



The effect of tree-diffusion in a mathematical model of Easter Island's population

Dedicated to Professor Tibor Krisztin on the occasion of his 60th birthday

Bálint Takács², Róbert Horváth^{1,3} and István Faragó^{✉2,3}

¹Department of Analysis, Budapest University of Technology and Economics
Egry J. u. 1., Budapest, H-1111, Hungary

²Department of Applied Analysis and Computational Mathematics, Eötvös Loránd University
Pázmány Péter sétány 1/C, Budapest, H-1117, Hungary

³MTA-ELTE NumNet Research Group

Received 13 June 2016, appeared 12 September 2016

Communicated by Eduardo Liz

Abstract. A number of theories have been constructed to explain the ecological collapse of the Easter Island. Basener and his co-authors proposed a mathematical model in the form of a system of ordinary differential equations. This system describes the change of the number of people, rats and trees in some subregions of the island. The movement of the human and rat populations was described by some diffusion parameters. They showed that the increase of the diffusion parameters of people and rats makes the system unstable. In the present paper we introduce a diffusion parameter for the tree population and show that this parameter has a stabilizing effect. Thus, it behaves oppositely to the other two diffusion parameters from the stability point of view. The results are demonstrated with some numerical calculations of the stability region.

Keywords: differential equations, stability of equilibria, population dynamics.

2010 Mathematics Subject Classification: 34D20, 92D25.

1 Introduction

Since its first discovery by Europeans in the 18th century, Easter Island (Rapa Nui) has always been the subject of speculations and theories. When in 1786 comte de Lapérouse, the first European, stepped on the island, he found only 2000 people with much less developed civilization than the one which would be required to build big monuments. In the following centuries, several theories tried to describe the events that could lead to the ecological collapse. One of them blames the irresponsible inhabitants and the reckless consumption of goods on the island. Since the increasing popularity of the conception of sustainable development, this theory gains even more recognition.

[✉]Corresponding author. E-mail: faragois@cs.elte.hu.

The last two authors were supported by the Hungarian Research Fund OTKA under grant no. K112157.

In the early 2000s historian Hunt arrived on the island to confirm these theories [3, 4]. However, he found no traces of the long decline of economy proposed in the original theory. The collected data showed a shorter and much drastic collapse that led Hunt to the realisation that some other factors could have had an effect on the events. Because of the numerous rat corpses and chewed seeds, he proposed a new model involving the Polynesian rats. These animals could have been originally brought to the island by the settlers themselves (some theories even suppose that these animals were transported to Easter Island for food – this concept was studied in [7], but we will neglect this effect). However, because the rats ate the seeds of the trees, the reproduction of trees was decreased so dramatically that the population of plants could not cope with the constant harvest done by the settlers.

The following spatial invasive species model was used in [1] by Basener et al. to represent the theories of Hunt:

$$\begin{aligned}\frac{dP}{dt} &= aP \left(1 - \frac{P}{T}\right) \\ \frac{dR}{dt} &= cR \left(1 - \frac{R}{T}\right) \\ \frac{dT}{dt} &= \frac{b}{1 + fR} T \left(1 - \frac{T}{M}\right) - hP\end{aligned}\tag{1.1}$$

where P , R and T denote the number of people, rats and trees, and $a > 0$, $c > 0$ and $b > 0$ are the reproduction rate of these groups, respectively. The parameter M denotes the maximum number of trees that could line on the island, h is the number of trees cut down by one person in a year, and $f > 0$ is the destructive effect of rats on the reproduction of trees.

