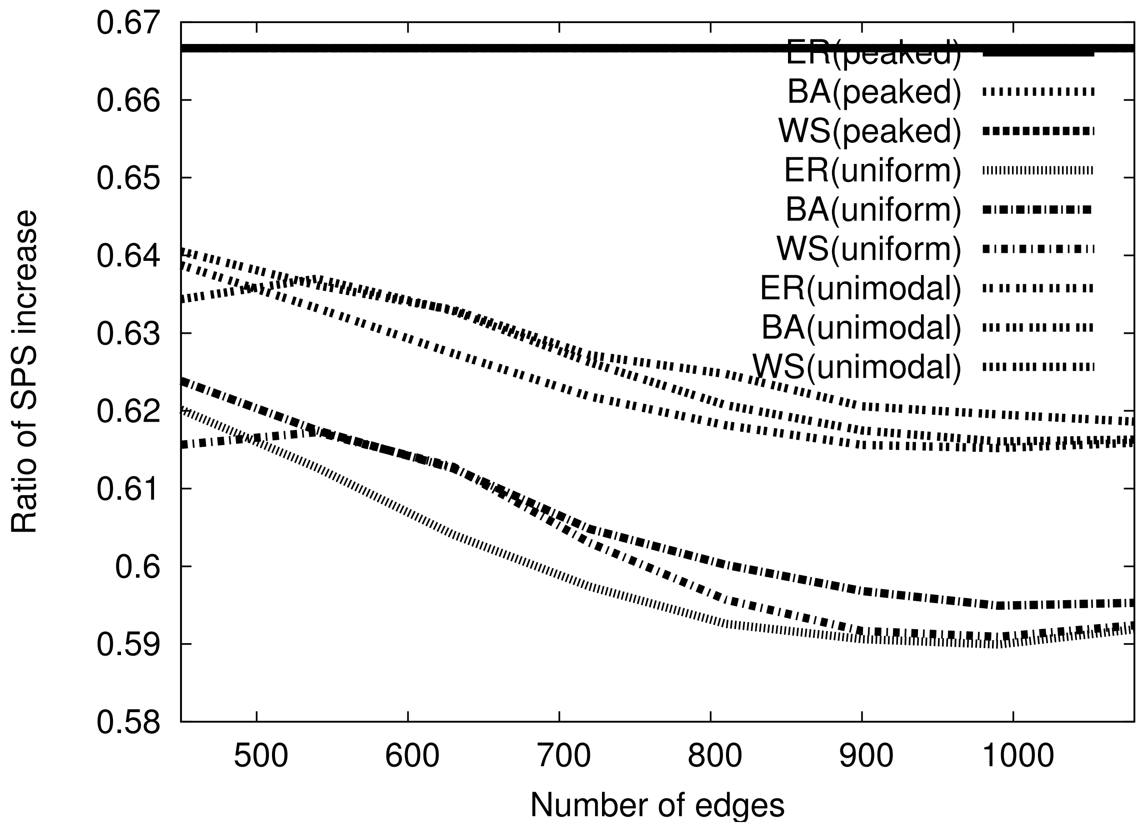


Fig. 19. Increasing shortest path sum ratio of delayed networks with different delay distributions: MC construction method. Increase ratio in shortest path sum under delay is shown as the initial number of edges increased. Each network contained 45 nodes. Delay distribution types are indicated in the parentheses. Three networks with uniformly randomly distributed delay showed the smallest increase in shortest path sum and WS had lower increase than BA in all delay distribution conditions.

struction methods. These results seem to explain the contradictory results in the two previous sections.

However, Fig. 21 shows that the smallworldness of WS is slightly lower than that of BA after the 720-edge point and I found that the implementation of WS model in Fig. 19 had an error. When we build a WS model, we first build a regular network. But in Fig. 19 I could not construct regular networks except for the 450-edge case because I fixed the neighborhood size to 10. For example, we can make a regular network with 45 nodes, 450 edges, and a neighborhood size of 10 but we cannot make it with 45 nodes, 510 edges, and a neighborhood size of 10. Therefore, I changed the neighborhood size proportional to the number of edges to get definitely higher smallworldness of WS as in Fig. 22. Fig. 23 shows the increase of shortest path sum based on smallworldness of Fig. 22 and increase of shortest path sum of WS was lower than that of BA at high node-to-edge ratio and higher than that of BA at low node-to-edge ratio. We can thus conclude that a sparse network with high smallworldness is less influenced and a dense network with high smallworldness is more influenced by the effect of delay.

