INTERNAL STRUCTURE OF PHYSICAL FIELDS

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The handedness, *i. e.*, the right and left asymmetry of elementary particles is explained in terms of the relativistic phase-space formalism. In this way a new development of our previous theory [2] is suggested. In the following the field equations are derived by variational method, the mass-spectrum of baryons are obtained and NOETHER's theorem is generalized. In fact, the transformations, defined usually in the isobaric spin space, appear as continuous transformations. Finally, the philosophical concept of space-time continuum as well as the interpretation of the method suggested is discussed.

§ 1. *Introduction*

The spin and the linear polarization of elementary particles, as well as their various internal attributes — such as baryon charge, isospin, hypercharge and parity — associated with the abstract concept of isospace (isobaric spin space) and its transformations indicate that there must be some additional intrinsic property of the fields, an additional degree of freedom, which has not been fairly considered in terms of the usual formalism of the theory of elementary particles. *E. g.*, the existence of the spin shows in itself that the point-model of elementary particles associated with the familiar local theory of fields does not provide a complete description of the properties of the particles, since, the rotational axis connected with the spin angular momentum cannot be explained in a natural manner. Furthermore, the linear polarization — *i. e.* the space-independent correlation of the momentum and the spin angular momentum of fermions — proves that this distinguished direction may be in close connection with the anisotropic internal structure of particles not properly considered previously. The intrinsic anisotropy of particles appears first of all in their handedness by which the asymmetry of right and left is expressed. Of course, on the bases of an extended particle-model or based on a rigid-body model this anisotropy could be characterized [1], but the relativistic formulation of such a theory would be rather difficult and the results of such theories would only be cumberously translated into the language of field theories. Moreover, taking the current methods of the theory of elementary particles into account, it can be noticed that these methods have, from certain point of view, two essential different features. Some of them are closely connected with geometry and obtain such physical laws as conservation of energy and momentum, *etc.*; the others, nevertheless, are rather based on the abstract concept of the isospace than current geometrical terms and result in such physical laws as the conservation of charge or that of baryon number, *etc.* In other words, some of the groups of transformations — like translation, rotations and inversions in the four-dimensional spacetime continuum — possess an immediate geometrical meaning, but some of the others — such as *e. g.* gauge transformation of first kind, charge conjugation, charge-symmetry and mesoparity transformation — possess none. However, the reality of these latter attributes of elementary particles indicate that the angles of isorotations as well as the planes of isoreflexions are not located in an abstract space, but within space-time itself. This is the reason that several investigations have been recently published to explain the internal degrees of freedom of physical fields as well as to interpret the isospace and its transformations in geometrical terms. These proposals are, of course, very different.

Our recent investigations [2] in this direction have been based on the supposition that in the anisotropic internal structure of the elementary particles the anisotropy of the space-time continuum would appear. Having FRÖHLICH's theory in mind [3—5], based on terms of the relativistic phase-space formalism the suggested theory can be developped as follows.

§ 2. *The Relativistic Phase-space and the Internal Structure of Physical Fields*

Let us provisionally suppose that the structure of the space-time continuum is pseudo-EUCLIDIAN, *i. e.*, its metrical fundamental tensor has the components

$$\gamma_{00} = 1, \quad \gamma_{0i} = 0, \quad \gamma_{ik} = -\delta_{ik}, \qquad (i, k = 1, 2, 3) \quad (2,1)$$

where δ_{ik} means the KRONECKER's tensor.

First of all, we would explain the handedness, *i. e.*, the right and left asymmetry of the elementary particles.

In the four-dimensional space-time continuum in a certain LORENTZ frame $\mathcal{K}$ the particle considered has the position coordinates $\{x^\mu\}$ and the components of its momentum four-vector are $\{p^\mu\}$ $(\mu = 0, 1, 2, 3)$ which fulfil the well-known relation:

$$p^\mu p_\mu = m_0^2 \geqq 0, \qquad (2,2)$$

together with the postulate that this definition should be invariant under continous LORENTZ transformation, where m_0 denotes the rest mass of the particle ($\hbar = c = 1$). Although, the components of the momentum have to fulfil condition (2,2), they are otherwise in all points of the space-time continuum quite arbitrary. Hence, the state of the particles can be easily characterized by introducing in every point $\{x^\mu\}$ of the *co-ordinate-space* a local *momentum-space* $\{p^\mu\}$ wherein the momentum four-vector p^μ can be regarded as radius vector. The position and momentum co-ordinates of the particle, *i. e.*, the rectangular coordinates $\{x^\mu\}$ and $\{p^\mu\}$ altogether determine the so-called *phase-space*, in which an arbitrary point $\{x^\mu, p^\mu\}$ refers to a special state of the particle considered.

Owing to the relation (2,2), however, the components of the momentum four-vector are not independent, namely, by the four numbers $\{p^\mu\} = \{p^0, p^k\}$ the relation

$$(p^0)^2 - \sum_{k=1}^{3} (p^k)^2 = m_0^2 \qquad (2,3)$$

has to be fulfilled. By the notation

$$\sum_{k=1}^{3}(p^k)^2 \equiv \mathbf{p}^2 \tag{2,4}$$

one can obtain that

$$p = \pm\{m_0^2+\mathbf{p}^2\}^{1/2}. \tag{2,5}$$

In fact, the local momentum-space has only three independent momentum co-ordinates. This means, of course, that in the co-ordinate-space $\{x^\mu\}$ by the radius vector of the momentum-space, *i. e.* by the four momentum components $\{p^\mu\}$, only a direction is determined. As a matter of fact, the phase-space has $4+3$ dimensions.

In the case of a co-ordinate transformation

$$\overline{x^\mu} = \overline{x^\mu}(x^\mu) \qquad \left(\Delta \equiv \det\left|\frac{\partial \overline{x^\mu}}{\partial x^\mu}\right| \neq 0\right) \tag{2,6}$$

the law of transformation of the momentum is given by

$$\overline{p^\mu} = \frac{\partial \overline{x^\mu}}{\partial x^\varrho} p^\varrho. \tag{2,7}$$

In the following, let us only consider the complete group of LORENTZ transformations which will be denoted by $\mathcal{G}_x$.

A LORENTZ transformation in momentum-space may be interpreted in two ways: either the co-ordinate axes are considered fixed and the radius vector has been changed according to (2,7) [the active interpretation of the group], or *vice versa* [the passive interpretation of the group]. The latter interpretation determines the possible frames of reference in the local momentum-space in terms of the frame $\mathcal{K}$ originally introduced in the co-ordinate-space $\{x^\mu\}$.

Due to the double sign of p^0 both of the radius vectors $\{p^0, p^k\}$ and $\{-p^0, p^k\}$ fulfil the relations (2,2) and (2,3), respectively. Of course, these radius vectors can be transformed in a continuous way into one an other, or into any radius vector of the momentum-space. Taking the complete group of LORENTZ transformation into account — as FRÖHLICH pointed out in [5] — these transformations may thus be considered as generating two „three-dimensional" momentum-spaces: one with $p^0>0$ and the other with $p^0<0$.

The frames of reference have certain features that can be chosen arbitrarily; *e. g.*, the frame $\mathcal{K}$ of the co-ordinate-space may be a right-handed system. In this case — having the sign of p^0 — in the momentum-space only the directions of space axes of the local frame of reference can be chosen arbitrarily. Let us suppose, for instance that its spatial axes form a right-handed system, then the local frame of reference of the momentum-space is right-handed if $p^0>0$ and left-handed if $p^0<0$.

Considering in the subspace $\{x^k\}$ the inversion in respect to the origin:

$$\overline{x^0} = x^0,\ \overline{x^1} = -x^1,\ \overline{x^2} = -x^2,\ \overline{x^3} = -x^3 \tag{2,8}$$

the momentum components p^k transform into $-p^k$. This means in terms of the passive interpretation of co-ordinate transformations that we have changed the handedness of the local frame of reference in the momentum-space, *i. e.* a right-handed frame in $\{p^k\}$ subspace (assuming that the original $\{x^k\}$ frame was right-handed) transforms into a left-handed one. It is obvious that the definitions (2,2) and (2,3) from which the properties of the momentum-space were originally defined do not permit distinction between right-handed and left-handed systems because (2,3) does not contain p^k but only $(p^k)^2$.

Owing to the above considerations to be explicitly expressed on the one hand that the momentum-space is three dimensional and on the other hand that the handedness of the local frame of reference in the momentum-space is arbitrary, let us introduce in the $\{x^\mu\}$ point of the co-ordinate-space three unit vectors λ_i^μ ($i=1, 2, 3$) which may be, without restriction of generality, by pairs orthogonal, *i. e.*

$$\gamma_{\mu\nu}\lambda_i^\mu\lambda_j^\nu = \delta_{ij}. \tag{2,9}$$

The three degrees of freedom of the momentum-space may be characterized by the angles

$$\vartheta_i = \arccos\{\gamma_{\mu\nu}\lambda_i^\mu p^\nu/m_0\}, \tag{2,10}$$

between the direction corresponding to the radius vector p^μ and the unit vectors λ_i^μ. The angles ϑ_i and the scalar quantities

$$\zeta_i \equiv \cos\vartheta_i = \gamma_{\mu\nu}\lambda_i^\mu p^\nu/m_0 = \lambda_i^\mu p_\mu/m_0, \tag{2,11}$$

respectively, in the case of any fixed direction of the unit vectors λ_i^μ determine unambigously the direction in the co-ordinate-space $\{x^\mu\}$ characterized by the radius vector p^μ. Call the three by pairs orthogonal unit vectors λ_i^μ in the following as λ-trieder. Of course, the λ-trieder as new local frame of reference and the ensemble of the quantities $\{\zeta_i\}$ as independent co-ordinates, or as the independent components of the radius vectors in the momentum-space, can be regarded. Indeed, in this manner any direction in the co-ordinate-space corresponding to the direction determined by the momentum vectors p^μ can unambigously be characterized.

