

METHODOLOGY AND ALGORITHM FOR ASPHALT SUPPLY CHAIN OPTIMIZATION

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Preliminary notes

In this paper authors suggest and test evolutionary algorithm of multiple criteria solver (MCS) for asphalt supply chain optimization. On the basis of the defined imperfection of the basic mathematical model authors made an algorithm and conducted simulations of transportation problems cases generated from the given realistic input intervals in order to determine the constraints and possibilities for improvement of the original model of the transportation problem of large amounts of asphalt mixture. Results have shown that the suggested model verifies and eliminates the lack of the original model. As well, it is shown in which scenarios of transportation problem and to which extent the suggested methodology contributes to the total transportation costs savings and insurance of the program's optimality.

Keywords: *asphalt; chain of processes; optimization; simulation; transportation*

Metodologija i algoritam za optimizaciju lanca procesa transporta asfalta

Prethodno priopćenje

U radu je predložen i testiran evolucijski algoritam za optimizaciju lanca procesa transporta asfaltne mješavine na gradilišta. Izrađen je algoritam (rješavatelj MCS) na temelju definiranog nedostatka postojećeg matematičkog modela te provedena simulacija slučajeva transportnog problema kako bi se utvrdila ograničenja i mogućnosti za poboljšanje originalnog modela za rješavanje transportnog problema velikih količina asfaltne mješavine. Rezultati su pokazali da predloženi model pridruživanja i isključivanja potencijalnih izvora potvrđuje i otklanja nedostatak originalnog modela. Pokazano je i u kojim slučajevima transportnog problema i u kojoj mjeri predložena metodologija doprinosi uštedi i osiguranju optimalnosti ponuđenog programa.

Ključne riječi: *asfalt; lanac procesa; optimizacija; simulacija; transport*

1 Introduction

Material network flow from and to construction sites is an everyday engineering task. For solving such problems, in the literature known as the transportation problems (TP), there are various mathematical (theoretical) models, deterministic, heuristic and metaheuristic optimization methods, as well as the computer programs (i.e. Solvers). Transportation problem is defined as a linear program for optimization of the transport of some goods (materials) from multiple origins (sources) to multiple destinations with usually one objective function: minimization of the total transportation costs [1÷4]. Linear programming, as well as the wider area to which it belongs (Operations Research), after a rapid upsurge in research in the 1970's stood on the developed mathematical models that had yet to be confirmed through the application [5, 6]. However, their application was limited by the lack of tools for efficient and quick usage, what is now finally made possible by the development in information technology (IT) sector [7, 8]. Nowadays, the mentioned models are again in the centre of attention of researchers, but practitioners as well. They are extended and modified through the application in solving various characteristic problems in various industries [9÷13]. Similar is happening with the models of the transportation problem, which is the motivation of this research.

In this paper authors have detected and defined the possibilities and limitations of the aforementioned basic TP mathematical model by testing it in simulations of transportation cases of hot asphalt mixture with known (given) constraints. Authors identified that the basic mathematical model of the transportation problem is insufficient tool to ensure optimal solution due to the task of fulfilling the basic assumption – balance of the model (total offer must satisfy the total demand) [3, 14, 15]. As a

response to the aforementioned basic model's imperfection authors have developed a model of inclusion and exclusion of sources based on the extended basic mathematical model of TP and tested it by numerical simulations. Generating the cases of TP in the form of 10 potential sources should supply 10 default sites with hot asphalt mixture. The main goals of the simulations are to determine the limitations of the basic TP model and to confirm the proposed model in different scenarios.

2 Research methodology

The research is divided into four interrelated parts. In the first part the authors expressed and formulated the imperfection of the basic TP model which was tested and proved by the simulation of the TP cases, as a continuation of authors' preliminary study [3]. In the second part authors made an algorithm for suggested model based on the extended model TP (MCS - Multiple Criteria solver) within the computer program Microsoft Excel for later numerical simulations. The third part of the research relates to the computer/numerical simulations of TP cases consisting of three rounds of simulated sets. In the first round of simulations (preliminary study) authors simulated 4 sets of realistic input parameters (i.e. nominal capacity of asphalt plants, construction sites demands and unit prices of asphalt transportation from plants to the sites). Each set consisted of 100 cases of TP in their initial form of 10 potential asphalt plants and had to supply 10 default sites. The input parameters were generated for each subsequent set of simulations from the determined intervals and by modifying the interval boundaries simulated different cases of TP. The first round of simulated cases is used to detect which parameters have a significant impact on the difference between the initial and optimal program. Based on the results of the aforementioned first round increment and decrement