In [2], Basener et al. modelled Easter Island as an island with an uninhabitable volcano in the middle, so the three groups only live on the coast in N regions. This way equation (1.1) gets the form

$$\begin{aligned}\frac{dP^s}{dt} &= aP^s \left(1 - \frac{P^s}{T^s}\right) + D_P(P^{s-1} - 2P^s + P^{s+1}) \\ \frac{dR^s}{dt} &= cR^s \left(1 - \frac{R^s}{T^s}\right) + D_R(R^{s-1} - 2R^s + R^{s+1}) \\ \frac{dT^s}{dt} &= \frac{b}{1 + fNR^s} T^s \left(1 - \frac{T^s}{M}\right) - hP^s\end{aligned}\tag{1.2}$$

where P^s , R^s and T^s denotes the number of people, rats and trees in region s , respectively ($s \in \{1, \dots, N\}$). The constant values D_P and D_R denote the diffusion coefficients of people and rats, respectively, that describe the movement of these subpopulations between the regions. It was found that the increase of either the D_P or D_R parameters leads to the instability of the system, that is of the nontrivial equilibrium point

$$P^* = R^* = T^* = \frac{1}{N} \frac{M(b-h)}{b + hMf}.\tag{1.3}$$

However, it is easy to notice that only the first two equations involve diffusion in (1.2), while the third does not. This means that our system is not symmetric, which can be the cause of the surprising results of [2]. Because Easter Island is a closed island, we cannot neglect the movement of trees, which is caused by the constant eastern wind on the island, or the movement of animals.

In this paper we extend the original model (1.2) with a non-zero diffusion of the trees. We will see that the equilibrium point (1.3) will be an equilibrium also of the new extended system. We will show that the tree-diffusion is able to stabilize the system.

The structure of the paper is as follows. In Section 2, we extend the system introduced in [2] by a tree-diffusion term and formulate the mathematical model of the population of Easter Island in the form of decoupled systems of ordinary differential equations. In Section 3, the stability of the system is investigated. The results are demonstrated in Section 4.

2 Mathematical model with tree-diffusion

Putting an additional diffusion term into the third equation of (1.2), we get the system

$$\begin{aligned}\frac{dP^s}{dt} &= aP^s \left(1 - \frac{P^s}{T^s}\right) + D_P(P^{s-1} - 2P^s + P^{s+1}) \\ \frac{dR^s}{dt} &= cR^s \left(1 - \frac{R^s}{T^s}\right) + D_R(R^{s-1} - 2R^s + R^{s+1}) \\ \frac{dT^s}{dt} &= \frac{b}{1 + fNR^s} T^s \left(1 - \frac{T^s}{M}\right) - hP^s + D_T(T^{s-1} - 2T^s + T^{s+1})\end{aligned}\tag{2.1}$$

where D_T is the diffusion coefficient of the trees.

Remark 2.1. Let us notice that the above model contains a number of simplifications. The landscape is circle symmetric unlike that of Easter Island. We chose a constant diffusion parameter in order to model the tree-diffusion (we think that, at the present state of the model, this simplification is at least as good as the approximation of the diffusion of people and rats). Moreover, instead of introducing a fourth class for the seeds of the trees, we considered the trees and the seeds together in the third equation of (2.1). The connection between the trees and their seeds is put into the relation between the two parameters b and D_T .

It can be checked easily that the equilibrium point (1.3), which is valid for all subregions $s \in \{1, \dots, N\}$, obtained in [2] will be an equilibrium also of our new extended system. After linearization at this equilibrium, we obtain the system

$$\begin{aligned}\begin{pmatrix} \frac{dP^s}{dt} \\ \frac{dR^s}{dt} \\ \frac{dT^s}{dt} \end{pmatrix} &= \begin{bmatrix} -a & 0 & a \\ 0 & -c & c \\ -h & \frac{-fMh(b-h)}{b(1+fM)} & \frac{fMh-b+2h}{1+fM} \end{bmatrix} \begin{pmatrix} P^s \\ R^s \\ T^s \end{pmatrix} \\ &+ \begin{pmatrix} D_P(P^{s-1} - 2P^s + P^{s+1}) \\ D_R(R^{s-1} - 2R^s + R^{s+1}) \\ D_T(T^{s-1} - 2T^s + T^{s+1}) \end{pmatrix}.\end{aligned}\tag{2.2}$$

