FACULTY OF ECONOMICS AND BUSINESS ADMINISTRATION

WORKING PAPER

BEYOND RATIONAL EXPECTATIONS: THE EFFECTS OF HEURISTIC SWITCHING IN AN OVERLAPPING GENERATIONS MODEL

Brecht Boone
Ewoud Quaghebeur

January 2018
2018/943

GHENT UNIVERSITY

Faculty of Economics and Business Administration
Campus Tweekerken, Tweekerkenstraat 2, 9000 Ghent - BELGIUM

D/2018/7012/01

Beyond Rational Expectations: The Effects of Heuristic Switching in an Overlapping Generations Model

Brecht Boone[a,b,*] and Ewoud Quaghebeur[a,b]

[a]*Ghent University, Department of Social Economics, Sint-Pietersplein 6, B-9000 Ghent, Belgium,*

[b]*Study Hive for Economic Research and Public Policy Analysis (SHERPPA), Belgium*

January 2018

Abstract

We explore the transitional dynamics in an Overlapping Generations framework with and without heuristic switching. Agents use simple heuristics to forecast the interest rate and the real wage. The fraction of agents using a specific heuristic depends on its relative forecasting performance. In the absence of heuristic switching, the results indicate that there is a lot of variation in the transitional dynamics over different parameter values and heuristics. They might even oscillate or diverge. Including heuristic switching has two advantages. First, it decreases the variation in the transitional dynamics significantly. Second, it has a stabilising effect on oscillating or diverging transitional dynamics.

Keywords: Heuristic Switching, Heterogeneous Agents, Fiscal Policy, Transitional Dynamics, Overlapping Generations

JEL classification codes: D83, D84, E60

*Corresponding author: brecht.boone@ugent.be. We are grateful to Freddy Heylen, Tim Buyse, Frédéric Docquier, Gerdie Everaert, Hans Fehr, Glenn Rayp, Dirk Van de gaer, Selien De Schryder, and the members of the Macro Research Group of Ghent University for valuable suggestions and comments. The computational resources (Stevin Supercomputer Infrastructure) and services used in this work were provided by the Flemish Supercomputer Center, funded by Ghent University, the Hercules Foundation, and the Economy, Science, and Innovation Department of the Flemish Government. Brecht Boone acknowledges financial support from the Research Foundation - Flanders. Any remaining errors are ours.

1 Introduction

This paper explores the stability and transitional dynamics of an Overlapping Generations model with heuristic switching. Accordingly, this paper contributes to a growing literature that goes beyond the rational expectations paradigm. After all, the transitional and equilibrium impact of public policy, for instance, on the macroeconomy depends crucially on the behavioural response of households to these policies. An important determinant of this behavioural response is the procedure households apply to form expectations regarding the future course of different variables. Moreover, these expectations themselves are typically a key determinant of the current realisation of these variables. Several authors have argued that assuming that economic agents have rational expectations (RE) is unrealistic. An alternative is provided by the learning literature (see e.g. Evans and Honkapohja, 2001). In this literature, boundedly rational agents form expectations using a perceived law of motion. In the lion's share of this literature, agents act as econometricians who update the coefficients of their perceived law of motion as new realisations of the variables of interest become available over time.

A number of papers studied the effects of least-squares adaptive learning within an Overlapping Generations framework (henceforth OLG) - see e.g. Schönhofer (1998), Adam (2003), Tuinstra (2003) and Tuinstra and Wagener (2007). In Schönhofer (1998), for example, it is shown that if one explicitly considers learning in a monetary OLG model, the dynamic system exhibits chaotic behaviour. Tuinstra (2003), on the other hand, introduces the notion of beliefs equilibria. These are equilibria where the belief of the agents best fits the time series data, which itself is generated by the model where agents have this belief. Although the learning dynamics might converge, the author shows that the corresponding inflation dynamics might be erratic. Different from these studies, Chen et al. (2008) study the dynamic behaviour of an OLG model with capital accumulation under three different types of expectations: rational, myopic and adaptive expectations. They conclude that the dynamics can be complex when using the latter two types. Moreover, the dynamic properties of the model crucially depend on the value of the intertemporal elasticity of substitution in consumption and, in the case of adaptive expectations, on the weight agents attach to past observations when forming expectations.

All these papers have enriched our knowledge of the properties that characterise OLG models when moving beyond the scope of rational expectations. The majority of the papers using an OLG framework, however, focus on the (mostly local) stability properties of the equilibrium, not on the transitional dynamics following policy shocks. A second shortcoming is that often only one forecasting rule is studied. If the agents act as econometricians, they constantly update the coefficients of the same equation, but they cannot distinguish or switch between different rules. Furthermore, even if multiple rules are studied simultaneously, it is virtually always assumed that all agents use the same rule at a given point in time. In these papers, the focus often lies exclusively with one-period-ahead forecasts as well, ruling out the possibility of multi-period-ahead forecasts.

It is, however, plausible that a fraction of the economic agents does not have the cognitive capacities to act as an econometrician. Just as it can not be ruled out a priori that different forecasting rules are being used by the economic agents at one point in time. So then, what is the macroeconomic impact if one would assume that economic agents use simple rules to forecast wages and interest rates? Additionally, how do the transitional dynamics behave when individuals have multiple rules to choose from? And are the transitional dynamics sensitive to the rule being used? Finally, how do they compare to the transitional dynamics in the rational expectations case?

In this paper, we assume that agents use such simple rules, heuristics, to forecast the future course of the interest rate and the real wage. Agents use these heuristics because, in general, they do not possess the cognitive abilities as assumed by the RE literature nor to act as econometricians. Instead, the agents have a certain number of different heuristics at their disposal. On a regular basis, they assess the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

The framework used to answer these questions is in line with the evidence provided by several laboratory experiments - see e.g. Adam (2007), Hommes (2011) , Heemeijer et al. (2012) and Hommes (2014). In Heemeijer et al. (2012), for example, the authors use a standard OLG framework to conduct an individual experiment in order to assess the ability of individuals to form expectations and the degree to which these individuals learn about the accuracy of their forecasts. In the experiment, participants are asked to submit fifty one-step-ahead forecasts for the inflation rate. Over time, the participants also observe the actual realisations of the inflation rate. These can be used by the participants to forecast the remaining future inflation rates. The authors argue that their experimental results cannot be explained using the rational expectations approach. Rather, they are consistent with the use of constant gain algorithms or average expectations. Their results also indicate that individuals switch between different heuristics according to the relative forecasting performance of these rules.

The aim of this paper is to contribute to the literature that goes beyond the rational expectations paradigm by exploiting the heuristic switching approach within an Overlapping Generations model. Triggered by a fiscal policy shock, the objective is to study the transitional dynamics of the model for a large number of settings including one or multiple heuristics and compare the behaviour of the dynamics with their rational expectations counterpart.

The simulations lead to three main findings. First, in a context without heuristic switching (i.e. a context where individuals only have one heuristic at their disposal to form expectations), the evolution of transitional dynamics can be substantially different from the rational expectations case, especially in the first periods of the transition. Rational expectations is thus not always a good approximation. Furthermore, there is a lot of variation in the dynamics over different parameter values and heuristics. This finding implies that if only heuristic is used, the macroeconomic impact of fiscal policy is highly sensitive to the heuristic being used. What is more, as the discount rate and the degree of risk aversion decrease, the model becomes unstable and the corresponding transitional dynamics oscillate or even diverge.

Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly. Consequently, the sensitivity of the transitional effects of fiscal policy is much lower now and its exact impact is thus less uncertain for policy makers.

Third and last, the heuristic switching has a stabilising effect on the transitional dynamics. For certain configurations of the parameter values for which the dynamics were very unstable in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

These findings are important. They imply that allowing individuals to choose from a wide range of forecasting rules is actually a better option than constraining them to use only forecasting rule. It allows them to select the rules that perform relatively better, a feature that not only enhances the stability of the model, but reduces the uncertainty in the transitional dynamics as well. It implies that going beyond rational expectations does not lead to a wide range of possible trajectories. Furthermore, it turns out that rational expectations is a better approximation when the switching mechanism is activated.

The remainder of this paper is structured as follows. Section 2 outlines the different model blocks. Section 3 focuses on the heuristic switching. In Section 4, we provide some details on the timing in the model. Section 5 describes the equilibrium of the model. The calibration and parameterisation of the model is described in Section 6. Section 7 consists of the description and a detailed look into the results of the different simulations. Section 8 concludes.

2 Model

We consider a closed economy in which time is discrete and runs from 0 to ∞. Each period lasts for 4 years. At each moment in time, the economy is populated by J overlapping generations. The model consists of three actors: heterogeneous agents, firms, and a fiscal government. Markets are incomplete meaning that individuals cannot explicitly insure themselves against productivity shocks.