relations of interval boundaries were determined for the simulation cases in the second round ($N = 100$), while fixing the boundaries of the nominal capacity. Upon the results of the second round, authors extracted the set that showed the highest average difference between the initial and optimal programs for the simulations in the third round where boundaries of unit prices and demand were constant while boundaries of the interval nominal capacity were changed. In this round authors simulated 1000 cases of TP to check and confirm the results from previous rounds.

3 Background of the basic TP model and formulation of the imperfection

Ever since Hitchcock [16, 17] structured a special case of the LP problem (i.e. transportation problem) and Dantzig [18] introduced the simplex method for solving such problems, practical and theoretical circles are working on its further application, modification and verification. Transportation problem depending on its form can be in the domain of the linear and non-linear programming [19, 20], or even in special cases when combined with routing problem noted to be NP-hard (non-polynomial time hard) [21]. Except linear or non-linear optimization problem, TP can be integer (ILP and INLP) and mixed-integer (MILP and MINLP) problem [22, 23]. Therefore, it is crucial to carefully and precisely define the problem which is subject of the optimization. Nevertheless, most of the transportation problems are considered to be linear and thus can be solved using the pallet of the deterministic optimization methods, as it is the case in this paper. In table 1 is shown the basic linear mathematical model [24] of TP in matrix form. The given number of the construction sites is denoted as $B_{1 \rightarrow n}$ with well-known demand of each site denoted as $b_{1 \rightarrow n}$, while number of potential sources is denoted as $A_{1 \rightarrow m}$ with the capacity of each source denoted as $a_{1 \rightarrow m}$ in given/estimated time period. The main assumption to keep the model solvable is that it has to be balanced: the total production (supply) has to match the total demand denoted in Tab. 1 as $\sum a_i = \sum b_j$.

Table 1 Matrix form of the basic model of TP

		Destinations				Supply
		B_1	B_2	B_3	B_n	
Sources	A_1	x_{11}	x_{12}	x_{13}	x_{1n}	a_1
	c_{11}	c_{12}	c_{13}	c_{1n}		
	A_2	x_{21}	x_{22}	x_{23}	x_{2n}	a_2
	c_{21}	c_{22}	c_{23}	c_{2n}		
A_3	x_{31}	x_{32}	x_{33}	x_{3n}	a_3	
c_{31}	c_{32}	c_{33}	c_{3n}			
A_m	x_{m1}	x_{m2}	x_{m3}	x_{mn}	a_m	
c_{m1}	c_{m2}	c_{m3}	c_{mn}			
Demand	b_j	b_1	b_2	b_3	b_n	$\sum a_i = \sum b_j$

Mathematical interpretation of the basic TP model is given by expressions for objective function (Eq. (1)) and belonging structural constraints (Eq. (2)):

$$\min Z = \sum_{j=1}^n \sum_{i=1}^m c_{ij} * x_{ij}, \tag{1}$$

$$\sum_{j=1}^n x_{ij} = a_i; \sum_{i=1}^m x_{ij} = b_j; x_{ij} \geq 0. \tag{2}$$

The problem arises with ensuring the mentioned assumption of balanced model. Due to the fact that when structuring the TP model by adding the potential sources (with the belonging capacities, to fulfil the total demand (to keep the model balanced) all the potential sources will be engaged in the optimal solution, which does not have to be true if the production time is not an objective. Time of the production could be an additional objective, or even constraint but it would have to include its dependence on the efficiency of both: transportation and paving operations on site. Suggested model elaborated in this paper takes into account the assumption that the time of production is a degree of freedom, while the incorporation of the time dependences among processes into the model will be a part of author's further research work.

The model is structured in the way which will allow testing the scenarios of solutions by excluding the potential sources in certain order and by certain criteria, but to ensure the balanced model. Therefore, with the suggested model the matter of optimization is not only the amount of asphalt mixture needed but the engaged asphalt plants from the initial entry as well.

