# NEW MEASURES AND EFFECTS OF STOCHASTIC RESONANCE

A Thesis

by

SWAMINATHAN SETHURAMAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2004

Major Subject: Electrical Engineering

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August 2004

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## ABSTRACT

New Measures and Effects of Stochastic
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In the case of wideband (aperiodic) signals, the classical signal and noise measures used to characterize stochastic resonance do not work because their way of distinguishing signal from noise fails. In a study published earlier (L. B. Kish, 1996), a new way of measuring and identifying noise and aperiodic (wideband) signals during strongly nonlinear transfer was introduced. The method was based on using cross-spectra between the input and the output. According to the study, in the case of linear transfer and sinusoidal signals, the method gives the same results as the classical method and in the case of aperiodic signals it gives a sensible measure. In this paper we refine the theory and present detailed simulations which validate and refine the conclusions reached in that study. As neural and ion channel signal transfer are nonlinear and aperiodic, the new method has direct applicability in membrane biology and neural science (S.M. Bezrukov and I. Vodyanoy, 1997).

## ACKNOWLEDGMENTS

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# 1. INTRODUCTION

## 1.1 What Is Stochastic Resonance?

Stochastic resonance has become an interesting field of study recently. It is common intuition that noise always plays the role of a spoiler and hinders the signal in being received and interpreted effectively. But surprisingly it was found that this need not always be true. There exist systems where noise actually aids or abets the passage of signal through the system. Such an effect is called as stochastic resonance and the systems which exhibit such a behavior are called stochastic resonators.

Stochastic resonance was first proposed to explain the periodic changes in long term climate of the earth and the onset of ice ages [1]. But later on it was found that the stochastic resonance effect had an ubiquitous nature being observed experimentally in such diverse systems as schmitt triggers, ring lasers, ionic channels, and mechanosensory pathways in arthropods and the complicated human sensory perception systems.

A simple system which illustrates this general phenomenon is as follows: Consider a bistable potential well as shown in Fig 1.1. The well has two stable states, Position 1 and Position 2, and a particle oscillating in the potential well. Consider a particle oscillating in this potential well at a frequency $F_s$ and a small external forcing of amplitude A (which plays the role of the input signal) which is smaller than the potential required to cross the potential barrier of the well. Let the output signal be the frequency $F_s$ at which the particle oscillates between the two equilibrium states Position 1 and Position 2.

If there is no noise, the output signal is zero as there is insufficient potential to cross the barrier. As the input noise strength increases, the output signal starts increasing as the particle moves between the two states. But if the noise strength is too high, it will swamp the signal as there will be no correlation between the forcing input and the output signal. Hence we can conclude that there is an optimal value of noise that is non zero at

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which the output signal is maximum. This phenomenon of noise induced signal transduction is called stochastic resonance (SR).

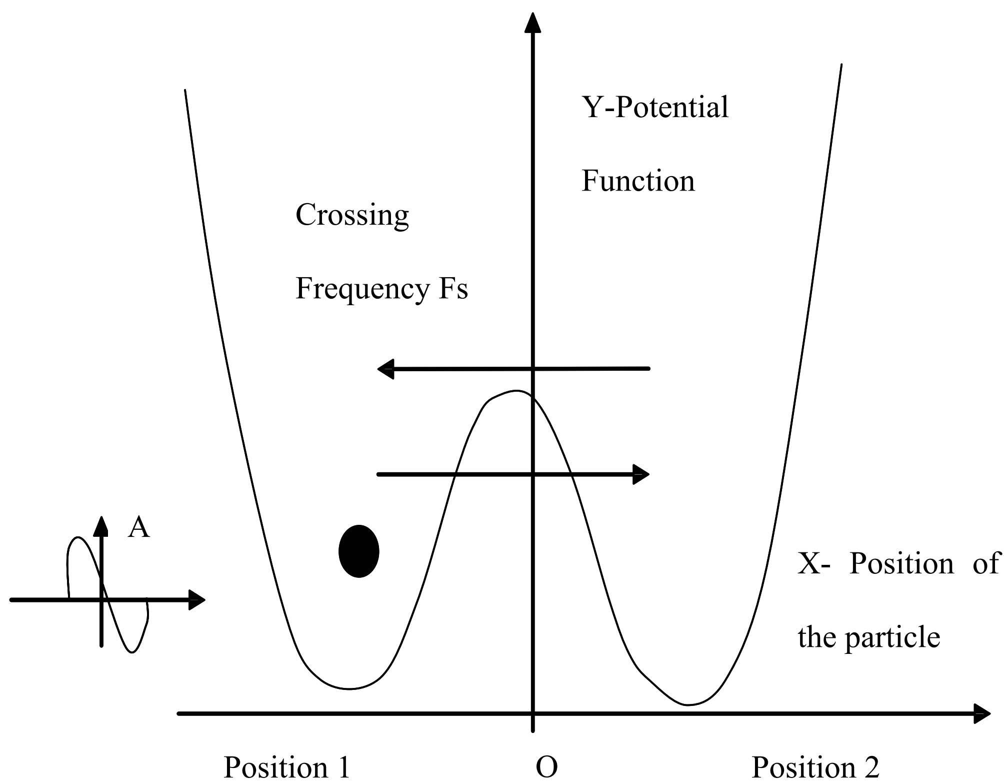


Fig 1.1. Illustration of the phenomenon of stochastic resonance in a simple double well potential system

1.2 Literature Review

Due to its relevance for biological information processing, in the recent times, the stochastic resonance (SR) effect has become one of the most promising phenomena taking place in non-linear systems driven by noisy periodic inputs [2-16]. It was shown by Bezrukov and Vodyanoy [17] that a reasonable model for neural signal transmission is the variable rate poisson model, where the neural system can be thought as a system which fires an output spike, if the input to the neuron exceeds a particular threshold in the positive direction. A similar model was found to be applicable for ion channels. This study of Bezrukov and Vodyanoy also showed that this particular neuron model exhibits the phenomenon of stochastic resonance.

Initially when SR effect was discovered, there was a widely held belief that it required a nonlinear dynamical system driven by a periodic input to observe this effect. But Kish, Moss and Zingl [15], showed that even non dynamical systems with a threshold like nonlinearity driven by aperiodic inputs can result in SR behavior. Further the non-dynamical system exhibited by them is a level crossing detector (LCD) which aptly captures the essential features of the neuron system described by Bezrukov and Vodyanoy.

The input of the stochastic resonators [14] has usually been excited by an additive Gaussian noise and a periodic signal with fundamental frequency $f_0$. As mentioned above, the SR effect is that, the output power spectral density shows a non-monotonic variation with respect to increasing the input noise power. That is, there exists an optimal strength of the input noise, where the system's output power density spectrum at the signal frequency $f_0$ has a maximal value. (See Fig 1.2)

The most important quantity of interest in SR systems is the "signal to noise ratio" (*SNR*), at the input ($SNR_{inp}$) and at the output ($SNR_{out}$) of the SR system. The *SNR* is defined as:

$$SNR = \frac{P_s}{S(f_0)} \tag{1.1}$$

In the above equation $P_s$ is the mean squared value of the (background corrected) Fourier component of the input voltage at frequency $f_0$ and $S(f_0)$ is the spectrum of background noise at $f_0$. Of particular interest to everyone in the field is whether there exist stochastic resonance systems that can *significantly* increase the *SNR* at the output. It was shown [18] that the "old dream" of achieving

$$SNR_{out} >> SNR_{inp} \quad (1.2)$$

can be achieved, in the strongly nonlinear response limit, if we use high bandwidth noise with strong subthreshold signal which has a spiky nature (small duty cycle).