We apply the same method as used in [2]. We decouple the equations using the Fourier transform. Let us denote the discrete Fourier transforms in the variable s of the functions P , R and T by x_r , y_r and z_r , respectively, where the parameter r is the variable of the transformed

functions. Since the transform of the first two equations can be found in [2], we have to calculate here only the transform of the third equation, which results in the expression

$$\begin{aligned}
\frac{dz_r}{dt} &= \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} \frac{dT^s}{dt} \\
&= \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} \left[-hP^s - \frac{fMh(b-h)}{b(1+fM)} R^s + \frac{fMh-b+2h}{1+fM} T^s + D_T(T^{s-1} - 2T^s + T^{s+1}) \right] \\
&= \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} \left[-hP^s - AR^s + BT^s + D_T(T^{s-1} - 2T^s + T^{s+1}) \right] \\
&= -h \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} P^s - A \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} R^s + B \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} T^s \\
&\quad + D_T \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} T^{s-1} - D_T \frac{2}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} T^s + D_T \frac{1}{N} \sum_{s=1}^N e^{-\frac{2\pi i r s}{N}} T^{s+1} \\
&= -hx_r - Ay_r + Bz_r + D_T e^{-\frac{2\pi i r}{N}} \frac{1}{N} \sum_{s=1}^N e^{\frac{-2\pi i r(s-i)}{N}} T^{s-1} - 2D_T z_r \\
&\quad + D_T e^{\frac{2\pi i r}{N}} \frac{1}{N} \sum_{s=1}^N e^{\frac{-2\pi i r(s+i)}{N}} T^{s+1} \\
&= -hx_r - Ay_r + Bz_r + D_T e^{-\frac{2\pi i r}{N}} z_r - 2D_T z_r + D_T e^{\frac{2\pi i r}{N}} z_r \\
&= -hx_r - Ay_r + \left[B - 2D_T \left(1 - \cos \frac{2\pi r}{N} \right) \right] z_r \\
&= -hx_r - Ay_r + \left[B - 4D_T \sin^2 \frac{\pi r}{N} \right] z_r
\end{aligned}$$

with the notations

$$\begin{aligned}
A &= \frac{fMh(b-h)}{b(1+fM)}, \\
B &= \frac{fMh-b+2h}{1+fM}.
\end{aligned}$$

Thus, the decoupled system has the form

$$\begin{pmatrix} \frac{dx_r}{dt} \\ \frac{dy_r}{dt} \\ \frac{dz_r}{dt} \end{pmatrix} = \begin{pmatrix} -[a + 4D_P \sin^2 \frac{\pi r}{N}] & 0 & a \\ 0 & -[c + 4D_R \sin^2 \frac{\pi r}{N}] & c \\ -h & -A & B - 4D_T \sin^2 \frac{\pi r}{N} \end{pmatrix} \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} \quad (2.3)$$

with $r = 1, \dots, N$. We will denote the matrix of this system by S . Albeit this is not indicated in the notation, the matrix depends both on the model parameters $a, b, c, f, h, M, D_P, D_R, D_T$ and the variable r of the Fourier transform. This shortening will not make any confusion in the sequel. In [1], the authors suggested the realistic parameter values $a = 0.03, b = 1, c = 10$,

$M = 12000$ and $h = 0.25$. With these values, the matrix in equation (2.3) has the form

$$S = \begin{pmatrix} -[0.03 + 4D_P \sin^2 \frac{\pi r}{N}] & 0 & 0.03 \\ 0 & -[10 + 4D_R \sin^2 \frac{\pi r}{N}] & 10 \\ -0.25 & \frac{-2250f}{1+12000f} & \frac{6000f-1}{2+24000f} - 4D_T \sin^2 \frac{\pi r}{N} \end{pmatrix}. \quad (2.4)$$

For example, if we choose the values $f = 0.001$ and $N = 10$, which were used for calculations in [2], we arrive at the more specific matrix

$$S = \begin{pmatrix} -[0.03 + 4D_P \sin^2 \frac{\pi r}{10}] & 0 & 0.03 \\ 0 & -[10 + 4D_R \sin^2 \frac{\pi r}{10}] & 10 \\ -0.25 & -\frac{9}{52} & \frac{5}{26} - 4D_T \sin^2 \frac{\pi r}{10} \end{pmatrix}. \quad (2.5)$$

Remark 2.2. We remark that the parameters A and B depend only on the product fM and not on the parameters f and M separately. Thus, we get the matrix (2.5) for all parameter choices where $fM = 12$.

