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Evaluation of conformity criteria for reinforcing steel properties

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Abstract: Quality inspection of strength properties has to ensure that the used materials comply with their specified requirements. Based on the average outgoing quality limit concept, conformity criteria can be designed and evaluated in an objective way. In this contribution, the conformity criteria for the yield strength of steel reinforcement bars given in the European Standard EN 10080 and the German Standard DIN 488-6 are evaluated based on this concept. It was shown that the current conformity parameters suggested in these standards yield OC-lines that cross the limiting boundary. Hence, subsequently, alternative values for the conformity control parameters are proposed. Finally, it was shown that conformity control of reinforcing steel has a beneficial influence on the reliability level of a reinforced concrete beam.

1 Introduction

Nowadays, the low risk associated with human casualties due to structural failure can be partly attributed to the high level of control efforts during the design and construction process of modern buildings. Quality inspection of the strength properties has to ensure that the used materials comply with their specified requirements. This is called quality or conformity control.

With the introduction of the European Standard EN 206-1, a practically applicable conformity control scheme of concrete is available. The extensive research done by Caspeele [1] and Taerwe [2] on this subject has resulted in a probabilistic framework regarding the evaluation of conformity criteria for concrete and the associated filtering effect of conformity control on concrete strength distributions and on the safety level of structures.

For tensile properties of reinforcing steel (e.g. the yield strength) no harmonized standard regarding conformity control is available. Hence, various types of conformity criteria and parameters are used in different countries. In this contribution, the conformity criteria for the tensile reinforcing steel properties that are currently used in the German Standard DIN 488-6 and the European Standard EN 10080 are evaluated and the effect of conformity control on the safety level of a reinforced concrete beam is investigated.

2 Basic principles of conformity control

2.1 Conformity criteria

The use of reinforcing steel in various applications requires specifications on the material properties of the reinforcing steel (e.g. for design calculations). Whether the produced reinforcing steel complies with the material properties that are specified has to be verified. This verification is called conformity control and is usually done by using so-called conformity criteria. A representative sample $\underline{x}$, consisting of n individual test results x_i, is taken from the production and the corresponding conformity criteria are generally given by the following expression:

$$z(\underline{x}) \geq a \tag{1}$$

where the value of 'a' is a threshold value and $z(\underline{x})$ is defined as the compliance function which is made up from a number of test statistics. The most common formulation of the compliance function is an estimation of the fractile of a strength distribution. Most conformity criteria can be classified based on the following four types, which are intended to assess the conformity of the characteristic value x_k of a variable X [2]:

$$\bar{x}_n \geq x_k + \lambda\sigma \tag{2}$$

$$\bar{x}_n \geq x_k + \lambda s_n \tag{3}$$

$$\bar{x}_n \geq x_k + k_1 \tag{4}$$

$$\bar{x}_{min} \geq x_k - k_2 \tag{5}$$

where $\bar{x}_n$ is the sample mean, σ is the standard deviation of the population, s_n is the standard deviation of the sample, x_{min} is the smallest value within the sample and λ, k_1 and k_2 are parameters that are used in the conformity criteria which are usually based on a certain fractile of the distribution of the variable X. The design of an appropriate control scheme is based on the choice of an appropriate value of the latter parameters. Note that criteria of type (4) can be rewritten in the shape of type (2):

$$\bar{x}_n \geq x_k + \left(\frac{k_1}{\sigma}\right)\sigma \qquad \text{or} \qquad \bar{x}_n \geq x_k + \lambda'\sigma \tag{6}$$

From this formulation, it can be seen that a value for the standard deviation σ is required for the analysis of the performance of the conformity criterion.

2.2 Operating characteristic for conformity criteria

For most design or construction purposes, the yield strength f_{yk} of reinforcing steel is used, in which case it is mostly defined as a characteristic value corresponding to the 5%-fractile of the considered strength distribution. In practice however, the specified characteristic yield strength will correspond to a fractile that is higher or lower than the 5%-fractile. In general, the fraction below the specified characteristic value f_{yk} is called the fraction defectives θ, given by:

$$P[Y \leq f_{yk}] = \theta \tag{7}$$

where Y represents the yield strength of the steel, considered as a random variable.

