DAMPING EFFECTS IN LETHARGIC GAIN

By

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The coherent amplification of a weak pulse by an inverted two-level medium is considered in the linear approximation. Analytic expressions are presented for the case of homogeneous and inhomogeneous broadening.

Considering the propagation of a coherent pulse, passing through a two level medium, one obtains relatively simple expressions for the transmitted amplitude, if one regards only the initial linear part of the process [1]. This means that one neglects the changes in the inversion — it is taken as a constant — during the interaction. This problem arises for instance at the so called induced superfluorescence [2, 3], where the initial polarization of the system is induced by a weak coherent pulse of small area. If damping processes are small, the signal, having passed a distance x in an inverted medium, will be amplified coherently as $e^{\sqrt{x}}$ instead of the usual e^x law. This effect is termed as lethargic gain [4], and has been observed in recent experiments [5]. In this paper we consider the effects of homogeneous and Lorentzian inhomogeneous broadening on this process.

The Maxwell—Bloch equations describing the interaction of an extended two-level system with a plane monochromatic wave are [6, 7]:

$$\frac{\partial A}{\partial x}+\frac{\partial A}{\partial t}=\int R(x,t,\nu)g(\nu)\,d\nu \tag{1a}$$

$$\frac{\partial R}{\partial t}=\left(-\frac{1}{T_2}+i\nu\right)R+2AZ \tag{1b}$$

$$\frac{\partial Z}{\partial t}=-(AR^*+A^*R) \tag{1c}$$

A system of units has been chosen where time is measured in units of $\Omega^{-1}=\frac{\hbar}{\mu}/\sqrt{2\pi N\hbar\omega_0}$, length in $c\Omega^{-1}$. A is proportional to the slowly varying amplitude of the electric field: $A=-iE/\sqrt{2\pi N\hbar\omega_0}$, R is the polarization, Z is half of the population difference between the upper and lower levels, N is the density of the active atoms, ω_0 is the carrier frequency of the field, μ is the dipole moment of the transition and $g(\nu)$ is the inhomogeneous line shape.

Assuming $Z=1/2$ during the whole process, we have two linear equations:

$$\frac{\partial A}{\partial x}+\frac{\partial A}{\partial t} = \int R(x, t, \nu)g(\nu)d\nu \tag{2a}$$

$$\frac{\partial R}{\partial t} = \left(-\frac{1}{T_2}+i\nu\right) R+A \tag{2b}$$

This approximation is valid for the initial part of the process until the inversion does not change considerably. A similar assumption has been used for investigating the delay time statistics of superfluorescence [8]. We shall see that this approximation remains valid for the whole process if homogeneous and inhomogeneous broadening are large enough to keep the pulse area small.

Eqs. (2) can be solved analytically for the case if g had the following Lorentzian form:

$$g(\nu) = \frac{T^*}{\pi\left(1+(T^*\nu)^2\right)} \tag{3}$$

Let the incoming amplitude at $x=0$ have the time dependence $A_0(t)$ with $A_0(0)=0$. The solution of Eqs. (2) can be obtained by a Laplace transformation technique:

$$A(x, t) = \int_t^x \left(\frac{dA_0(t-\tau)}{dt}+\frac{1}{T}A_0(t-\tau)\right) I_0\left(\sqrt{2x(\tau-x)}\right) e^{-(\tau-x)/T} d\tau \tag{4}$$

Here I_0 is the modified Bessel function [9] and:

$$\frac{1}{T} = \frac{1}{T_2}+\frac{1}{T^*} \tag{5}$$

Thus Lorentzian inhomogeneous broadening has the same effect as T_2, they equally reduce the amplification. The polarization is given by

$$R(x, t, \nu) = \int_0^t A(x, \tau)\, e^{\left(-\frac{1}{T_2}+i\nu\right)(t-\tau)} d\tau \tag{6}$$

The amplification of a Gaussian input pulse of width $\sigma=0.2\ \Omega^{-1}$ is showing in Fig. 1. at the distance $x=1$. We see that for small T the pulse area is not growing unlimitedly, which means that $Z=0.5$ is approximately valid through the whole process if the initial area is small. For larger T-s the rising amplitude at the right end is the radiation of the induced dipole moment. This nonlinear part can be treated only numerically [3].

We shall in some extent investigate the special case, when the incoming pulse is a step function, switched on at $t=0$: $A_0(t)=A_0\Theta(t)$. For $T=\infty$ (unbroadened case)

$$A(x, t) = A_0 I_0\left(2\sqrt{x(t-x)}\right)\Theta(t-x) \tag{7}$$

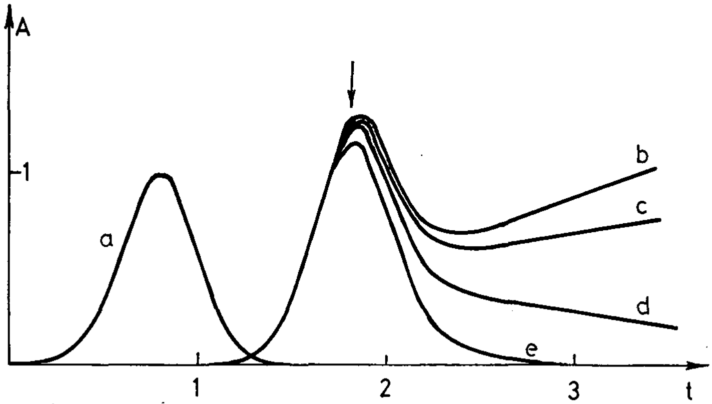

Fig. 1. The amplification of a Gaussian input pulse of width $\sigma=0.2\,\Omega^{-1}$ (amplitude) at a distance $x=1(c\Omega^{-1})$: a. input amplitude; b. amplified amplitude for $T=\infty$; c. $T=5$; d. $T=1$; e. $T=0.238$ The arrow shows the point where the peak would be translated by light speed

For a given $t-x$ and $x\gg 1$, $A(x)=A_0 e^{\sqrt{x}}$ which is the case of lethargic gain [4]. If we have a finite T, the step-like incoming pulse will be transformed as

$$A(x,t) = A_0 I_0(2\sqrt{x(t-x)})e^{-(t-x)/T}\Theta(t-x)+\frac{A_0}{T}\int_x^t I_0(2\sqrt{x(\tau-x)})e^{-(t-x)/T}d\tau \quad (8)$$

For t large enough, the first term goes to zero, the second to a constant. To determine this constant we choose the upper limit of the integral $t=\infty$ and obtain [9]:

$$A(x,t=\infty) = A_0 e^{Tx}$$

This is the usual Beer law for incoherent amp]ification. Eq. (8) shows the time dependence of achieving this limiting stationary case at a given x.

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ЭФФЕКТЫ ЗАТУХАНИЯ ПРИ ЛЕТАРГИЧЕСКОМ УСИЛЕНИИ

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Рассматривается когерентное усиление слабого импульса в двухуровневой системе в линейном приближении. Получены аналитические выражения для случая однородного и неоднородного уширения.