Due to the definition (2,11) of the quantities $\{\zeta_i\}$ one sees immediately that they are invariant under any LORENTZ transformation; *e. g.*, in the case of the inversion (2,8) the quantities ζ_i do not change, since the spatial components of the λ_i^μ and p^μ vectors change their sign simultaneously. Considering this property of the quantities $\{\zeta_i\}$ the doubling of the momentum-space appears again. The inversion (2,8) transforms, namely, the λ-trieder $\{\lambda_i^0, \lambda_i^k\}$ into $\{\lambda_i^0, -\lambda_i^k\}$ and this transformation indicates simultaneously the change of $\{p^0, p^k\}$ into $\{p^0, -p^k\}$. In fact, the inversion (2,8) changes the handedness of the λ-trieder in the spatial subspace of the momentum-space. However, the quantities depend linearly on the base vectors λ_i^μ of the new local frame of reference of the momentum-space, therefore, the sign of the quantities $\{\zeta_i\}$ depends on the handedness of the λ-trieder. For the sake of simplicity denote the λ-trieder with base vectors $\{\lambda_i^0, \lambda_i^k\}$ as λ^+-trieder, and the other with base vectors $\{\lambda_i^0, -\lambda_i^k\}$ as λ^--trieder.

For the sake of appropriateness let the directions of the λ-trieder axes be chosen in a special way. Considering the fact that by the introduction of the local momentum-space those properties of the particles have to be characterized which are independent of their translatory motions, it seems to be suitable for the direction of the trieder axes to take into account the rest frame of reference $\mathcal{K}^0$ of the particle in which its momentum components are $\{p_{(0)}^\mu\} = \{p_{(0)}^0, 0, 0, 0\}$. As matters stand, $\mathcal{K}^0$ is distinguished among the other frame of references $\mathcal{K}$ that the direction of its time axis is directed into the direction of the momentum of the particle. Let the directions of the new local frame of reference be fixed in the system $\mathcal{K}^0$ in any way, then the transformations of the LORENTZ group transform the momentum four-vector $p^{\mu}_{(0)}$ into the different radius vectors of the momentum-space, and, simultaneously, the quantities $\{\zeta_i\}$ run over the values $-1 \leqq \zeta_i \leqq +1$ $(i=1, 2, 3)$.

To express the doubling of the momentum-space more simply, let us suppose that the by pairs orthogonal λ^{μ}_i unit vectors are orthogonal to the momentum vector of the particles. This means, however, that in the rest frame of reference $\mathcal{K}^0$ the relations

$$\zeta_i^{(0)} = \underset{(0)}{\lambda^{\mu}_i}\, p_{(0)\mu}/m_0 = 0 \qquad (i=1, 2, 3) \tag{2,12}$$

have to be fulfilled. In fact, the components of the λ^{μ}_i-trieder axes in $\mathcal{K}^0$ are $\{\underset{(0)}{\lambda^{\mu}_i}\} = \{0, \vec{\lambda}_i\}$, where $\vec{\lambda}_i = \{\underset{(0)}{\lambda^{k}_i}\}$ denote the spatial components of the unit vectors $\underset{(0)}{\lambda^{\mu}_i}$. Due to (2,9), the $\vec{\lambda}_i$ three-vectors are, indeed, by pairs orthogonal, too, otherwise they can be directed in the $\{x^k\}$ subspace arbitrarily. Of course, we will suppose that the direction of the $\vec{\lambda}_i$-vector coincides with that of the i-th axis of $\mathcal{K}^0$. In terms of the original local frame of reference of the momentum-space this means that its time-axis is determined by the four-vector $p^{\mu}_{(0)}$ and its spatial axes coincide with the vectors $\vec{\lambda}_i$. But, in this way also the handedness of the local frame of reference is determined: it is right-handed if $p^0_{(0)} > 0$ and left-handed if $p^0_{(0)} < 0$. In the case of $p^0_{(0)} > 0$ and λ^+-trieder or $p^0_{(0)} < 0$ and λ^--trieder the local frame of reference is right-handed, as well as in the case of $p^0_{(0)} < 0$ and λ^+-trieder or $\lambda^0_{(0)} > 0$ and λ^--trieder it is left-handed. This means, indeed, that the doubling of the momentum-space be characterized by the handedness of the λ-trieder itself.

It is obvious that by fixing the trieder axes in $\mathcal{K}^0$ the λ^{μ}_i vectors are be unambiguously determined in all frame of reference. This is the reason that for the sake of simplicity our argumentation will be only developped in the $\mathcal{K}^0$ system — the co-ordinates of which will be denoted instead of $\{x^{\mu}_{(0)}\}$ by $\{x^{\mu}\}$ — namely, our results can be transformed into all frames of reference without any difficulty. Let us, however, emphasize again that the system is distinguished by physical terms: it is the rest system of the particle and it can be, *e. g.*, supposed that the $\vec{\lambda}_3$ axis of the λ-trieder coincides with the direction of rotational axis associated with the spin angular momentum of the particle.

Owing to the definition (2,11) of the quantities $\{\zeta_i\}$, the ζ_i-s are dependent on the momentum conponents $\{p^{\mu}\}$ and on the direction of the λ-trieder. We have emphasized several times that the quantities ζ_i are invariant under any LORENTZ transformation of the group $\mathcal{G}_x$. However, intrinsic transformations of the local momentum-space can be introduced by changing the directions of the trieder axes which do not induce any change of the $\{x^{\mu}\}$ coordinates. These transformations, denoted in the following by $\mathcal{G}_{\zeta}$, may be identified with the intrinsic motions of the particles referring to the internal degrees of freedom of physical fields. Therefore, let the „co-ordinates" $\{\zeta_i\}$ of the local momentum-space be called as *internal co-ordinates* and the „co-ordinates" $\{x^{\mu}\}$ of the co-ordinate-space as *external co-ordinates* of the physical fields $\psi = \psi(x^{\mu}, \zeta_i)$.

It is obvious that the group $\mathcal{G}_{\zeta}$, *i. e.*, the group of internal transformations can be generated by the rotations of the λ-trieder around its origin and by the reflexions in respect of the trieder. Consider, first of all, the rotations of the λ-trieder which can be characterized by the EULERIAN angles $\{\varphi, \psi, \vartheta\}$:

$$\vec{\lambda}_i' = M_i^k \vec{\lambda}_k, \tag{2,13}$$

where the well-known matrixelements $M_i^k = M_i^k(\varphi, \psi, \vartheta)$ fulfil the orthogonality relations

$$M_i^k M_j^k = \delta_{ij} \quad \text{and} \quad M_i^r M_i^s = \delta^{rs}. \tag{2,14}$$

Due to the definition (2, 11) of the internal co-ordinates their transformation law can be obtained as follows:

$$\zeta_i' = \lambda_i'^{\mu} p_{\mu}/m_0 = M_i^k \lambda_k^{\mu} p_{\mu}/m_0 = M_i^k \zeta_k. \tag{2,15}$$

It is obvious that under the rotation (2, 13) the trieder axes remain orthogonal to the momentum four-vector $p_{(0)}^{\mu}$, therefore the rotations of the $\mathcal{G}_{\zeta}$ group are identical with the EUCLIDIAN rotations of the three-dimensional space, *i. e.*, they are isomorphic to the rotations of the spatial subspace of the four-dimensional co-ordinate-space which is orthogonal to $p_{(0)}^{\mu}$. This can be proved as follows: Let us introduce the notation

$$\delta^{ij}\zeta_i\zeta_j = \zeta_1^2 + \zeta_2^2 + \zeta_3^2 \equiv \zeta^2, \tag{2,16}$$

then, due to (2, 15),

$$\zeta'^2 = \delta^{ij}\zeta_i'\zeta_j' = \delta^{ij} M_i^r M_j^s \zeta_r \zeta_s = M_i^r M_i^s \zeta_r \zeta_s = \delta^{rs}\zeta_r\zeta_s = \zeta^2 \tag{2,17}$$

can be obtained, *i. e.*, ζ^2 remains invariant under the rotations (2, 13), which is just the definition of the EUCLIDIAN rotations. In fact, one can establish a mapping of the rotations of $\mathcal{G}_{\zeta}$ into the motions of the unit sphere in the local momentum-space which has the equation in its parametric form:

$$\begin{aligned} \zeta_1/\zeta &= \sin\vartheta \cos\varphi \\ \zeta_2/\zeta &= \sin\vartheta \sin\varphi \\ \zeta_3/\zeta &= \cos\vartheta \end{aligned} \tag{2, 18}$$

where (ϑ, φ) mean the polar angles in the momentum-space. Furthermore, one can immediately see that the reflexions of the group $\mathcal{G}_{\zeta}$ are isomorphic to those of the three-dimensional reflexions, therefore, the group $\mathcal{G}_{\zeta}$ is isomorphic to the three-dimensional rotary-reflexion group.