Assuming a certain strength distribution, one can determine the probability P_a that a steel batch – corresponding to a certain value of θ – is accepted for a given conformity criterion. The probability P_a is generally denoted as the probability of acceptance. This probability can be expressed as a function of the fraction defectives θ. The function $P_a(\theta)$ is called the operation characteristic or OC-line for the associated conformity criterion. Depending on the type of conformity criterion used for the conformity control, either an analytical expression for the OC-line can be derived or the acceptance probability can be found through numerical simulation by means of random numbers [2].

2.3 Design of conformity criteria

As mentioned before, the design of a certain conformity criterion consists of the determination of the parameters λ, k_1 and k_2. Different approaches to do this are available in literature, see e.g. [1], [2]. In this contribution, the design approach based on the average outgoing quality limit (AOQL) concept [3] is used. The average outgoing quality (AOQ) curve is a graphical representation of the expected fraction defectives in accepted lots after quality inspection. The curve shows how the outgoing quality depends on the incoming quality. It depicts the expected fraction defectives in the accepted products (after inspection) as a function of the assumed fraction defectives in the lot that is submitted for inspection. The AOQ curve makes the assumption that rejected lots are completely and thoroughly inspected and that all defectives can be considered as removed. Hence, The AOQ curve can be calculated as given in equation (8):

$$AOQ(\theta) = \theta \cdot P_a(\theta) + 0 \cdot P_r(\theta) = \theta \cdot P_a(\theta) \qquad (8)$$

The first term in equation (8) represents the accepted lots for which the fraction defectives is θ and the probability that these lots are accepted is $P_a(\theta)$. The second term indicates that the rejected lots are all completely screened and will be returned as perfect lots, thus free from defective units.

The AOQ curve reaches a maximum value for a certain fraction defectives θ. This maximum value is called the average outgoing quality limit (AOQL). The AOQL denotes the maximum possible fraction defectives for the considered quality control scheme. If the AOQL concept is used for the design of conformity criteria for the variable X, the specified characteristic value X_k can be fixed as the AOQL. Designing the conformity criteria based on this AOQL value will result in an average fraction defectives in the outgoing lots that is lower than the prescribed characteristic value. When this characteristic value is for example defined as the 5% fractile of the strength distribution, two regions in the P_a-θ diagram can be described. The 'unsafe' region in the P_a-θ diagram has the following boundary:

$$\theta \cdot P_a = 0.05 \quad \text{for} \quad \theta \geq 0.05 \qquad (9)$$

On the other hand, a boundary line for the 'uneconomic' region in the P_a-θ diagram was suggested by Taerwe [2]:

$$\frac{\theta}{1-P_a} = 0.05 \quad \text{for} \quad \theta \leq 0.05 \qquad (10)$$

In the next section, the conformity criteria that are currently used in the European Standard EN 10080 and in the German Standard DIN 488-6 are evaluated according to the average outgoing quality limit (AOQL) method. Monte Carlo simulation techniques are used to generate the OC-curves in order to evaluate the performance of these conformity criteria. Analysis of a large set of results of consecutive tensile tests on reinforcement bars revealed that there is an undeniable autocorrelation. Based on these test results, an appropriate AR(2)-model was derived:

$$u_i = \phi_1 u_{i-1} + \phi_2 u_{i-2} + \varepsilon_i \qquad (11)$$

with $\phi_1 = 0.35$, $\phi_2 = 0.25$ and $\varepsilon_i : N(0, 0.9)$. This autoregressive model was considered when generating the random samples required to derive the OC-curves. It was observed that autocorrelation has a significant influence on these OC-curves.