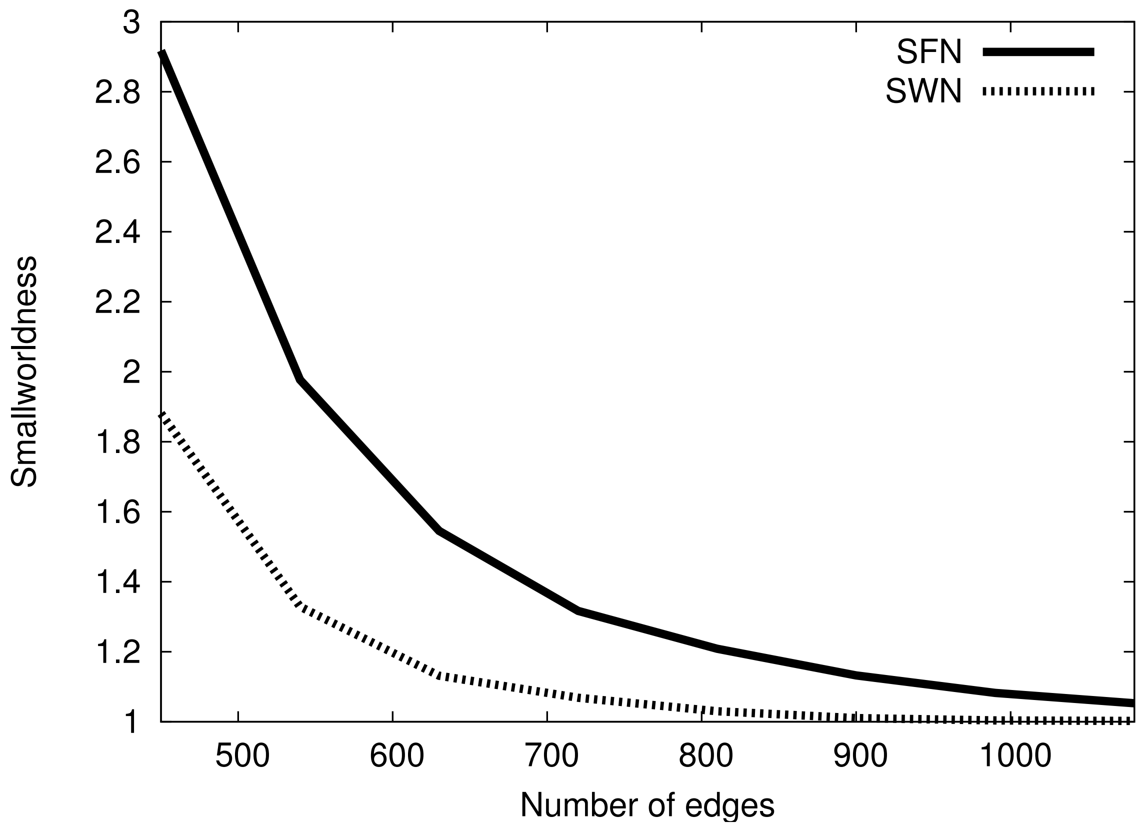


Fig. 20. Smallworldness in naive construction method. The smallworldness of SFN is higher than that of SWN, which was an unexpected finding.

