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Application of a multi-level probabilistic framework for the risk-based robustness assessment of a RC frame structure

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Abstract: Despite the increased interest and research about structural robustness, one has to notice that no practical framework is available yet to quantify and assess the robustness of structures which takes into account both local structural behavior of the elements under large deformations and the uncertainties of the acting loads and materials. In this contribution advanced calculation methods and risk-based quantification approaches for robustness are combined by a multi-level calculation scheme which is applied for two alternative designs. The developed approach is able to quantify the reliability and structural robustness of planar reinforced concrete frames in an objective way while using a conditional risk-based robustness index and taking into account the developed membrane action. Additionally the assessment and influence of the direct and indirect costs on risk-based robustness quantification are studied.

1 Introduction

The importance of structural robustness has been underlined by numerous failures in the past decades, such as the notable failures at Ronan Point (1968), the Murrah Federal building (1985) and the World Trade Center (2001). Typical for these past failures, which are unfortunately a worldwide recurring issue, is the fact that due to the lack of structural robustness, a local event with a very low probability of occurrence resulted in very large and disproportionate consequences [1]. Although designing a structure to withstand these exceptional events such as human error and terrorist attack is impracticable and uneconomical, a beneficial strategy is to allow the development of alternate load paths to redistribute the loads and reduce the extent of damage as much as possible in case of an exceptional local failure. Based on numerous recent experimental findings and numerical studies on reinforced concrete beams and slabs, it is clear that these elements have a large potential to develop alternate load paths in RC structures due to the development of membrane action [11], [15-16]. On the other hand research has also been focusing on theories to quantify structural robustness by robustness indicators [2]. An important next step is to combine the experimental and numerical results of the structural elements with the available probabilistic techniques and robustness indicators to assess the reliability and structural performance of structures in case of exceptional events. Moreover due to the development of membrane action, large membrane forces are generated which have to be taken into account when assessing the stability of the remaining structure. To achieve the latter, a multi-level probabilistic framework is illustrated in this paper for two alternative designs of an office building which takes the behavior of the complete structures into account. With this framework it is possible to quantify the reliability of RC frames in case of the notional removal of load-bearing columns. Next, the computed failure probabilities can be combined with a risk-based robustness quantification which is illustrated at the end of this contribution.

2 Multi-level probabilistic framework

2.1 Structural design of illustrative cases

In order to illustrate the developed calculation scheme, two office buildings have been designed according to EN 1992-1-1 [8]. Both buildings have the same useful office space but a different structural layout (Figure 1). The first building consists of 6 bays in both orthogonal directions and has 6 floors. The second building on the other hand consists of 6 and 12 bays in the X- and Y-direction respectively and has 3 floors. The height of each floor and the span of each bay for both buildings is 3 m and 6 m respectively. The floor consists of precast hollow core concrete slabs which carry the loads in one direction (i.e. the X-direction) to the frames placed in the Y-direction. Hence as a conservative simplification, the analysis can be performed by considering 2D frames and no 3D effects are considered. The permanent load applied on the beams consists of the self-weight g_k of the concrete floors of 6.25 kN/m^2 and the variable load for office buildings consists of the service load q_k of 3 kN/m² in accordance with EN 1991-1-1 [7]. An illustration of both buildings subjected to a notional column removal can be found in Figure 1. The dimensions of the beams and columns as well as the reinforcement details are given in Table 1. It is noted that the reinforcement amount which is calculated according to the regular design requirements is sufficient to act as horizontal and vertical ties according to EN 1991-1-7 [7].