3 Conformity control in current European Standards

The use of conformity criteria for the quality control of reinforcing steel is mentioned in both the German standard DIN 488-6 and the European Standard EN 10080. In this section, an overview is given of the conformity criteria related to the yield strength R_e of reinforcing steel bars in both standards. Both standards make a distinction between factory production control, initial type testing and the assessment of the long-term quality level. In this contribution only the conformity criteria related to factory production control are evaluated, as they are used for continuous quality assessment of reinforcing steel.

3.1 German Standard DIN 488-6

The factory production control for produced reinforcing steel bars is the permanent quality verification executed by the steel-producing factory itself. For the verification of the yield strength R_e of reinforcing steel bars one test piece per 30 t with a minimum of three test pieces per test unit and nominal diameter is required. Hence, a sample size n = 3 will be considered for the assessment of the conformity criterion.
The assessment of conformity based on the n test results for yield strength R_e is done using the following compound conformity criterion:

$$\begin{cases} \bar{x}_n \geq C_v + k_1 \\ x_i \geq C_v + k_2 \end{cases} \tag{12}$$

The numerical values for k_1 and k_2 are equal to 30 MPa and 5 MPa respectively, according to DIN 488-6. The AOQL for the yield strength is assumed to be 5%.
For this type of compound conformity criterion an assumption of the standard deviation σ is required (as indicated in section 2.1). DIN 488-6 indicates that the conformity criteria may be applied in case the standard deviation σ is less than or equal to 30 MPa. It should be noted that this value for the standard deviation σ = 30 MPa is also given in the JCSS Probabilistic Model Code [5] and originates from the sum of squares of three contributing standard deviations, as shown in the following equation:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} = \sqrt{19^2 + 22^2 + 8^2} \cong 30\ MPa \tag{13}$$

where σ_1 represents the variation between different steel producers, σ_2 represents the variation between different batches (for a certain steel producing factory) and σ_3 represents the variation in one specific batch.

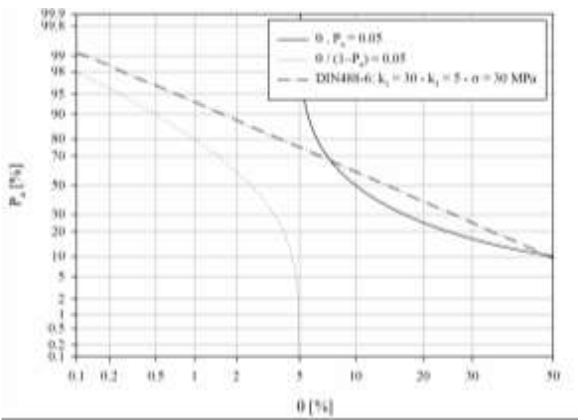


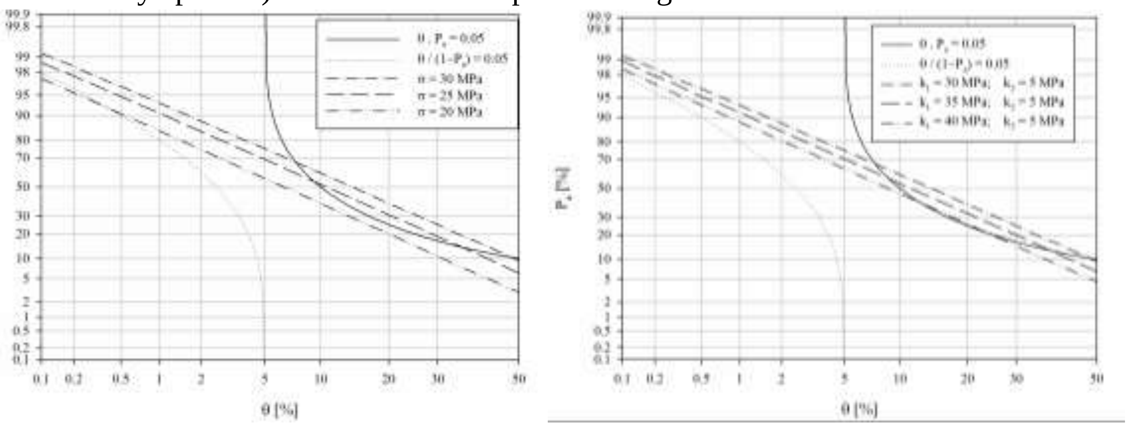
Figure 1: OC-curve corresponding to the compound conformity criterion for factory production control for R_e as given in DIN488-6 (σ = 30 MPa)