2.1 Demographics

At the beginning of each period, a continuum of new agents with measure one enters the model. Individuals have an uncertain lifespan. They face an age-specific survival probability φ_j between the age of j and $j+1$. The demographics of the model are exogenous and given by:

$$N_{j+1,t+1} = \varphi_j N_{j,t} \tag{1}$$

where $N_{j,t}$ represents the number of individuals of age j at time t. Every individual who survives J periods will die with certainty after the J-th period.

2.2 Individuals

Individuals enter the model at the age of 18. Ex-ante, before any decisions are made, individuals only differ with respect to the heuristic they apply to form expectations. Furthermore, the economic agents face idiosyncratic income risk during their active period of life. At any given point in time, individuals are characterised by a state vector (j, a, η, h), where j is the age of the agent, a the accumulated non-human wealth at the beginning of period t, η the productivity shock and h the heuristic that the agents is currently applying. Let $\Phi_{j,t}(a, \eta, h)$ denote the share of agents aged j of type (a, η, h) at date t. For each t and j we have $\int \Phi_{t,j}(da \times d\eta \times dh) = 1$.

Individuals choose sequences of (n, c, a'), i.e. labor supply, consumption and accumulated non-human wealth, to maximise their expected lifetime utility. The latter is given by

$$U = E\left\{ \sum_{j=1}^{J} \beta^{j-1} \frac{\left({c_j}^{(1-\mu)}(1-n_j)^{\mu}\right)^{1-\theta}}{1-\theta} \right\}. \tag{2}$$

The share of consumption is given by $1 - \mu$. The degree of relative risk aversion is governed by θ. The time discount factor is denoted by β. Individuals reaching the age of J_R retire.

The dynamic budget constraint of an individual aged $j < J_R$ with state (a, η, h) at time t is given by

$$(1+\tau_c)c_j + a'_j = w_t \eta \varepsilon_j n_j (1-\tau) + (1 + r_t(1-\tau_k))(a_j + Tr_t) \tag{3}$$

He or she earns an after-tax wage of $w_t \varepsilon_j \eta n_j (1-\tau)$, where w_t is the real wage per unit of effective labour at time t, ε_j is an age-specific productivity parameter, η is the labour productivity shock and τ is the average tax rate on labour income. The consumption tax rate is τ_c. The real interest rate is given by r_t. Individuals pay taxes on capital income where the capital tax rate is denoted by τ_k. Individuals enter the model without wealth and leave no intentional bequests. Due to accidental bequests, individuals receive a transfer Tr_t from the government. The accumulated non-human wealth at the end of the period is denoted by a'. We impose that individuals are not able to borrow: $a' \geq 0$. This individual maximises the following recursive problem:

$$V(j, a, \eta, h) = \max_{c_j, n_j, a'_j} U(c_j, n_j) + \beta \varphi_j \sum_{\eta'} \pi(\eta'|\eta) V(j+1, a', \eta', h) \tag{4}$$

The stochastic process regarding the labor productivity shock is denoted by $\pi(\eta'|\eta)$. The heuristic used is given by h. From the age of J_R onwards, individuals receive a public pay-as-you-go pension. Their budget constraint for the ages $j \geq J_R$ is given by

$$(1+\tau_c)c_j + a'_j = (1 + r_t(1-\tau_k))(a_j + Tr_t) + pp_t \tag{5}$$

The maximisation problem is now given by

$$V(j,a,h) = \max_{c,a'} U(c_j) + \beta\varphi_j V(j+1,a',h) \tag{6}$$

A final note on the basic PAYG-pension pp_t received by the retired households in the model. For simplicity, it is assumed that all individuals receive the same pension, i.e. a fraction b_p of the average after-tax wage in the economy.

2.3 Firms

The production function of the representative firm is given by

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{7}$$

where A is the level of technology that assumed to be constant over time, K_t is the capital used by the firm, and L_t is given by

$$L_t = \sum_{j=1}^{J_R-1} N_{j,t} \int n_j(a,\eta,h)\eta\varepsilon_j \Phi_{t,j}(da \times d\eta \times dh) \tag{8}$$

2.4 Government

Government expenditures on goods and public pensions are financed by taxes on labour, capital and consumption. The fraction of output that is devoted to government consumption g_c is adjusted such that the government budget is balanced every period. Formally, the government budget constraint is given by

$$G_{c,t} + P_t = T_{n,t} + T_{c,t} + T_{k,t} \tag{9}$$

$$\text{with: } \begin{cases} G_{ct} = g_c Y_t \\ P_t = \sum_{j=j_R}^{J} N_{j,t} \int pp_t \Phi_{j,t}(da \times dh) \\ T_{kt} = \tau_k r_t K_t \\ T_{nt} = \tau \sum_{j=1}^{J_R-1} N_{j,t} \int w_t n_j(a,\eta,h)\eta\varepsilon_j \Phi_{j,t}(da \times d\eta \times dh) \\ T_{ct} = \tau_c \sum_{j=1}^{J} N_{j,t} \int c_j(a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) \end{cases}$$

3 Heuristic switching

In this paper, we take the view that economic agents have limited cognitive capabilities. In such a world, individuals use simple rules, heuristics, to forecast the evolution of aggregate macroeconomic variables and to form expectations. Notwithstanding their limited cognitive capabilities, economic agents are willing to learn from their mistakes. To this end, we combine the use of heuristics with a trial-and-error learning approach[1]. More specifically, economic agents have different heuristics at their disposal and they endogenously select the heuristic or forecasting rule that performed the best in previous periods. On a regular basis, individuals assess the predictive power of the heuristic they are currently using vis-à-vis the predictive power of the other rules. If the current rule performs well, the probability that an individual will keep on using the same rule is higher. If not, there is a higher probability that he or she will switch.

The different heuristics at the disposal of an individual to form expectations are the following[2]:

$$r^{1,e}_{t+s,t} = r^{1,e}_{t,t-1} + \psi_1(r_t - r^{1,e}_{t,t-1}) \text{ (Adaptive (1))} \tag{10}$$

$$r^{2,e}_{t+s,t} = r_t + \psi_2(r_t - r_{t-1}) \text{ (Trend (1))} \tag{11}$$

$$r^{3,e}_{t+s,t} = \frac{1}{\psi_3} \sum_{j=0}^{\psi_{3-1}} r_{t-j} \text{ (Average (1))} \tag{12}$$

$$r^{4,e}_{t+s,t} = \sum_{j=0}^{\psi_4-1} \phi_j r_{t-j} \text{ (Average (2))} \tag{13}$$

$$r^{5,e}_{t+s,t} = r_t \text{ (Myopic)} \tag{14}$$

with $s \in \{1, ..., J\}$. In these equations, both actual realisations (r_t) and expected values ($r^{h,e}_{t+s,t}$) of the interest rate are given. The subscript t in r_t denotes the historical period t in which the realisation occurred. On the other hand, $r^{h,e}_{t+s,t}$ is the expectation at time t of the interest rate in period $t+s$ using heuristic h. For example, $r^{1,e}_{t,t-1}$ is the expectation at time $t-1$ using the first heuristic of the interest rate at time t.

The first heuristic boils down to an adaptive expectations approach to form forecasts. It states that the expectation of an individual regarding the evolution of the interest rate equals $r^{1,e}_{t,t-1}$, the expectation of the interest rate in the current period t made at time $t-1$, and a fraction ψ_1 of the forecast error, i.e. the difference between the actual realisation of the interest rate r_t and $r^{1,e}_{t,t-1}$. The second one is a trend rule. Here, $r^{2,e}_{t+1,t}$ equals the actual realisation of the interest rate r_t plus a fraction ψ_2 of the difference between the current and previous realisation of the interest rate. Agents expect higher interest rates in the future when the current interest rate r_t is higher than the previous interest rate r_{t-1} and vice versa. When ψ_1 and ψ_2 are low, individuals are less inclined to adjust

[1]For an in-depth analysis of the use of heuristics and heuristic switching, we refer the reader to De Grauwe (2012), Heemeijer et al. (2012) and Hommes (2014).

[2]In this section, we only provide the different heuristics for the evolution of the interest rate. Note that agents form expectations about wages as well using the same heuristic, and the heuristics that they use to do so are equivalent to the ones stated in this section. Furthermore, these heuristics are based on the heuristics provided in Hommes (2014).

their expectations. When ψ_1 and ψ_2 are high, individuals will be more inclined when adjusting their expectations. The third heuristic implies that the expected interest rate for the next period equals an unweighted average of the last ψ_3 realisations of the interest rate, while in the fourth heuristic $r^{4,e}_{t+1,t}$ is determined using a weighted average of the last ψ_4 realisations of the interest rate. Finally, the fifth heuristic is equivalent to the use of myopic expectations. Individuals simply assume that the interest in the next period will equal the current realisation of the interest rate.