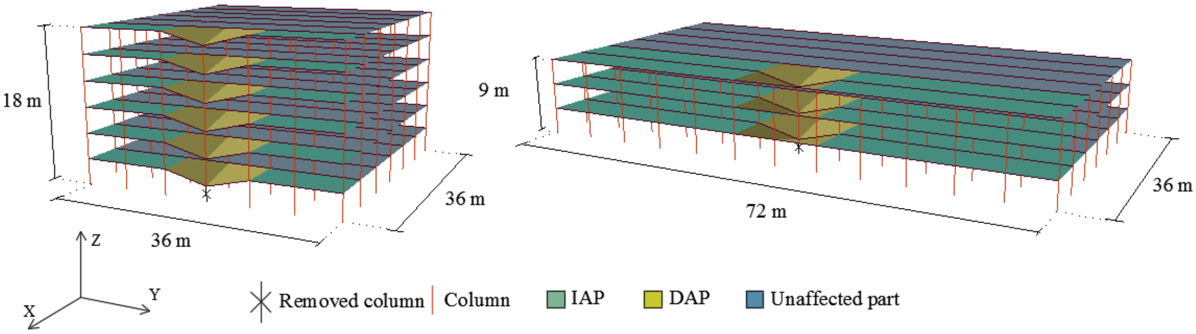

Figure 1: Overview of the illustrative cases: Building 1 subjected to the notional removal of an edge column (left) and Building 2 subjected to the notional removal of an internal column (right)

*Table 1: Summary of geometrical properties of the considered illustrative cases**

			Building 1		Building 2	
Frame			Internal	Edge	Internal	Edge
Columns	Dimensions b x h		450 x 420	450 x 420	350 x 420	350 x 420
	Reinforcement**	Floors 1-2	12 ∅ 18 12 ∅ 14***	8 ∅ 14	12 ∅ 14	8 ∅ 14
		Floors 3-6	12 ∅ 14	8 ∅ 14	12 ∅ 14	8 ∅ 14
Beams	Dimensions b x h		420 x 450	420 x 450	420 x 450	420 x 450
	Top Reinforcement		4 ∅ 25	4 ∅ 18	4 ∅ 25	4 ∅ 18
	Bottom Reinforcement		3 ∅ 20	2 ∅ 18	3 ∅ 20	2 ∅ 18
	Shear Reinforcement		∅ 10	∅ 10	∅ 10	∅ 10