The OC-curve for the conformity criterion according to DIN 488-6 is depicted in Figure 1 in case σ = 30 MPa is considered. It is shown that the OC-curve crosses the boundary of the unsafe region defined by $\theta \cdot P_a = 0.05$. Since the OC-curve is influenced by the parameters k_1 and k_2 and the assumed standard deviation σ, a safe proposal for the conformity criterion can be obtained by adapting the values for these parameters. The compound conformity criterion (12) is used to verify the performance of the factory production control of one specific steel producing factory. Hence the influence of σ_1 in (13) could be omitted and a more appropriate value of the standard deviation can be found as:

$$\sigma = \sqrt{\sigma_2^2 + \sigma_3^2} = \sqrt{22^2 + 8^2} \cong 25\ MPa \qquad (14)$$

Therefore, OC-curves for the conformity criterion given in the German Standard DIN 488-6 corresponding with standard deviation σ = 25 MPa and σ = 20 MPa are also generated and depicted in Figure 2a. It can be concluded that a decrease in the standard deviation σ causes a downward shift of the OC-curve towards the safe region. For a standard deviation σ = 22 MPa the OC-curve will be tangent to the safe boundary and therefore performing optimally.

The German Standard DIN 488-6 states that the conformity criteria can be used in case the standard deviation is less than or equal to 30 MPa. Therefore, the influence of parameter k_1 on the conformity criteria has been investigated in order to end up with an optimized criterion for σ = 30 MPa. The value of k_2 has not been altered since its influence on the OC-curve is rather limited and its value is usually related to a certain fractile of the strength distribution (which can be country-specific). The results are depicted in Figure 2b.



(a) *(b)*

Figure 2: (a) Influence of the standard deviation σ on the OC-curve of DIN 488-6; (b) Influence of the parameter k_1 of DIN 488-6 (σ = 30 MPa)

It can be concluded that in case a lower standard deviation (i.e. σ = 22 MPa) is adopted, the parameters which are currently proposed in DIN 488-6 are optimal. However, in case the choice for σ = 30 MPa remains, the parameter k_1 should be adjusted to k_1 = 40 MPa in order to end up with a conformity criterion which is optimal.

3.2 European Standard EN 10080

For the verification of the tensile properties, according to the European Standard EN 10080, one test piece per 30 t with a minimum of three test pieces should be used. The assessment of the test results for the yield strength R_e is done using the following compound conformity criterion:

$$\begin{cases} \bar{x}_n \geq C_v + k_1 \\ x_i \geq C_v - k_2 \end{cases} \qquad (15)$$

In addition to the European Standard 10080, the Belgian Annex NBN A 24-301 prescribes the characteristics and technical requirements for the tensile properties for two classes of weldable ribbed reinforcing steel - B500A and B500B (corresponding with the two different ductility classes for reinforcing steel bars) - as well as the numerical values for the parameters k_1 and k_2 that are used in the conformity criteria mentioned in the European Standard EN 10080. For both classes the values for C_v, k_1 and k_2 are 500 MPa, 10 MPa and 15 MPa respectively. Furthermore, the AOQL for the yield strength is 5%.

The OC-curve for the conformity criterion according to EN 10080 is depicted in figure 3, considering σ = 30 MPa.