4 Model of source inclusion and exclusion for solving and optimization TPs

To enable the exclusion of potential sources from the model, the basic model has to be expanded (modified) by certain criteria. Authors have expanded the basic model by adding three criteria of input parameters (Tab. 2): nominal capacity of each source (NC_i), mean and median value of the transportation costs per unit (mean and median c_{ij}).

Table 2 Matrix form of the expanded TP

Sources	Nominal capacity (t/h)	Destinations				Supply a_i (t)	Mean value c_{ij} (€/t)	Median value c_{ij} (€/t)
		B_1	B_2	B_3	B_n			
A_1	NC_1	x_{11}	x_{12}	x_{13}	x_{1n}	a_1	c_{1j}^{mean}	c_{1j}^{median}
		c_{11}	c_{12}	c_{13}	c_{1n}			
A_2	NC_2	x_{21}	x_{22}	x_{23}	x_{2n}	a_2	c_{2j}^{mean}	c_{2j}^{median}
		c_{21}	c_{22}	c_{23}	c_{2n}			
A_3	NC_3	x_{31}	x_{32}	x_{33}	x_{3n}	a_3	c_{3j}^{mean}	c_{3j}^{median}
		c_{31}	c_{32}	c_{33}	c_{3n}			
A_m	NC_m	x_{m1}	x_{m2}	x_{m3}	x_{mn}	a_m	c_{mj}^{mean}	c_{mj}^{median}
		c_{m1}	c_{m2}	c_{m3}	c_{mn}			
		$\sum NC_i$	b_1	b_2	b_3	b_n	$\sum a_i = \sum b_j$	
		Demand b_j (t)						

When the expanded model of TP is structured sources can be sorted in the wanted order for their exclusion from the simulation by the mentioned criteria. Exclusion of the sources from the initial model one by one is conducted till the model when two sources have to match the demand of the given constant number of destinations (sites). By solving those models (Eqs. (3) to (5)) output will be the matrix of solutions (total transportation costs) sorted by the criteria and model (Tab. 3).

$$Z_{m-1} = \sum_{j=1}^n \sum_{i=1}^{m-1} c_{ij} * x_{ij}; \sum_{j=1}^n x_{ij} = a_i; \sum_{i=1}^{m-1} x_{ij} = b_j; x_{ij} \geq 0, \quad (3)$$

$$Z_{m-2} = \sum_{j=1}^n \sum_{i=1}^{m-2} c_{ij} * x_{ij}; \sum_{j=1}^n x_{ij} = a_i; \sum_{i=1}^{m-2} x_{ij} = b_j; x_{ij} \geq 0, \quad (4)$$

$$Z_{m=2} = \sum_{j=1}^n \sum_{i=1}^2 c_{ij} * x_{ij}; \sum_{j=1}^n x_{ij} = a_i; \sum_{i=1}^2 x_{ij} = b_j; x_{ij} \geq 0. \quad (5)$$

Table 3 Matrix form of the solutions gained by exclusion of sources

		Criteria		
		Mean c_{ij}	Median c_{ij}	Nominal capacity NC_i
Models	m/n	Z_i		
	$m-1/n$	Z_{m-1}^{mean}	Z_{m-1}^{median}	Z_{m-1}^{NCi}
	$m-2/n$	Z_{m-2}^{mean}	Z_{m-2}^{median}	Z_{m-2}^{NCi}

	$2/n$	$Z_{m=2}^{mean}$	$Z_{m=2}^{median}$	$Z_{m=2}^{NCi}$

The optimal solution is provided by the model with the minimal total transportation costs $\rightarrow Z_{min}$. Authors have developed a computer program (Solver) MCS using the algorithm described above by the mathematical expressions and shown in Fig. 1. With this approach influence on the optimal result by structuring the initial model will be avoided.