Agents using heuristic h at t use this heuristic to form $r^{h,e}_{t+s,t}$ ($s \in \{1,..,J-j\}$, J denoting the maximum age an individual can reach and j denoting the actual age), i.e. the expected values at time t of the interest rate and the real wage in the remaining periods of their life. In period $t+1$, they will update their expectations of these values as new information becomes available. For the last three heuristics (Equations (12-14)), we assume that the agents using these heuristics expect that the new value applies for the remainder of their life. Thus, for example, if the individual has at most three more periods to live after the current period, expectations for these periods held at time t are the following: $r^{h,e}_{t+1,t} = r^{h,e}_{t+2,t} = r^{h,e}_{t+3,t}$. In the next period, however, the updated expected values for the last two periods of life ($r^{h,e}_{t+2,t+1} = r^{h,e}_{t+3,t+1}$) might differ from the expected values for these periods at time t ($r^{h,e}_{t+2,t} = r^{h,e}_{t+3,t}$). In other words, agents assume that the expected value at time t applies for the remainder of their life. In time $t+1$, the expected value itself might change, but they still assume that this new expected value applies for the remainder of their life.

For the first two heuristics, we use two different versions. In one version, we use the same assumption as for the last three heuristics, namely that the forecast for the next period $r^{h,e}_{t+1,t}$ equals the forecast for the subsequent periods as well. In the second version, though, to form expectations for the periods $t+2, t+3, t+4, ...$ the heuristics are adjusted as follows:

$$r^{6,e}_{t+s,t} = r^{6,e}_{t+s-1,t} + \frac{\psi_1}{\psi_5 s}(r_t - r^{6,e}_{t,t-1}),\ s \in \{1,...,J-j\}\ ,\ \text{(Adaptive (2))} \tag{15}$$

$$r^{7,e}_{t+s,t} = r^{7,e}_{t+s-1,t} + \frac{\psi_2}{\psi_6 s}(r_t - r_{t-1}),\ s \in \{1,...,J-j\}\ ,\ \text{(Trend (2))} \tag{16}$$

where s is the number of remaining periods of life. Heuristic 6 implies that the individual observes the forecast error $r_t - r^{6,e}_{t,t-1}$ and acknowledges that he or she might make forecasting errors in the future. Using the forecast error, he or she iterates forward to form expectations for the interest rate in period $t+j$. The term $\psi_5 s$ then captures to what extent he or she thinks that they will make the same forecasting error in the future. For the last heuristic, the basic idea is the same, but now it captures to what extent an individual thinks that the trend will continue in the future.

In total agents have seven different heuristics at their disposal. Consistent with the literature on Heuristic Switching Models, the performance of heuristic $h \in (1,2,3,4,5,6,7)$ is measured by the squared prediction error of that specific rule $\Theta_{t,h}$ in a specific period t:

$$\Theta_{t,h} = -(r_t - r^{h,e}_{t,t-1})^2 + \upsilon\Theta_{t-1,h} \tag{17}$$

All that is left to be specified is the fraction of agents using a specific heuristic h. Using a discrete choice model, this is given by

$$\Gamma_{h,t} = \xi\Gamma_{h,t-1} + (1-\xi)\frac{exp(k\Theta_{t-1,h})}{\sum_{h=1}^{7} exp(k\Theta_{t-1,h})} \tag{18}$$

Here, $\Gamma_{h,t}$ measures the fraction of individuals using heuristic h at period t. This means that $\sum_{h=1}^{7} \Gamma_{h,t} = 1$. Furthermore, $\upsilon \in$[0,1] is a parameter measuring the memory of the economic agents. The lower υ, the less economic agents take past periods into account when comparing heuristics. Furthermore, $k \geq 0$ is the intensity of choice. The larger k, the faster agents switch between heuristics. The last parameter is $\xi \in [0, 1]$, measuring inertia. If this parameter is low, economic agents switch less to other heuristics even if they clearly perform better. In other words, the habit of using a certain heuristic is stronger.

4 Timing

Each period, a number of decisions have to be made by the individuals populating the economy. These sequential steps are:

1. Given their expectations, individuals decide on the amount of labour they want to supply to the labour market.

2. Based on K_t and L_t, r_t and w_t are determined. These values of w_t and r_t do not change the value of n. Labour has already been supplied to the labour market.

3. Individuals receive their labour (based on n as determined in step 2) and capital income. Afterwards, they decide on c and a'.

4. They evaluate the heuristic they are applying using Equation (17). Afterwards, the heuristic switching takes place.

5. Having observed the actual realisation of the interest rate and the wage, individuals update their expectations about the future values of the interest rate and the real wage rate using the heuristic they are using.

5 Definition of Equilibrium

Let $\Phi_{j,t}(a, \eta, h)$ denote the share of agents aged j at time t with state (a, η, h). For each j and t we have $\int \Phi_{j,t}(da \times d\eta \times dh) = 1$.

Definition 1 Given an initial capital stock K_0, a given vector of exogenous fiscal policy variables $\{\tau_{c,t}, \tau_{k,t}, \tau_t, b_{p,t}\}_{t=1}^{\infty}$ and initial measures $\{\Phi_{j,0}\}_{j=1}^{J}$, an intertemporal equilibrium consists of sequences of value and policy functions $\{V_t, a'_{j,t}, c_{j,t}, n_{j,t}\}_{t=0}^{\infty}$, sequences of transfers $\{Tr_t\}_{t=0}^{\infty}$, sequences of prices $\{w_t, r_t\}_{t=0}^{\infty}$, sequences of expectations $\left\{r_{t+k,t}^{h,e}, w_{r+k,t}^{h,e}\right\}_{t=1}^{\infty}$, sequences of taxes, social security policies, tax aggregates and government spending $\{G_c, T_{n,t}, T_{k,t}, T_{c,t}, pp_t\}_{t=1}^{\infty}$, aggregate variables $\{Y_t, K_t, L_t\}_{t=0}^{\infty}$ and sequences of measures $\{\Phi_{j,t}\}_{t=0}^{\infty}$ such that:

1. Given expectations $r_{t+k,t}^{h,e}$ and $w_{t+k,t}^{h,e}$ and government policies, $n(j,a,\eta,h)$ is the optimal labour supply of an individual with state $n(j,a,\eta,h)$.

2. Given prices r_t and w_t, government policies, expectations $r_{t+k,t}^{h,e}$ and $w_{t+k,t}^{h,e}$, and the labour supply $n(j,a,\eta,h)$ as decided in the beginning of period t, V_t satisfies the Bellman equations as given in subsection 2.2 for all states (j,a,η,h), where $\{a'_{j,t}, c_{j,t}\}$ are the related policy functions.

3. Interest rates and wages are given by:

$$r_t = \alpha A \left(\frac{L_t}{K_t}\right)^{(1-\alpha)} - \delta \tag{19}$$

and

$$w_t = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{20}$$

4. The pension pp_t is determined by:

$$pp_t = b_{p,t} \left(\frac{\sum_{j=1}^{J_{R-1}} L_{j,t} \int w_t n(j,a,\eta,h)\eta\varepsilon_j(1-\tau)\Phi_{j,t}(da \times d\eta \times dh)}{\sum_{j=1}^{J_{R-1}} L_{j,t}} \right) \tag{21}$$

where $b_{p,t}$ is the net replacement rate for the individuals.

5. The transfer Tr_{t+1} is determined by:

$$Tr_{t+1} = \left(\frac{\sum_{j=1}^{J} (1-\varphi_j) N_{j,t} \int a'(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh)}{\sum_{j=1}^{J} N_{j,t+1}} \right) \tag{22}$$

6. Expectations $r_{t+k,t}^{h,e}$ and $w_{t+k,t}^{h,e}$ are updated using Equations (10-16).

7. The performance of the heuristic and the fraction of individuals using a certain heuristic is determined via Equations (17-18).