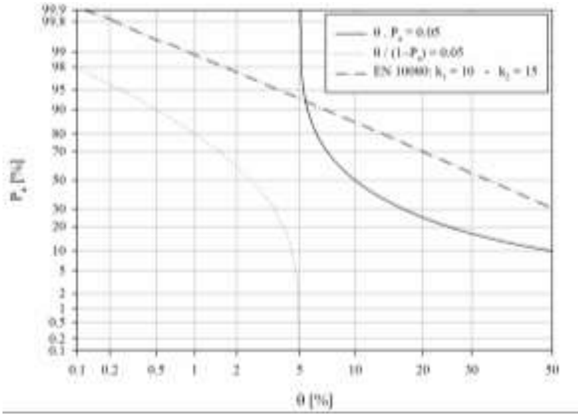
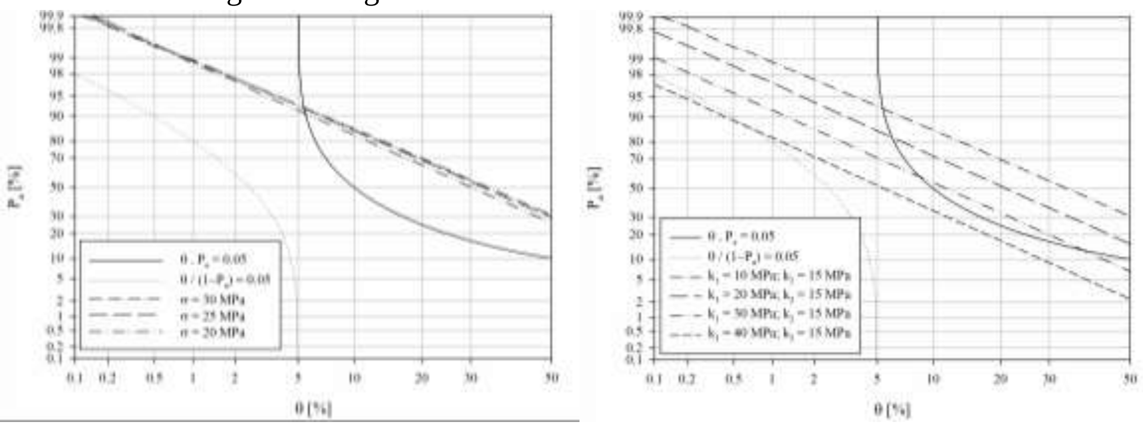


Figure 3: OC-curve corresponding to the compound conformity criterion for factory production control for R_e as given in EN 10080 (σ = 30 MPa)

From Figure 3 it can be concluded that the conformity criterion for the yield strength R_e with parameters as currently specified in the European Standard EN 10080 is at the unsafe side. Therefore, the influence of the standard deviation σ and the parameter k_1 on the OC-curve was investigated in order to end up with a safe alternative for the currently specified parameters. The result is given in Figure 4a and 4b.



(a) *(b)*

Figure 4: (a) Influence of the standard deviation σ on the OC-curve of EN 10080; (b) Influence of the parameter k_1 on the OC-curve of EN 10080 (σ = 25 MPa)

The value for the standard deviation σ = 30 MPa that is currently used in the conformity criterion for R_e as stated in the European Standard NBN EN 10080 is, as explained in section 3.1, too conservative. From Figure 4a it can be seen that lowering the value of the standard deviation for the analysis of the performance of the conformity criterion results in a downward shift and rotation of the OC-line. However, the influence is much less significant than for the equivalent compound criterion given in DIN 488-6 due to the different values for k_1 and k_2.

The influence of k_1 on the OC-curve can been seen in figure 4b in case a more appropriate standard deviation is adopted, i.e. $\sigma = 25$ MPa. One can see that the value of k_1 should be significantly increased in order to end up with an OC-line which does not cross the unsafe region. According to Figure 4(b), the optimal choice for the conformity criterion specified in EN 10080 would be: $k_1 = 40$ MPa, $k_2 = 15$ MPa and considering $\sigma = 25$ MPa.