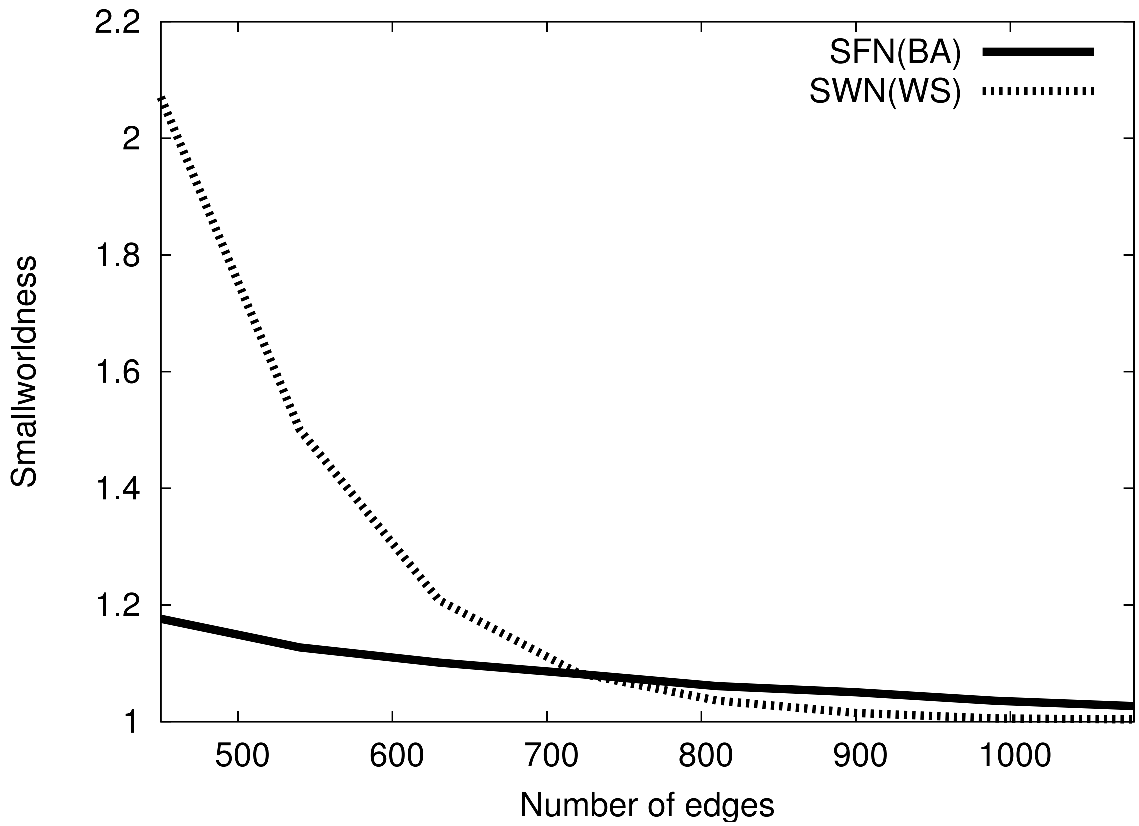


Fig. 21. Smallworldness of MC construction method. The smallworldness of WS is higher than that of BA at less than 720 edges and slightly lower than that of BA at more than 720 edges.

