MEASUREMENT OF DIFFUSIVITY, LIFETIME AND SURFACE RECOMBINATION VELOCITY IN SEMICONDUCTORS BY THE FLYING SPOT METHOD*

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Detailed discussion of the solution of the continuity equation with surface recombination is given, when generation by a flying light spot takes place. The solution is of a convenient form for experimental studies and renders possible the simultaneous determination of bulk lifetime, ambipolar diffusivity and surface recombination velocity. The reliability of the method is proved by measurements carried out on Ge specimens.

Photoelectric measurements in semiconductor investigations are very widespread. Both steady-state and non-stationary effects may give a number of important informations concerning the behaviour of electrons and holes injected. One of the non-stationary methods is the so-called Flying Spot Method, proposed by ADAM [1]. This method renders possible to measure simultaneously both bulk-lifetime (τ) and ambipolar diffusivity (D_0)[1] in a semiconducting crystal, even, as ADAM did, neglecting end effects, *i. e.* considering the case of zero surface recombination velocity ($s=0$). To avoid this restriction SOROKIN [2] has given a more general solution of the corresponding continuity equation (1) (see below). As this solution in reference [2] has the form of FOURIER-series and requires the knowledge of the imaginary roots of a trigonometric equation, it seemed to be reasonable to find a solution of a more convenient form when generating by a flying light spot.

In the present work the solution is obtained by using a generalization of the well-known integral transform of LAPLACE, the generalization for RIEMANN—STIELTJES integrals, as is described by van ROOSBROECK [3]. The physical picture of the present investigation is also similar to that of van ROOSBROECK.

According to this a semi-infinite semiconductor is assumed ($z \leqq 0$).

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[1] $D_0 = \dfrac{n_0 + p_0}{p_0/D_e + n_0/D_h}$, here D_e, D_h are diffusion constants and n_0, p_0 are the equilibrium concentrations of electrons and holes, respectively.

The source of excess carriers is a normalized[2], steady line source (at x_s, z_s) moving along the x-axis with uniform velocity c. The small signal differential equation describing the carrier concentration added (δp) has the following form (trapping neglected):

$$\partial\,\delta p/\partial t = D_0 \text{ divgrad } \delta p - \delta p/\tau. \tag{1}$$

To take end effects into consideration, the fulfilment of the condition

$$D_0 \partial\,\delta p/\partial z = s\delta p \quad \text{(in the plane } z=0) \tag{2}$$

is required.

In order to simplify Eqs. (1) and (2) we introduce dimensionless variables

$$\delta P = \delta p/(n_0 - p_0),\ U = t/\tau,\ X = x/L,\ Y = y/L,\ Z = z/L,\ L = (D_0\tau)^{1/2}$$

and $S = s/v_D$, $v_D = L/\tau$ (denoting the drift velocity by v_D).

The continuity equation has then the form:

$$\partial\,\delta P/\partial U = \text{divgrad } \delta P - \delta P \tag{3}$$

with

$$\partial\,\delta P/\partial Z = S\delta P, \quad \text{when} \quad Z = 0. \tag{4}$$

The solution of (3) for an infinite semiconductor (ignoring for a moment the motion of the source) is, as follows:

$$\delta P = (4\pi U)^{-1} \exp[-U - (X - X_s)^2/4U] \exp[-(Z - Z_s)^2/4U], \tag{5}$$

where the co-ordinates X_s and Z_s correspond to x_s and z_s.

The solution of the semi-infinite problem with surface recombination differs from (5) only in its Z-dependence, thus the condition (4) may be fulfilled when conveniently altering the last term in (5).

The method applied makes use of the following RIEMANN—STIELTJES integral representation of (5):

$$\delta P = (4\pi U)^{-1} \exp[-U - (X - X_s)^2/4U] \int_{-\infty}^{+\infty} \exp[-(Z - \zeta)^2/4U]\, dF(\zeta), \tag{6}$$

in which the integrator function, $F(\zeta)$, is chosen that, for $\zeta \geqq 0$, (6) is equivalent to (5) and for other ζ values, $F(\zeta)$ is determined that (4) should be fulfilled. The solution so determined has the form, [3] (written as the GREEN's function corresponding to the system of equations (3), (4)):

$$\begin{aligned} &G(X, Z, U; X_s, Z_s) \\ &\quad = (4\pi U)^{-1} \exp[-U - (X - X_s)^2/4U] \{\exp[-(Z - Z_s)^2/4U] \\ &\quad + \exp[-(Z + Z_s)^2/4U] - 2S \int_0^\infty \exp[-S\zeta - (Z + Z_s + \zeta)^2/4U]\, d\zeta\}. \end{aligned} \tag{7}$$

[2] $(n_0 - p_0)L^2$ carrier pairs per length L (L denotes the diffusion length).