* Distances, dimensions and diameters in mm

** Total reinforcement of the columns, placed symmetrically in the columns

*** Inner columns of frame 12 ∅ 18, outer columns of frame 12 ∅ 14

2.2 Development of the multi-level calculation scheme

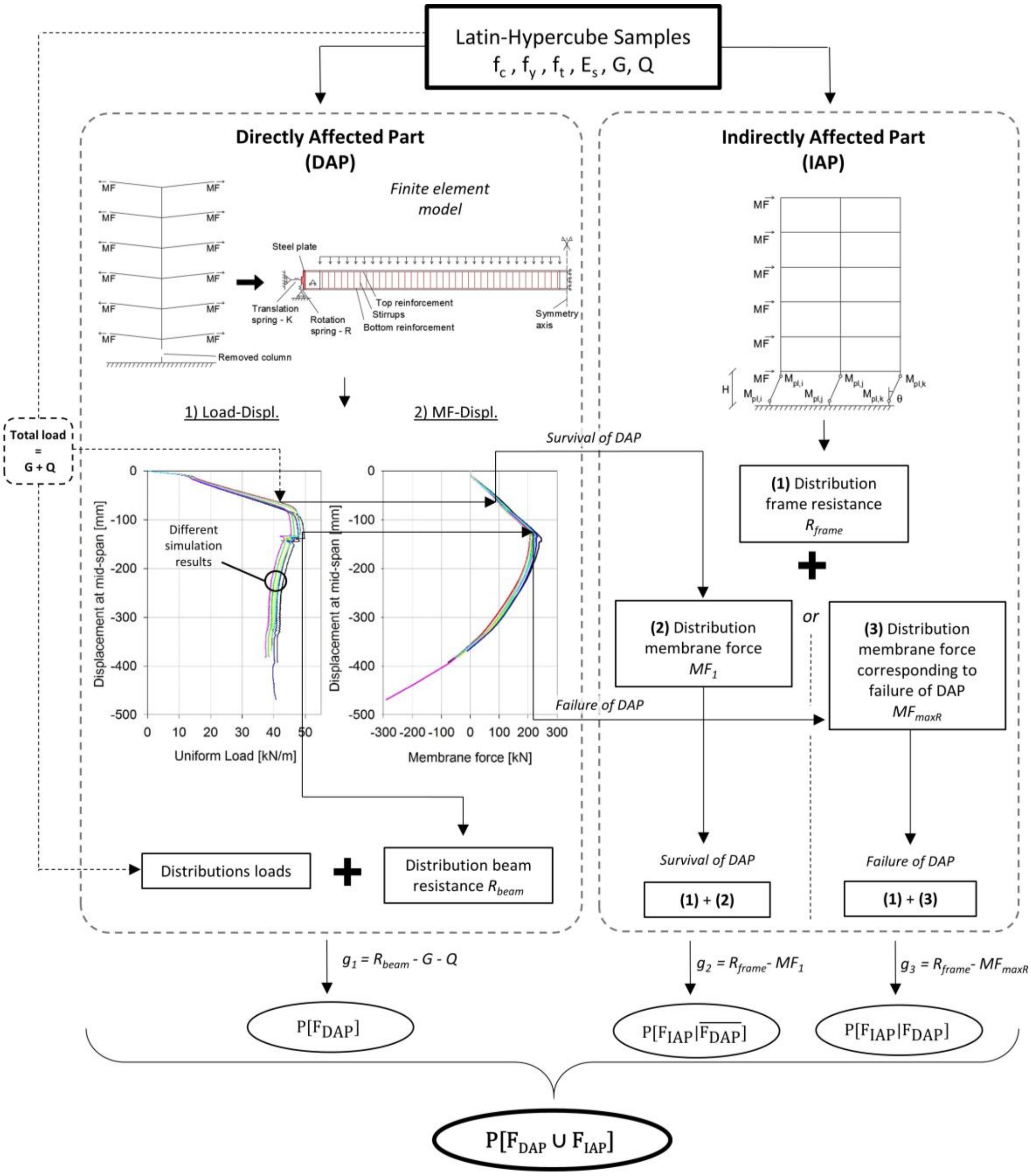

Figure 2: Developed multi-level calculation scheme

2.2.1 Distinction between the directly and indirectly affected part of the structure

To obtain an event-independent robustness assessment, the notional removal of load-bearing columns (edge and internal column) at the ground-floor are considered. Consequently the cause and probability of occurrence of the exceptional event is not explicitly considered and the robustness indices which are obtained at the end of the calculations are conditional on the considered damage and the initiating event. As large deformations are expected for the considered accidental states, it is required to perform non-linear analyses which consider both geometrical and material non-linear behavior. However as this type of analyses is computationally demanding, especially in case of reinforced concrete, the real structure is subdivided in different parts which can be idealized each to reduce the computational efforts. Regarding the notional removal of the columns, the considered structure is subdivided into three parts (Figure 1). The first part is the unaffected part which is not considered further in the calculations as this part is assumed to be unaffected by the considered accidental states. The two other parts are the directly (DAP) and indirectly affected part (IAP) which are located in the same bay where the column is removed. The DAP is located immediately above the removed column, and hence this part will be subjected to large deformations for which detailed numerical calculations are necessary. The IAP on the other hand is situated next to the DAP and defines the boundary conditions of the DAP. The IAP is subjected to smaller deformations, hence less detailed calculation methods are used for this part. More information on the implemented modelling simplifications can be found in [4]. The following steps of the multi-level calculation scheme to quantify the reliability of the damaged structure are applied for both buildings in the following subsections and are presented in Figure 2 for Building 1.