8. The capital market, the labour market and the goods market clear every period t:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} \int a'(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) \tag{23}$$

$$L_t = \sum_{j=1}^{J_{R-1}} N_{j,t} \int n(j,a,e,\pi,\eta,H)\varepsilon_j \eta \Phi(da \times d\eta \times dh) \tag{24}$$

$$Y_t = \sum_{j=1}^{J} N_{j,t} \int c(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) + G_{c,t} + (K_{t+1} - (1-\delta)K_t) \tag{25}$$

9. Government policies $\{G_c, T_{n,t}, T_{k,t}, T_{c,t}\}$ are determined using Equation (9) and the fraction of Y_t used for government spending (g_c) is endogenously determined such that the government budget is balanced each period:

$$g_c = \frac{T_{n,t} + T_{c,t} + T_{k,t} - PP_t}{Y_t} \tag{26}$$

10. Y_t is determined by equation (7).

11. $\Phi_{j+1,t+1} = Z_{j,t}(\Phi_{j,t})$ where $Z_{j,t}$ is the law of motion induced by the exogenous mortality rates, the exogenous Markov process for labour productivity, the endogenous asset accumulation and the heuristic switching regime.

6 Data and calibration

In this section, the parameterisation and calibration of the model is outlined in detail. The lion's share of the calibration is in line with the literature on quantitative OLG models with idiosyncratic risk. The model is calibrated to Belgium for the period 2000-2007.

6.1 Demographics

Agents enter the economy at the age of 18 (model age = 1), retire at the age of 66 (model age = 13) and live at most until the age of 94 years. Each period in the model lasts for four years. The conditional survival probabilities $\{\varphi_j\}$ are taken from the Human Mortality Database and are for 2000.

6.2 Technology and employment

The parameters regarding technology are $\{\alpha, A, \delta\}$. The capital share in production α equals 0.36. The level of technology A is constant and normalised such that the equilibrium real wage rate in the benchmark model w is equal to 1. The depreciation rate δ is calibrated using a target for the equilibrium annual real interest rate of $r = 4.5\%$.

6.3 Labour productivity shocks and parameters

In the model, an individual of age j and idiosyncratic shock η who works n hours will earn a pre-tax wage of

$$w n_j \eta \varepsilon_j \tag{27}$$

We use the specification reported by Cournède and Gonand (2006) to calibrate the age-specific productivity profile ε_j. The resulting profile is hump-shaped.

The productivity shock η can take three values: $\eta \in \{\eta_1, \eta_2, \eta_3\}$. The Markov transition matrix is a (3x3)-matrix:

$$\Omega = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix} \tag{28}$$

where ρ_{ij} is the probability $Pr(j|i)$ to end up in state j in the next period given state i in the current period. Taking all this information together, the states of the Markov chain $\{\eta_1, \eta_2, \eta_3\}$ and the Markov transition matrix Ω still have to be determined. For the labor productivity states $\{\eta_1, \eta_2, \eta_3\}$ in the labor earnings process, we use a discretised Markov chain for a continuous AR(1)-process with persistence ζ_s and variance σ_η^2. The persistence is chosen to be 0.969 and the variance 0.01 (Krueger and Ludwig, 2013).

The markov transition matrix for the idiosyncratic productivity risk is then given by

$$\Omega = \begin{bmatrix} 0.8851 & 0.1113 & 0.0035 \\ 0.0557 & 0.8887 & 0.0557 \\ 0.0035 & 0.1113 & 0.8851 \end{bmatrix}$$

while the values for η_1, η_2 and η_3 are respectively 0.6029, 1 and 1.6587.

6.4 Preferences

The instantaneous utility function of the individuals is given by Equation (2)[3]. The parameters to be calibrated are $\{\beta, \mu, \theta\}$. As in Conesa and Krueger (2006) and Krueger and Ludwig (2013), θ is chosen to be 4. The relative weight on leisure μ, on the other hand, is determined such that Belgian employed individuals work on average $1/3$ of their time. Using the values of μ and θ, a coefficient of relative risk aversion of approximately 2 is obtained. Finally, β is set to 0.96.

[3]This functional form is often used in the quantitative OLG literature with idiosyncratic risk: see e.g. Conesa and Krueger (2006) and Krueger and Ludwig (2013)

Table 1: Calibration summary

Parameter values			
Weight on leisure in utility function	μ	0.6164	Average fraction of time spent working = 1/3
Discount factor in utility function	β	0.96	
Coefficient of risk aversion	θ	4	Conesa and Krueger (2006); Krueger and Ludwig (2013)
Level of technology	A	4.478	$w = 1$
Capital share in production	α	0.36	
Depreciation rate	δ	0.36	$r = 4.5\%$
Age-specific component of wages	ε_j		Cournède and Gonand (2006)
Tax rate on labour	τ	52.2%	Heylen and Van de Kerckhove (2013)
Tax rate on consumption	τ_c	13.4%	Heylen and Van de Kerckhove (2013)
Tax rate on capital	τ_k	26.8%	McDaniel (2007)
Net replacement rate pension	b_p	0.631	OECD, Pensions at a Glance (2005)

6.5 Fiscal policy variables

The government in the model finances spending on goods and PAYG pensions with taxes on consumption, capital and labour. For the tax rates τ_c and τ, we use the same data as Heylen and Van de Kerckhove (2013). For details on the construction of these fiscal policy variables, we refer to Heylen & Van de Kerckhove (2013, their Appendix 1). The value for the capital tax rate τ_k is determined using the tax series constructed by Cara McDaniel. we use the average between 2000-2007[4]. Regarding the basic PAYG-pension received by the retired households in the model, we use data on the average net replacement rate after retirement obtained from the OECD (Pensions at a Glance, 2005).

7 Simulations

7.1 Set-up

The goal of this paper is go beyond the rational expectations paradigm and use heuristics to study the transitional dynamics following fiscal policy shocks. In this section, we explore the effects of an unanticipated permanent labour tax decrease financed by government spending. We proceed in two steps. In the first step, we assume that economic agents have only one of the available heuristics to form expectations at their disposal. Thus, at this point there is no heuristic switching. In a second step, we perform the same analysis, however, now we do use the heuristic switching framework: i.e. we assume that agents evaluate the performance of the heuristic they are currently using. According to its relative performance to other heuristics, individuals might switch to a different heuristic. All the results discussed in the following subsections are for the scenario in which τ is reduced from 52.2% to 42.2%.

[4]The updated tax series can be downloaded from www.caramcdaniel.com/researchpapers. The methodology is discussed in McDaniel (2007).

7.1.1 No heuristic switching

We perform the analysis in step 1 for different parameter values for the discount factor β, the coefficient of relative risk aversion θ and the parameters ψ_1, ψ_2, ψ_5 and ψ_6 governing the adaptive and trend heuristics. For β, we use two different and rather extreme values: 0.98 and 0.86, whereas θ takes the values 2 and 4[5]. The lower (higher) the value of θ, the higher (lower) the Frish elasticity of labour supply and the more (less) individuals respond to different real wage rates. For ψ_1 and ψ_2, the two values are 0.1 and 0.9. In the latter case, individuals are much faster in adjusting their expectations to current evolutions. To conclude, the values used for ψ_5 and ψ_6 are 1 and 2. Higher values for the latter two variables indicate that current evolutions matter less in the future. The values for ψ_3, ψ_4 and ϕ_j remain constant over all scenarios. For the first two, we attach the value of 5. That means that individuals take a total period of 20 years to average. The weights ϕ_j are determined using the formula $\frac{9-2j}{25}$, meaning that the weights are 9/25 for the current period, 7/25 for the previous period, and 5/25, 3/25 and 1/25 for the other periods.

Thus, we end up with 4 parameter combinations of β and θ. For each of these combinations, we have 4 different combinations for each of the adaptive expectations heuristics (Equations (10) and (15)), 4 different combinations for each of the trend rule heuristics (Equations (11) and (16)) and 1 single specification for the other three heuristics (Equations (12-14)), leading to a total number of 76 scenarios.

7.1.2 Heuristic switching

Of course, not all individuals use the same heuristic to form expectations about the future realisations of the interest rate and the real wage rate. It is more realistic to assume that individuals have more than one heuristic at their disposal to form expectations. Therefore, a next step is activating the heuristic switching framework. For these simulations, we use the same range of parameter values for β, θ, ψ_1, ψ_2, ψ_5 and ψ_6. The values for ψ_3, ψ_4 and ϕ_j are the same as well. Furthermore, we have to choose values for the parameters υ, ξ and k. For the first parameter υ, measuring the degree to which individuals take the past performance of heuristics into account, the values 0.2 and 0.9 are used. In the first (last) case, individuals attach low (high) weight to errors in the past. For the second parameter ξ, measuring the degree of inertia in the switching of individuals between different heuristics, the values 0.1 and 0.9 are used. The latter value indicates that the fraction of individuals using a specific weight will change slowly, even if its performance is especially good or bad. The last parameter k indicates the intensity of choice. We use two values: 0.1 and 100. The higher, the more people are inclined to switch to the best performing heuristic.