Solution of continuity equation in the system of co-ordinates fixed to the source, determination of D_0 and τ

To obtain the solution in the case of the Flying Spot Method, we introduce a new system of co-ordinates, moving together with the source, on the base of $\bar{x} = x - ct$, $\bar{y} = y$, $\bar{z} = z$, $\bar{x}_s = \bar{z}_s = 0$:

$$\bar{X} = X - (c/L)t = X - (c\tau/L)U \equiv X - \alpha U,\ \bar{Y} = Y,\ \bar{Z} = Z,\ \bar{X}_s = \bar{Z}_s = 0. \quad (8)$$

In this system of co-ordinates (7) has the following form:

$$\begin{aligned} G(\bar{X}, \bar{Z}, U; 0, 0) &= (2\pi U)^{-1} \exp[-U - (\bar{X} + \alpha U)^2/4U] \\ &\quad \cdot \{\exp[-\bar{Z}^2/4U] - S\int_0^\infty \exp[-S\zeta - (\bar{Z} + \zeta)^2/4U]\, d\zeta\} \\ &= (2\pi U)^{-1} \exp[-(1 + \alpha^2/4)U - (\alpha/2)\bar{X} - \bar{X}^2/4U] \\ &\quad \cdot \{\exp[-\bar{Z}^2/4U] - S\int_0^\infty \exp[-S\zeta - (\bar{Z} + \zeta)^2/4U]\, d\zeta\}. \end{aligned} \quad (9)$$

In this new system of co-ordinates we evidently encounter the conditions of a steady state. Thus, to obtain steady-state solutions from (9) we have to integrate (9) respect to U from zero to infinity. The integration results in the following expression, having a detector in the $Z = 0$ plane:

$$\begin{aligned} \bar{G}(\bar{X}, 0; 0, 0) & \\ = \pi^{-1} \exp[-\alpha/2)\bar{X}]\{K_0[|\bar{X}|(1 + \alpha^2/4)^{1/2}] - S\int_0^\infty e^{-S\zeta} K_0[(\bar{X}^2 + \zeta^2)^{1/2}(1 + \alpha^2/4)^{1/2}]\, d\zeta\} \end{aligned} \quad (10)$$ [3]

The form (10) is the generalization of the solutions of ADAM and van ROOSBROECK, as, for $S = 0$ it is reduced to the result of ADAM and, for $\alpha = 0$ (*i.e.* $c = 0$) to that of van ROOSBROECK.

Equation (10) may be simplified on integrating it by parts as the integrated term vanishes at the boundaries:

$$\begin{aligned} \bar{G}(\bar{X}, 0; 0, 0) & \\ = \pi^{-1} \exp[-(\alpha/2)\bar{X}] \int_0^\infty e^{-S\zeta} K_1[(\bar{X}^2 + \zeta^2)^{1/2}(1 + \alpha^2/4)^{1/2}] \frac{\zeta}{(\bar{X}^2 + \zeta^2)^{1/2}}\, d\zeta. \end{aligned} \quad (11)$$

[3] Using the $(1 + \alpha^2/4)U = V$ substitution and the integral representation

$$K_\nu(z) = \frac{1}{2}(z/2)^\nu \int_0^\infty V^{-(\nu+1)} \exp[-V - z^2/4V]\, dV$$

of the BESSEL functions of the second kind for imaginary argument [4].