4 Effect of conformity control on the reliability of structural elements

4.1 Filter effect of conformity control

The filtering effect of conformity control on reinforcing steel properties (or other material properties in general) originates from the acceptance or rejectance - due to the conformity assessment - of a sample set of reinforcing steel bars. The fraction defectives in a certain population of reinforcing steel bars after conformity control decreases when compared to the fraction defectives of the same population before conformity control, because reinforcing steel lots with insufficient quality are rectified. Hence, conformity control has a filtering effect on the strength distribution of reinforcing steel.

4.2 Quantitative analysis of the filter effect of conformity control

An analytical formulation of the filter effect can be derived, based on the following model for the yield strength Y of steel. Assume that the yield strength of steel is given by two additive contributions Y_m and Y_l, with Y_m representing the variation of the yield strength between different lots, i.e. $Y_m : N(\mu_m , \sigma_m)$, and Y_l representing the variation of the yield strength inside one lot, i.e. $Y_l : N(0 , \sigma_l)$. The additive model of the yield strength Y of reinforcing steel is then given by (16):

$$Y = Y_m + Y_l \tag{16}$$

This model corresponds to the model suggested in the JCSS Probabilistic Model Code [5].
Based on the notation 'i' for incoming or offered reinforcing steel lots and 'o' for outgoing or accepted reinforcing steel lots, the mean and the standard deviation of the incoming lots are given by (17) and (18), respectively, considering Y_m and Y_l both normally distributed:

$$\mu_i = \mu_m \tag{17}$$

$$\sigma_i = \sqrt{\sigma_m^2 + \sigma_l^2} \tag{18}$$

In [6] a posterior predictive distribution for Y, based on a strength model of type (16), is given. Including conformity control, the following expression for the posterior density function for Y_m can be derived based on Bayes theorem:

$$f_o(y_m) = \frac{\frac{1}{\sigma_m}\phi\left(\frac{y_m-\mu_m}{\sigma_m}\right)P_a(y_m|\dots)}{\int_{-\infty}^{+\infty}\frac{1}{\sigma_m}\phi\left(\frac{y_m-\mu_m}{\sigma_m}\right)P_a(y_m|\dots)dy_m} \tag{19}$$

with $P_a(y_m|\dots)$ the probability that a certain lot of reinforcing steel bars with mean strength y_m is accepted by the given conformity criteria. Furthermore, Taerwe [2] derived a posterior predictive distribution for Y, given by:

$$f_o(y) = \int_{-\infty}^{+\infty} f(y|y_m,\sigma_l)\, f_o(y_m)dy_m = \int_{-\infty}^{+\infty}\frac{1}{\sigma_l}\phi\left(\frac{y-y_m}{\sigma_l}\right) f_o(y_m)dy_m \tag{20}$$

Only for specific cases of conformity criteria, an analytical expression can be derived for the posterior distribution (see e.g. [2]). For complex situations, e.g. for compound conformity criteria, Caspeele [1] developed a numerical algorithm in order to calculate the filter effect. More information regarding this numerical algorithm for the calculation of the filter effect as well as a brief summary of the implementation of the algorithm can be found in [1].

4.3 Filter effect of conformity criteria in DIN 488-6

The previously mentioned computational method for the evaluation of the filter effect of conformity criteria is used to investigate the filter effect for the compound conformity criterion (12) suggested by the German Standard DIN 488-6 (with k_1 = 30, k_2 = 5) considering σ = 25 MPa and for the optimized compound conformity criterion (15) suggested by EN 10080 (with k_1 = 40, k_2 = 5) considering σ = 25 MPa.
Under the assumption that the standard deviation of the incoming strength distribution is σ_i = 25 MPa and that the ratio σ_l/σ_m = 8/22 = 0.36 (see section 3.1), the numerical algorithm is used to determine the ratios μ_o/μ_i and σ_o/σ_i for the compound conformity criteria (12) and (15) for R_e. Figure 9 depicts the filter effect (i.e. the ratios μ_o / μ_i and σ_o / σ_i) associated with the described situation. In general, it can be seen that the mean of the outgoing strength distribution increases with respect to the incoming strength distribution, whereas the standard deviation of the outgoing strength distribution decreases with respect to the incoming strength distribution. Quantitatively, the described effect increases for increasing fraction defectives θ_i in the incoming population offered for conformity control. Furthermore, it can be seen that the filter effect of the optimized conformity criterion suggested in EN 10080 is slightly more pronounced. The difference, however, is very limited.