We use all the combinations of these parameters for 10 different initial distributions of weights, i.e. initial fractions of individuals using the different heuristics. These are given in Table 2. For example, in (1), all agents start by using Adaptive (1) to form expectations. When one heuristic is

[5] We performed the analysis as well for $\theta = 10$ but the results are highly similar to $\theta = 4$.

Table 2: Different initial distributions for the weights (fractions of individuals using the specific heuristics).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\Gamma_{1,0}$ (Adaptive (1))	1	0	0	0	0	0	0	1/7	1/4	0
$\Gamma_{2,0}$ (Adaptive (2))	0	1	0	0	0	0	0	1/7	1/4	0
$\Gamma_{3,0}$ (Trend (1))	0	0	1	0	0	0	0	1/7	1/4	0
$\Gamma_{4,0}$ (Trend (2))	0	0	0	1	0	0	0	1/7	1/4	0
$\Gamma_{5,0}$ (Average (1))	0	0	0	0	1	0	0	1/7	0	1/3
$\Gamma_{6,0}$ (Average (2))	0	0	0	0	0	1	0	1/7	0	1/3
$\Gamma_{7,0}$ (Myopic)	0	0	0	0	0	0	1	1/7	0	1/3

used extensively in the first periods of the transition period, this could lead to different dynamics compared to when a second heuristic is used heavily in the beginning.

7.2 Results

7.2.1 With vs without heuristic switching for $\beta = 0.98$ and $\theta = 4$

The results for $\beta = 0.98$ and $\theta = 4$ with and without heuristic switching are given in Figure 1[6]. The three panels on the left concern the scenarios without heuristic switching. They display the median transitional dynamics for output, the real wage rate and the forecast errors for all the different heuristics over all the different parameter values for ψ_1, ψ_2, ψ_5 and ψ_6 as discussed in subsection 7.1.1. The three panels on the right concern the results with heuristic switching for $\beta = 0.98$ and $\theta = 4$. For each combination of respectively ξ, υ and k, they show the median transitional path over all the combinations for ψ_1, ψ_2, ψ_5 and ψ_6 and over all initial distributions as given in Table 2. Furthermore, for the panels with heuristic switching, we also give an indication of the variation in the transitional dynamics over all different scenarios. Note that the RE case is displayed using the dashed line.

We start with the discussion for the results without heuristic switching. In the first period, the difference between output in the rational expectations case (dashed line) and the heuristic scenarios is most pronounced. In the RE case, the value for w_t is lower than the value in the old steady state due to the higher labor supply. For the different heuristics, however, the expectation for the wage equals the value for the real pre-tax wage rate in the old steady state. As the individuals base their labour supply on their expectation for w, they supply more labour relative to the RE case leading to a higher level of output. Afterwards, output decreases as employment decreases, but the increase in K_t counteracts this decrease to some extent. These panels indicate that substantial output gains can be achieved compared to the rational expectations case. In all heuristic cases, output overshoots its

[6]The permanent decrease in the labour tax occurs in period 1. The value for output in the old steady state is 16.6. The initial steady state level for the pre-tax wage is 1. The results for $\beta = 0.86/\theta = 4$, $\beta = 0.98/\theta = 10$, $\beta = 0.86/\theta = 10$ and $\beta = 0.98/\theta = 2$ are similar and are therefore not displayed in this text. The conclusions mentioned in the text apply to these configurations as well. The results for $\beta = 0.86/\sigma = 2$ are discussed in the next subsection.

RE counterpart. This effect is most pronounced in the adaptive expectations case, since expectations are lagging behind the most. When individuals use the trend heuristic, there is some oscillation in the beginning. Overshooting is the largest when average expectations are used.

Unsurprisingly, the dynamics of the actual real wage rate are closely related to the heuristic used to form expectations. At first, the real wage rate is lower than its RE counterpart, but it quickly catches up. The panels in the last row show the forecast errors. The forecast error is negative when the expected value is bigger than the actual value and positive otherwise. The oscillation following the trend heuristic is also clear from the panels in the last row of Figure 1. The forecast error changes in sign compared to the period before and gradually it converges to zero. In the adaptive expectations case, forecast errors are always negative. Average expectations lead to a negative forecast error in the beginning. Around the fifteenth period, they become positive for some periods.

Most importantly, these panels indicate that even for very common parameter values for β and θ, the resulting dynamics, both for output and wages but also for all the other variables, are substantially different from the dynamics in the rational expectations case. Second, they show that the resulting dynamics depend on the heuristic that is used and the parameterisation of that heuristic. Thus, there is a certain degree of variation in the dynamics.

As for the results with heuristic switching, one can see that the distance between the values of respectively + 1 standard deviation and -1 standard deviation from the mean value in each period is small and quickly becomes smaller as time goes by. Thus, in a framework without rational expectations and with heuristic switching, the effects of a labour tax decrease become much more predictable and monotonic even though a lot of individuals have different ways of forming expectations and even considering different initial distributions. The heuristic switching framework enables policy makers to better anticipate the effects of tax changes.

Figure 2 displays the boxplot of the value of Equation (17) for each period for each heuristic over the transition. This figure gives a lot of information on how the different heuristics are performing over the transition period. The two adaptive heuristics and the two average heuristics perform the best in the beginning, as they are overall the slowest ones to adjust their expectations. The trend rules are the worst performers in the beginning. Over time, the myopic expectations heuristic gradually becomes the best performing heuristic based on the median value of Equation (17). Another aspect worth mentioning is the fact that for the adaptive and trend heuristics, the range of the values for Equation (17) is much bigger compared to the others. This means that in some cases, especially when the adjustment factors ψ_1 and ψ_2 are high, these heuristics perform extremely bad. For the adaptive heuristics, this is typically after the tenth period.

The main message here is that after the heuristic switching regime has been activated, the variation in the transitional dynamics decreases significantly, meaning that the transitional effects of a decrease in labour taxes become more predictable and monotonic over all different parameter values in the specifications of the heuristics (both extreme and moderate) and of the parameters in Equations (17) and (18) and over all possible initial distributions. Thus, allowing for alternative expectations and a lot of heterogeneity in terms of initial fractions does not lead to a wide range of possible transitional paths, but decreases in fact the range in which the transitional dynamics are located.

7.2.2 With vs without heuristic switching for $\beta = 0.86$ and $\theta = 2$

In the previous subsection, we discussed the results for β =0.98 and $\theta = 4$. As mentioned previously, we also performed the same analysis for $\beta = 0.86$ and $\theta = 2$. We will, however, discuss the results in a different way as we want to highlight a different feature of the heuristic switching approach compared to the previous subsection.

Figure 1 showed that in all of the scenarios, the transitional dynamics converge to the steady state. This is not always the case, however. In Figure 3, we include the transitional dynamics for output for all the different heuristics but without heuristic switching when $\beta = 0.86/\theta$=2 and over all the different parameter values as discussed in subsection 7.1.1. For a lot of the scenarios, the transitional dynamics oscillate around the steady state or even diverge. The oscillations are very large in some cases. Not surprisingly, in all the adaptive and trend scenarios in which individuals adjust their expectations faster (ψ_1 and ψ_2 equal to 0.9), the dynamics diverge or experience large oscillations. For smaller weights, the dynamics oscillate to a smaller extent. The variation in the dynamics is thus very big.

With heuristic switching, the conclusion is different. Figure 4 reveals that the heuristic switching approach has a stabilising effect on the dynamics. Only when the agents using the adaptive heuristic and the ones using the trend heuristic are simultaneously aggressive in adjusting their expectations ($\psi_1 = \psi_2 = 0.9$), the dynamics oscillate in some cases. In all other cases, the heuristic switching approach stabilises the dynamics.

Figures 5 and 6 further examine the sensitivity of the results to the memory parameter (υ), the noise parameter (k) and the inertia parameter (ξ) for $\beta = 0.86$ and $\theta = 2$. First, the results for $\upsilon = 0.2$. Only when $k = 100$ and $\xi = 0.9$ do all the different simulations over the parameter values for the specification of the heuristics and the initial distributions converge. So even when $\psi_1 = \psi_2 = 0.9$, the scenario which appeared to be problematic in the previous figure. When the noise is low, the economic agents will be more able to detect the better performing heuristics. Combined with high inertia, meaning that economic agents don't quickly switch from heuristic, this leads to a stabilising effect for all cases. The value of k is more important than ξ. Even for low inertia, if there is not a lot of noise, the lion's share of the simulations converge.

When the memory parameter is higher, the conclusions from the previous figure are even stronger. In this case, whenever $k = 100$, the dynamics will converge to the steady state, even for low inertia. The oscillations in the cases where $k = 0.1$ are also smaller than their counterparts in Figure 5.