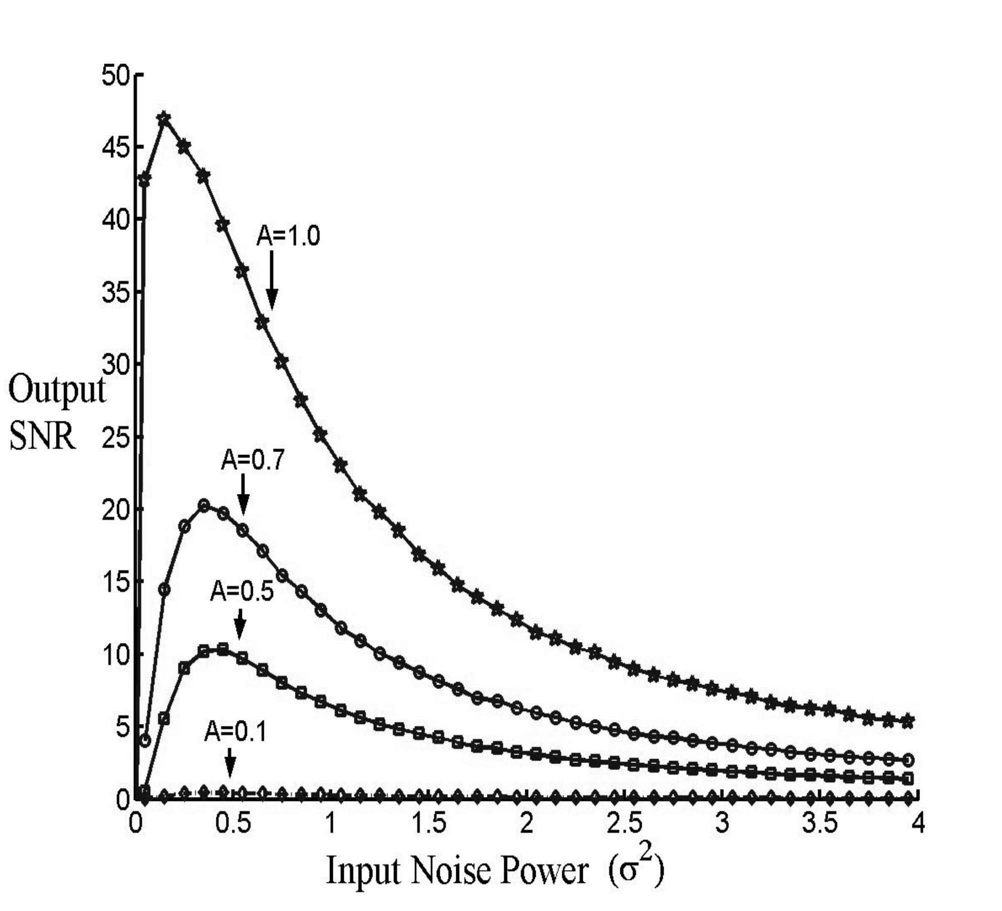


Fig 1.2. Demonstration of the stochastic resonance effect in the asymmetric LCD system

## 1.3 Aim of the Thesis

It is an interesting and a practical problem to determine the accuracy of these claims. But, to truly evaluate the accuracy of this claim, we need a proper measure of the *SNR*, which works under *all* circumstances, not merely in the linear response case. This is because high *SNR* gains are achieved at a strongly nonlinear limit where the spectrum of the background noise is shaped by the input signal. In other words there is an interaction between signal and noise at the output and hence the signal and noise components are no longer independent. This means that we cannot measure the noise power when there is no input signal and take that as the noise component at the output. This clearly necessitates a need for a general measure for *SNR* valid in all cases. The total failure of classical suggestions for *SNR* measures becomes most obvious in the case of *wideband aperiodic signals*, which have been shown in [18] to include the case when significant *SNR* gain is achieved. It is important to emphasize that *all neural and ion channel signals belong to this class*. The aim of this thesis is the following:

1. To present and discuss a measure of Signal to Noise ratio which is applicable under different scenarios

    a) Nonlinear signal transfer through a system

    b)When the input is stochastic rather than deterministic

    c)When there are no strong periodic components in the input compared to the duration of observation. e call the new measure as the Cross Spectral measure of SNR.

2. To present detailed simulations to substantiate the claim that the cross spectrum method used to determine the *SNR* is indeed a valid and the most general method which works under all circumstances i.e. nonlinear limit and wideband input signals.

3. Discuss the applications of the new measure under different circumstances and present examples and simulations, mostly related to models applied to the study of neurons.

Section 2 describes the new cross spectrum method for determining the output *SNR* and presents some theoretical analysis. In Section 3 we give a description of the SR system used in the simulations and then present the simulation results. Section 4 gives an account of a new and interesting phenomenon observed in asymmetric LCD systems with large output spike width and Section 5 ends with conclusions and suggestions for future work.

## 2. THE CROSS SPECTRAL MEASURE OF SNR*

The signal to noise (SNR) is a quantity which tells how much of the total power is contributed by the signal and how much the noise component is. In general we have a system with an input signal say x(t) and an output signal say y(t) as in Figure 2.1.

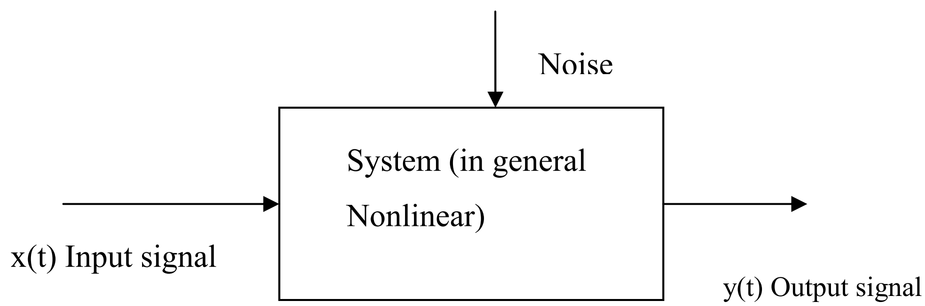


Fig 2.1. Schematic diagram to illustrate some terminology used

We can decompose the output y(t) into its spectral components and ask how much of the power in each spectral component comes from the signal $S_y(f)$ and how much from noise $S_n(f)$. $S_y(f)$ is called as the output signal spectral component and $S_n(f)$ is called as the output noise spectral component. The ratio $S_y(f)/S_n(f)$ is called the Signal to Noise ratio.