The intrinsic anisotropy of elementary particles with non-vanishing spin momentum, characterized by the longitudinal polarization of the particle, means in terms of the theory suggested that only those elements of the local momentum-space have to be considered which form a constant angle with the rotational axis associated with the spin of the particles. If we suppose that this rotational axis coincides in the rest system $\mathcal{K}^{\circ}$ of the particle with the $\vec{\lambda}_3$ axis of the λ^+-trieder, then the intrinsic anisotropy may be characterized by the relation:

$$\zeta_3 \equiv \cos\vartheta_3 = \lambda_3^{\mu} p_{\mu}/m_0 = \text{const.} \tag{2,19}$$

So far the momentum-space was three-dimensional corresponding to the three internal degrees of freedom, but the relation (2,19) reduces the internal degrees of freedom by one and the internal space becomes only two-dimensional. This means that the adequate directions can be characterized by the internal co-ordinates $\{\zeta_1, \zeta_2\}$.

The characterization of the intrinsic anisotropy of the particles is not yet explicitly covariant. However, it can be easily reformulated in such a way that the direction of the distinguished rotational axis mentioned above is rather characterized by the anisotropy of the co-ordinate-space than explicitly by the $\vec{\lambda}_3$ axis of the λ-trieder.

Let the longitudinal polarization of the particle be denoted by $\mathscr{P}$ and consider the surface in the momentum-space in its parametric form

$$\begin{aligned}\zeta_1/\zeta &= (1+\mathscr{P}\cos\vartheta)\sin\vartheta\cos\varphi \\ \zeta_2/\zeta &= (1+\mathscr{P}\cos\vartheta)\sin\vartheta\sin\varphi \\ \zeta_3/\zeta &= (1+\mathscr{P}\cos\vartheta)\cos\vartheta\end{aligned} \tag{2,20}$$

instead of the unit sphere (2,18), the points of which determine the different directions corresponding to the radius vectors of the local momentum-space. It is obvious that (2,20) is an equation of a rotational surface which distinguishes the direction of the λ_3^μ axis of the λ-trieder.

Let the polar angles (ϑ, φ) be eliminated, then with the abbreviation $y^k = \zeta_k/\zeta$ we have instead of (2,20):

$$\{1+\mathscr{P}y^3[(y^1)^2+(y^2)^2+(y^3)^2]^{-1/2}\}^{-2}[(y^1)^2+(y^2)^2+(y^3)^2] = 1. \tag{2,21}$$

Introducing the metrical fundamental tensor

$$g_{ik} = -\delta_{ik}\{1+\mathscr{P}y^3[(y^1)^2+(y^2)^2+(y^3)^2]^{-1/2}\}^{-2} \qquad (i, k = 1, 2, 3) \tag{2,22}$$

in the $\{y^k\}$ space, (2,21) may be written in the form

$$-g_{ik}y^i y^k = 1. \tag{2,23}$$

The surface (2,20) or (2,22) is defined in the momentum-space, *i. e.*, in all points of the co-ordinate-space $\{x^\mu\}$. Therefore, it can also be defined in the following way:

Consider the „unit vectors" l^μ by re-definition of the metrical structure of the co-ordinate-space. Let the components of the new metrical fundamental tensor be given by

$$g_{00} = 1,\ g_{0i} = 0,\ g_{ik} = -\delta_{ik}\{1+\mathscr{P}y^3[(y^1)^2+(y^2)^2+(y^3)^2]^{-1/2}\}^{-2} \tag{2,24}$$

which depend on the directions determined by the radius vectors in the momentum-space. Then, the unit vectors l^μ directed in the direction of p^μ may be defined as follows:

$$l^\mu = p^\mu/F, \tag{2,25}$$

where

$$F \equiv \{g_{\mu\nu}p^\mu p^\nu\}^{1/2} \tag{2,26}$$

means the new metrical fundamental function of the co-ordinate-space. One can immediately observe that the components $g_{\mu\nu}$, of the new metrical fundamental tensor as well as the new unit vectors l^μ are homogeneous functions of the $\{p^\mu\}$ „direction co-ordinates" of zero order.

The new metrical structure of the co-ordinate-space may be covariantly characterized by the surface:

$$F(x^\mu, l^\mu) = 1 \tag{2,27}$$

which, as the indicatrix of the space, can be regarded. Its explicit form is

$$(l^0)^2 + g_{ik} l^i l^k = 1. \tag{2,28}$$

Denote the co-ordinates of the end-points of l^μ by y^μ, then we have

$$(y^0)^2 + g_{ik} y^i y^k = 1. \tag{2,29}$$

Comparing (2,29) to (2,23) one observes that the surface (2,23) can be deduced from the indicatrix (2,29) by cutting it by the hyper plane

$$y^0 = \sqrt{2}. \tag{2,30}$$

In fact, the intrinsic anisotropy of the co-ordinate-space due to the internal structure of the particles had been experimentally expressed by the longitudinal polarization of the particles, which can be covariantly characterized by the indicatrix (2,29) of the space. This indicatrix distinguishes a direction in the spatial subspace, namely, its rotational axis, and the geometrical and physical quantities defined in the phase-space are dependent on the directions in respect to this distinguished axis. If the longitudinal polarization of the particle vanishes ($\mathcal{P} = 0$), the metrical fundamental tensor (2,24) is reduced to the metrical fundamental tensor of the pseudo-EUCLIDIAN space (2,1), the adequate section of the indicatrix is instead of (2,23) or (2,20) the sphere (2,18) and the phase space is isotropic.

The metrical fundamental tensor plays an important role in the definition of the scalar product of vectors, therefore, in anisotropic spaces not only the length, but also the angle of inclination of vectors depends on the direction. At the definition of the local frame of reference of the momentum-space as well as that of the EUCLIDIAN rotation of the λ-trieder were apparently an important supposition that the trieder axes were orthogonal in EUCLIDIAN sense. Therefore, the problem occurs whether in the case of the new metric (2,24) our previous results concerning the definition of the group $\mathcal{G}_{\mathfrak{f}\zeta}$ remained unchanged or not? It will, however, be proved that in the case of the metrical fundamental tensor (2,24) the condition of orthogonality (2, 9) remained valid, *i. e.*, if the trieder axes $\vec{\lambda}_i$ and $\vec{\lambda}_j$ were orthogonal in EUCLIDIAN sense, then they remain orthogonal also in the sense of the new metric. But, we have to mention that the length of the vectors will generally be changed.

Owing to the introduction of the new metric, one has to write the orthogonality relations in the form:

$$g_{\mu\nu} \lambda_i^\mu \lambda_j^\nu = 0, \quad \text{if} \quad i \neq j. \tag{2,31}$$

This means in the rest system of the particle ($\mathcal{K}^\circ$) that

$$g_{rs} \lambda_i^r \lambda_j^s = 0, \quad \text{if} \quad i \neq j, \tag{2,32}$$

where due to (2,24)

$$g_{rs} = -\delta_{rs} m(p). \; [m(p) \equiv \{1 + \mathscr{P} p^3 [(p^1)^2 + (p^2)^2 + (p^3)^2]^{-1/2}\}^{-2} \neq 0] \quad (2,33)$$

Hence, (2,32) may be written as

$$m(p) \delta_{rs} \lambda_i^r \lambda_j^s = 0, \quad \text{if} \quad i \neq j, \quad (2,34)$$

and, indeed, one immediately observes that the EUCLIDIAN definition of orthogonality remains valid. Nevertheless, the length of the vectors $\vec{\lambda}_i = \{\lambda_k^i\}$ depends on the direction determined by $\{p^\mu\}$ and the vectors with the components

$$\tilde{\lambda}_i^r = \lambda_i^r / \sqrt{m(p)} \qquad (i = 1, 2, 3) \quad (2,35)$$

are the new unit vectors.

Of course, the definition (2,10) or the angles ϑ_i will be changed, too, and their new definition is

$$\theta_i \equiv \arccos \{g_{\mu\nu}(p) \tilde{\lambda}_i^\mu l^\nu\} \quad (2,36)$$

as well as instead of ζ_i we introduce

$$\xi_i \equiv \cos \theta_i = g_{\mu\nu} \lambda_i^\mu l^\nu \quad (2,37)$$

as new internal co-ordinate. The further results discussed above do not formally change, but in reality in the case of non-vanishing longitudinal polarization all geometrical and physical quantities depend on the direction in respect to the distinguished rotational axis corresponding to the spin angular momentum.

The space of co-ordinates $\{x^\mu, p^\mu\}$, *i. e.*, in our previous terms: the phase-space with anisotropic metric, are usually called in geometry as line-element space which is an ensemble of line-elements, or in other words: an ensemble of all directions $\{p^\mu\}$ in the different points of the space $\{x^\mu\}$. The $\{x^\mu\}$ are the position co-ordinates and the $\{p^\mu\}$ the homogeneous direction coordinates of the line-elements. The line-element geometry with the metrical fundamental tensor (2,24) is a special case of the general line-element geometry previously suggested [6—8]. In this case the angles θ_i and the quantities ξ_i as inhomogeneous direction co-ordinates can be regarded.