For the cases of S sufficiently large, the integral in (11) will converge rapidly, thus the approximation $(\bar{X}^2+\zeta^2)^{1/2}\approx\bar{X}$ and $\zeta(\bar{X}^2+\zeta^2)^{-1/2}\approx\zeta/\bar{X}$ is valid, and (11) will be of the following simple form:

$$\bar{G}(\bar{X},0;0,0)\approx(\pi\bar{X}S^2)^{-1}\exp\left[-(\alpha/2)\bar{X}\right]K_1[|\bar{X}|(1+\alpha^2/4)^{1/2}],$$

or using an approximation for $K_1(z)\approx(\pi/2z)^{1/2}e^{-z}$ (independently of ν) [5], valid for large z, and rewriting the quantities having dimensions, δp will be proportional to

$$\delta p\sim(\bar{x})^{-3/2}\exp\left[-(c/2D_0)\bar{x}+|\bar{x}|((c/2D_0)^2+1/(D_0\tau))^{1/2}\right]. \qquad (12)$$

This equation, just as the corresponding one in reference [1], renders possible the simultaneous determination of D_0 and τ, as the logarithmic derivative of (12) at a great distance before and behind the source has the following form[4]:

$$1/L_1\equiv\left|\frac{d(\log\delta p)}{d\bar{x}}\right|_1=c/2D_0+[(c/2D_0)^2+1/(D_0\tau)]^{1/2}$$

and

$$1/L_2\equiv\left|\frac{d(\log\delta p)}{d\bar{x}}\right|_2=-c/2D_0+[(c/2D_0)^2+1/(D_0\tau)]^{1/2}.$$

The quantities D_0,τ (and also L) then may be calculated, as is well known, on base of the simple relations:

$$L=(D_0\tau)^{1/2}=(L_1L_2)^{1/2},$$
$$D_0=\frac{c}{1/L_1-1/L_2}, \qquad \tau=\frac{L_2-L_1}{c}. \qquad (13)$$

According to these, the quantities in interest may be calculated by measuring the slopes before and behind the source of the curve *logarithm of detector response* versus $\bar{x}=x-ct$.

Figures 1a and 1b show the theoretical curves for several light spot velocities and surface recombination velocities cerresponding to Eq. (11). The numerical values of D_0 and τ (also in the forthcoming Fig. 2.) were chosen as $10^2\,\text{cm}^2\,\text{sec}^{-1}$ and 10^{-4} sec. (f denotes the rate of generation per unit length.)

Solution of continuity equation in the system of co-ordinates fixed to the crystal, determination of S

As the problem is essentially not a steady-state one, it is useful to return to the laboratory system of co-ordinates. In this system the state is time-dependent. On the base of (7) the corresponding solution is of the form

[4] $\log(\bar{x})^{-3/2}$ is supposed to be approximately constant.

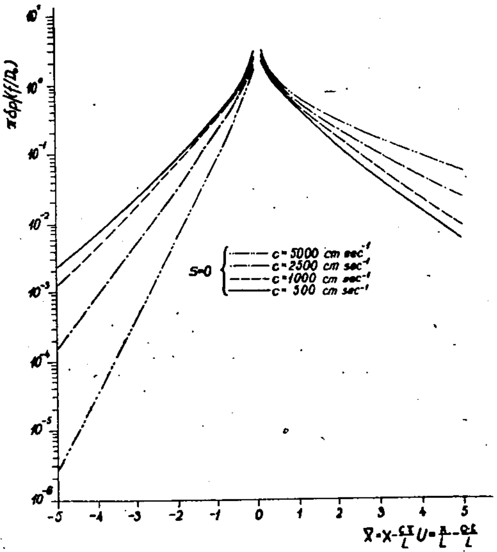

Fig. 1a. Theoretical curves, corresponding to Eq. (11)

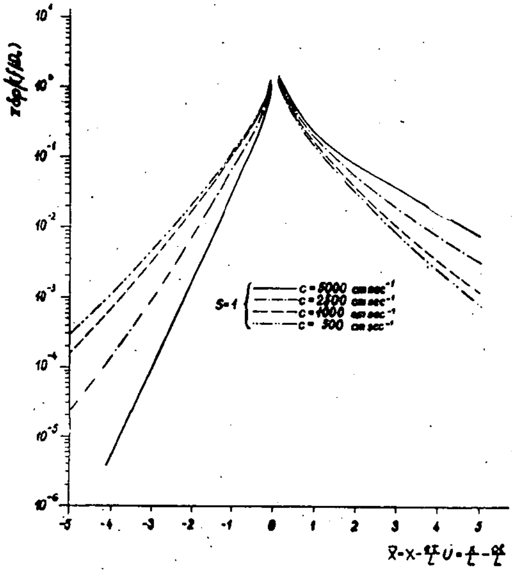

Fig. 1b. Theoretical curves, corresponding to Eq. (11)