### 2.1 Some Definitions

The cross correlation function of two signals x(t) and y(t) which are real , stationary, ergodic and of finite power ( that is $\int E[x^2(t)]dt < \infty$ ) is defined as follows:

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$$R_{xy}(\tau) = E[X(t)Y(t+\tau)] \tag{2.1}$$

From this taking y(t)=x(t) we get the autocorrelation function of a signal x(t) namely $R_{xx}(\tau)$. The cross power spectral density (CSD) of two such signals, $x(t)$ and $y(t)$ is given by the Fourier transform of the cross correlation of $x$ and $y$, $R_{xy}(t)$, as :

$$S_{xy}(f) = \int_{-\infty}^{+\infty} R_{xy}(\tau)\exp(-j2\pi\tau)d\tau \tag{2.2}$$

Similarly the power spectral density of a signal (PSD) x(t) is given as the Fourier transform of its autocorrelation function, $R_{xx}(\tau)$.

$$S_{xx}(f) = \int_{-\infty}^{+\infty} R_{xx}(\tau)\exp(-j2\pi\tau)d\tau \tag{2.3}$$

The following highly useful approximations for $S_{xx}(f)$ and $S_{xy}(f)$ can be applied when x(t) and y(t) are ergodic and stationary.

$$\begin{aligned} X(f) &= \lim_{T \longrightarrow \infty} \left(\frac{1}{T}\right) \int_{-T}^{T} x(t)\exp(-2i\pi ft)dt \\ Y(f) &= \lim_{T \longrightarrow \infty} \left(\frac{1}{T}\right) \int_{-T}^{T} y(t)\exp(-2i\pi ft)dt \\ S_{xx}(f) &= E[X(f)X^*(f)] \\ S_{xy}(f) &= E[X(f)Y^*(f)] \end{aligned} \tag{2.4}$$

The above approximate formulae are accurate only when the limits of integrations are infinite. But in practice the limits range from –T to +T, where T is taken sufficiently large for all practical purposes. Note the we have dodged numerous issues while defining X(f) and Y(f). A proper rigorous treatment needs measure theoretic framework which is beyond the scope of this thesis. Hence the above definitions should be taken with a grain of salt and the engineer's dictum that what works in practice could be used with caution! The reason for the above approximations is that X(f) and Y(f) could be computed efficiently using FFT and prove convenient in simulations. Further in simulations one always works with sampled discrete data and hence many of the tricky convergence issues requiring measure theory can be avoided.

## 2.2 Classical Methods of Determining the SNR

The simplest method which is still widely adopted [9-14, 19, 20], is the following: The output noise spectrum is computed when there is no input. That is the input is switched off so to speak (x(t) = 0) and the output power spectral density $S_{yy}(f)$ is computed. And this $S_{yy}(f)|_{x(t)=0}$ is taken as the output noise spectral component even when x(t) is non zero. Summarizing in terms of equations,

$$\begin{aligned} S_n(f) &= S_{yy}(f)|_{x(t)=0} \\ S_y(f) &= S_{yy}(f) - S_n(f) \\ SNR(f) &= S_y(f) / S_n(f) \end{aligned} \quad (2.5)$$

The disadvantage of this method is that although it works well in the linear response limit, it fails badly when the signal transfer is non linear. This is because, the presence of signal influence the output noise spectrum when the transfer is nonlinear. This is illustrated through a simulation result in Section 3.

We now describe a correlation coefficient based method. In general, at nonlinear signal transfer, the output background noise cannot be determined by measuring the output noise spectral component when there is no input signal. In the case of nonlinear transfer, there are extra cross modulation product terms between the input signal and noise. Therefore this leads to an output noise which has a strong dependence on the input signal. Collins and coworkers [21] proposed a method that takes the correlation between input and output into account which is called as the correlation coefficient method. Here the SNR measure is given by the cross correlation coefficient between the input x(t) and output y(t), that is $SNR = E[x(t)y(t)]$.

But this quantity becomes zero when the input and the output are sinusoids shifted in phase by $90^0$. This is illustrated in Section 3. Hence the right idea would be to use the entire cross correlation function when defining the SNR. An intelligent choice would be to use the Fourier transform of the cross correlation function that is, the CSD.

The next classical method is the method based on continuity argument (see [15]). This method is applicable only for input signals containing periodic components. In such cases the total output power spectrum $S_{yy}(f)$ has sharp spikes at multiples of the fundamental frequency of the input periodic signal. We know that the background output noise spectrum (the output noise spectral component) is a continuous function of frequency. Hence its value at the multiples of the fundamental frequency can be obtained by interpolation at the nearby frequencies. Thus one obtains the output noise spectral component $S_n(f)$. The output signal spectral component $S_y(f)$ is then obtained as $S_{yy}(f)-S_n(f)$. The ratio $S_y(f)/S_n(f)$ gives the desired SNR. This is illustrated in Fig 2.2.

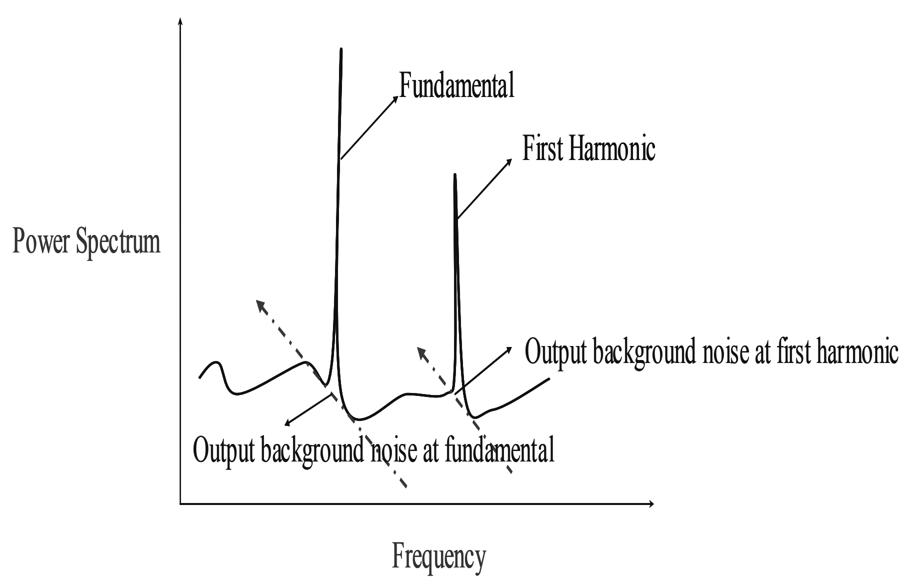


Fig 2.2. Illustration of computation of SNR using continuity arguments

The major drawback of this method is that it works only with periodic input signals. Further it is rather cumbersome and prone to errors.