Figure 1: Transitional dynamics after a labour tax decrease for different heuristics and under rational expectations scenario

θ = 4 - β = 0.98: Without Heuristic Switching | **θ = 4 - β = 0.98: With Heuristic Switching**

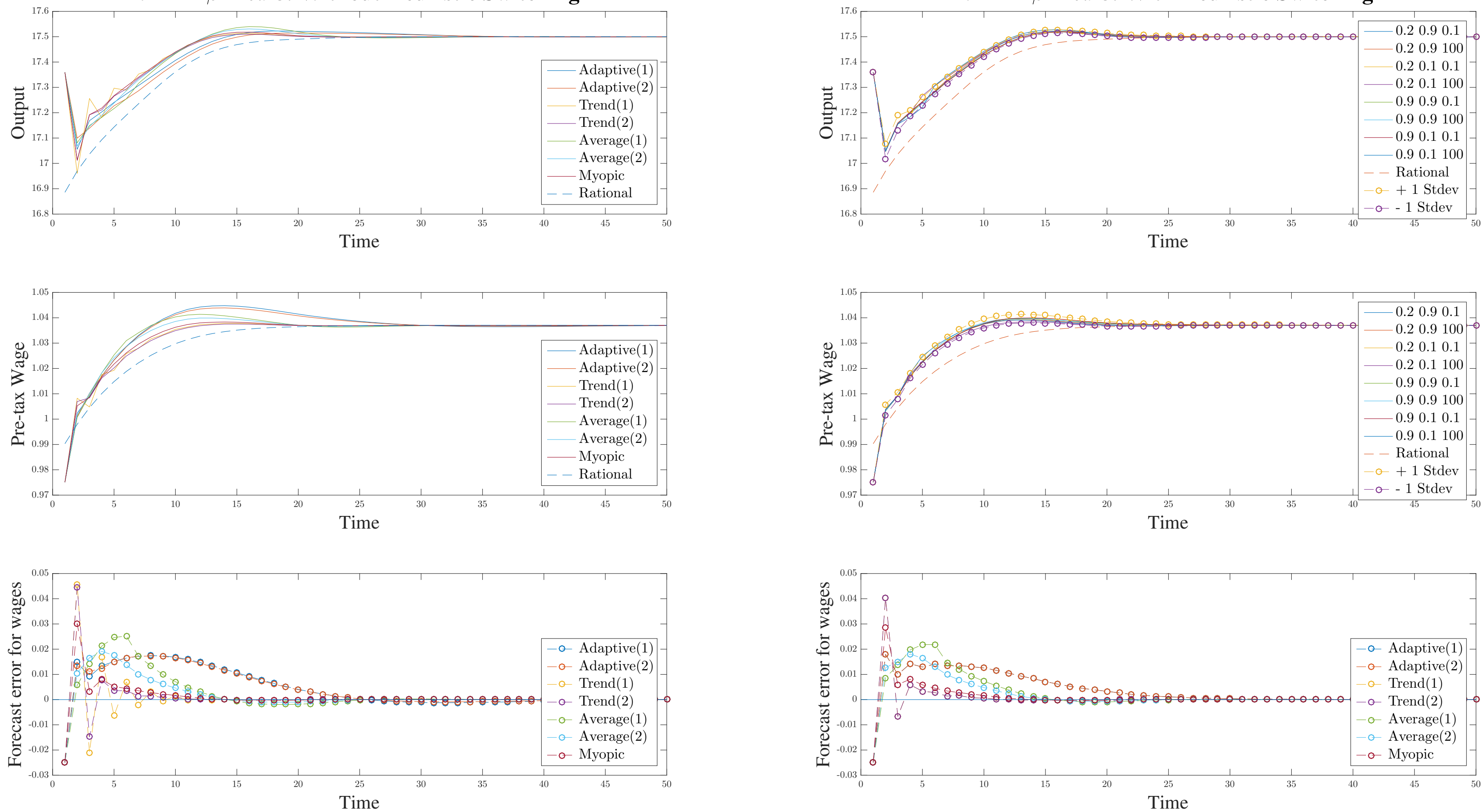
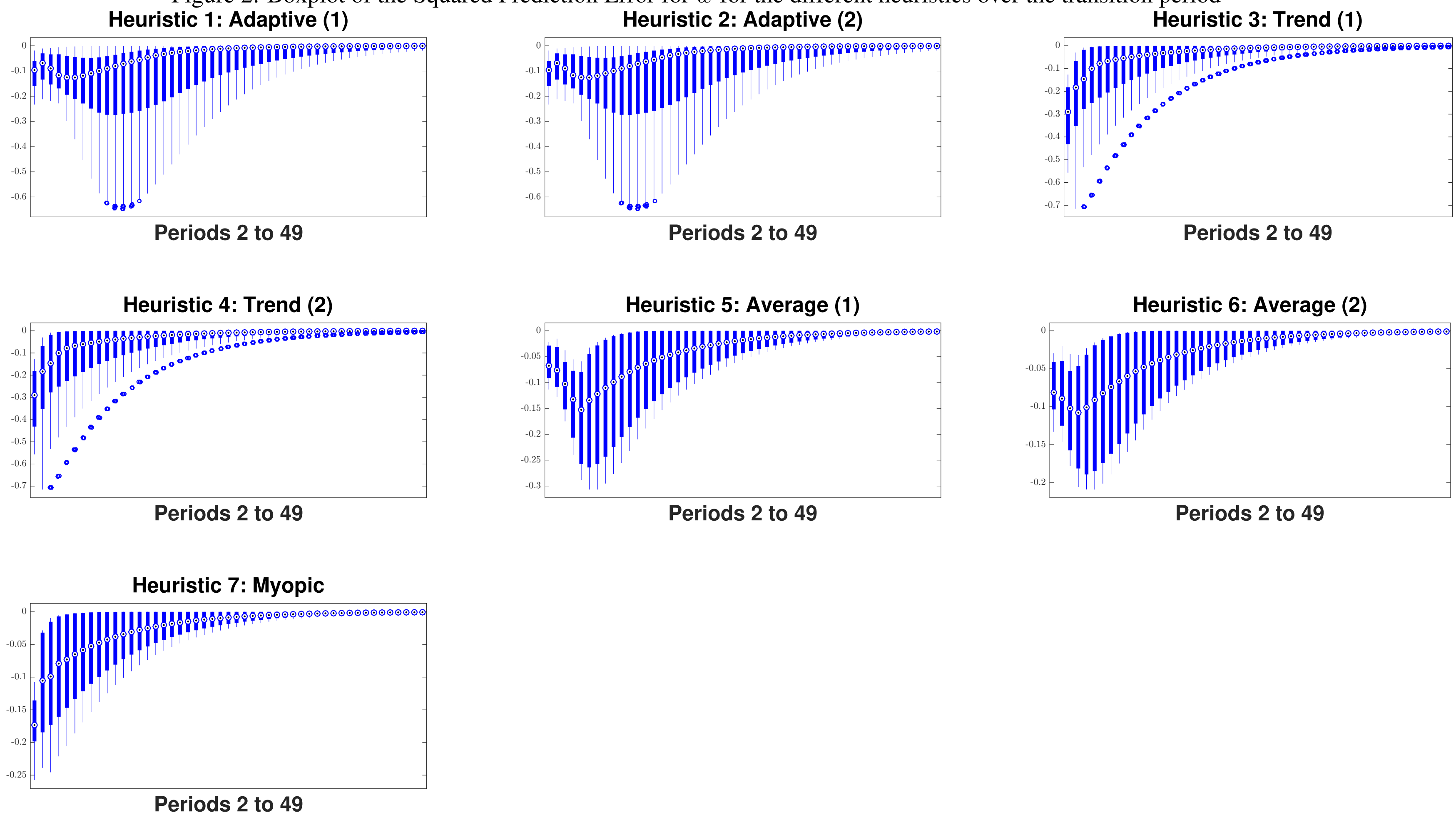


Figure 2: Boxplot of the Squared Prediction Error for w for the different heuristics over the transition period



Note: these boxplots are constructed using the Squared Prediction Error for a given heuristic in a given period over all parameter values discussed in Section 5.7.1.2.