The stability of the coexistence equilibrium of the investigated model is equivalent with the condition that the above matrices S are stable for all $r = 1, \dots, N$, that is all of their eigenvalues have negative real parts. In the next section we will investigate the effect of a non-zero tree-diffusion on the stability of the system.

3 The effect of tree-diffusion on the stability of the system

As it was mentioned in the introduction, the increase of the diffusion parameters D_P and D_R leads to the instability of the system [2]. In this section we show that the introduction of the tree-diffusion has a stabilizing effect. Thus, it behaves oppositely to the other two diffusion parameters from the stability point of view.

The stability of square matrices can be guaranteed by the necessary and sufficient Routh–Hurwitz criterion [5,6]. For 3×3 matrices the criterion can be formulated as follows. A matrix $S \in \mathbb{R}^{3 \times 3}$ is stable if and only if the three conditions

1. $\det(S) < 0$ ($\det(S)$ denotes the determinant of S),
2. $\text{tr}(S) < 0$ ($\text{tr}(S)$ denotes the trace of the matrix S),
3. $\text{tr}(S) \cdot \text{pm}_2(S) < \det(S)$ ($\text{pm}_2(S)$ denotes the sum of the three 2×2 principal minors of S)

are fulfilled. For the sake of simplicity, let us introduce the following notations: let S_+ be the matrix S in (2.3) with positive tree-diffusion $D_T > 0$, and let S_0 denote the same matrix as S_+ but here D_T is set to be zero (the other parameters are kept fixed). Moreover, let us set $C_r = 4 \sin^2(\pi r/N)$.

Theorem 3.1. *Let us suppose that the model parameters satisfy the condition*

$$B^2 - Ac - ah < 0. \quad (3.1)$$

Then, if system (2.3) is stable for $D_T = 0$ then it is stable for all positive D_T values.

Proof. We have to show that, under the condition (3.1), to the stability of S_+ it is enough to guarantee the stability of S_0 . A simple calculation shows the identities

$$\begin{aligned}\det(S_+) &= \det(S_0) - D_T C_r (a + D_P C_r) (c + D_R C_r), \\ \operatorname{tr}(S_+) &= \operatorname{tr}(S_0) - D_T C_r, \\ \operatorname{pm}_2(S_+) &= \operatorname{pm}_2(S_0) + (c + D_R C_r) D_T C_r + (a + D_P C_r) D_T C_r \\ &= \operatorname{pm}_2(S_0) + (a + c + (D_R + D_P) C_r) D_T C_r.\end{aligned}\tag{3.2}$$

It can be seen from the nonnegativity of the model parameters that if S_0 satisfies the first two of the Routh–Hurwitz conditions then the matrix S_+ will satisfy these conditions too. Let us check the third condition. We have

$$\begin{aligned}\operatorname{tr}(S_+) \cdot \operatorname{pm}_2(S_+) - \det(S_+) &= (\operatorname{tr}(S_0) - D_T C_r) \cdot (\operatorname{pm}_2(S_0) + (a + c + (D_R + D_P) C_r) D_T C_r) \\ &\quad - (\det(S_0) - D_T C_r (a + D_P C_r) (c + D_R C_r)) \\ &= \operatorname{tr}(S_0) \cdot \operatorname{pm}_2(S_0) - \det(S_0) \\ &\quad - D_T C_r (a + c + (D_R + D_P) C_r) D_T C_r \\ &\quad + \operatorname{tr}(S_0) (a + c + (D_R + D_P) C_r) D_T C_r \\ &\quad - D_T C_r (\operatorname{pm}_2(S_0) - (a + D_P C_r) (c + D_R C_r)).\end{aligned}\tag{3.3}$$