## 2.3 The Cross Spectral Method

It is clear that $S_{xx}(f)$ and $S_{yy}(f)$ measure the total power in each frequency component at the input and the output respectively. Intuitively, the signal spectral component at the output is that part of the output signal which is correlated with the input. Also reasoning similarly, we can conclude that the output noise spectral component is that part which is statistically independent of the input signal. The cross spectral density (CSD) $S_{xy}(f)$ however measures the correlation between the spectral components at the input and output. The squared modulus of the CSD is hence a suitable candidate for the signal spectral component at the output. The only refinement is that this quantity must be normalized by $S_{xx}(f)$. Also the CSD is in general complex and hence retains the phase information and is robust phase errors between the input and the output. This intuition leads to the following equations. First we define the generalized amplification factor,

$$K(f) = \frac{S_{xy}(f)}{S_{xx}(f)} \tag{2.6}$$

Note that in nonlinear systems, *K(f)* can depend not only on the frequency, but also on the input signal and on the input noise. Now the output signal spectral component, $S_y(f)$ is obtained as follows,

$$S_y(f) = S_{xx}(f)\left|\frac{S_{xy}(f)}{S_{xx}(f)}\right|^2 = \frac{\left|S_{xy}(f)\right|^2}{S_{xx}(f)} \tag{2.7}$$

The definition of the noise power in the output signal is now straight forward consequence of the above. Accordingly, we define the output noise spectral component

$S_n(f)$ as the difference between the total output power spectrum $S_{yy}(f)$ and the output signal spectral component $S_y(f)$.

$$S_n(f) = S_{yy}(f) - S_y(f) \tag{2.8}$$

Note that the above definitions restore the validity of the old definitions in the limit of small sinusoidal input signal (linear transfer and sinusoidal excitation, see Fig 3.2). Moreover the new definitions work at arbitrary conditions and the only pre requirement is the stationarity of the input noise, input signal and the stochastic resonator.

2.4 The Analysis of the Cross Spectral Method

We now describe, how under simplifying assumptions the cross spectral method leads to intuitively satisfying results and reduces to classical definitions. In the case of deterministic signals, the above definition simplifies as follows: The signal power becomes $E[|Y(f)|]^2|$ and the noise power is nothing but the variance of $|Y(f)|$. This leads to an intuitively satisfying view of the output signal power and noise power. This in the linear limit reduces to the classical definition. This is shown in the equations below,

$$\begin{aligned}
S_y(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)} &= \frac{|E[X(f)Y^*(f)]|^2}{|E[X(f)X^*(f)]|} \\
&= \frac{|E[Y^*(f)]|^2 X(f)X^*(f)}{X(f)X^*(f)} \\
&= |E[Y^*(f)]|^2 \\
&= |E[Y\ \ (f)]|^2
\end{aligned} \tag{2.9}$$

This simplification was possible because x(t) being deterministic implied that E[X(f)]=X(f) and E[Y(f)X(f)]=E[Y(f)]X(f) and so on. The output noise simplifies to the following,

$$S_n(f) = S_{yy}(f) - S_y(f) = E[Y(f)Y^*(f)] - \left|E[Y(f)]\right|^2 = \mathrm{var}(Y(f)) \qquad (2.10)$$

This leads to the pleasing interpretation of the signal power as the mean squared value of Y(f) and the noise power as the variance of Y(f).

Now we take the case of linear systems. In linear systems the output y(t)=H[x(t)+n(t)], where x(t) is the input signal and n(t) is the noise which is uncorrelated to the signal and H is a linear transformation. In this case the output signal spectral density reduces to $S_{xx}(f)|H(f)|^2$ and the output noise spectral density becomes $S_{nn}(f)|H(f)|^2$ where H(f) is the system transfer function. Hence the SNR becomes,

$$SNR(f) = \frac{S_{xx}(f)}{S_{nn}(f)} \qquad (2.11)$$

Hence the input signal does not change the output background noise. So we could use the classical definition of switching off the signal and measuring the output power spectrum to determine the background noise spectrum.

# 3. SIMULATION RESULTS*

Before describing the simulation results we describe the level crossing detector (LCD) system which was used in simulations where nonlinear transfer was required. The reason for choosing the LCD system is that the LCD is a simple non dynamical system with threshold nonlinearity. It is one of the simplest examples of a nonlinear system which occurs in a wide variety of situations including neuron models and ion channel models.

## 3.1 Description of the LCD system

The suitability of the cross-spectra measure for SNR is demonstrated using a Level Crossing Detector (LCD) setup. The LCD is a suitable candidate for study as it has a threshold like non-linearity, which is ubiquitous in most SR systems. Further extensive experimental study show that the level crossing dynamics of the Gaussian noise inherently contains the SR effect (see Fig 1.2). In this paper we use the LCD systems as described in [18] (see [23] for a fuller account).

First we describe the asymmetric LCD system. The asymmetric system consists of an LCD of the following kind: whenever the input amplitude of the input excitation (noise and signal) crosses the positive threshold level $U_t$ in increasing direction, the LCD produces a positive, short pulse with amplitude A and duration $\tau_0$ at its output. The resulting output response of the system is a random time-sequence u(t) of uniform, positive pulses.

The symmetric system consists of an LCD of the following kind: whenever the input amplitude crosses the positive threshold level $U_t$ in increasing direction, the LCD produces a positive, short pulse with amplitude A and duration $\tau_0$ at its output. On the other hand, whenever the input amplitude crosses the negative threshold level $-U_t$ in

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decreasing direction, the LCD produces a negative, short pulse with amplitude -A and duration $\tau_0$ at its output. The resulting output response of the system is a random time-sequence u(t) of uniform, positive and negative pulses with zero time average.(See Fig 3.1).

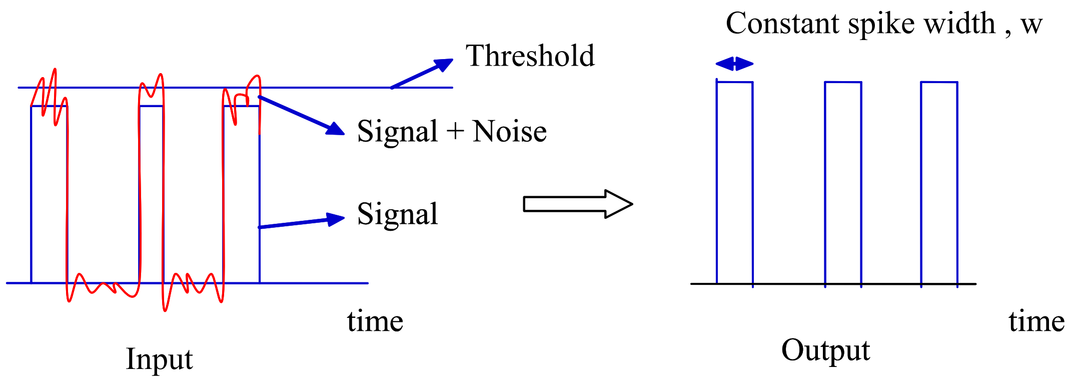


Fig 3.1. Illustration of the Asymmetric LCD setup

3.2 Comparison of the Classical and the New Method

First we compare the two systems in the linear response limit. In the case of classical definitions, the signal component at the output is defined to be the square of the frequency component of the total output power spectrum at the frequency of the input signal, so that the output noise power at this frequency is subtracted. The output noise is the total output AC voltage in the case of no signal. In the simulations the input was a sinusoidal signal of a fixed frequency and the output was the input signal corrupted with additive white gaussian noise of a fixed variance. Both the classical and the new definition were tested by MATLAB simulations. And in the case of sinusoidal signals with linear transfer the two values agreed which is a pleasant confirmation. The results are shown in Fig 3.2. The input signal was a pure sinusoidal signal of amplitude 0.5 V and frequency 5 Hz and the output signal was the input signal corrupted with additive white Gaussian noise of variance 1. The threshold $U_t$, of the asymmetric LCD was set to