Figure 3: Transitional dynamics after labour tax decrease for $\beta = 0.86$ and $\theta = 2$ without heuristic switching

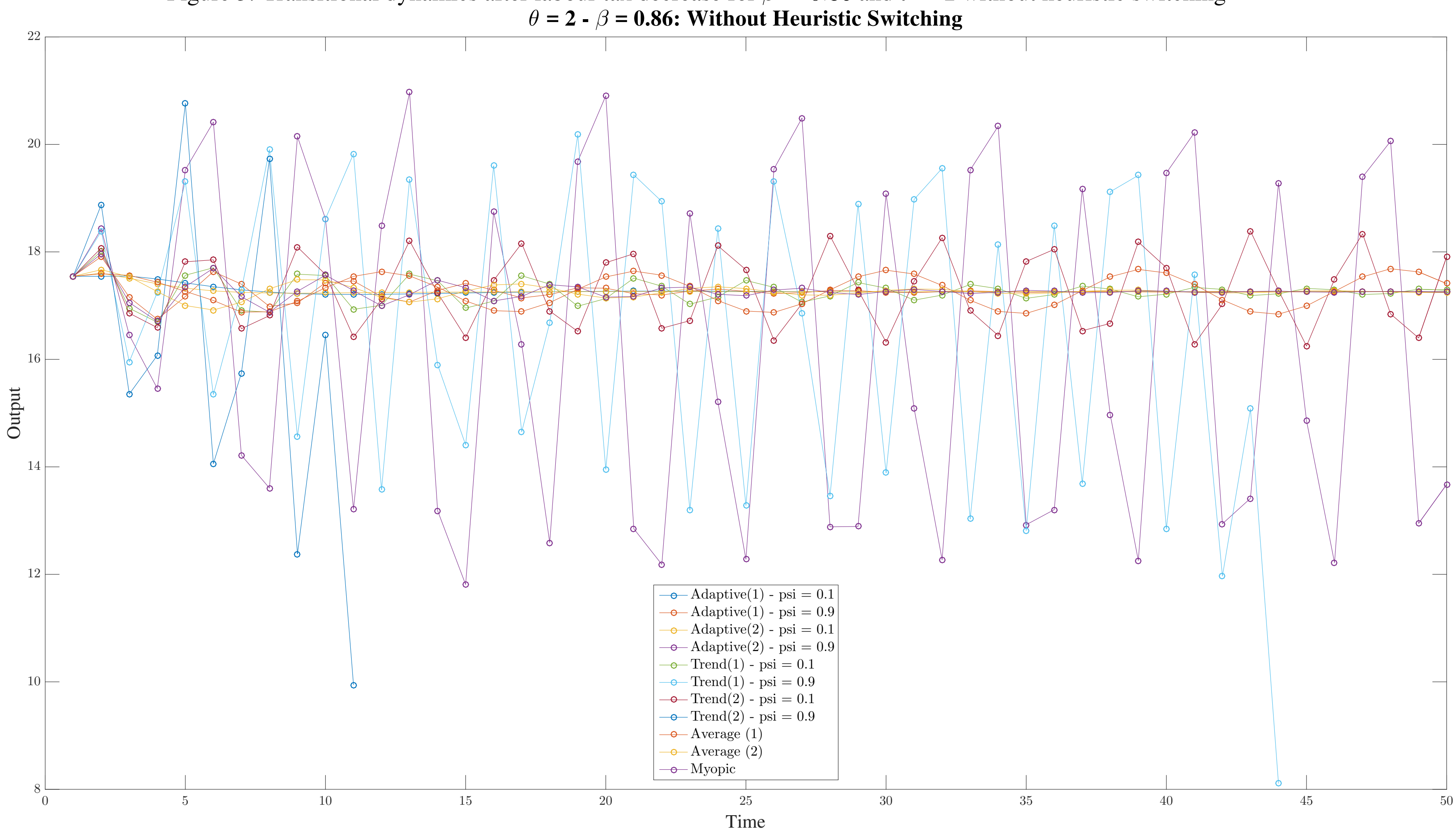
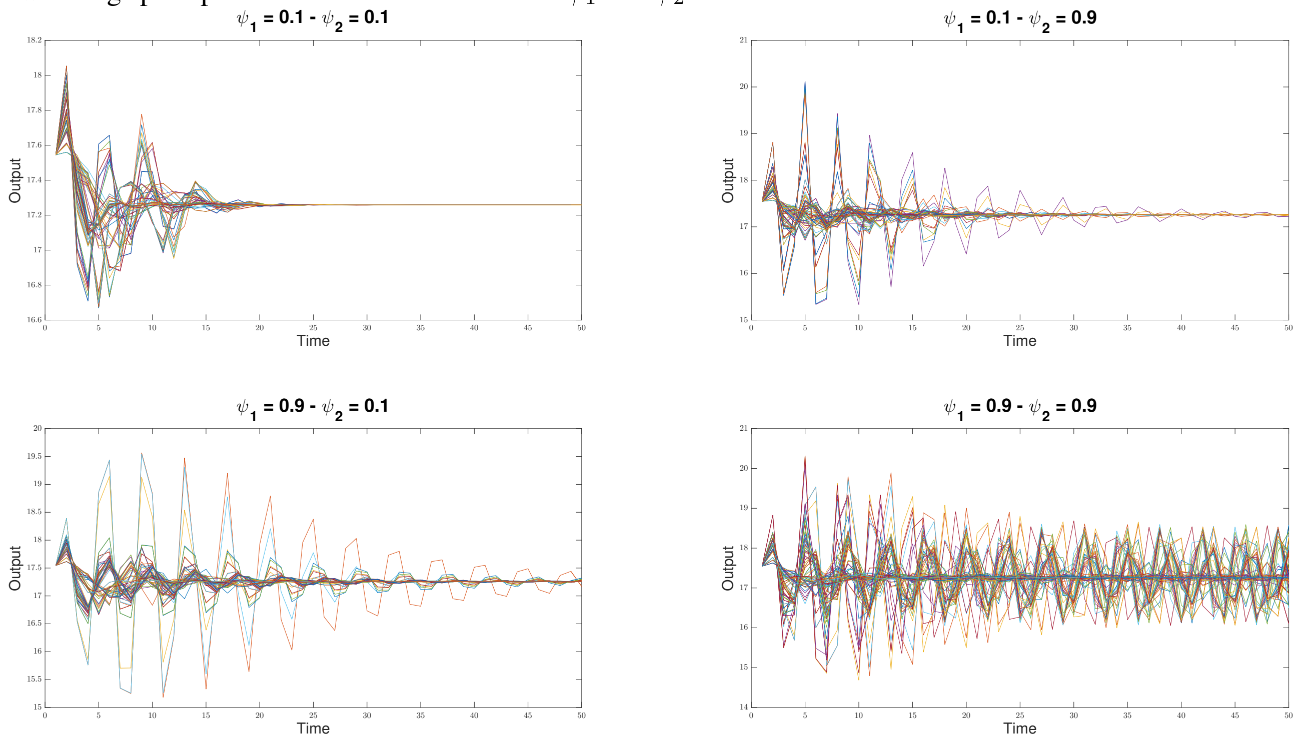


Figure 4: Transitional dynamics after labour tax decrease for $\beta = 0.86$ and $\theta = 2$ with heuristic switching split up for different combinations of ψ_1 and ψ_2



8 Conclusion

This paper explores the stability and transitional dynamics of an overlapping generations model with heuristic switching. Accordingly, this paper contributes to a growing literature that goes beyond the rational expectations paradigm. We assume that agents use simple rules, heuristics, to forecast the future course of the interest rate and the real wage. Agents use these heuristics because, in general, they do not possess the cognitive abilities as assumed by the RE literature nor to act as econometricians. Instead, the agents have a certain number of different heuristics at their disposal. On a regular basis, they assess the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

The aim of this paper is to contribute to the literature that goes beyond the rational expectations paradigm by exploiting the heuristic switching approach within an Overlapping Generations model. Triggered by a fiscal policy shock, the objective is to study the transitional dynamics of the model for a large number of settings including one or multiple heuristics and compare the behaviour of the dynamics with their rational expectations counterpart.