2.2.2 Numerical model for the directly affected part

For the detailed numerical analysis of the DAP, the advanced finite element software DIANA 10.0 is used. The model is based on the experimentally verified numerical 2D plane stress model developed by Botte et al. [3] which was modified and extended to the considered situation. The established numerical model was also validated with the experimental results obtained by NIST on RC beams [17]. The model takes into account geometrical and material nonlinearities and applies a load-controlled procedure with a uniform load in combination with an arc-length algorithm to simulate some softening behavior. The boundary conditions of this model which represent the partially clamped connection with the IAP are modelled by multi-linear elastic springs which take into account possible failure of the IAP. Due to symmetry only half of the beam in the accidental state is modelled. The calculation procedure is continued till failure of the beam which is governed by crushing of the concrete and finally by rupture of the top reinforcement at the partially clamped support or by failure of its boundary conditions (i.e. IAP).

2.2.3 Analytical model for the indirectly affected part

Considering the presumed failure mode of the IAP, i.e. a soft-story at ground-level, a plastic calculation procedure is implemented which considers the development of plastic hinges in the columns at the ground-floor. Next, based on the principle of conservation of energy the following equation can be obtained which can be solved for the frame resistance MF of the IAP:

$$2\left(\sum_{i-1}^{n} M_{pl,Ci}\,\theta\right) = N(H\theta MF) \tag{1}$$

- $M_{pl,Ci}$ [kNm] is the plastic moment of the respective columns of the IAP at ground-level;
- n is the number of columns of the IAP at ground level at one side of the IAP;
- N is the number of floors of the building under consideration;
- θ [radians] is the rotation angle of the deformed frame;
- MF [kN] is the membrane force acting at each floor level;
- H [m] is the height of the first floor.

The plastic moment capacity $M_{pl,Ci}$, the rotation capacity $\phi_{pl,Ci}$ of the columns and the corresponding yielding spring stiffness K of the IAP for the multi-linear translation spring of the DAP-model are calculated based on the work of Monti and Petrone [18].

2.2.4 Input parameters for the Latin-Hypercube samples

In order to determine the failure probabilities of the DAP, IAP and system, the Latin Hypercube Sampling (LHS) technique is combined with the established numerical and analytical models.

LHS allows to limit the number of (FEM) calculations necessary for a probabilistic assessment to a reasonable amount [20]. A set of four key variables related to the material properties of the model are selected for the LHS procedure (Table 2). These material properties are considered to have the most influence on the load bearing capacity, which in this case is governed by compressive membrane action (CMA). Other material properties to model the concrete material are based on formulas found in the *fib*-model code [10] and EN 1992-1-1 [8]. The ultimate strain of the reinforcement is considered to be deterministic, i.e. ε_u = 7.5% (ductility class C), as this parameter has little influence on the ultimate capacity in case of CMA. Apart from these four material properties, also the permanent load *G* and the variable load *Q* are included in the LHS procedure. The probabilistic models for the parameters are adopted based on the experimental observations of Gouverneur et al. [11], the suggestions provided in the Probabilistic Model Code from the JCSS [13] and the recommendations provided by Holicky et al. [12]. In total 256 Latin-Hypercube sample sets are used for each building and each damage scenario. A summary of the parameters for the LHS procedure is given in Table 2.

Table 2: Probabilistic models for the most important model parameters

Variable		Distribution	Mean	COV
Concrete compressive strength	f_c	Lognormal	38.8 MPa	0.10
Reinforcement yield strength	f_y	Lognormal	555 MPa	0.03
Reinforcement tensile strength	f_t	Lognormal	605 MPa	0.03
Reinforcement Young’s Modulus	E_s	Lognormal	200 GPa	0.08
Volumetric weight of concrete	G	Normal	24 kN/m^3	0.04
Service load	Q	Gumbel	0.6 kN/m^2	1.10

2.3 Structural reliability calculations

Combining the LHS samples with the developed models, the failure probability of the DAP, IAP and the probability of some progressive collapse after the notional removal of a load-bearing column can be determined using FORM calculations.