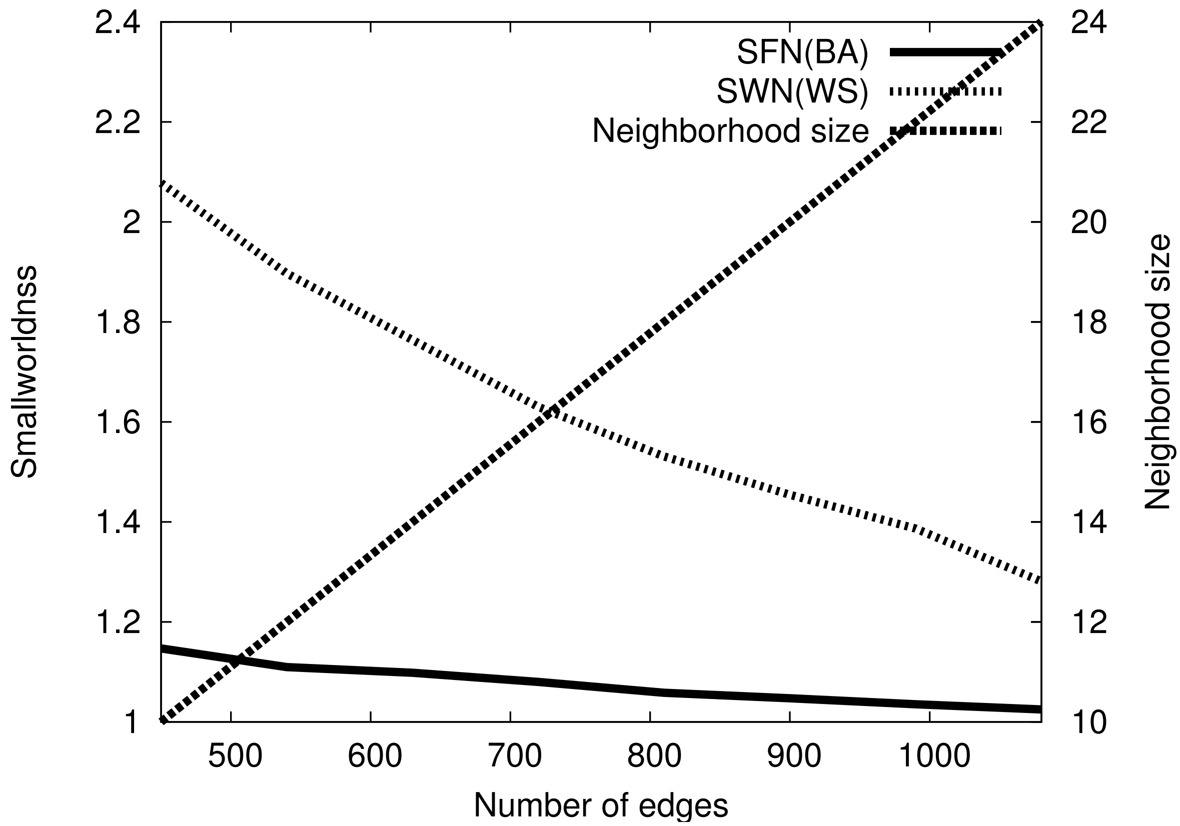


Fig. 22. Smallworldness of networks constructed by the MC method. Neighborhood size influences the smallworldness of WS. The smallworldness of WS is consistently better than BA, when the neighborhood size in WS is increased as the initial number of edges is increased.

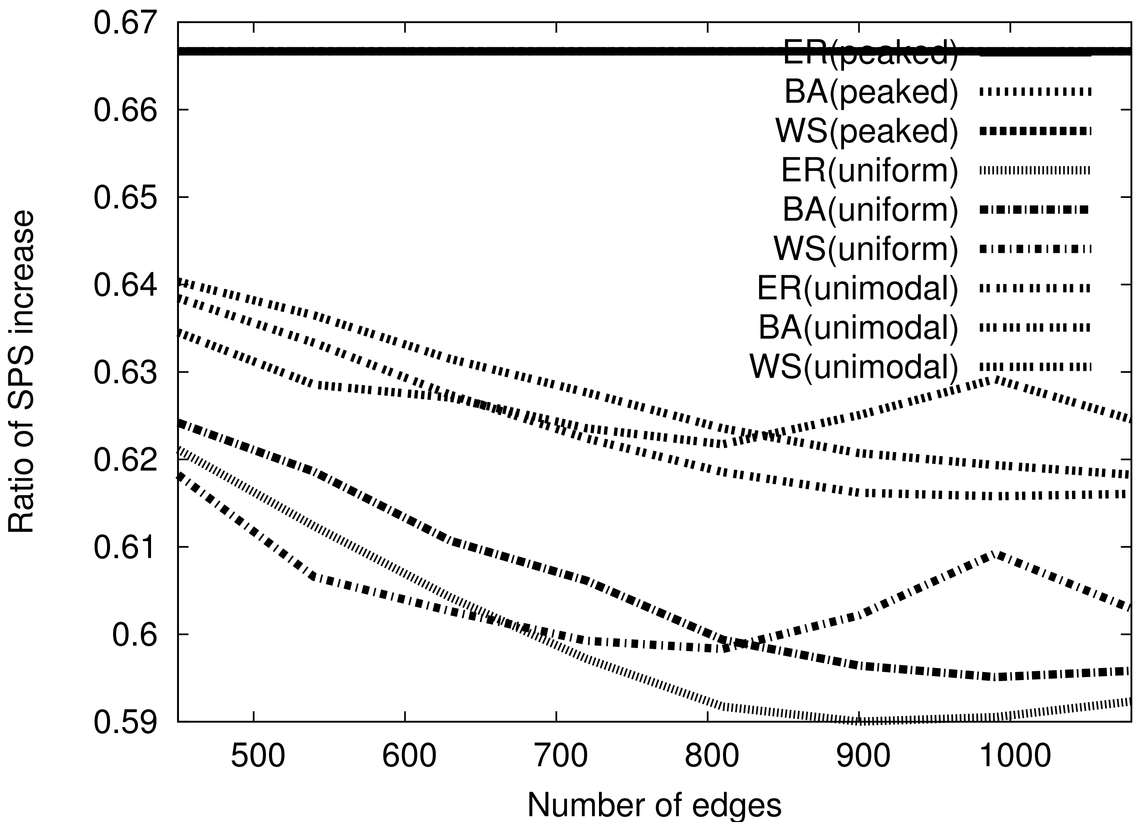


Fig. 23. Increase in shortest path sum in delayed networks with different delay distributions constructed by the MC method. The increase ratio in shortest path sum as the initial number of edges increase is shown. Each network contained 45 nodes. Increase of shortest path sum in WS is smaller than that of BA at low node-to-edge ratio (number of edges below 650) and larger than that of BA at high node-to-edge ratio. Note that here, for WS, neighborhood size was increased as in Fig. 22

CHAPTER V

DISCUSSION

This chapter discusses the main contribution of the current study with respect to relevant works by other researchers, followed by a discussion on the limitations and future directions.