The simulations lead to three main findings. First, in a context without heuristic switching (i.e. a context where individuals only have one heuristic at their disposal to form expectations), the evolution

Figure 5: Transitional dynamics after labour tax decrease for $\beta = 0.86$, $\theta = 2$ and $\upsilon = 0.2$: sensitivity of results to different values of the inertia and the noise

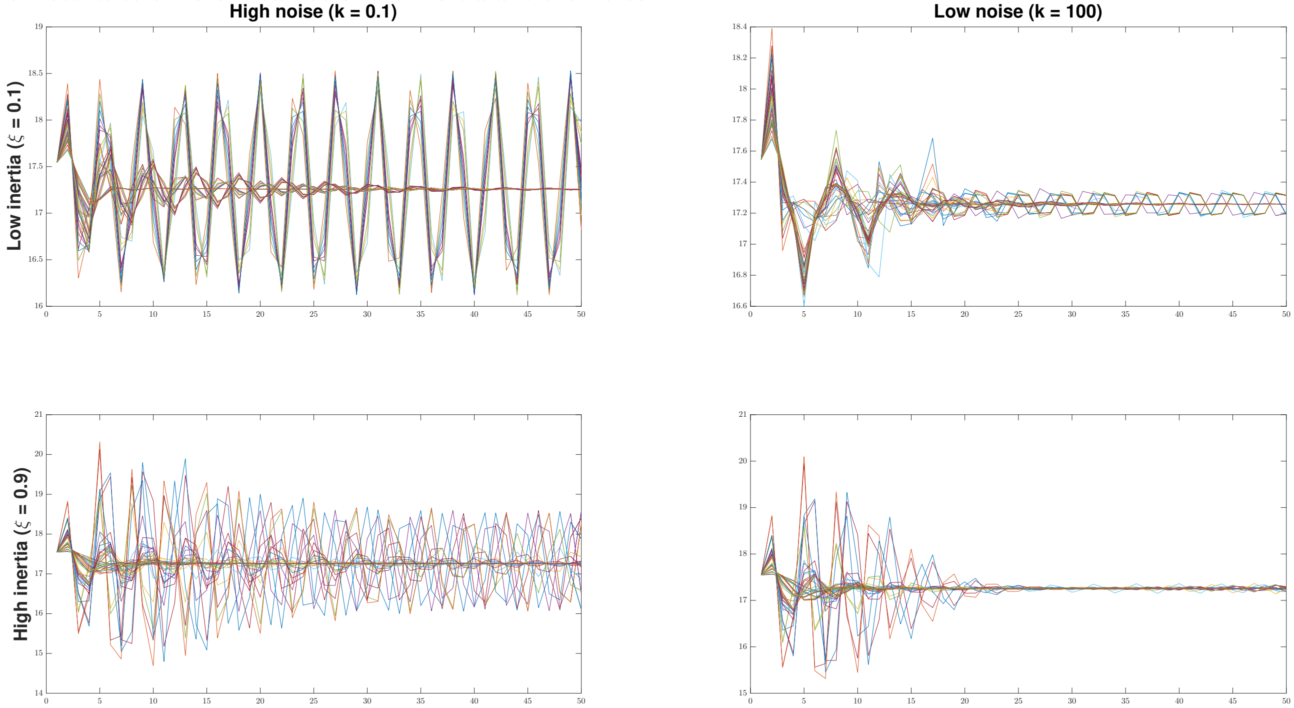
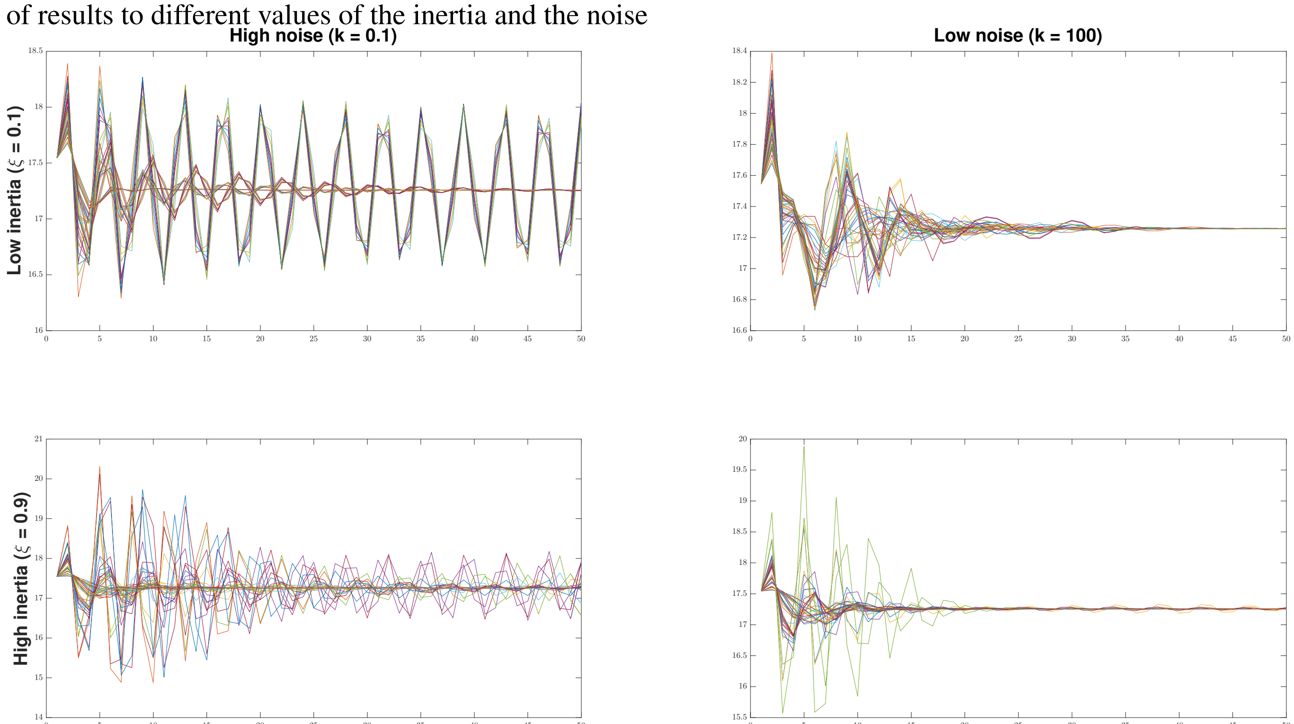


Figure 6: Transitional dynamics after labour tax decrease for $\beta = 0.86$, $\theta = 2$ and $\upsilon = 0.9$: sensitivity of results to different values of the inertia and the noise



of transitional dynamics can be substantially different from the rational expectations case, especially in the first periods of the transition. Furthermore, there is a lot of variation in the dynamics over different parameter values and heuristics. This finding implies that if only heuristic is used, the macroeconomic impact of fiscal policy is highly sensitive to the heuristic being used. What is more, as the discount rate and the degree of risk aversion decrease, the model becomes unstable and the corresponding transitional dynamics oscillate or even diverge. Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly. Consequently, the sensitivity of the transitional effects of fiscal policy is much lower now and its exact impact is thus less uncertain for policy makers. Third and last, the heuristic switching has a stabilising effect on the transitional dynamics. For certain configurations of the parameter values for which the dynamics were very unstable in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

These findings are important. They imply that allowing individuals to choose from a wide range of forecasting rules is actually a better option than constraining them to use only forecasting rule. It allows them to select the better performing rules, a feature that not only enhances the stability of the model, but reduces the uncertainty in the transitional dynamics as well.

References

Adam, K. (2003). Learning and equilibrium selection in a monetary overlapping generations model with sticky prices. *Review of Economic Studies*, 70:887–907.

Adam, K. (2007). Experimental Evidence on the Persistence of Output and Inflation. *Economic Journal*, 117(520):603–636.

Chen, H., Li, M., and Lin, Y. (2008). Chaotic dynamics in an overlapping generations model with myopic and adaptive expectations. *Journal of Economic Behavior and Organization*, 67:48–56.

Conesa, J. and Krueger, D. (2006). On the optimal progressivity of the income tax code. *Journal of Monetary Economics*, 53:1425–1450.

Cournède, B. and Gonand, F. (2006). Restoring fiscal sustainability in the euro area: raise taxes or curb spending. *OECD Economics Department Working Papers*, No. 270.

De Grauwe, P., editor (2012). *Lectures on Behavioral Economics*. Princeton: Princeton University Press.

Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press, Princeton, New Jersey.

Heemeijer, P., Hommes, C., Sonnemans, J., and Tuinstra, J. (2012). An experimental study on expectations and learning in overlapping generations models. *Studies in Nonlinear Dynamics and Econometrics*, 16(4).

Heylen, F. and Van de Kerckhove, R. (2013). Employment by age, education, and economic growth: effects of fiscal policy composition in general equilibrium. *B. E. Journal of Macroeconomics*, 13(1):49–103.

Hommes, C. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, 35(1):1 – 24.

Hommes, C. H. (2014). Behaviorally rational expectations and almost self-fulfilling equilibria. *Review of Behavioral Economics*, 1:75–97.

Krueger, D. and Ludwig, A. (2013). Optimal progressive taxation and education subsidies in a model of endogenous human capital formation. *American Economic Review: Papers and Proceedings*, 103:496–501.

McDaniel, C. (2007). Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003. *Working Paper*.

Schönhofer, M. (1998). Chaotic learning equilibria. *Journal of Economic Theory*, 89:1–20.

Tuinstra, J. (2003). Beliefs equilibria in an overlapping generations model. *Journal of Economic Behavior and Organization*, 50:145–164.

Tuinstra, J. and Wagener, F. (2007). On learning equilibria. *Economic Theory*, 30:493–513.