5 Numerical simulation - experiment

5.1 Input parameters and generating the round of TP simulation cases

All sets of simulated TP cases had the same initial form: 10 potential asphalt plants supply 10 construction sites (initial 100 variables). Previous researches [25, 26] of the available optimization solvers have pointed out that Microsoft Excel add-in Solver is applicable and successful optimization tool for problems with up to 200 variables. As well, authors [22] noted that in that for small case matrix models of the TP the difference between

solution gained by linear and non-linear programming is not significant. Thus, MCS model presented in this paper is LP based, using the Microsoft Excel Solver add-in. Input parameters in the experiment (i.e. nominal capacities of the asphalt plants and truck transportation costs per unit) are realistic even though the main objectives of the experiment are expressed as quotients and percentages. Different sets were obtained by generating the input parameters in the defined intervals (equations 6 to 8) for each set: nominal capacities of asphalt plants labelled as NC_i , the values of the unit transport costs of asphalt marked with c_{ij} and sites' demands marked with b_j .

$$NC_i \in [\min NC; \max NC] \rightarrow qNC_i = \frac{\max NC}{\min NC}, \quad (6)$$

$$c_{ij} \in [\min c; \max c] \rightarrow qc_{ij} = \frac{\max c}{\min c}, \quad (7)$$

$$b_j \in [\min b; \max b] \rightarrow qb_j = \frac{\max b}{\min b}. \quad (8)$$

For structuring different simulation sets authors are using quotients of input parameters (i.e. quotient on maximal and minimal transportation costs qc_{ij} , quotient of maximal and minimal nominal capacities of asphalt plants qNC_i , quotient of maximal and minimal value of sites demand for asphalt mixture qb_j).

The results of all simulations sets were recorded as comparison of the programme gained by initial (10/10) (Z_i) and optimal models (Z_{min}) denoted as ΔZ (Eq. (9)), as well the percentage of cases when the optimal program was gained by the initial model (Eq. (10)).

$$\Delta Z = \frac{Z_i - Z_{min}}{Z_i}, \quad (9)$$

$$Z_{min} = Z_i \rightarrow \frac{Z_i}{N} 100 \%. \quad (10)$$

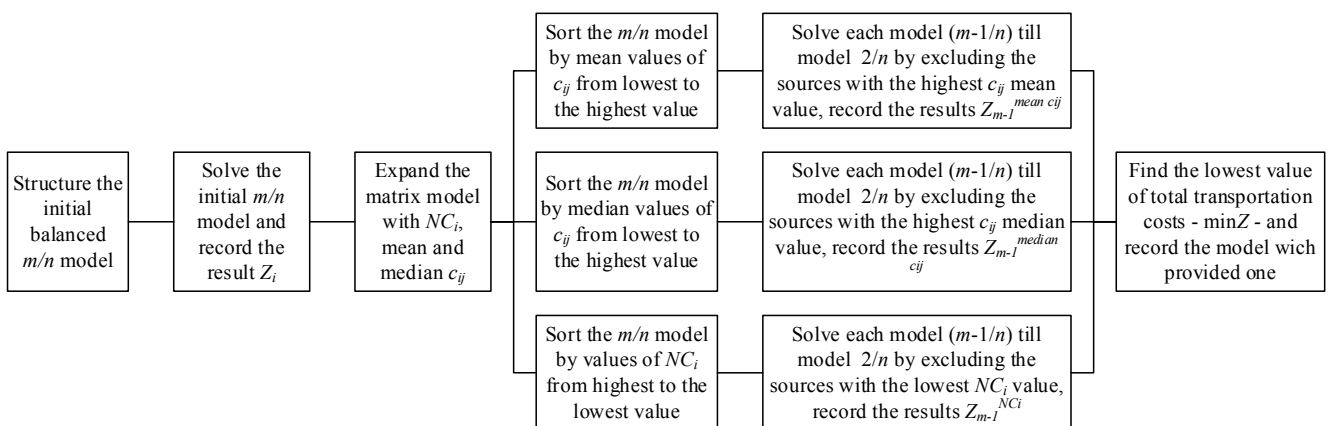


Figure 1 Algorithm of the MCS model

5.2 1st round of simulation

In the first round of the simulation TP cases (preliminary research) first set of cases was the base from

which all of the following sets of simulation were generated. The first round of simulations consists of a total 4 sets with 100 cases of TP (Tab. 4). This preliminary study was conducted to determine which

parameters have the greatest influence on the difference between the initial decision and optimal program, and in which percentage of the simulated cases initial model offered the optimal solution.