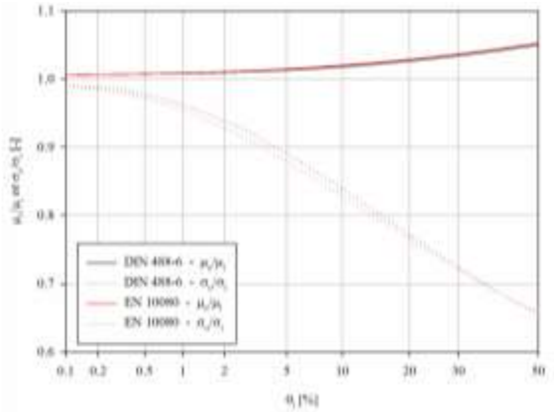


Figure 5: Filter effect associated with the optimized conformity criteria from DIN 488-6 and EN 10080

4.4 Effect of conformity control on structural reliability – case study

In this section, a case study regarding the effect of conformity control on the safety level of a reinforced concrete beam subjected to bending is provided using a FORM analysis.
EN 1990 suggests the following fundamental load combination for the design of structural elements:

$$r_d = max\{\gamma_g G_k + \psi_0 \gamma_Q Q_k \,;\, \xi \gamma_G G_k + \gamma_Q Q_k\} \tag{21}$$

with r_d the design value of the resistance, γ_G =1.35 the partial factor for the permanent load, G_k the characteristic value of the permanent load effect, $\psi_{0,Q}$ = 0.7 the combination value of the imposed load effect, γ_Q =1.5 the partial factor for the imposed load, Q_k the characteristic value of the imposed load effect and ξ = 0.85 a reduction factor for the permanent load.
In order to be able to cover possible combinations of actions, the load ratio χ ($\chi = Q_k/G_k + Q_k$) is introduced. For a given permanent load and load ratio χ , the required design resistance r_d can be obtained from (21) [7].
The reliability of the considered reinforced concrete beam under bending is determined through a FORM analysis. The limit state function for the case under consideration is given

by:

$$g(\underline{X}) = K_R \rho b (h-a) f_y \left[h - a - 0.5\rho(h-a)\frac{f_y}{f_c}\right] - K_E(G - Q_{50}) \quad (22)$$

with K_R the model uncertainty regarding the structural resistance, ρ the reinforcement ratio, b the width of the beam, h the height of the beam, a the distance between the axis of the reinforcement and the surface of the beam, f_y the yield strength of the reinforcement, f_c the concrete compressive strength, K_E the model uncertainty regarding the load effect and Q_{50} the imposed load, for this specific case related to a reference period t_{ref} of 50 years. The probabilistic models for all basic variables used in this case study are given in Table 1.
Furthermore, the design resistance of the reinforced concrete beam is given by:

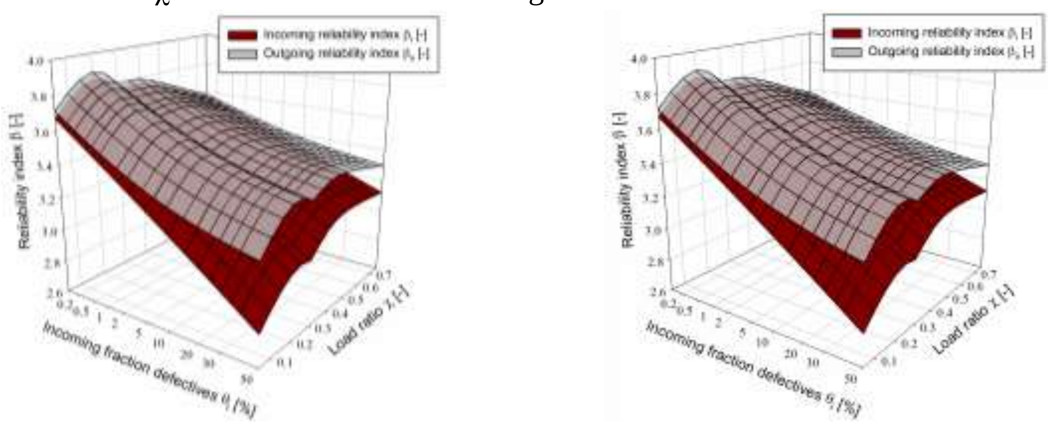
$$r_d(\rho) = \rho b (h-a)\frac{f_{yk}}{\gamma_s}\left[h - a - 0.5\rho(h-a)\frac{f_{yk}/\gamma_s}{f_{ck}/\gamma_c}\right] \quad (23)$$

with γ_s the partial factor for steel and γ_c the partial factor for concrete.

Table 1: Probabilistic models for the basic variables (based on [5])

X	**Description of X**	**Dist.**	**Unit**	**γ_X**	**μ_X / X_k**	**COV**
G	Permanent load	N	MN	1.35	1	0.1
Q_{50}	Imposed load	GU	MN	1.5	0.6	0.35
K_R	Resistance model uncertainty	LN	-	-	1	0.06
K_E	Load effect model uncertainty	LN	-	-	1	0.1
f_c	Concrete compressive strength	LN	MPa	1.5	40/30	0.15
f_y	Yield strength of steel reinforcement	N	MPa	1.15	μ_i/500 or μ_o/500	σ_i/μ_i or σ_o/μ_o
h	Height of the beam (0.5 m)	N	m	-	1	0.02
b	Width of the beam (0.3 m)	N	m	-	1	0.02
a	Position of reinforcement axis (0.03 m)	GA	m	-	1	0.17

The filter effect of conformity control according to DIN 488-6 and EN 10080 on the reliability index β is calculated based on FORM analyses. The resulting prior reliability index β_i and the posterior reliability index β_o for a reinforced concrete beam are shown in Figure 6 for different load ratios χ as well as different incoming fraction defectives.



(a) *(b)*

Figure 9: Influence of conformity control according to (a) DIN 488-6 and (b) EN 10080 on the reliability index of a concrete beam subjected to bending

The following observations can be made:

- The dependency of the reliability index β corresponding to a certain incoming fraction defectives θ_i increases with decreasing load ratio χ;

- The filter effect of the considered conformity criterion on the reliability index β increases with an increasing incoming fraction defectives θ_i;
- Conformity control of steel has a favourable effect on the reliability index β of a reinforced concrete beam. For this specific case, the reliability index β can (for θ_i = 50%) be considered to be 0.4 higher when the fact that conformity control was performed is adequately taken into account. The difference between the effect associated to the two standards (DIN 488-6 and EN 10080) is negligible.

5 Conclusions

The conformity criteria for the quality assessment of the yield strength of reinforcing steel bars that are currently used in the German Standard DIN 488-6 and in the European Standard EN 10080 were evaluated based on the AOQL concept and by using Monte Carlo simulations to derive the OC-curves. It was observed that the currently used criteria yield OC-curves which cross the boundary of the unsafe region. Consecutively, he conformity control parameters were optimized.

Furthermore, the effect of conformity control of reinforcing steel bars on the reliability index of a reinforced concrete beam was investigated. It was shown that conformity control has a significant beneficial influence on the reliability level and hence it is beneficial to take into account the fact that conformity control was executed when performing reliability calculations.

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