1 (see Fig 3.1). The signal to noise ratios were computed by the two methods and the theoretical value was also computed. The three values show that they all agree in the linear limit. This establishes that the new SNR measure gives the same value as the classical measure in the linear limit. (See Table 3.1)

Table 3.1: Comparison of SNR obtained by classical, new and continuity methods at the linear limit

| Method Used | SNR value |
|---|---|
| Classical SNR | **3.7167** |
| New Method | **3.7065** |
| Theoretical value` | **3.7500** |

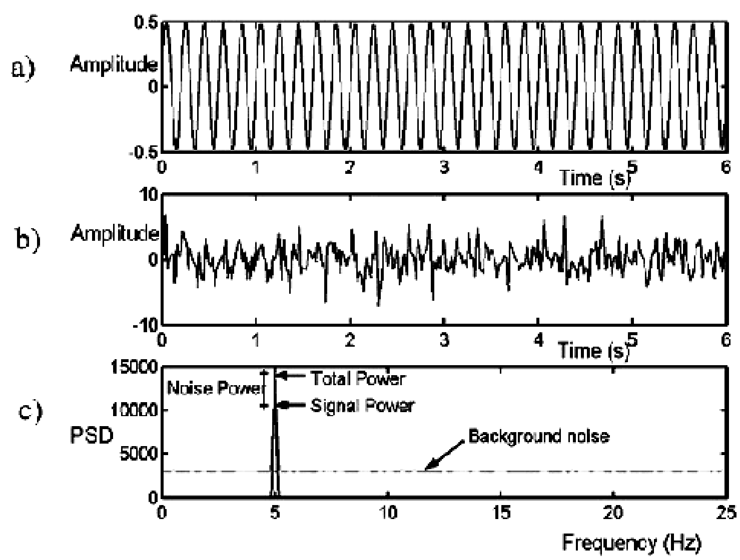


Fig 3.2. Linear response limit (a) sinusoidal signal of amplitude 0.5 V and frequency 5 Hz (b) corrupted in gaussian noise σ=1V (c) total output power and the signal and noise power components

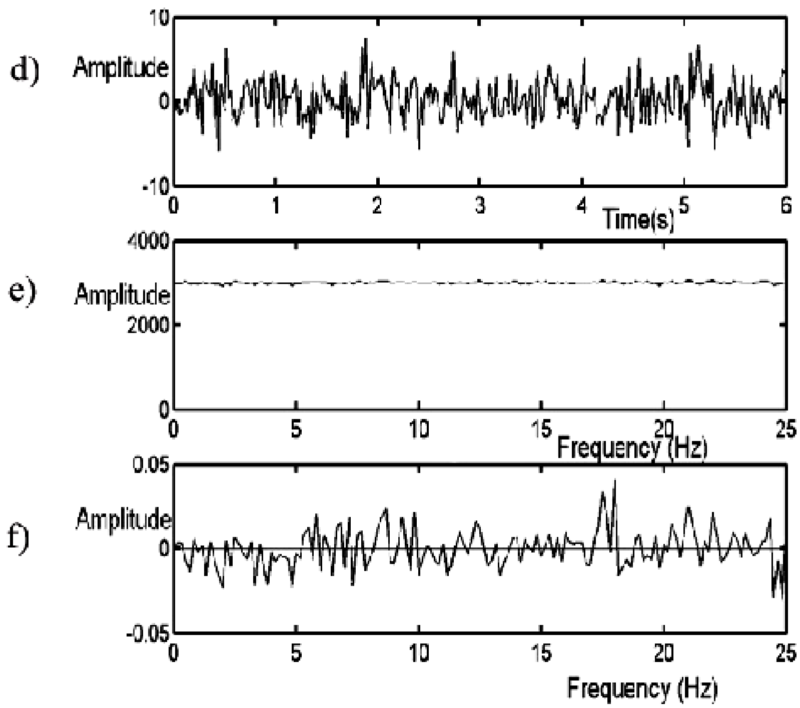


Fig 3.2. (cont.) (d) no input signal (e) background noise spectrum when the signal is absent. (f) the difference (very negligible) of the background noise between the without signal and with signal cases with sampling time = 20 ms

Next we take the case of nonlinear response limit. In the case of nonlinear response and periodic signals, the classical and the new method differ remarkably. Why the classical measure fails even for very strong periodic input signals is because then the output noise can be suppressed due to saturating the resonator by the signal. In the test simulations (Fig 3.3) the input signal was a pure sinus of strong amplitude (i.e. comparable to the noise variance), corrupted by an additive white Gaussian noise of variance 1. This signal was passed through the asymmetric LCD described in Section 2 ($U_t$.=1). The background output spectrum is compared to the background spectrum when only the input noise is present. It is clear that the presence of the signal definitely has an effect on the shape of the output noise spectrum (see Fig 3.4). The signal to noise ratio is now computed by both the classical and new methods. There is a significant difference between the two. The output noise given by the classical method is higher. On the other hand, the background noise spectrum must be continuous with frequency. Hence one can compute the noise power at the signal frequency by measuring the height of the periodic spike in the output spectrum. This value agrees with the value given by the new method as shown in Table 3.2, clearly showing that the spike method works well. However, the spike reading method can only be used for sinusoidal input signal and so for wideband signals only the new method works.

Table 3.2: Comparison of SNR obtained by classical, new and continuity methods at the nonlinear limit

| Signal amplitude | Classical SNR | New Method | SNR by continuity argument |
|---|---|---|---|
| A=0.5 | 11.1490 | 12.5911 | 12.6090 |
| A=1.0 | 30.9393 | 44.7374 | 44.7333 |
| A=1.5 | 31.8721 | 60.4989 | 60.5106 |

These results unambiguously confirm the validity and effectiveness of the new method. Moreover, the value given by the continuity argument can be unreliable because the height of the noise power is determined manually from the plot, where the area below the spectral spike has to be determined for that. The cross spectral method has not only a better reliability but also can be employed in a straightforward mathematical formulation.

As we mentioned above, a measure for the output signal power using the cross correlation coefficient between the signal and the output was proposed [21] by Collins and coworkers and recently by Stocks and coworkers [22]. Though the Collins method works nicely in systems with sinusoidal signals and no phase shift between the input and the output, it fails in the presence of phase shift or frequency dependent transfer and wideband signal. For example if the input is a sinusoid whose phase is unknown, then a 90 degree phase shift between the actual and assumed phase will result in the correlation coefficient being zero. Our cross spectral measure does not suffer this drawback as it is shown by the simulation results in Fig 3.4. The input is a sinusoidal signal of unknown phase and the stochastic resonator shifts the phase by $90^{o}$. Still the output signal does not change. The imaginary part of the cross spectrum can be used to compute the phase difference of the output signal with respect to the original signal.