In the first set of simulated cases the quotient (qc_{ij}) of maximum and minimum value of transportation costs per unit is 3,0, while the quotient of demand (qb_j) is equal to 2,205 and the quotient of maximal and minimal nominal capacity (qNC_i) is equal to 1,67. In Tab. 4 authors gave a graphic representation of the simulated sets of cases, as well as the output statistical data - simulation results. It is evident that in the first set the average difference between the initial and optimal program (ΔZ) is 3,15%. In the second set of cases the quotient of unit prices was kept the same as well as the quotient of the demand, while the quotient of nominal capacity was reduced to 1,2. This set of simulated cases presents the cases where the capacities of asphalt plants are uniformly distributed. The results

showed an increase in the average difference between the initial and optimal program (4,19 %). In the third set of cases the quotient of prices of transportation per unit was reduced to 1,2; while the other two quotients were held as in the first set. Results showed the presence of the mean differences, but it was significantly lower (0,56 %). Furthermore, in the fourth set the quotient of the unit prices of transport was 1,2; the quotient of the demand was reduced to 1,2; and the quotient of nominal capacity was as in the first set. The mean difference was still recorded but it was even lower than in the previous set.

Preliminary research (the first round of simulations) was concluded with the statement that in further study and simulations authors should consider the relationship of quotient of transportation prices per unit and the quotient of nominal capacity, and to investigate the quotient of demand and its contribution to the average difference.

Table 4 Preliminary sets of simulated cases of TP

Simulation set	Graphical representation of the simulated TP cases	Input parameters	Results – summary statistics of ΔZ
1 st set		Transportation costs c_{ij} generated from the interval [7 €/t; 21 €/t] $\rightarrow qc_{ij} = \max c_{ij} / \min c_{ij} = 3,0$ Demand by each destination b_j generated from the interval [680 t; 1500 t] $\rightarrow qb_j = \max b_j / \min b_j = 2,205$ Source nominal capacity NC_i generated from the interval [90 t/h; 150 t/h] $\rightarrow qNC_i = \max NC_i / \min NC_i = 1,67$	Mean = 0,031529 \rightarrow 3,15 % Median = 0,027973 \rightarrow 2,89 % Std. Dev. = 0,023961 Range = 0,106656 \rightarrow 0 \div 10,67 % Quartile Range = 0,0316 \rightarrow 1,58 \div 4,74 %
2 nd set		Transportation costs c_{ij} generated from the interval [7 €/t; 21 €/t] $\rightarrow qc_{ij} = \max c_{ij} / \min c_{ij} = 3,0$ Demand by each destination b_j generated from the interval [680 t; 1500 t] $\rightarrow qb_j = \max b_j / \min b_j = 2,205$ Source nominal capacity NC_i generated from the interval [100 t/h; 120 t/h] $\rightarrow qNC_i = \max NC_i / \min NC_i = 1,2$	Mean = 0,041945 \rightarrow 4,19 % Median = 0,036469 \rightarrow 3,64 % Std. Dev. = 0,035791 Range = 0,174568 \rightarrow 0 \div 17,47 % Quartile Range = 0,05110 \rightarrow 1,2 \div 6,32 %
3 rd set		Transportation costs c_{ij} generated from the interval [14 €/t; 16,8 €/t] $\rightarrow qc_{ij} = \max c_{ij} / \min c_{ij} = 1,20$ Demand by each destination b_j generated from the interval [680 t; 1500 t] $\rightarrow qb_j = \max b_j / \min b_j = 2,205$ Source nominal capacity NC_i generated from interval [90 t/h; 150 t/h] $\rightarrow qNC_i = \max NC_i / \min NC_i = 1,67$	Mean = 0,005664 \rightarrow 0,56 % Median = 0,004404 \rightarrow 0,44 % Std. Dev. = 0,005148 Range = 0,02888 \rightarrow 0 \div 2,89 % Quartile Range = 0,0056 \rightarrow 0,20 \div 0,76 %
4 th set		Transportation costs c_{ij} generated from the interval [14 €/t; 16,8 €/t] $\rightarrow qc_{ij} = \max c_{ij} / \min c_{ij} = 1,20$ Demand by each destination b_j generated from the interval [1000 t; 1200t] $\rightarrow qb_j = \max b_j / \min b_j = 1,2$ Source nominal capacity NC_i generated from the interval [90 t/h; 150 t/h] $\rightarrow qNC_i = \max NC_i / \min NC_i = 1,67$	Mean = 0,004637 \rightarrow 0,46 % Median = 0,004015 \rightarrow 0,40 % Std. Dev. = 0,003741 Range = 0,013789 \rightarrow 0 \div 0,14 % Quartile Range = 0,00597 \rightarrow 0,13 \div 0,72 %

5.3 2nd round simulation

In the second round of simulations the first set was structured by modification of the first set of the first round, where the quotient of demand was increased from

2,205 to 3 (Fig. 2). Thus, authors simulated cases where the demand was more dispersed among construction sites. It has been shown that increasing the quotient of demand has no significant contribution to average difference between the initial and optimal program.