§ 3. *The Field Equations*

In current field theories the physical fields are characterized by one or several space-time functions: $\psi(x^\mu)$, $\psi_\alpha(x^\mu)$ *etc.* — fulfilling certain partial differential equations, the so called field equations — having definite laws of transformations under the co-ordinate transformations (2,6). Physical field defined in anisotropic spaces are analogously characterized by such quantities fulfilling the field equations, nevertheless, these functions depend on the line-elements $\{x^\mu, p^\mu\}$, *i. e.*, the field components are: $\psi(x^\mu, p^\mu)$, $\psi_\alpha(x^\mu, p^\mu)$ *etc.* being homogeneous functions of zero degree of the direction co-ordinates $\{p^\mu\}$, of course. Instead of the homogeneous direction co-ordinates let us introduce the inhomogeneous direction co-ordinates θ_i or rather the internal co-ordinates ξ_i.

The field components have to satisfy definite laws of transformations again, however, in this case their characters of transformation are doubled. They have definite laws of transformation under the transformations of the group $\mathcal{G}_x$ as well as those under the transformations of the group $\mathcal{G}_\xi$. These two kinds of transformation laws are independent. This point will be discussed in the following in details (§ 7.).

Suppose that the LAGRANGIAN of the field depends on the metrical fundamental tensor, on the field components and on their derivatives. For the sake of appropriateness, the symbols:

$$\psi_{\alpha,\mu} \equiv \partial_\mu \psi_\alpha \equiv \frac{\partial \psi_\alpha}{\partial x^\mu} \quad \text{and} \quad \psi_{\alpha|i} \equiv \partial_i^* \psi_\alpha \equiv \frac{\partial \psi_\alpha}{\partial \xi_i} \tag{3,1}$$

will be introduced. Then, the LAGRANGIAN can be implicitly written as follows:

$$\mathfrak{L} = |g|^{-1/2} |\gamma|^{-1/2} L(\psi_\alpha, \psi_{\alpha,\mu}, \ldots, \psi_{\alpha|i}, \ldots), \tag{3,2}$$

where g denotes the determinant of the metrical fundamental tensor of the external space:

$$g \equiv \det |g_{\mu\nu}| \tag{3,3}$$

and γ denotes that of the internal space:

$$\gamma \equiv \det |\gamma_{ik}|, \tag{3,4}$$

the latter with the law of transformation

$$\tilde{\gamma}_{ik} = \frac{\partial \tilde{\xi}_i}{\partial \xi_r} \frac{\partial \tilde{\xi}_k}{\partial \xi_s} \gamma_{rs}, \tag{3,5}$$

where

$$\tilde{\xi}_i = \tilde{\xi}_i(\xi_r) \qquad \left(\Delta^* \equiv \det \left| \frac{\partial \tilde{\xi}_i}{\partial \xi_r} \right| \neq 0 \right) \tag{3,6}$$

means the transformations of the internal space being elements of the group $\mathcal{G}_\xi$. From a geometrical point of view we have, of course, no *a priori* restrictions for the structure of the internal space and its structure may be determined by physical factors. It seems, however, that it can provisionally be assumed that the metrical structure of the internal space is EUCLIDIAN, *i. e.*,

$$\gamma_{ik} = \delta_{ik}. \tag{3,7}$$

Owing to these considerations the integral of action can be written in the form:

$$\mathfrak{I} = \int_\Omega \int_{\Omega^*} \mathfrak{L} \, d^4x \, d^i\xi, \tag{3,8}$$

where $d^i\xi$ means the two-, or three-dimensional volume element of the internal space according to its dimension. The domain of integration for the external coordinates is a four-dimensional domain Ω, and for the internal co-ordinates Ω^* with the restriction $-1 \leqq \xi_i \leqq +1$ $(i = 1, 2, 3)$.

The integral of action (3,7) has to be invariant under any transformation of the external and internal space, respectively; *i. e.*, it is an invariant of the general group of transformations

$$\mathcal{G} = \mathcal{G}_x \times \mathcal{G}_\xi. \tag{3,9}$$

In respect to the derivation of the field equations be only mentioned that the variation of $\mathfrak{L}$ has to vanish:

$$\delta\mathfrak{J} = \int\limits_{\Omega}\int\limits_{\Omega^*} \sum_{\alpha} \left\{ \frac{\partial\mathfrak{L}}{\partial\psi_\alpha}\delta\psi_\alpha + \frac{\partial\mathfrak{L}}{\partial\psi_{\alpha,\mu}}\delta\psi_{\alpha,\mu} + \ldots + \frac{\partial\mathfrak{L}}{\partial\psi_{\alpha|i}}\delta\psi_{\alpha|i}\ldots \right\} d^4x\, d^i\xi = 0. \quad (3,10)$$

In fact, as usual $\delta\psi_\alpha$ has to vanish at the limit of the integration domains, therefore, by partial integration

$$\int\limits_{\Omega}\int\limits_{\Omega^*} \sum_{\alpha} \left\{ \frac{\partial\mathfrak{L}}{\partial\psi_\alpha} - \partial_\mu \frac{\partial\mathfrak{L}}{\partial\psi_{\alpha,\mu}} - \ldots - \partial_i^* \frac{\partial\mathfrak{L}}{\partial\psi_{\alpha|i}} - \ldots \right\} \delta\psi_\alpha\, d^4x\, d^i\xi = 0 \qquad (3,11)$$

can be obtained for all variation of the field components $\delta\psi_\alpha$, and, finally, we get the field equations:

$$\frac{\partial\mathfrak{L}}{\partial\psi_\alpha} - \partial_\mu \frac{\partial\mathfrak{L}}{\partial\psi_{\alpha,\mu}} - \ldots - \partial_i^* \frac{\partial\mathfrak{L}}{\partial\psi_{\alpha|i}} - \ldots = 0. \qquad (3,12)$$

For the sake of simplicity it has been considered a LAGRANGIAN depending only on the first derivatives of the field components. The general cases were previously discussed in details [6—8].

§ 4. The Mass-spectrum of Fermions

In order to apply the theory in a particular case, consider the two-component fermion fields investigated by FEYNMAN [9], GELLMANN [10] and MARX [11]; *i. e.*, let us suppose that the LAGRANGIAN has the following explicit form:

$$\mathfrak{L} = -|g|^{-1/2}|\gamma|^{-1/2} \sum_A g^{\mu\nu}\partial_\mu\overline{\psi}_A\partial_\nu\psi_A + \gamma^{ik}\partial_i^*\overline{\psi}_A\partial_k^*\psi_A + \varepsilon\overline{\psi}_A\psi_A\}, \qquad (4,1)$$

where the index A distinguishes the two components of the fermion field corresponding to different spin states ($A = \uparrow, \downarrow$). Substituting this LAGRANGIAN into the integral of action by variation in respect to ψ_A the field equations

$$\partial^\mu\partial_\mu\psi_A + \partial^{*i}\partial_i^*\psi_A - \varepsilon\psi_A = 0 \qquad (4,2)$$

can be obtained, where

$$\partial^\mu \equiv g^{\mu\nu}\partial_\nu \quad \text{and} \quad \partial^{*i} \equiv \gamma^{ik}\partial_k^*. \qquad (4,3)$$

As a matter of fact, if we suppose that

$$\psi_A(x^\mu, \xi_i) = X_A(x^\mu)\Xi_A(\xi_i) \qquad (4,4)$$

Eq. (4, 2) can be separated:

$$X_A^{-1}\{\partial^\mu\partial_\mu X_A\} = -\Xi^{-1}\{\partial^{*i}\partial_i^*\Xi_A - \varepsilon\Xi_A\}, \qquad (4,5)$$

and, finally, if the constant of separation is denoted by $\varkappa^2$ we have

$$\begin{aligned} &\{\partial^\mu\partial_\mu - \varkappa^2\}X_A = 0, \\ &\{\partial^{*i}\partial_i^* + (\varkappa^2 - \varepsilon)\}\Xi_A = 0. \end{aligned} \qquad (4,6)$$

One can immediately observe that the result obtained is very analogous to that derived previously in the case of bilocal theory of fields reformulated it in terms of the general line-element geometry [6—8].

The second equation of (4,6) is the well-known differential equation of an eigenvalue problem for the constant of separation $\varkappa^2$, the spectrum of which — taking into account its meaning in the first equation of (4,6) — gets the mass-spectrum of fermions under consideration ($\hbar = c = 1$).

The internal space has in the case of fermions two dimensions, so that the eigenvalue problem corresponding to the second equation of (4,6) can explicitly be written as follows:

$$\{\partial_1^{*2} + \partial_2^{*2} + (\varkappa^2 - \varepsilon)\} \Xi_A = 0. \qquad (4,7)$$

This equation is, indeed, the differential equation of the eigenvalue problem of the two-dimensional rotator.