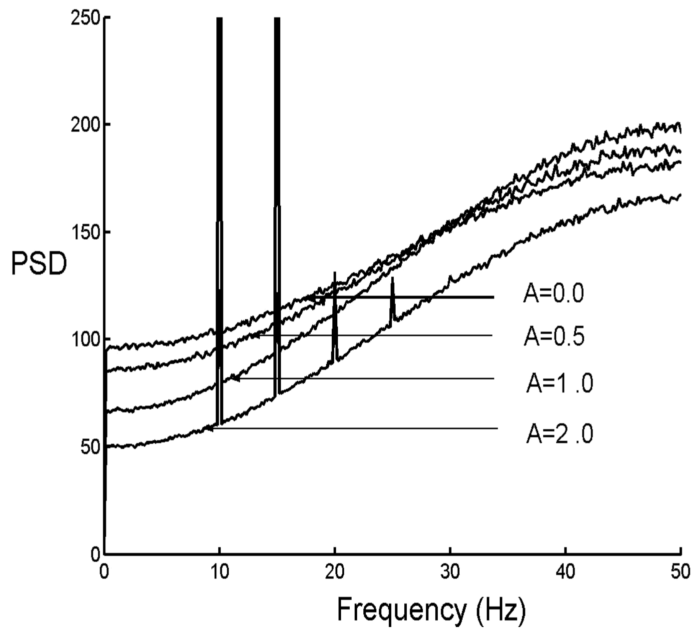


Fig 3.3. Plot of the output background noise by the new method

Now we present a few more comparisons [23]. For wideband aperiodic signal with phase shift or frequency dependent transfer, it is obvious from the above results and considerations that, presently, the only method able to provide usable results is the cross spectral method. That means, many biophysical applications have no other choice, so far, than to use cross spectra.

In Fig 3.5, further comparisons between the SNR determined by the classical and the cross spectral methods is shown. Here the input signal was a sinusoid corrupted by Gaussian noise. The simulations were carried out for different values of the amplitude of the sinusoid. Clearly at the non linear limit (higher signal amplitudes), the classical method is inadequate. The results shown above (Figs 3.2, 3.3 and 3.4) clearly indicate that the cross spectrum method is a consistent measure at all ranges. The plot in Fig 3.5 gives us an estimate of the error made by the classical method in the strongly nonlinear limits and also high values of input noise. Thus all the simulations presented here indicate that the new SNR measure is beyond doubt both a correct and a convenient one to use under a wide variety of circumstances of practical importance.

The comparisons are presented at different values of the input noise power. In the linear limit there is a close agreement and in the nonlinear limit the error made by the classical method is quite substantial. The error of the classical method reaches one order of magnitude.

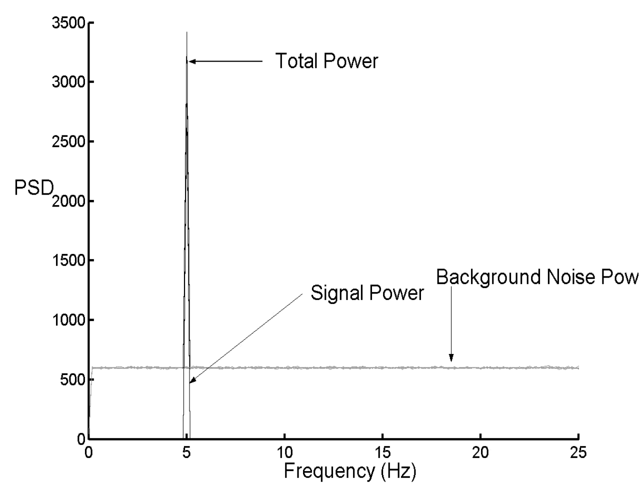


Fig 3.4. Cross spectrum measure in the case of unknown phase of the input signal

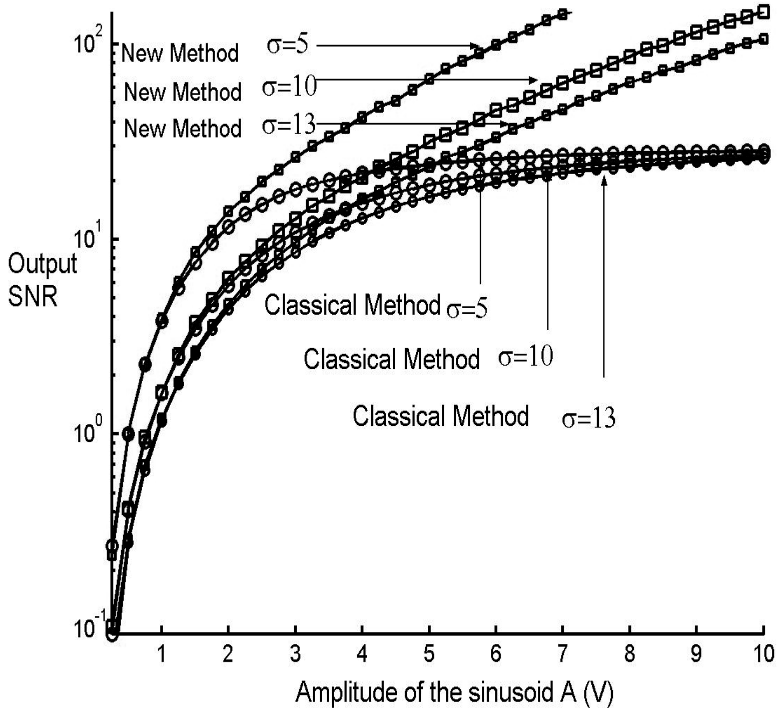


Fig 3.5. Comparison of the SNR determined by the classical and the new methods as the amplitude of the input sinusoidal signal varies

This section has shown simulations results to show the efficiency of cross spectra measure for signal and noise in the case aperiodic spiky and other wideband signals in the strongly nonlinear limit. The results show that the cross-spectral identifications of output signal and noise are sensible measures and that they work for arbitrary signals and noise, for both the linear and nonlinear cases. As the neural and ion channel signal transfers are nonlinear and aperiodic, the new method has direct applicability in biophysics and neural science.

# 4. BLUE SHOT NOISE*

This section presents an account of an interesting phenomenon [24] which arises while investigating the output power spectral density of an asymmetric LCD when the input noise is very high and the width of the output spike of the LCD is also increased.