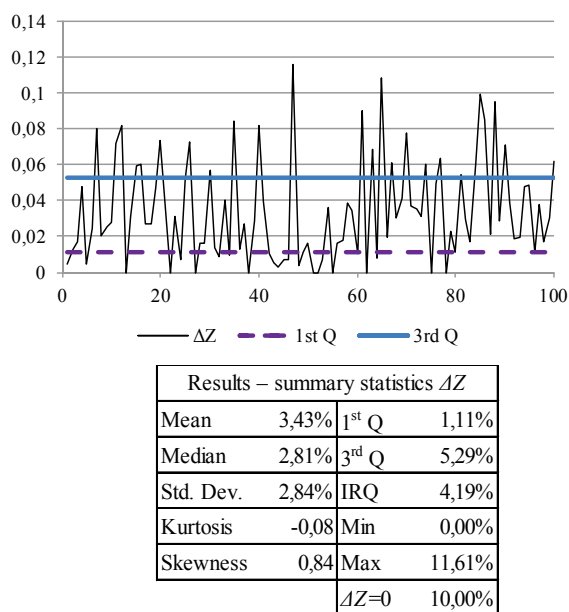


Figure 2 Results of the 1st set in second round of simulations

In the next set (Fig. 3) authors simulated cases where the quotient of demand is less than previous (1,5), when the demand among construction sites is more balanced. It has been shown that change of the quotient has no significant impact on the average difference between the initial and optimal program, and that in further sets of the simulations researchers should examine cases where the quotient of unit price increases.

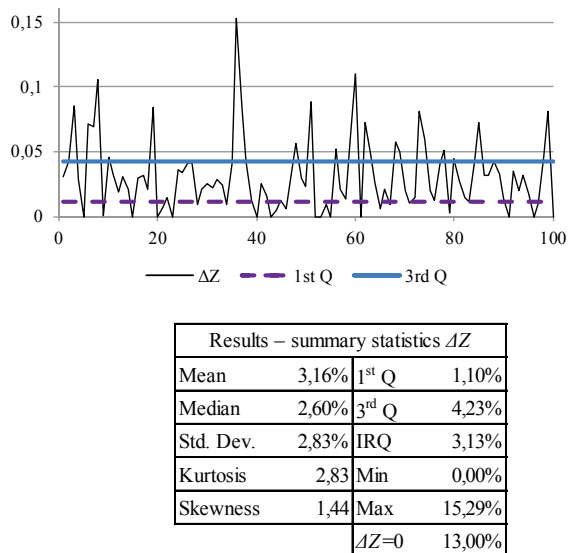


Figure 3 Results of the 2nd set in second round of simulations

Fig. 4 shows the results of the set of cases where the quotient of unit prices was increased to 4,0; while the other quotients of input parameters remained the same as in the first set of simulations in the first round. The average difference was significantly increased (4,92 %), which led to the conclusion that authors should keep this quotient of transportation costs per unit in the third (final) round of simulation and should examine the impact of the value of quotient qb_j of 2,2; and to reduce the quotient of nominal capacity of asphalt plants (qNC_i).