2.3.1 Directly affected part (DAP)

Implementing the set of LHS samples in the established numerical models, a set of load-displacement and membrane force – displacement graphs are calculated. Next from the load load-displacement graphs the beam resistance R_{beam} is determined for each sample and a lognormal distribution is fitted to the obtained set of beam resistances (Figure 3). Note that for most of the samples, the beams reach their maximal resistance in the compressive membrane phase without failure of the IAP, while they fail during the transient phase by rupture of the top reinforcement at the partially clamped support (i.e. $P[F_{DAP}|\overline{F_{IAP}}]$). No catenary phase is developed due to the limited rotation capacity of the beams. Failure of the DAP can also be initiated when the developed membrane forces attain the maximal resistance of the multi-linear translation spring or in other words failure of the IAP occurs $P[F_{DAP}|F_{IAP}]$. For the latter situation the corresponding uniform load is determined as a sample point for the dataset of R_{beam}.
Next applying following limit state equation (2), the failure probability of the DAP $P[F_{DAP}]$ can be determined:

$$g_1 = R_{beam} - G - Q \tag{2}$$

In this equation the terms G and Q are multiplied with the respective deterministic dimensions to obtain the imposed load in kN/m.

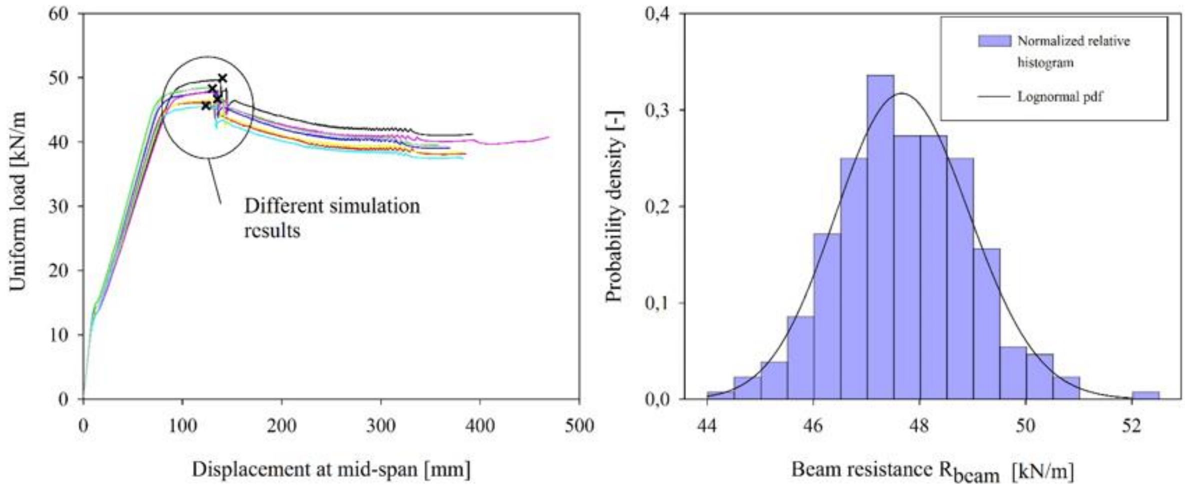

Figure 3: Determination of the beam resistance R_{beam}

2.3.2 Indirectly affected part (IAP)

In a similar way the reliability of the IAP is determined by combining the LHS samples with the implemented plastic calculation model. To determine the membrane force acting on the IAP, first the total load $(G+Q)$ acting on the DAP is calculated for each sample. Next two situations can be distinguished:

1. No failure of the DAP: $G+Q \leq R_{beam}$
   The membrane force corresponding to the total load $(G+Q)$ is derived from the membrane force-displacement graph (Figure 2). This results in samples for the membrane force MF_1 acting on the IAP for which a lognormal distribution is fit. The failure probability of the IAP conditional on the survival of the DAP $P[F_{IAP}|\overline{F_{DAP}}]$ is then calculated by following limit state equation:

$$g_2 = R_{\mathrm{IAP}} - MF_1 \tag{3}$$

2. Failure of the DAP: $G+Q > R_{beam}$
   If the total load $(G+Q)$ is larger than the beam resistance R_{beam}, then the membrane force MF_{maxR} corresponding to the beam resistance R_{beam} of the DAP is considered. This situation corresponds to failure of the DAP and, as a consequence, the membrane force cannot increase further. Again this results in samples for the membrane force MF_{maxR} acting on the IAP for which a lognormal distribution is fit. The failure probability of the IAP conditional on the failure of the DAP $P[F_{IAP}|F_{DAP}]$ is then calculated by following limit state equation:

$$g_3 = R_{\mathrm{IAP}} - MF_{\mathrm{maxR}} \tag{4}$$

Finally the failure probability of the IAP can be calculated as follows:

$$\mathrm{P[F_{IAP}] = P[F_{IAP}|F_{DAP}].P[F_{DAP}] + P[F_{IAP}|\overline{F_{DAP}}].(1 - P[F_{DAP}])} \tag{5}$$

2.3.3 Structural reliability calculations for the system

The probability of some follow-up damage or progressive collapse in the frame or system after some (relatively small) localized damage, i.e. the removal of one column at the ground floor, can be calculated by combining previous obtained results according to equation (6).

$$\mathrm{P[F_{DAP} \cup F_{IAP}] = P[F_{DAP}]+P[F_{IAP}]-P[F_{DAP} \cap F_{IAP}]=P[F_{DAP}]+P[F_{IAP}]-P[F_{IAP}|F_{DAP}].P[F_{DAP}]} \tag{6}$$

In Table 3, the calculated failure probabilities for the DAP, IAP and the probability for progressive collapse are given for each building and for both considered accidental states. Note that these failure probabilities are conditional on the considered damage state 'D' (i.e. removal of a column) and the exposing event 'E' (unknown).

Table 3: Calculated failure probabilities for the DAP, IAP and progressive collapse

Design	Accidental situation	$P[F_{DAP}]$	$P[F_{IAP}]$	$P[F_{DAP} \cup F_{IAP}]$
Building 1	Inner column	0.254	1.12 E-06	0.254
	Edge column	0.106	0.009	0.106
Building 2	Inner column	0.173	1.00 E-08	0.173
	Edge column	0.041	0.001	0.041

Based on these results one can conclude the following:

- In general the probability of progressive collapse is governed by $P[F_{DAP}]$. Still the authors would like to indicate the importance to take into account the developed membrane forces to evaluate the potential of progressive collapse. In case it is possible to develop tensile membrane action (TMA), for instance in steel frames, larger membrane forces will be introduced into the structure. For the considered RC beams TMA cannot develop due to the limited rotation capacity of the beams.
- Building 2 has smaller failure probabilities than Building 1 as Building 2 has more bays in the Y-direction. More horizontal bays results in a larger horizontal stiffness which enhances the compressive membrane action in the DAP and increases the resistance of the IAP.
- Removal of an edge column in Building 2 results in the smallest probability of progressive collapse as the edge frames are loaded by smaller loads than an inner frame.