In terms of polar co-ordinates

$$\xi_1 = r\cos\varphi, \quad \xi_2 = r\sin\varphi \qquad (r = \sqrt{2J} = \text{const}). \qquad (4,8)$$

Eq. (4,7) can be written into the form

$$\left\{\frac{1}{2J}\frac{d^2}{d\varphi^2} + (\varkappa^2 - \varepsilon)\right\} \Xi_A = 0, \qquad (4,9)$$

where J means the moment of inertia. Considering, of course, the usual condition of periodicity $0 \leqq \varphi \leqq 2\pi$, the following eigenvalues and eigenfunctions can be obtained:

$$\varkappa_n^2 = \varepsilon + \frac{1}{2J} n^2 \qquad (n = 0, \pm 1, \pm 2, \ldots) \qquad (4,10)$$

$$\Xi_A^{(n)} = \frac{1}{\sqrt{2\pi}} \exp\{in\varphi\}. \qquad (4,11)$$

The case $n=0$ has to be excluded, namely for $n=0$ the eigenfunction $\Xi_A^{(0)}$ would not depend on the internal co-ordinates which is in contradiction with the general supposition that all physical quantities have to depend on the internal co-ordinates, too. In fact, to express this circumstance explicitly, let the notation

$$n = S + 1 \qquad (S = 0, 1, 2, \ldots) \qquad (4,12)$$

be introduced.

Unfortunately, in the expression (4,10) of $\varkappa_S^2$ the constant ε and J are unknown and we have not yet any possibility to get their *a priori* values which would only be expected in the frame of a non-linear theory where also the interactions of fields are taken into account. Nevertheless, one can immediately observe that for $\varkappa_S^2$ in the case $S \geqq 2$ the relation

$$\varkappa_3^2 = \varkappa_0^2 + \frac{1}{3}\{(S+1)^2 - 1\}[\varkappa_1^2 - \varkappa_0^2] \qquad (4,13)$$

can be obtained. In fact, for a family of particles if the masses of the first two isodoublets are known, the masses of the heavier isodoublets can be calculated by means of (4,13). This is the case for baryons as it will be shown in the next paragraph.

§ 5. The Mass-spectrum of Baryons

Owing to our previous result reviewed in § 4. of this paper, the fermions form isodoublets. In fact, this result does not agree with the usual supposition according to which the Σ hyperons form an isotriplet and Λ° particle is an isosinglet. It is, however, in full agreement with the hypothesis of the global baryon-pion interaction suggested by GELL-MANN [12]. In formulating this principle, one has to consider that the pion interaction of hyperons of first order ($\Lambda^\circ, \Sigma^+, \Sigma^\circ, \Sigma^-$) shows threedimensional isotropic invariance both if the four particles are divided into two doublets as well as if they form a singlet and triplet. This remarkable fact has been emphasized also by SCHWINGER [13] who at the same time proposed a possible explanation in the frame of the four-dimensional isospace. In order to overcome the difficulty of SCHWINGER's scheme that in the case of baryons of even order (p, n, Ξ°, Ξ^-) an other subgroup of the six-parametric symmetry group must be identified with the three-dimensional symmetry group of kaons as in the case of baryons of odd order ($\Lambda^\circ, \Sigma^+, \Sigma^\circ, \Sigma^-$), GELL-MANN's idea has been more recently reinvestigated by KÁROLYHÁZY and MARX [14] whose theory reproduces the important results of GELL-MANN, SCHWINGER and others, but, is free from this difficulty.

The theory of KÁROLYHÁZY and MARX has been built up on a four-dimesional mathematical scheme proposed for particles being in strong interactions with each other. To describe the pions and the nucleons they need three and two independent components, respectively; therefore, the former are represented by the spinor $\pi_\mu{}^\nu = \pi_\nu{}^\mu$ and the latter by a spinor B_α. For the description of the hyperons of first order ($\Lambda^\circ, \Sigma^+, \Sigma^\circ, \Sigma^-$) a spinor $B_{\alpha\dot{\nu}}$ was suggested. In fact, pions and nucleons have nothing to do with dotted indices, hence, it can be supposed that the number of dotted indices is related to the absolute values of the strangeness. This means, however, that the doublet of kaons is represented by the spinor $K_{\dot{\mu}}$, furthermore, the baryons are described by spinors with one undotted and so many dotted indices as is the order (*i. e.*, absolute value of the strangeness) of the baryon which brings KRÓLIKOWSKY's theory in mind:

$$\begin{aligned} |S| &= 0 : B_\alpha \quad : p, n \\ |S| &= 1 : B_{\alpha\dot{\sigma}} \quad : \Lambda^\circ, \Sigma^+, \Sigma^\circ, \Sigma^- \\ |S| &= 2 : B_{\alpha\dot{\sigma}\dot{\tau}} : \Xi^\circ, \Xi^-, \Omega^+, \Omega^\circ, \Omega^-, \Omega^{--} \\ &\dots \qquad \dots \qquad \dots \end{aligned} \tag{5,1}$$

As a matter of fact, the baryons are split into doublets:

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} p \\ n \end{pmatrix}; \begin{pmatrix} B_{1\dot{1}} \\ B_{1\dot{2}} \end{pmatrix} = \begin{pmatrix} \Sigma^+ \\ 1/\sqrt{2}(\Lambda^0 - \Sigma^0) \end{pmatrix}; \begin{pmatrix} B_{2\dot{1}} \\ B_{2\dot{2}} \end{pmatrix} = \begin{pmatrix} 1\sqrt{2}(\Lambda^0 + \Sigma^0) \\ \Sigma^- \end{pmatrix}; \dots \ etc. \tag{5,2}$$

Antibaryons are represented by the complex conjugate of the corresponding spinors, B_α^*, *etc.*

Bearing the classification (5, 1) of baryons in mind, one observes that *the quantum number S of our isorotator introduced in the last paragraph as the absolute value of the strangeness can be interpreted and the exitation of the isorotator $n = S + 1$ agrees with the number of isodoublets of type* (5,2). Furthermore, one can observe that due to the double sign of the quantum number n in Eq. (4,10) and (4,11), respectively, $\pm(|n|-1)$ — with $n\neq 0$ — immediately with the strangeness of the particles can be identified, and the double degeneracy of isorotator states may be associated with the well-known property of the scheme of GELL-MANN—NISHIJIMA that the strangeness of baryons and antibaryons have opposite signs.

So far, as only pion-interactions are considered, the masses of the different isodoublets are the same and the mass-spectrum of baryons can be described by the relation (4,13). As a matter of fact, let us suppose that $\varkappa_0^2$ and $\varkappa_1^2$ equal with the averages of the masses of nucleons and of that of the hyperons of first order [15], then due to Eq. (4,13) the mass-average of the hyperons of second order can be obtained. Our calculations are summarized in Table I. (the unit of mass is the mass of electrons). In fact, the agreement is satisfactory. The calculated average

Table I.

Elementary particle	Observed mass	Mass-average	
		observed	calculated
p n	$1836{,}03\pm 0{,}02$ $1938{,}56\pm 0{,}02$	1837	—
Λ^0 Σ^+ Σ^0 Σ^-	$2182{,}39\pm 0{,}24$ $2327{,}4\ \pm 0{,}69$ $2329\ \{^{+1,8}_{-3,7}$ $2342\ \pm 1$	2295	—
Ξ^0 Ξ^-	$2585\ \pm 1$ $2595\ \pm 39$	2590	2901,92

value of the mass of hyperons of second order differs by 12% from that of xions. As matters stand, this difference may be reasonable, namely, the mass-average of these hyperons must be somewhat larger than the mass-avarege of xions expected. Indeed, in calculating the mass-average of the hyperons of second order — mentioned as the observed mass-average of xions in Table I. — the masses of the hypothetical Ω particles could not be considered.

The splitting of the degenerate baryon states into isomultiplets will be performed first of all by kaon interactions ($\Lambda-\Sigma$ mass-splitting). Electromagnetic interactions will go a step further and distinguish the $\vec{\lambda}_3$ axis of the λ-trieder and it remains only the invariance with respect to the rotations about this axis. Indeed, the electromagnetic interactions cause further mass-splitting: $p-n$, $\Sigma^+-\Sigma$, $\Xi^\circ-\Xi^-$, *etc.*

The Ω particles are hypothetical ones, the mass of which is about the sum of the xion- and pion-mass. If actual, the $\Omega-\Sigma$ mass difference is larger than the Ξ mass, then the Ω-hyperons decay in a very short time (about 10^{-22} sec) into xions, and is practically unobservable. Nevertheless, it cannot be omitted from our scheme, as the Σ° hyperon — having a lifetime longer by only a few orders — plays also decisive role.

Of course, it is an interesting problem whether the other isodoublets suggested for $S\geqq 2$ by the formula (5,13) will be observed in the future or not.

In order to find also the fine structure of the mass-spectrum, one has to consider the interactions of fields, too. However, this will be discussed in the future.