The last factor in the last term can be written in the form

$$\begin{aligned}\operatorname{pm}_2(S_0) - (a + D_P C_r) (c + D_R C_r) &= [(a + D_P C_r) (c + D_R C_r) - B(c + D_R C_r) + Ac - B(a + D_P C_r) + ah] \\ &\quad - (a + D_P C_r) (c + D_R C_r) \\ &= -B(c + D_R C_r) + Ac - B(a + D_P C_r) + ah \\ &= -B(a + c + (D_R + D_P) C_r) + Ac + ah,\end{aligned}$$

moreover $\operatorname{tr}(S_0) = -(a + c + (D_R + D_P) C_r) + B$. For the sake of brevity let us introduce the notation $X = a + c + (D_R + D_P) C_r$. The value of X is always positive. With this notation (3.3) can be rewritten as

$$\begin{aligned}\operatorname{tr}(S_+) \cdot \operatorname{pm}_2(S_+) - \det(S_+) &= \operatorname{tr}(S_0) \cdot \operatorname{pm}_2(S_0) - \det(S_0) \\ &\quad + D_T C_r [-X D_T C_r + X(B - X) - (Ac + ah - BX)] \\ &= \operatorname{tr}(S_0) \cdot \operatorname{pm}_2(S_0) - \det(S_0) \\ &\quad + D_T C_r [(-X^2 + 2BX - (Ac + ah)) - X D_T C_r] \\ &= \underbrace{\operatorname{tr}(S_0) \cdot \operatorname{pm}_2(S_0) - \det(S_0)}_{\text{part I}} \underbrace{- X D_T C_r^2}_{\text{part II}} \\ &\quad + \underbrace{D_T C_r [-X^2 + 2BX - (Ac + ah)]}_{\text{part III}}.\end{aligned}$$

Let us suppose that S_0 is stable. Then the first part of the above expression is negative. The non-positivity of the second part is valid because of the non-negativity of the factors. Up to this point we have not used the condition (3.1) of the theorem. We need the condition to show the non-positivity of the third part. The condition (3.1) implies that the factor $-X^2 + 2BX -$

$(Ac + ah)$ is negative for all real X because the discriminant of the polynomial is negative. Thus $\text{tr}(S_+) \cdot \text{pm}_2(S_+) - \det(S_+) < 0$, that is the third Routh–Hurwitz condition is satisfied provided that S_0 was stable. This completes the proof of the theorem. \square

Remark 3.2. It can be shown that if

$$\max\{B^2, (B - 4D_T^*)^2\} - Ac - ah < 0 \quad (3.4)$$

for some $D_T^* > 0$ then the stability of S in (2.3) for D_T^* implies the stability for all $D_T > D_T^*$. The proof is similar to the proof of the previous theorem. We have to change simply the parameter B in the proof to the new parameter $B - D_T^*C_r$ and use the estimate $(B - D_T^*C_r)^2 \leq \max\{B^2, (B - 4D_T^*)^2\}$. Let us notice that condition (3.4) can be valid only for sufficiently small D_T^* values.

Remark 3.3. Now we check whether the matrix (2.4) fulfils the sufficient condition (3.1). The condition requires the negativity of

$$\begin{aligned} B^2 - Ac - ah &= \left(\frac{6000f - 1}{2 + 24000f} \right)^2 - \frac{22500f}{1 + 12000f} - \frac{1}{4} \cdot \frac{3}{100} \\ &= -\frac{1}{400} \frac{10483200000f^2 + 10272000f - 97}{(1 + 12000f)^2}, \end{aligned}$$

which can be guaranteed by choosing f to be greater than

$$-\frac{107}{2184000} + \frac{1}{291200} \sqrt{282} \approx 8.6751 \times 10^{-6}.$$

In fact, the value fM must be greater than 1.0410×10^{-1} to the negativity (Remark 2.2). This result shows that the matrix (2.5) is stable for $D_T > 0$ provided that it is stable for $D_T = 0$.

The next theorem shows the stabilizing effect of the tree-diffusion parameter from another point of view.

Theorem 3.4. *Let us suppose that the model parameters satisfy the conditions*

$$\begin{aligned} B &< \min\{a + c, A + h\}, \\ B(Ac + ah + (a + c)^2) &< c^2(A + a) + B^2(a + c) + a^2(c + h). \end{aligned} \quad (3.5)$$

Let the diffusion of the people D_P and the diffusion of the rats D_R be two fixed positive numbers. Then there is a positive number \tilde{D}_T such that the system (2.3) is stable for all $D_T > \tilde{D}_T$.