3 Structural robustness quantification

After obtaining the respective failure probabilities, these failure probabilities can be used to quantify the robustness of the buildings by risk-based robustness indicators. Moreover as single exceptional events are considered, i.e. the notional removal of load bearing columns, conditional risk-based robustness indicators can be used as proposed by Baker et al. [2]. These robustness indicators are conditional on a certain damage state 'D' which is caused by a certain exposure event 'E' and are calculated as:

$$I_{rob}|D,E = \frac{R_{dir}}{R_{dir} + R_{ind}} = \frac{C_{dir}}{C_{dir} + P[F_{DAP} \cap \overline{F_{IAP}}].C_{DAP} + P[F_{IAP}].C_{IAP}} \tag{7}$$

with

- R_{dir} and R_{ind} the direct and indirect risks respectively;
- C_{dir} the direct consequences associated with the examined accidental states;
- C_{DAP} and C_{IAP} the consequences in case of failure of the DAP and IAP respectively;
- $P[F_{DAP} \cap \overline{F_{IAP}}]$ the probability of the event corresponding to failure of the DAP and survival of the IAP;
- $P[F_{IAP}]$ the failure probability of the IAP.

3.1 Assessment of failure costs

For the robustness indicator, the consequences are expressed as costs because a quantitative risk assessment is carried out. Moreover to enable decision making on the basis of generalized data, the associated costs can be expressed relative to the total building costs C_{tot}.

$$I_{rob}|D,E = \frac{C_{dir}/C_{tot}}{C_{dir}/C_{tot} + P[F_{DAP} \cap \overline{F_{IAP}}].C_{DAP}/C_{tot} + P[F_{IAP}].C_{IAP}/C_{tot}} \tag{8}$$

Note that a distinction is made between costs associated to failure of the DAP only and to failure of the IAP only. One should also note that in reality in case the IAP fails the DAP will fail as well due to the removal of its boundary conditions. As a consequence the costs associated to failure of the DAP are included in the failure costs of the IAP.
To assess the ratio of the direct costs and total building cost, three values are considered: 0.001, 0.01 and 0.1 as it is difficult to assess all the costs involved with the notional removal of the column. Values with the same order of magnitude for this ratio were also applied by Narasimhan and Faber [19]. The relative costs associated to the failure of the DAP and IAP on the other hand are estimated based on the work by Faber et al. on the failure of the World Trade Center in 2001 [9] and the research by Kanda on the failure cost evaluation for office buildings [14]. A summary of the values found for the considered relative failure costs in case of total collapse of the building are given in Table 4. Due to several uncertainties a low and high scenario value is given by each author. In this paper only the low scenarios of each author are implemented as a first indication of the robustness of each Building.

Table 4: Assessment of relative failure costs (values relative to total building cost in %)

	M.H. Faber et al. [9]		J. Kanda et al. [14]	
Scenario	Low	High	Low	High
Rescue & Clean-up * $C_{\text{rescue \& clean-up}}/C_{tot}$	36.2	36.2	-	-
Structure $C_{\text{structure}}/C_{tot}$	100.0	100.0	60	130
Inventory of building $C_{\text{inventory}}/C_{tot}$	55.3	55.3	10	200
Fatalities ** $C_{\text{fatalities}}/C_{tot}$	117.0	117.0	0	200
Environment and cultural aspects * $C_{\text{env.}}/C_{tot}$	2.1	2.1	-	-
Impact to economy *** $C_{\text{economy}}/C_{tot}$	193.6	1408.5	0	50
Total failure cost	504.2	1719.1	70	580

* Not considered separately by J. Kanda [14]

** No uncertainties considered on the fatalities as the number of fatalities is known for the WTC towers. Still the authors would like to emphasize that quantifying the cost of a human life is a very difficult and subjective task which is under discussion and is subjected to many assumptions.

*** Including business interruption, loss of infrastructure and loss of rents

Next to calculate the associated failure costs of the IAP and DAP, the relative failure costs of Table 4 are multiplied with the relative building volume of the IAP and DAP respectively. Only the relative failure cost regarding impact to the economy is not adapted as it is assumed that in case of failure of the DAP or IAP, the building has to stop all its business affairs due to the large extent of the damage.