§ 6. *Mass-spectrum in the Case of Three-dimensional Isospace*

In the case of three-dimensional internal space the second equation of (4,6) has the explicit form:

$$\{\partial_1^{*2}+\partial_2^{*2}+\partial_3^{*2}+(\varkappa^2-\varepsilon)\}\Xi = 0. \qquad (6,1)$$

This is the differential equation of a three-dimensional rotator. By introducing the polar co-ordinates

$$\xi_1=\xi\sin\vartheta\cos\varphi,\ \xi_2=\xi\sin\vartheta\sin\varphi,\ \xi_3=\xi\cos\vartheta \quad (\xi=\{\xi_1^2+\xi_2^2+\xi_3^2\}^{1/2}=\sqrt{2M}) \qquad (6,2)$$

we have

$$\left\{\frac{1}{2M}\left[\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right)+\frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right]+(\varkappa^2-\varepsilon)\right\}\Xi(\vartheta,\varphi) = 0. \qquad (6,3)$$

Considering those solutions $\Xi(\vartheta, \varphi)$ which are finite, continuous and monovalent in the domains $0\leqq\vartheta\leqq\pi$ and $0\leqq\varphi\leqq 2\pi$, they are

$$\Xi^{(l,m)}(\vartheta,\varphi)=Y_{lm}(\vartheta,\varphi), \qquad (l=0,1,2,\ldots\ m=0,\pm1,\pm2,\ldots\pm l) \qquad (6,4)$$

where $Y_{lm}(\vartheta, \varphi)$ mean the spherical functions corresponding to the eigenvalues

$$\varkappa_l^2 = \varepsilon+\frac{1}{2M}l(l+1). \qquad (6,5)$$

The case $l=0$ has to be excluded again, namely, $\Xi^{(0,0)}=$const. would be in contradiction to our fundamental supposition that Ξ has to depend on the internal coordinates.

So far, the mass-spectrum (6,5) cannot be discussed as well as the constant ε and M cannot be determined in this case, namely, isotriplets with strangeness $|S|>1$ and with vanishing spin are unknown. However one has to conclude that such particles are possible in this approximation. Considering the interactions of the fields may be motivated while these particles have such a short life time that they are unobservable.

§ 7. *The Generalization of* NOETHER'*s Theorem*

In fact, the field components depend on the external and internal co-ordinates: $\psi_\alpha=\psi_\alpha(x^\mu, \xi_i)$. Investigate how in this case NOETHER's theorem has to be generalized.

Consider any infinitesimal variation of the co-ordinates:

$$\bar{x}^\mu = x^\mu+\delta x^\mu \quad \text{and} \quad \tilde{\xi}_i = \xi_i+\delta^*\xi_i. \qquad (7,1)$$

As matters stand, the internal co-ordinates $\{\xi_i\}$ are invariant under any change of the external co-ordinates $\{x^\mu\}$ so that $\delta^*\xi_i$ is induced by the change of the direction of the λ-trieder which involves no change of the external co-ordinates. This means that δx^{μ} and $\delta^{*}\xi_{i}$ are independent. Denoting the variation of the field component ψ_{α} induced by δx^{μ} by $\overline{\psi}_{\alpha}$, and that induced by $\delta^{*}\xi_{i}$ by $\tilde{\psi}_{\alpha}$, the total variation of ψ_{α} is defined as

$$\delta\psi_{\alpha} \overset{\text{def}}{=} \tilde{\overline{\psi}}_{\alpha}(\overline{x^{\mu}}, \tilde{\xi}_{i}) - \psi_{\alpha}(x^{\mu}, \xi_{i}). \tag{7,2}$$

The transformations (7,1) are elements of the group $\mathcal{G}$ defined by $\mathcal{G} = \mathcal{G}_{x} \cdot \mathcal{G}_{\xi}$.

The local variation of the field components ψ_{α} may be defined as follows:

$$\delta^{0}\psi_{\alpha} \overset{\text{def}}{=} \tilde{\overline{\psi}}_{\alpha}(x^{\mu}, \xi_{i}) - \psi_{\alpha}(x^{\mu}, \xi_{i}). \tag{7,3}$$

This means, however, in first order of δx^{μ} and $\delta^{*}\xi_{i}$ that

$$\delta\psi_{\alpha} = \delta^{0}\psi_{\alpha} + \tilde{\overline{\psi}}_{\alpha}(x^{\varrho} + \delta x^{\varrho}, \xi_{r} + \delta^{*}\xi_{r}) - \tilde{\overline{\psi}}_{\alpha}(x^{\varrho}, \xi_{r}) = \delta^{0}\psi_{\alpha} + \psi_{\alpha,\varrho}\delta x^{\varrho} + \psi_{\alpha|r}\delta^{*}\xi_{r}. \tag{7,4}$$

The total variation of ψ_{α} does not commute with the derivation according to the co-ordinates, nevertheless, the local variation does. Otherwise, we have:

$$\delta\psi_{\alpha,\mu} = \partial_{\mu}\delta^{0}\psi_{\alpha} + \psi_{\alpha,\mu\varrho}\delta x^{\varrho} + \psi_{\alpha,\mu|r}\delta^{*}\xi_{r} = \partial_{\mu}\delta\psi_{\alpha} - \psi_{\alpha,\varrho}\partial_{\mu}\delta x^{\varrho}, \tag{7,5}$$

$$\delta\psi_{\alpha|i} = \partial_{i}^{*}\delta^{0}\psi_{\alpha} + \psi_{\alpha,\varrho|i}\delta x^{\varrho} + \psi_{\alpha|ir}\delta^{*}\xi_{r} = \partial_{i}^{*}\delta\psi_{\alpha} - \psi_{\alpha|r}\partial_{i}^{*}\delta^{*}\xi_{r}. \tag{7,6}$$

The total variation of the integral of action — being an invariant of the group $\mathcal{G}$ — has to vanish under the transformations (7,1)

$$\begin{aligned} \delta\mathfrak{J} = \int\limits_{\overline{\Omega}}\int\limits_{\tilde{\Omega}^{*}} \tilde{\overline{L}}[\tilde{\psi}_{\alpha}(\overline{x^{\mu}}, \tilde{\zeta}_{i}), \tilde{\overline{\psi}}_{\alpha,\mu}(\overline{x^{\mu}}, \tilde{\xi}_{i}), \tilde{\overline{\psi}}_{\alpha|i}(\overline{x^{\mu}}, \tilde{\xi}_{i})]\, d^{4}\overline{x}\, d^{i}\tilde{\xi} - \\ - \int\limits_{\Omega}\int\limits_{\Omega^{*}} L[\psi_{\alpha}(x^{\mu}, \xi_{i}), \psi_{\alpha,\mu}(x^{\mu}, \xi_{i}), \psi_{\alpha|i}(x^{\mu}, \xi_{i})]\, d^{4}x\, d^{i}\xi = 0, \end{aligned} \tag{7,7}$$

where we have implicitly supposed that the metrical fundamental tensors of the external and internal spaces remain unchanged as well as we considered that $|g| = |\gamma| = 1$. The general case was previously investigated in details in [7]. $\delta\mathfrak{J}$ may, however, be written in the form:

$$\begin{aligned} \delta\mathfrak{J} = \int\limits_{\Omega}\int\limits_{\Omega^{*}} \delta^{0}L\, d^{4}x\, d^{i}\xi + \int\limits_{\overline{\Omega}}\int\limits_{\tilde{\Omega}^{*}} L(\tilde{\overline{\psi}}_{\alpha}, \tilde{\overline{\psi}}_{\alpha,\mu}, \tilde{\overline{\psi}}_{\alpha|i})\, d^{4}\overline{x}\, d^{i}\tilde{\xi} - \\ - \int\limits_{\Omega}\int\limits_{\Omega^{*}} L(\psi_{\alpha}, \psi_{\alpha,\mu}, \psi_{\alpha|i})\, d^{4}x\, d^{i}\xi. \end{aligned} \tag{7,8}$$

Supposing that the LAGRANGIAN L is form invariant, the local variation $\delta^{\circ}L$ vanishes and we have

$$\delta\mathfrak{J} = \int\limits_{\overline{\Omega}}\int\limits_{\tilde{\Omega}^{*}} L(\tilde{\overline{\psi}}_{\alpha}, \tilde{\overline{\psi}}_{\alpha,\mu}, \tilde{\overline{\psi}}_{\alpha|i})\, d^{4}\overline{x}\, d^{i}\tilde{\xi} - \int\limits_{\Omega}\int\limits_{\Omega^{*}} L(\psi_{\alpha}, \psi_{\alpha,\mu}, \psi_{\alpha|i})\, d^{4}x\, d^{i}\xi. \tag{7,9}$$

The form invariance of the LAGRANGIAN is defined by several authors directly by this equation [15].