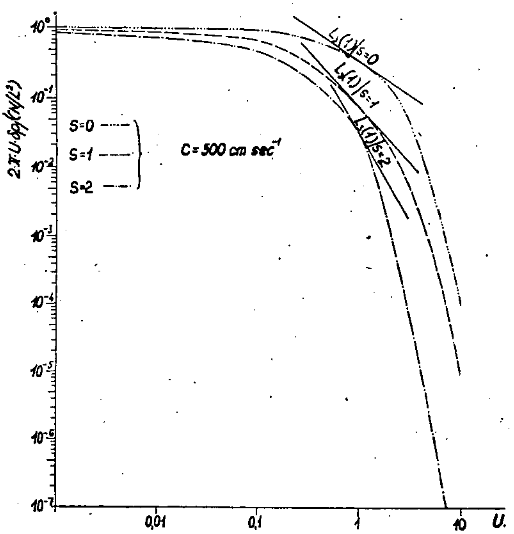

Fig. 2a. Theoretical curves, corresponding to Eq. (14)

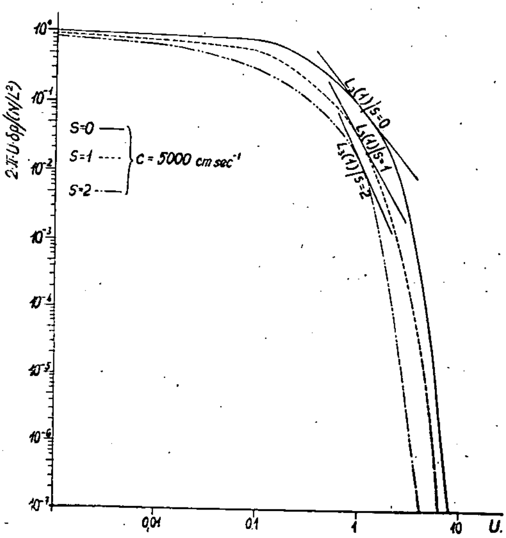

Fig. 2b. Theoretical curves, corresponding to Eq. (14)

(with a detector at $X=Z=0$):

$$\delta p/(N/L^2)=G(0,0,U;\ \alpha U,0)$$

$$=(4\pi U)^{-1}\exp[-U-\alpha^2 U^2/4U]\cdot\{2-2S\int_0^\infty \exp[-S\zeta-\zeta^2/4U]d\zeta\} \quad (14)$$

$$=(2\pi U)^{-1}\exp[-(1+\alpha^2/4)U]\cdot\{1-S\pi^{1/2}U^{1/2}e^{S^2U}[1-\mathrm{erf}(SU^{1/2})]\},$$

where N the number of pairs generated and $\mathrm{erf}(x)=2\pi^{-1/2}\int_0^x e^{-\beta^2}d\beta$. Eq. (14) has been obtained by using integral tables to evaluate the integral $\int_0^\infty \exp[-S\zeta-\zeta^2/4U]dU$, [6].

It is convenient to calculate the derivative of the function $UG(0,0,U;\alpha U,0)$, as the slope of this curve can be easily obtained from oscillograms. To get a curve, independent of scale factors, for the determination of S it is useful to plot the *product of U and detector response* versus U, both in logarithmic scale, as the slope of this curve, $L_3(V)$, at $V=\log U$ has the form (for $Se^{V/2}>1$) [7]:

$$L_3(V)\equiv\frac{\partial \log[e^V G(0,0,e^V;\ \alpha e^V,0)]}{\partial V}$$

$$=-\left\{(1+\alpha^2/4)e^V+\frac{\pi^{1/2}Se^{V/2}e^{S^2e^V}[1-\mathrm{erf}(Se^{V/2})]\left(\frac{1}{2}+S^2e^V\right)-S^2e^V}{1-\pi^{1/2}Se^{V/2}e^{S^2e^V}[1-\mathrm{erf}(Se^{V/2})]}\right\}$$