## 4.1 Motivation

Threshold crossing problems of gaussian noise are at the core of many stochastic phenomena. They play also a determining role in non-dynamical stochastic resonators in which were first experimentally studied by Frank Moss [15]. In this paper, we show colored noise effects called blue noise in a level crossing detector (LCD) system which was proposed by Moss to model simple neural responses. When a noise spectrum is constant versus frequency, the noise is called *white noise*. Following this fashion, a noise with $1/f^2$ spectral shape is called *red noise*, due to the strong weight of the lower frequencies and the 1/f noise is called *pink noise*. Thus a noise having an increasing spectrum versus frequency is *bluish* or it can simply be called *blue noise*. In this paper, we show that a level-crossing detector with wideband input noise and wide output pulse width generates a blue noise which we call the *blue shot noise* because of the similarity of this response to shot noise. This situation is very similar to neural response and it follows that under certain conditions neural response can also produce *blue noise*. Although such blue noise effect can be seen in stochastically driven harmonic oscillators, such a system is dynamical and is governed by differential equations which can simulate differentiation and hence observing blue noise effect in dynamical systems is not surprising. However its existence in non-dynamical systems like a LCD is not a trivial problem. By observing this effect we suspect that the LCD has "time derivative" capability under certain conditions.

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* The material in this section is reprinted with permission from "Blue noise effects in a non-dynamical neural model system" by S.Sethuraman, L. B. Kish, March 2004, Fluctuation and Noise Letters, Vol 4, No 1, L179-L183. 

4.2 Simulations

Computer simulations were carried out simulating an asymmetric level crossing detector (LCD) with the following conditions: whenever the input amplitude at the LCD crosses a fixed threshold level from below, an output pulse of width *w* is generated. The fixed width w of output pulse corresponds to a fixed time-integral of the pulse and this corresponds to the case of a shot noise pulse. The correlation time τ of the band-limited white noise driving the input was one computer step. In Fig 4.1, the blue shot noise effect can be seen.

Figure 4.1 shows the output noise spectrum of the LCD when driven by only noise, for different values of the width of the output spike, in units of sampling time $T_s=1/F_s$ where the sampling frequency $F_s = 65$ Hz. The threshold of the LCD was 1 V. The input was driven by a white gaussian noise of variance 1 V and the input signal was absent.

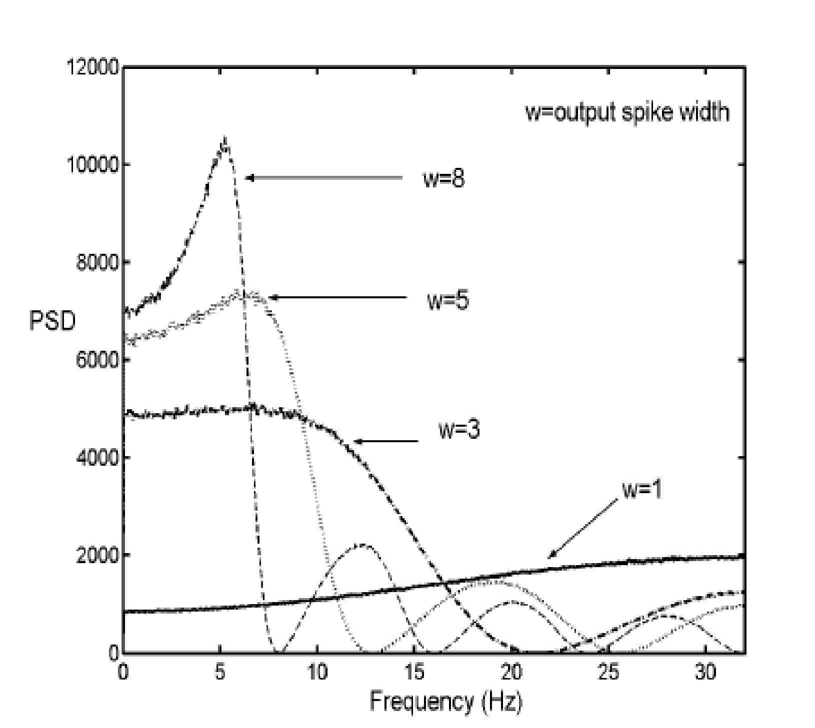


Fig 4.1. Illustration of the blue noise effect

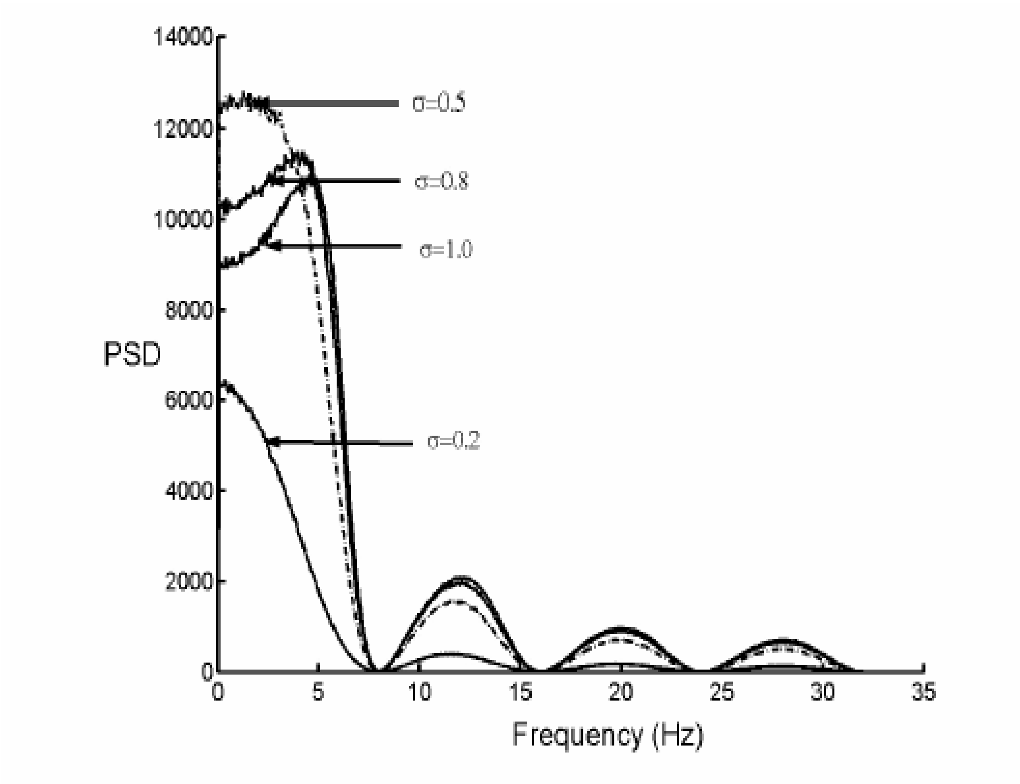


Fig. 4.2. Background noise spectrum at different values of the input noise level where the sampling frequency = 65 Hz with the width of the output spike kept constant at $5T_s$

In Fig. 4.2, at fixed pulse width *w* = 5, the dependence of the blue noise effect on the strength of the input noise is shown. Apparently, the stronger the noise the more emphasized the blue noise effect is.

In Fig. 4.3, at fixed pulse width *w* = 5, the dependence of the blue noise effect on the strength of additive sinusoidal input signal is shown. Apparently, the stronger the signal the more emphasized the blue noise effect is.