Owing to (7,2) — due to (7,5) and (7,6) — we have

$$L(\tilde{\bar{\psi}}_\alpha, \tilde{\bar{\psi}}_{\alpha,\mu}, \tilde{\bar{\psi}}_{\alpha|i}) = L(\psi_\alpha + \delta\psi_\alpha, \psi_{\alpha,\mu} + \delta\psi_{\alpha,\mu}, \psi_{\alpha|i} + \delta\psi_{\alpha|i}) =$$

$$= L(\psi_\alpha, \psi_{\alpha,\mu}, \psi_{\alpha|i}) + \frac{\partial L}{\partial\psi_\alpha}\delta\psi_\alpha + \frac{\partial L}{\partial\psi_{\alpha,\mu}}\delta\psi_{\alpha,\mu} + \frac{\partial L}{\partial\psi_{\alpha|i}}\delta\psi_{\alpha|i} = \tag{7,10}$$

$$= L + \frac{\partial L}{\partial\psi_\alpha} + \frac{\partial L}{\partial\psi_{\alpha,\mu}}\partial_\mu\delta\psi_\alpha - \frac{\partial L}{\partial\psi_{\alpha,\mu}}\psi_{\alpha,\varrho}\partial_\mu\delta x^\varrho +$$

$$+ \frac{\partial L}{\partial\psi_{\alpha|i}}\partial_i^*\delta\psi_\alpha - \frac{\partial L}{\partial\psi_{\alpha|i}}\psi_{\alpha|r}\partial_i^*\delta^*\xi_r .$$

Considering

$$\partial_\mu\left(\frac{\partial L}{\partial\psi_{\alpha,\mu}}\delta\psi_\alpha\right) = \partial_\mu\left(\frac{\partial L}{\partial\psi_{\alpha,\mu}}\right)\delta\psi_\alpha + \frac{\partial L}{\partial\psi_{\alpha,\mu}}\partial_\mu\psi_\alpha, \; etc., \tag{7,11}$$

one gets

$$L(\tilde{\bar{\psi}}_\alpha, \tilde{\bar{\psi}}_{\alpha,\mu}, \tilde{\bar{\psi}}_{\alpha|i}) = L + \left[\frac{\partial L}{\partial\psi_\alpha} - \partial_\mu\frac{\partial L}{\partial\psi_{\alpha,\mu}} - \partial_i^*\frac{\partial L}{\partial\psi_{\alpha|i}}\right]\delta\psi_\alpha +$$

$$+ \partial_\mu\left(\frac{\partial L}{\partial\psi_\alpha} - \frac{\partial L}{\partial\psi_{\alpha,\mu}}\psi_{\alpha,\varrho}\delta x^\varrho\right) + \partial_\mu\left(\frac{\partial L}{\partial\psi_{\alpha,\mu}}\psi_{\alpha,\varrho}\right)\delta x^\varrho + \tag{7,12}$$

$$+ \partial_i^*\left(\frac{\partial L}{\partial\psi_\alpha} - \frac{\partial L}{\partial\psi_{\alpha|i}}\psi_{\alpha|r}\delta^*\xi_r\right) + \partial_i^*\left(\frac{\partial L}{\partial\psi_{\alpha|i}}\psi_{\alpha|r}\right)\delta^*\xi_r .$$

The JACOBIANS of the transformations (2,6) and (3,6) in the case of the infinitesimal transformations (7,1) in first order of δx^μ and $\delta^*\xi_i$ have the explicit form:

$$\Delta = 1 + \partial_\varrho\delta x^\varrho \quad \text{and} \quad \Delta^* = 1 + \partial_r^*\delta^*\xi_r , \tag{7,13}$$

respectively. This means, however, that

$$\delta\mathfrak{J} = \int_\Omega\int_{\Omega^*}\{L(\psi_\alpha + \delta\psi_\alpha, \psi_{\alpha,\mu} + \delta\psi_{\alpha,\mu}, \psi_{\alpha|i} + \delta\psi_{\alpha|i})(1 + \partial_\varrho\delta x^\varrho)(1 - \partial_r^*\delta^*\xi_r) - \tag{7,14}$$

$$- L(\psi_\alpha, \psi_{\alpha,\mu}, \psi_{\alpha|i})\}\, d^4x\, d^i\xi .$$

Taking into account that

$$\partial_\varrho L = \frac{\partial L}{\partial x^\varrho} + \frac{\partial L}{\partial\psi_\alpha}\psi_{\alpha,\varrho} + \frac{\partial L}{\partial\psi_{\alpha,\mu}}\psi_{\alpha,\mu\varrho} + \frac{\partial L}{\partial\psi_{\alpha|i}}\psi_{\alpha,\varrho|i}, \; etc., \tag{7,15}$$

furthermore, that δx^μ are independent of ξ_i and $\delta^*\xi_r$ are independent of x^μ, respectively, it can, finally, be obtained by simple calculations that

$$\delta\mathfrak{J} = \int_\Omega\int_{\Omega^*}\{\partial_\mu\mathfrak{f}^\mu + \partial_i^*\mathfrak{f}^{*i}\}\, d^4x\, d^i\xi = 0 \tag{7,16}$$

where the abbreviations are introduced:

$$\mathfrak{f}^{\mu} \overset{\text{def}}{=} \left(L\delta^{\mu}_{\varrho} - \frac{\partial L}{\partial \psi_{\alpha,\mu}} \psi_{\alpha,\varrho} \right) \delta x^{\varrho} - \frac{\partial L}{\partial \psi_{\alpha,\mu}} \psi_{\alpha|r} \delta^{*} \xi_{r} + \frac{\partial L}{\partial \psi_{\alpha,\mu}} \delta \psi_{\alpha}; \tag{7,17}$$

$$\mathfrak{f}^{*i} \overset{\text{def}}{=} \left(L\delta^{i}_{r} - \frac{\partial L}{\partial \psi_{\alpha|i}} \psi_{\alpha,r} \right) \delta^{*} \xi_{r} - \frac{\partial L}{\partial \psi_{\alpha|i}} \psi_{\alpha,\varrho} \delta x^{\varrho} + \frac{\partial L}{\partial \psi_{\alpha|i}} \delta \psi_{\alpha}. \tag{7,18}$$

In fact, the integration domains Ω and Ω^*, respectively, — the latter within the unit sphere of the $\{\xi_i\}$ space — are arbitrary, so that due to (7,16) the generalization of the contonuity equation:

$$\partial_{\mu} \mathfrak{f}^{\mu} + \partial^{*}_{i} \mathfrak{f}^{*i} = 0 \tag{7,19}$$

can be obtained. Denoting by $\vec{\mathfrak{f}} = \{\mathfrak{f}^k\}$ and by $\vec{\mathfrak{f}}^* = \{\mathfrak{f}^{*k}\}$, respectively, the spatial components of $\mathfrak{f}^{\mu}$ and $\mathfrak{f}^{*k}$, as well as by $\vec{\nabla}_{(x)} = \{\partial_k\}$ and by $\vec{\nabla}_{(\xi)} = \{\partial^*_k\}$ the corresponding nabla operators, then (7,19) can be written in the form:

$$\partial_0 \mathfrak{f}^0 + \vec{\nabla}_{(x)} \vec{\mathfrak{f}} + \vec{\nabla}_{(\xi)} \vec{\mathfrak{f}}^* = 0. \tag{7,20}$$

Let V be the projection of Ω onto the spatial part $\{x^{\mu}\}$ of the co-ordinate space, surrounded by the closed surface F, then due to GAUSS' theorem

$$\int_V \partial_k \mathfrak{f}^k d^3x = \int_V \vec{\nabla}_{(x)} \vec{\mathfrak{f}} d^3x = \oint_F \mathfrak{f}^k n_k df \tag{7,21}$$

can be obtained where $\{n_k\}$ means the unit normal vector of the surface element df of F. At the spatial limit $F \to \infty$ this integral vanishes, if one supposes — as it is usual — that the field components and their derivatives vanish at the infinity.

The integral in the internal space may analogously be transformed. Now, let Ω^* be in this case the internal part of the unit sphere Γ^* in the internal space, then due to GAUSS' theorem we have again:

$$\int_{\Omega^*} \partial^*_k \mathfrak{f}^{*k} d^i\xi = \int_{\Omega^*} \vec{\nabla}_{(\xi)} \vec{\mathfrak{f}}^* d^i\xi = \oint_{\Gamma^*} \mathfrak{f}^{*k} n^*_k d\Gamma^* \tag{7,22}$$

where $\{n^*_k\}$ means the unit normal vector of the surface element $d\Gamma^*$ of Γ^*. This integral vanishes too, namely, the internal part $\Xi(\xi_i)$ of the field components introduced in Eq. (4,4) is periodical on Γ^*.

Introducing the quantity

$$\mathscr{F}^0 \overset{\text{def}}{=} \int_V \int_{\Omega^*} \mathfrak{f}^0 \, d^3x \, d^i\xi =$$

$$= \int_V \int_{\Omega^*} \left\{ \left(L\delta^0_{\varrho} - \frac{\partial L}{\partial \psi_{\alpha,0}} \psi_{\alpha,\varrho} \right) \delta x^{\varrho} - \frac{\partial L}{\partial \psi_{\alpha,0}} \psi_{\alpha|r} \delta^{*} \xi_{r} + \frac{\partial L}{\partial \psi_{\alpha,0}} \delta \psi_{\alpha} \right\} d^3x \, d^i\xi, \tag{7,23}$$

due to the above considerations, the conservation rule associated with the continuity equation (7,19) can be written in its integral form:

$$\frac{d}{dt} \mathscr{F}^0 = 0, \tag{7,24}$$

i. e., $\mathscr{F}^0$ is a constant of motion.

Summing up, we arrived, at the following important result: any continuous symmetry group induces a conservation law for a certain physical quantity $\mathcal{F}^{\circ}$ which can be derived for any given system according to (7,23), once the LAGRANGIAN is known. Since, the LAGRANGIANS of the fields are bilinear in the field components ψ_{α} and their derivatives, the same will hold for the constants of motion belonging to free fields.

The connection between continuous symmetry groups and conservation laws was first recognized in full by E. NOETHER [14] and in current field theories — where the internal degrees of freedom have not be taken into account — were discussed by several authors in details [15].