Proof. The conditions in (3.5) assure the stability of the matrix (2.3) for $r = N$. In this case $C_r = 0$ and the matrix is independent of the diffusion parameters.

Let us fix D_P and D_R and assume that $r \in \{1, \dots, N - 1\}$, which implies that $C_r > 0$, and recall the equalities from the proof of the previous theorem

$$\begin{aligned} \det(S_+) &= \det(S_0) - D_T C_r (a + D_P C_r) (c + D_R C_r), \\ \text{tr}(S_+) &= \text{tr}(S_0) - D_T C_r, \\ \text{tr}(S_+) \cdot \text{pm}_2(S_+) - \det(S_+) &= \text{tr}(S_0) \cdot \text{pm}_2(S_0) - \det(S_0) \\ &\quad - X D_T^2 C_r^2 + D_T C_r [-X^2 + 2BX - (Ac + ah)]. \end{aligned} \quad (3.6)$$

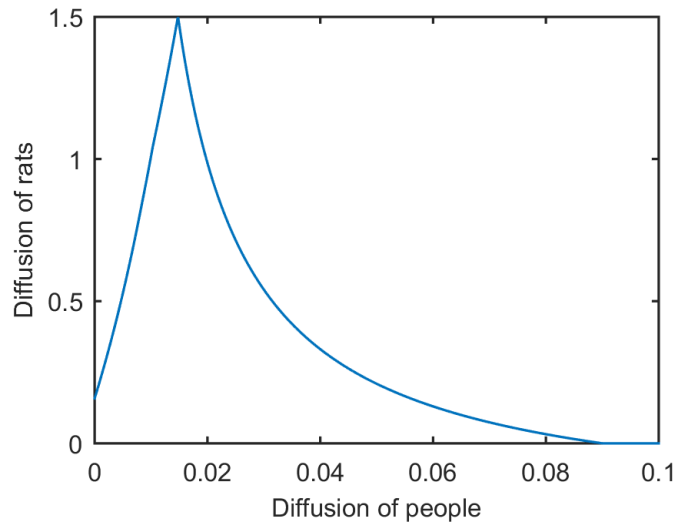


Figure 4.1: Stability bound in the case $D_T = 0$. The stable region is below the graph.

Here the values $\det(S_0)$, $\text{tr}(S_0)$ and $\text{pm}_2(S_0)$ are independent of D_T . In order to prove the statement of the theorem, we have to show that the expressions in (3.6) are negative for sufficiently large D_T values. According to the Routh–Hurwitz criterion this is enough to the stability of the system. In the first two expressions the coefficients of D_T are negative. This gives that $\det(S_+)$ and $\text{tr}(S_+)$ are negative for sufficiently large D_T . In view of $X > 0$ the expression

$$-XD_T^2C_r^2 + D_TC_r[-X^2 + 2BX - (Ac + ah)] = -D_T^2 \left(XC_r^2 - \frac{C_r[-X^2 + 2BX - (Ac + ah)]}{D_T} \right)$$

tends to $-\infty$ if D_T tends to $+\infty$. This shows that $\text{tr}(S_+) \cdot \text{pm}_2(S_+) - \det(S_+)$ is negative for sufficiently large D_T values, that is the third Routh–Hurwitz condition is also satisfied. This completes the proof. \square

Remark 3.5. It can be checked easily that condition (3.5) is satisfied for the matrix (2.5). Thus this system can be stabilized for arbitrary (D_P, D_R) pairs by choosing D_T to be sufficiently large.

4 Numerical demonstration of the results

In this section we demonstrate the results of the previous section by calculating the stability bounds numerically. We carry out the calculation for the matrix (2.5).

For the case without tree-diffusion ($D_T = 0$) we get the same bounds like in [2] (see Figure 4.1). In the figure, the horizontal axis is D_P and the vertical axis is D_R . The stable points are below the graph, and the points on it and above are unstable. Note that if $D_P = 0$ then the system gets unstable if $D_R > 0.15$ and if $D_R = 0$ then the same happens if $D_P > 0.09$.