$$=-(1+\alpha^2/4)e^V-\frac{4S^4e^{2V}-12S^2e^V-15/2}{4S^4e^{2V}-6S^2e^V+15}, \quad (15)$$

when using the first four terms of the asymtotic expansion of $\mathrm{erf}(x)$ for large arguments [8][5]. This simple form makes possible to calculate S by help of the slope $L_3(V)$, as the root of (15), having physical significance, is of the following form:

$$S=\left\{\frac{3a+[9a^2+60(a-1)b]^{1/2}}{4e^V(a-1)}\right\}^{1/2},$$

where $$a=L_3(V)+(1+\alpha^2/4)e^V+2 \quad (16)$$

and $$b=-L_3(V)-(1+\alpha^2/4)e^V+\frac{1}{2}.$$

Figures 2a and 2b show theoretical curves for some light-spot velocities and for several surface recombination velocities corresponding to (14). The

[5] The expansion is valid for $Se^{V/2}\gg 1$, but when already $Se^{V/2}=2$ it gives rise of an error less then 5 *per cent.*

slopes, $L_3(V)$, at $V=0$, belonging to different surface recombination velocities, show that the quantity $L_3(V)$ — as demanded in the experimental work — depends on S appreciably.

Experimental part

Measurements on Ge samples has been carried out to prove the reliability of the method. The apparatus used has been analogous to the one described by ADAM. For the sake of consistency, the apparatus is sketched here also, as follows (Fig. 3).

A micromanipulator served for holding the specimen and the point contact detector. The light of an incandescent lamp (L_1) falls on a rotating metal-mirror, driven by a synchron motor, the flying image of the slit (S) is thus produced on the surface of the specimen. The photocell (C_1) served for time-scale calibration and the revolution of the motor has been controlled by a stroboscope. The amplifier is followed by a suitably synchronized (by the lamp L_2 and photocell C_2) cathode ray oscilloscope (CRO).

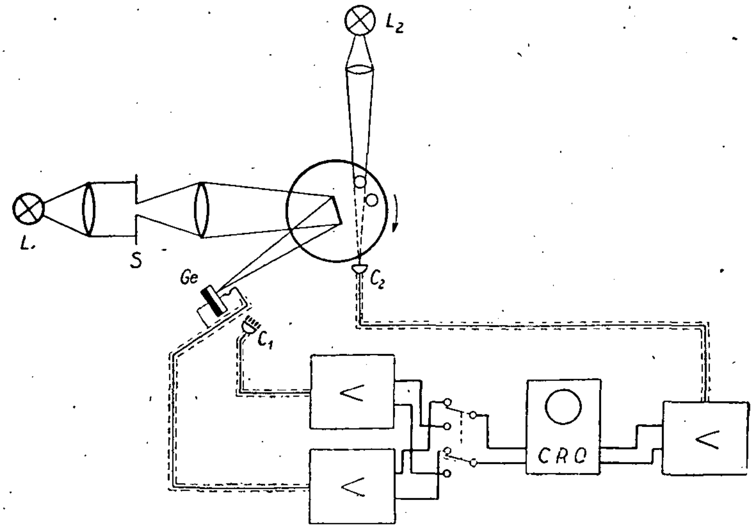

Fig. 3. Measuring apparatus

Oscillograms were taken for several light spot velocities and for several samples (Fig. 4).

The results calculated on the base of (13) and (16) for samples 1 and 2 (surface polished and etched with $H_2O_2 + NaOH$) are to be seen on tables I and II.

The $S(\bar{L}/\bar{\tau})$ [and $s(\bar{L}/\bar{\tau})$] are calculated on using the average values $(\bar{L}, \bar{\tau})$ of L and τ, respectively.