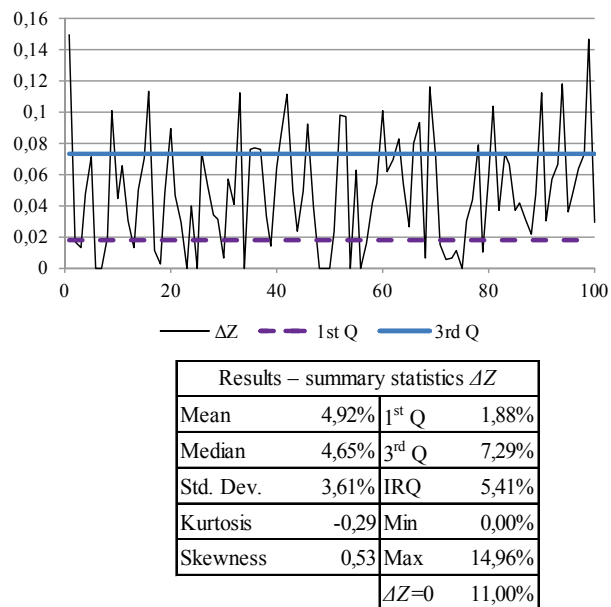


Figure 4 Results of the 3rd set in second round of simulations

5.4 3rd round of simulation

Previous rounds of simulated sets showed the influence of quotient of transportation costs per unit and quotient of sites' demand, and yet it was to be examined what would happen by the reduction of quotient of nominal capacity of asphalt plants. In order to confirm the sample population of previous simulations, in this set authors increased the number of simulation cases to $N=1000$. Due to the sample increase there was a larger dissipation of the results than in previous simulations, which is evident in Fig. 5. As expected, this round with the mentioned quotients has reported the highest average difference between initial and optimal program (5,29 %).

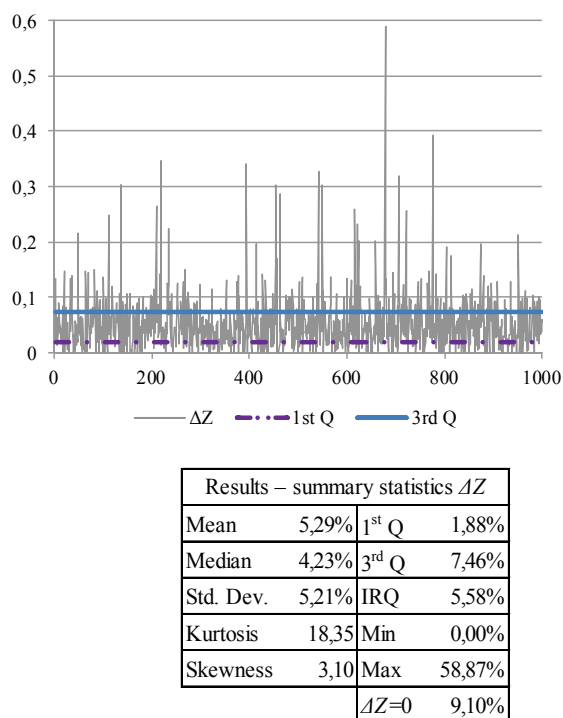


Figure 5 Results of the third round of simulations

5.5 Interpretation of the results

Experiment proved that the basic model of the TP is not a sufficient tool to ensure optimal program of transportation. Differences between optimal and initial program depending on the input parameters are between 1,0 and 5,0 %; whereas in only 10 % of the simulated cases the initial model (which engages all the potential sources) offered the optimal transportation program. This confirms the hypothesis of the imperfection of basic (original) TP model, and also verifies the proposed method, especially in cases where the input parameters are within the limits that have shown the greatest deviation from the initial program.

6 Conclusions and further research

The proposed methodology and algorithm elaborated in this article eliminate the imperfection of the basic mathematical model for solving the transportation problem of delivery of large quantities of asphalt mix on construction sites. The main disadvantage of the original model is that in the phase of structuring the initial TP model user affects the optimal program. While the proposed method of inclusion and exclusion of the potential sources resolves this issue as well as the danger of sub optimality of the adopted program. The methodology in this study was tested on the transportation problem of asphalt mixtures, but in the future work it will be tested to solve transportation problem of concrete on larger matrix forms. MCS solver is the starting point which authors will use for solving optimization of the integrated chain of production, supply and asphalt paving. The algorithm on which the MCS is based in the future researchers work will be expanded by impacts and dependencies between the components in the chain of processes, as well as the specificity of “just-in-time” production and asphalt mixtures paving operations. This will also remove the assumption, which the proposed methodology adopted, that the production time is a degree of freedom. Time of the asphalt production is conditioned by the efficiency of paving operations on site. Future work will acknowledge the fact that the time of transportation of asphalt mixture is determined by the temperature drop from the production temperature to the minimum temperature for paving in order to ensure the wanted quality of the final product.

7 References

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