If we increase the D_T value, we get the bounds in Figure 4.2. The graphs were drawn bottom-up with the values $D_T = 0, 0.015, 0.03, \dots, 0.15$, respectively. As we can see, the area of stability gets larger as the diffusion of the trees increases.

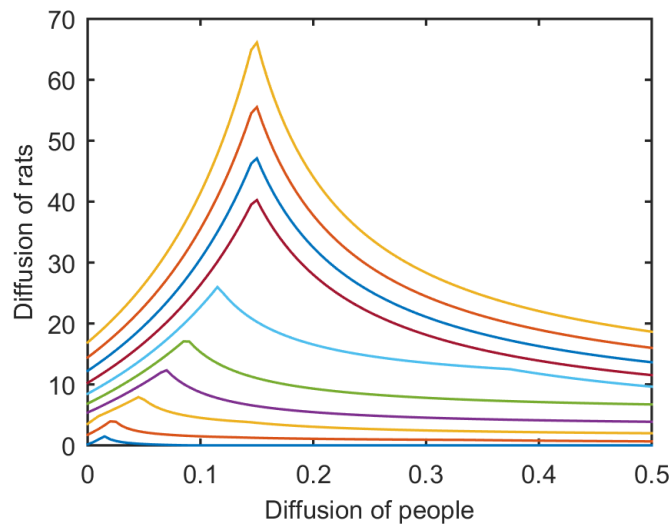


Figure 4.2: The increasing stability bounds for the values $D_T = 0, 0.015, 0.03, \dots, 0.15$.

In order to confirm the statement of Theorem 3.4, we examine those D_T points for which the system changes stability at a certain (D_P, D_R) pairs. We choose the values $D_R = 50k^2$ ($k = 1, \dots, 10$) and calculate the critical D_T values as a function of D_P . If D_T is greater than the critical value then the system is stable, otherwise it is unstable. The result can be seen in Figure 4.3. The critical values seem to converge for all fixed D_P as D_R tends to infinity. Thus, we can suspect from the figure that a certain D_T value (probably about 0.51) can make the system stable for all possible realistic D_R and D_P parameters.

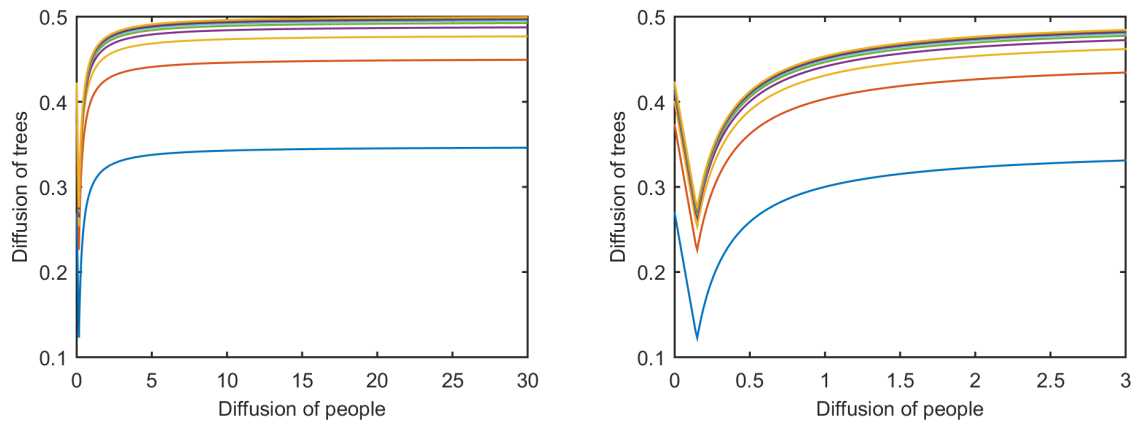


Figure 4.3: Left: the critical D_T values as the function of D_P on the interval $[0, 30]$. The graphs were drawn bottom-up with the values $D_R = 50k^2$ ($k = 1, \dots, 10$). The stability region is located above the graphs. Right: the same but zoomed in on the interval $[0, 3]$.