3.2 Results of the risk-assessment in case of the illustrative examples

A summary of the calculated conditional risk-based robustness indicator $I_{rob}|d,e$ is given in Figure 4 for both structures, for all cost assumptions and for both accidental scenarios. Logically the robustness indicator is much smaller when adopting the relative costs proposed by Faber as the total relative failure costs are larger by Faber than by Kanda. In general Building 2 results in a larger robustness index as the failure probability of this building is smaller than for Building 1. Considering the results for the different values of C_{dir}/C_{tot} very large differences can be found in the obtained robustness index. However in most situations the conditional robustness indices are regarded relatively to each other and the exact value of the failure costs are less important as long as the same approach and assumptions for all possible designs are applied when comparing different designs.

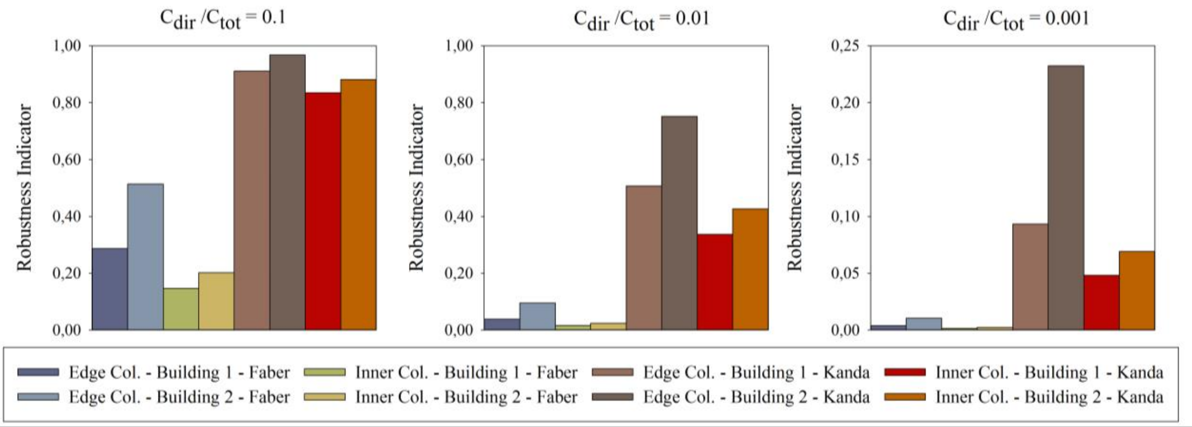

Figure 4: Variation of the index of robustness with the different scenarios, designs and costs

4 Discussion

In this contribution several assumptions and simplifications have been made which are discussed below:

- The previous analyses are related to specific damage situations in a specific structural configuration. Hence, the established results and high failure probabilities should be treated with care and should be considered as indicative only. For the considered cases a clear distinction is made between the DAP and IAP, which in reality is not always possible.
- No three-dimensional effects by the RC slabs to redistribute the loads are considered, i.e. the slabs are considered in the most conservative way as they do not take part in the load redistribution by membrane action and the development of alternate load paths.

5 Conclusions

In this paper a computer efficient calculation scheme is illustrated for two RC frames to calculate the probability of some progressive collapse in a RC frame in case of the notional removal of a load-bearing column. By subdividing the structure in different parts, i.e. the unaffected part, the IAP and DAP, and considering the interaction between the different parts in an elegant way, the reliability of the system is calculated efficiently while combining detailed numerical analyses with plastic calculation methods and the LHS technique. From the results one concludes that the probability on progressive collapse in a RC frame is governed by failure of the DAP in case of a notional column removal due to the limited rotation capacity of RC beams. Next the obtained failure probabilities are combined with a risk-based robustness index to quantify and compare the structural robustness of different designs. Based on the computed robustness indices it is clear that the considered cost scenarios for the direct and indirect costs have a significant influence on the final robustness index. However as long as the respective robustness indices are compared relatively to each other, it is possible to compare different designs in an objective way.

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entitled “Structural reliability and robustness assessment of existing structures considering membrane action effects and Bayesian updating of test information”.

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