A. Main contribution

Structures of network and delay are essential issues in network connectivity analysis and have been studied respectively. From the previous works, we were aware of several facts about network structure and delay. For example, the functional structure depends on anatomical structure [2] and broader delay distribution yields simpler dynamics [19]. However, there was no systematic study that looked at the relationship between network structure and delay. The studies about network structure did not consider delay (i.e. many researchers assumed homogeneous or zero delay) and studies about delay did not consider the various network structures. In this thesis, I proposed a method to evaluate the effect of delay in three classes of networks using graph theory and temporal augmentation. I compared multiple network structures in a graph context and simulated delay using temporal augmentation. Through experiments on these network, the role of the number of hubs in SFNs and that of the neighborhood size in SWNs became apparent. Chapter IV showed that both the number of hubs and the neighborhood size (in SWN) affect the shortest path sum of the networks. Influence of the neighborhood size was very significant in SWNs, while that of the number of hubs was negligible in SFNs. Using two different network construction methods, I found the factors that reduce the shortest path sum of temporally augmented networks; smallworldness and neighborhood size of non-augmented network affected the effects of delay. Because smallworldness and neighborhood size depend on the network structure, we can say that network structure also influences the effects of delay.

From the last experiment in chapter IV, I only changed the neighborhood size proportional to number of edges to get higher smallworldness as in Fig. 22. This figure shows that smallworldness of WS is higher than that of BA. However, the increase of the shortest path sum of WS in Fig. 23 was smaller than that of BA at low node-to-edge ratio and larger than that of BA at high node-to-edge ratio. We can speculate from these results that there may exist a proper neighborhood size for a certain node-to-edge ratio that reduces the shortest path sum the most.

In [28], Lim and Choe proposed there may exist delay compensation mechanism in the brain because neuronal delay can cause severe problems. It is known that the brain has a small-world topology and it can respond in almost real time. Therefore, there may be delay compensation mechanisms in the brain and/or we can assume that small-world structure itself provides robustness to delay. The result in this thesis showed partial evidence that special structure of neural networks may exist that are robust to delay. Previous studies are supportive of this hypothesis because they showed the brain, which is a sparse network, has a small-world structure and it seems to operate in real time [4, 11, 5, 12, 13, 14]. Moreover, the result that a network with uniformly randomly distributed delay was less affected by added delay can be related to the fact that the brain has distributed axonal conduction delay (or speed of signal transmission) as in Fig. 24. This link may partly explain why the brain is robust to delay.

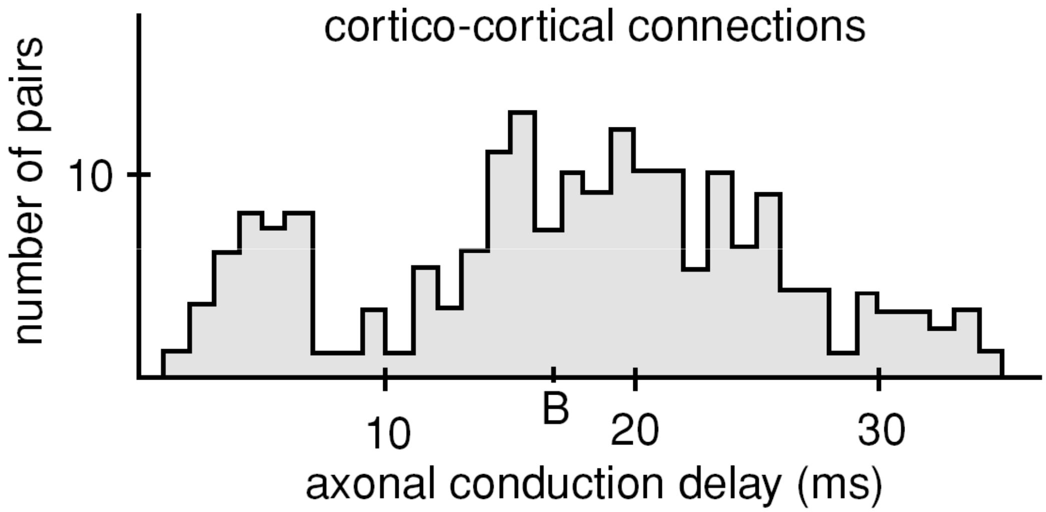


Fig. 24. Axonal conduction delay distribution. Distribution of experimentally measured conduction delays of cortical axons running through the corpus collosum is shown. The distribution turns out to be broad. Adapted from [6].