We can also use a three-dimensional surface to represent the region of stability. Figure 4.4 shows the border of the stability region. The vertical axis is D_R , the one going left is D_P and the right one is D_T . As we can see, as we increase D_T , the stability region gets bigger and bigger. If we increase D_R , the system becomes unstable, or it remains stable for all D_R if D_T

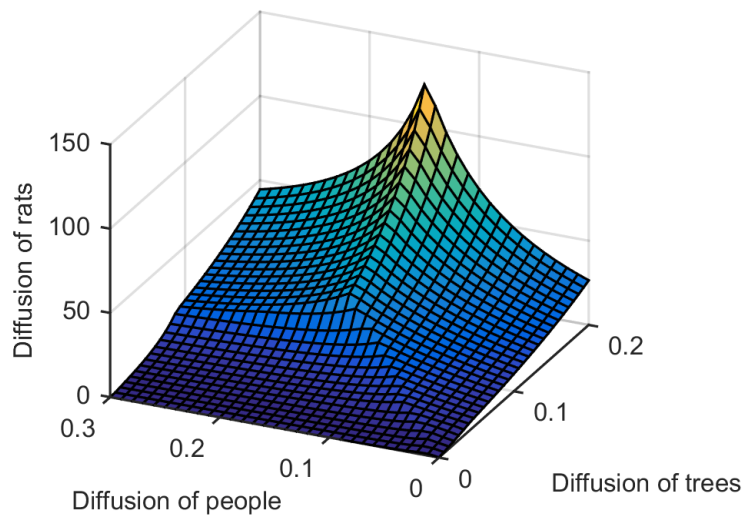


Figure 4.4: 3D graph of the stability region.

is sufficiently large. The case of D_p is more complicated. There are certain (D_R, D_T) pairs for which the change of the diffusion of the people will have no effect on the stability, because it is stable (e.g. the pair $(10, 0.2)$ fulfills this condition) or it is unstable (in the case of the pair $(50, 0.01)$) for every D_p value. At the same time, with other parameter choices the system may become stable but increasing D_p further the stability can be lost (e.g. in the case of the pair $(100, 0.2)$).

The above numerical results support the statements of the theorems of the previous section.

5 Conclusions and future work

We extended a mathematical model constructed by Basener and his co-authors to describe the ecological collapse of Easter Island. The original system describes the change of the number of people, rats and trees in some subregions of the island. We introduced a tree-diffusion parameter into the model and investigated that how the increase of this parameter affects the stability of the system. We have found that the parameter can stabilize the system. The results were confirmed with some numerical calculations of the stability region.

The investigated model is a relatively simple model of the Easter Island. The model does not take into account, for instance, the realistic landscape of the island and the real wind direction. Our future plans are to formulate a more realistic model. This can be done using a system of reaction diffusion equations for the three species and solving the system with some numerical methods. This new model will require new mathematical tools in the investigations.

References

- [1] W. BASENER, B. BROOKS, M. RADIN, T. WIANDT, Rat instigated human population collapse on Easter Island, *Nonlinear Dyn. Psychol. Life Sci.* **12**(2008), No. 3, 227–240.

- [2] W. BASENER, B. BROOKS, M. RADIN, T. WIANDT, Spatial effects and Turing instabilities in the invasive species model, *Nonlinear Dyn. Psychol. Life Sci.* **15**(2011), No. 4, 455–464. [MR2883515](#)
- [3] T. HUNT, Rethinking the fall of Easter Island. New evidence points to an alternative explanation for a civilization's collapse, *Am. Sci.* **94**(2006), 412–419. [url](#)
- [4] T. HUNT, Rethinking Easter Island's ecological catastrophe, *J. Archaeol. Sci.* **34**(2007), 485–502. [url](#)
- [5] A. HURWITZ, Ueber die Bedingungen, unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt (in German), *Math. Ann.* **46**(1895), No. 2, 273–284. [MR1510884](#)
- [6] E. J. ROUTH, *A treatise on the stability of a given state of motion: particularly steady motion*, Macmillan, 1877.
- [7] G. SEBESTYÉN, I. FARAGÓ, Invasive species model with linear rat harvesting on Easter Island, *J. Appl. Computat. Math.* **4**(2015), 1–6. [url](#)