As a new and perhaps important result of our above considerations not the formal generalization of NOETHER's theorem has to be held according to which the internal motion of the physical systems induces new constants of motion too, however, the following recognition seems to be more remarkable: Owing to the connection between the local co-ordinate system of the momentum-space and the original frame of reference $\mathcal{K}$ in the co-ordinate-space, mentioned above in connection with the passive interpretation of co-ordinate transformations, in fact, continuous LORENTZ transformations of the co-ordinate-space correspond also to the discontinuous internal transformations (such as reflexions and inversions in respect to the λ-trieder). Indeed, the internal inversion (in other words the change of the handedness of the λ-trieder), *e. g.*, may be reached by continuous rotations in the four-dimensional co-ordinate-space. This means, however, that if the internal space can be interpreted in physical terms, then new constants of motion can be derived corresponding to the intrinsic properties of physical fields.

§ 8. *Discussions*

Collect the most important geometrical properties of the internal space pointed out above:

(a) The internal space is three-dimensional;

(b) The group $\mathcal{G}_{\xi}$ of the transformations of the internal space — *i. e.*, the group of transformations of the λ-trieder considered — is isomorphic with the three-dimensional rotary-reflexion group;

(c) The constants of motion corresponding to the symmetry transformations of the group $\mathcal{G}_{\xi}$ determine internal attributes of elementary particles and that of the associated physical fields, respectively;

(d) The constants of motion referring to the internal attributes of elementary particles as well as the mass-spectrum of free fields prove the reality of the internal space and that of the internal degrees of freedom.

The results *(a)—(c)* suggest the connection, and — what is more — the identification of the internal space with the threedimensional isospace. Indeed, our more recent investigation [2] proved that the isotransformations can be interpreted in geometrical terms and several laws of conservation (such as *PC, PZ, PP'* theorems *etc.*) were justified.* The interpretation of isotransformations and the intrinsic

* In our paper [2] Eq. (12) means too radical condition for the field component ψ which may be fulfilled only by very special functions. This condition can, however, be omitted, namely, it was not used in the following.

constants of motion based on NOETHER's theorem will be investigated in the next future.

Now, consider a special property of physical fields induced in a line-element space $\{x^\mu, p^\mu\}$ or $\{x^\mu, \xi_i\}$. As it has been emphasized above several times, the field components are depending on the line-elements. Due to the physical interpretation of the homogeneous direction co-ordinates in the case of hyper quantization the operators $\mathbf{p}^\mu$ associated with the momentum p^μ, and the operators $\mathbf{x}^\mu$ associated with the co-ordinates x^μ, as well as the operators $\psi_\alpha(x^\mu, \mathbf{p}^\mu)$ do not commute by pairs. This means, however, that beside the usual commutators of the current field theories also the commutator $[\mathbf{x}^\mu, \psi_\alpha]$ has to be defined. This can analogously be done by BORN's reciprocity transformation [17]:

$$x^\mu \to p^\mu, \quad p^\mu \to -x^\mu \tag{8,1}$$

as it is well-known in the case of YUKAWA's bilocal theory of fields [6—8]. In fact, this analogy refers to an intrinsic connection between the suggested theory and that of YUKAWA previously emphazised.

Finally, the problem arises how the anisotropy of the phasespace has to be interpreted philosophically? In order to carry out a possible interpretation, let us discuss shortly the philosophical cencept of the space-time continuum.

Independent of their concrete material content, all events of the material world take place *in space* (side by side) and *in time* (one after another) as well. This means, however, that the events of the material world can be characterized by four objective parameters: by three data mapping their side-by-sideness (place) and by one determining their succession (time-point). As a matter of fact, the whole of material events can be regarded as a four-dimensional ensemble of events denoting it in terms of A. D. ALEXANDROV [18] as *the space of events*, or rather in the more usual terms of the theory of relativity — also considering that, in fact, all of the material events are continuously dependent on each other — as the *space-time continuum*. From this point of view *the space of events must be the absolute existential form of the material world.*

Nevertheless, the real physical events can only be truly mapped in this way if the geometrical connections among the „points" of the space of events — which are realized in the geometrical structure of the space-time continuum — by objective connections among the corresponding events, *i. e.*, by real material interactions, are determined. This means, however, that the spacetime continuum, or rather its geometrical structure depends on the concrete material content, *i. e.*, on special physical interactions, of the material world. As matters stand, the space-time continuum and its geometrical structure, respectively, which correspond to the whole of material events and their objective interactions as well, are unified in the dialectical unity of form and content. From this point of view *the space of events is relative;* indeed, its structure is determined by the concrete features of matter.

Due to these considerations — to be summarized — the space of events and the space-time continuum, respectively, is the objective existential form of the material world and *by the philosophical cathegory of space-time continuum the absolute and relative features of space and time are represented.* This can also be expressed by saying that the space-time continuum — in spite of the previous metaphysical concept of space and time according to which the EUCLIDIAN character of space and the absoluteness of time would be an *a priori* cathegory of human mind — cannot be a bare passive and from the matter independent geometrical background of physical processes; but, its structure is determined by objective interactions. J. BOLYAI was the first scientist who already hundred years ago suggested this idea; then it was recalled by RIEMANN and finally, as a principal idea of EINSTEIN's theory of gravitation has been scored its revolutional success in macro-physics. So far, this point of view is generally accepted in up-to-date physics.

Nevertheless, the gravitational interactions can be neglected in micro-physics. Therefore, in the case of elementary particles — owing to EINSTEIN'theory — it is usually supposed that the structure of the space-time would be pseudo-EUCLIDIAN. Hence, if we take seriously into account the suggested point of view, one can say that the pseudo-EUCLIDIAN character of the space-time world, *i. e.* its homogenity and isotropy, is rather a consequence of the special symmetry properties of the actual interactions than an *a priori* feature of space-time. In fact, if *e. g.*, the violation of parity conservation can be regarded as a special property of weak interactions, it seems that from the anisotropy of these interactions also the anisotropy of the space may be concluded. The reason that the structure of the space-time world is in most of the cases isotropic, seems to be that the anisotropy of the weak interactions are overlapped by the electromagnetic and strong interactions which have higher or at least another symmetry character. This can also be expressed by saying that the strict insistence of the *a priori* EUCLIDIAN (or pseudo-EUCLIDIAN) structure of the space-time world can be regarded as a rest of the metaphysical concepts.

Our recent investigations in this direction have been based on the supposition that in the anisotropic internal structure of the elementary particles the anisotropy of the space-time continuum would appear. This supposition may be illustrated in simple terms as follows:

In anisotropic spaces the structure of the space is characterized not only by its curvature, but by its torsion too. If the general idea could be accepted that the anisotropy of the space-time world is determined by the anisotropy of the interactions, it should be supposed, of course, that the longitudinal polarization of the particles may be induced by the torsion of the anisotropic space-time. Consider the following analogy: In the case of the gravitational field the photon with zero rest mass is the most adequate test particle which moves on a geodetical line of the space-time world. As a matter of fact, the deflexion of light in the neighbourhood of the Sun, *e. g.*, proves curvature of the space-time continuum. Analogously, the neutrino seems to be a similar test particle to the photon among the fermions to observe the torsion of the space-time. Indeed, its rest mass is zero — so that during its motion adapts perfectly itself to the structure of the space-time — and its longitudinal polarization is a maximal one, so far that the two-component theory of neutrino is aware of one kind of neutrinos with helicity (−1). In these terms one can say that the longitudinal polarization of elementary particles demonstrates the space-time anisotropy.

This was the reason that we have recently suggested the unfamiliar idea that the strict adherence to the *a-priority* of the pseudo-EUCLIDIAN space-time structure would be responsible for the problems connected with the violation of parity conservation predicted by LEE and YANG [19]. In other words, it may be supposed that the structure of our physical world as a consequence of anisotropic interactions seems to be richer than it was previously supposed [20].

In terms of the new development of the suggested theory one can say that in the anisotropy of the phase-space the anisotropy of the space-time is reflected, determined by special physical interactions. For the sake of simplicity it was, however, supposed that the dynamism of the local change of the space-time structure due to the anisotropic interactions has provisionally not to be investigated; but, in fact, only the consequences of the actual anisotropy of the space-time world — in the case of different but specialized fields — were discussed. Furthermore, it was supposed that the anisotropy of the space-time can be characterized by the longitudinal polarization of the field quanta which can be regarded as a constant anisotropy parameter $\mathscr{P}$. Nevertheless, this means only a provisional supposition. Indeed, in a field theory — considering also the interactions of fields — this constant anisotropy parameter has to be changed by an anisotropy parameter which depends on space and time: $\mathscr{P}=\mathscr{P}(x^{\mu})$ determined by the interactions of fields to be considered. This problem has, however, to be discussed in the future in detail.

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ВНУТРЕННЯЯ СТРУКТУРА ФИЗИЧЕСКИХ ПОЛЕЙ

Я. И. Хорват

Свилеватость элементарных частиц, значит правая и левая анизотопия были объяснены с помощью формализма относительного фазового пространства. Таким образом предложено новое обоснование теории, описанной в предыдущей работе, кроме того выведены уравнения пространства из принципа вариации, дан масс-спектр барионов, обобщена теорема *Нётера*. Преобразования, определяемые в пространстве изоспина, можно заменить непрерывными преобразованиями в координатном пространстве. Наконец были описаны философские понятия континуума пространства-времени и интерпретация предложенного метода.