B. Limitations

The main goal of this thesis was to investigate the effects of varying delay distribution on different network classes. For that, I used graph theory and temporal augmentation. To reduce complexity of the experiments, I modeled networks as directed graphs that are connected, i.e. there was no isolated node. These constraints pose some limitations: real neural networks may not be connected, i.e. there may be isolated networks, and there exists bidirectional connections.

When I generated networks and calculated their shortest path sum, it took a long time because BA model has a 'growth' step and the shortest path sum was derived from the matrix powers of the adjacency matrix. Therefore, I could not simulate large scale networks. The adjacency matrix representation also required a huge mount of memory. The computation can be made much more efficient using an adjacency list with weighted edges, and Dijkstra's shortest path algorithm [29].

I used the shortest path sum as the only measurement of the effect of delay. This measure determines only the degree of integration in networks. Therefore, we need other measurements to measure the effects of delay. Clustering coefficient is one such measure to evaluate the locality of networks, but it was previously used in non-delay topologies. Therefore, we need other measurements to check the locality in delayed networks (or weighted networks). Vragovic et al. proposed "efficiency" for weighted graphs. Local efficiency shows similar patterns such as clustering coefficient C [24]. Therefore, we can use the local efficiency measure when we treat temporally augmented distance matrices as weighted graphs.

Delay is related to dynamics. Complexity of a network is a measure of network dynamics and it depends on the function of the network [1, 20, 23]. However, I did not specify the functions of the nodes or the weights of the edges. We could not get dynamic patterns in a graph-based context without such information.

Lastly, I used three types of delay distributions while maintaining the total delay of each network. Experiments with more delay distribution profiles may show more meaningful results.

C. Future work

In this thesis, only smallworldness and neighborhood size were discovered as factors affecting the delay in networks. In addition, the variation in delay distribution was found to be capable of altering the effects of delay. However, we still do not know how these factors give rise to these effects. Moreover, other factors of network structure may influence the effects of delay. Therefore, to expand on the experiments in this thesis, we may loosen the network constraints and investigate large scale networks. Then, we should perform experiments with other measurements such as complexity or locality. Mathematical analysis will also help further explain the results of this thesis. I simply showed the results of the experiments and interpreted the results.

When I represented the graphs, I used adjacency matrices. However, it is not efficient because it is hard to simulate large scale networks using such a matrix. Thus, it is better to use another representation such as adjacency list or sparse matrices, and use more efficient methods to calculate the shortest path sum (e.g. [29]).

CHAPTER VI

CONCLUSION

Delay in network analysis is an important issue. Neuronal delay could have caused severe problems for animals in motion, but their reaction seems to be in real time. Based on the observation, some researchers suggested that there exists delay compensation mechanisms in the brain network. I began this thesis with a simple question: How can network structure and delay distribution influence a network's ability to cope with delay? I supposed that special network structures and specific delay distributions would be robust to delay. I compared three representative classes of networks under different delay distribution conditions. To compare these networks with delay, I constructed networks in a graph-based context and used temporal augmentation to represent delay.

I found that the role of the number of hubs in SFNs has a marginal effect, while the neighborhood size a more significant effect in SWNs. That is, the shortest path sum of temporally augmented networks (delayed network) is increased according to the increase of the neighborhood size. On the other hand, I found that small-world property in non-augmented network is crucial in reducing or increasing the shortest path sum of temporally augmented networks. Sparse networks with small-world structure are robust to delay and neighborhood size of small-world network was found to be a critical parameter.

In addition, when the same amount of delay exists in the same type of networks, the network with uniformly randomly distributed delay will be less affected by delay. Based on these results, I carefully conclude that sparse networks with a small-world structure and uniformly randomly distributed delay are robust to delay and these results provide partial explanation of the fact that the brain has small-world network and broad axonal delay distribution while being robust to delay. We were able to extend our understanding of network structures and delay based on this thesis, but we are still far from a complete understanding.

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