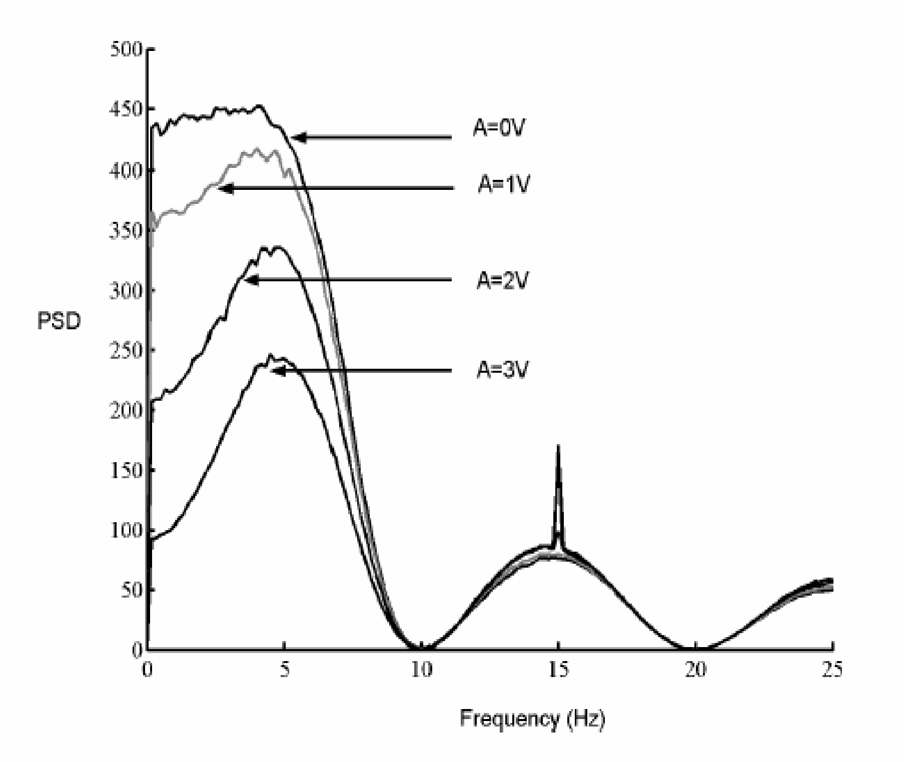


Fig. 4.3. The output noise spectrum at LCD threshold 1 V for different values of the signal strength with fixed width of output spike = $5T_s$ and sampling frequency $F_s = 50$ Hz

## 4. 3 Explanation

When the level crossing frequency $f_L$ (which can be evaluated from the Rice formula [25]) of the threshold level by the noise is much greater than 1/w, the output time function is a roughly periodic spike train with mean repetition frequency $1/(w+1/f_L)$. The spike duration is fluctuating and its mean value is $1/f_L = \langle q \rangle$ and in the limit $f_L \rightarrow \infty$ , the spike train would be periodic with period time $w$, so the first harmonic would be at frequency $1/w$. As at finite $f_L$ the period time and the pulse width are slightly fluctuating, in a random fashion, the harmonic spikes will not be sharp and they will have sidebands. The lowest side band is the blue shot noise.

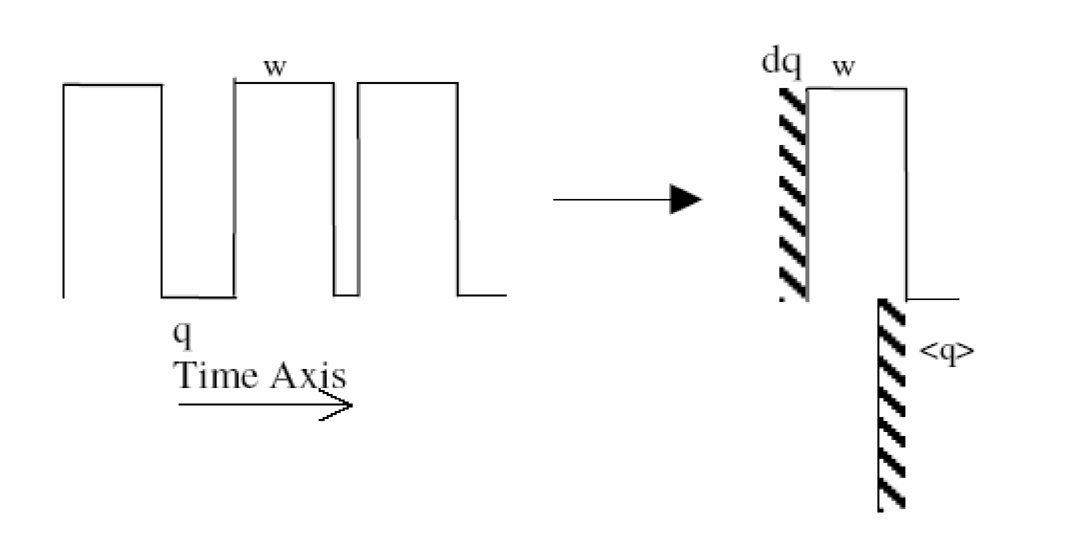


Fig. 4.4. Illustration of the time derivative characteristics of the saturated system

The shattered area, in Fig 4.4, which is similar to the time-derivative of a single square pulse, is the difference between the original output pulse and the delayed one. As we mentioned above, the blue noise effect suggests a time-derivative characteristics of the system. At the first look it is not obvious how a time-derivative characteristics could arise in such a non-dynamical rigid system as an LCD. As seen in the above diagram, each variable-width pulse (w + q range) can be represented as a fixed-width pulse (w + <q> pulse) added to a derivative pulse (shown as shattered). In this way, even a nondynamical system can simulate blue noise effect.

## 5. CONCLUSION AND RECOMMENDATIONS

This thesis has shown simulations results and some theoretical analysis to show the efficiency of cross spectra measure for SNR for a wide variety of signals under mild restrictions of stationarity and in particular for neural and biological signals. Further work would be to try and come up other interesting measures of SNR using higher order statistics which deals with higher order spectra of signals. Also the proposed method of SNR could be tested and applied to many other systems of interest [26] which arise in practice where stochastic or wideband input signals are encountered in the presence of nonlinear transfer where the classical methods are shown to fail.

We can also consider another angle. It was shown in [15], using cross spectral measure that SNR gain greater than one is indeed possible using wideband aperiodic signal and gaussian noise as input to an asymmetric LCD described in Section 3. Using detection theory one can obtain theoretically by maximizing aposteriori probability, an optimal estimator or detector for any signal corrupted with noise. Often this involves non linear optimization problem which often turns out to be intractable. The optimal detector often has SNR gain. Above discussions indicate that sub optimal detector using SR systems can also provide gains and can be used with effectiveness in cases where the optimal solution is intractable.

It would be interesting to compare the performance of a simple LCD detector followed by a matched filter with that of the optimal (but with high computational complexity) algorithm obtained using detection theory. We conjecture that the simple LCD system can indeed come closer to the optimal algorithm in many cases. It would be a profitable future exercise to prove this theoretically.

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