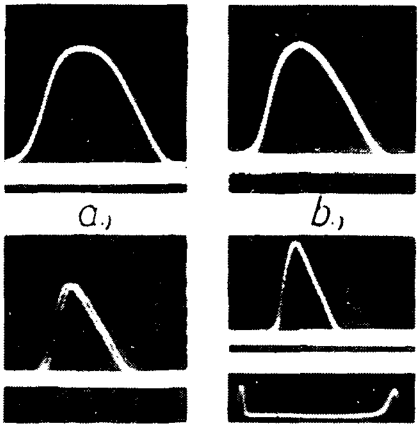

Fig. 4. Oscillograms taken on sample 1. a) $c = 785$ cm sec^{-1}; b) $c = 1000$ cm sec^{-1}; c) $c = 1500$ cm sec^{-1}; d) $c = 2000$ cm sec^{-1}; e) oscillogram for time-scale calibration (the distance of peaks covers a time-interval of 434 μsec)

Table I

Sample 1

c cm sec^{-1}	D_0 cm^2 sec^{-1}	τ μsec	L cm	μ_p cm^2 volt^{-1} sec^{-1}	S	s cm sec^{-1}	$S(\bar{L}/\bar{\tau})$	$s(\bar{L}/\bar{\tau})$ cm sec^{-1}
785	47	103	0,071	1838	0,953	656	1,077	892
1000	65	100	0,081	2512	1,153	934	1,066	883
1500	81	133	0,104	3134	1,573	1230	1,573	1302
2000	83	62	0,071	3219	1,560	1811	4,011	3321
mean values	69,6	99,5	0,081	2677	1,310	1158	1,932	1600

Table II

Sample 2

c cm sec^{-1}	D_0 cm^2 sec^{-1}	τ μsec	L cm	μ_p cm^2 volt^{-1} sec^{-1}	S	s cm sec^{-1}	$S(\bar{L}/\bar{\tau})$	$s(\bar{L}/\bar{\tau})$ cm sec^{-1}
785	96	83	0,089	3692	1,976	2118	1,771	1849
1000	107	70	0,087	4115	2,719	3380	1,813	1893
1500	101	94	0,097	3884	2,026	2091	1,853	1935
2000	97	114	0,105	3730	1,485	1368	2,232	2330
mean values	100,2	90,2	0,094	3855	2,051	2239	1,917	2002

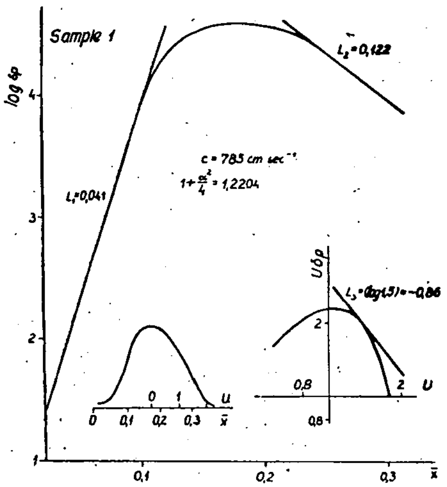

Fig. 5. The determination of the three slopes needed in calculations of D_0, τ and S, sample 1

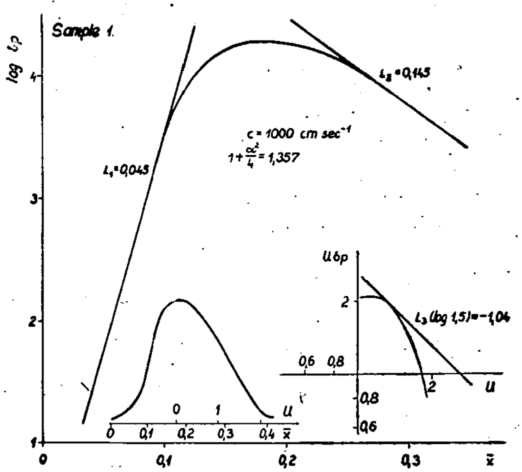

Fig. 6. The determination of the three slopes needed in calculations of D_0, τ and S, sample 1

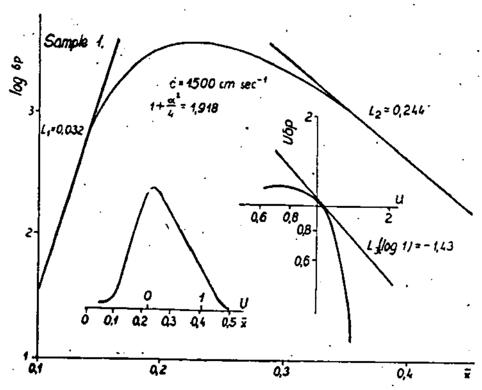

Fig. 7. The determination of the three slopes needed in calculations of D_0, τ and S, sample 1

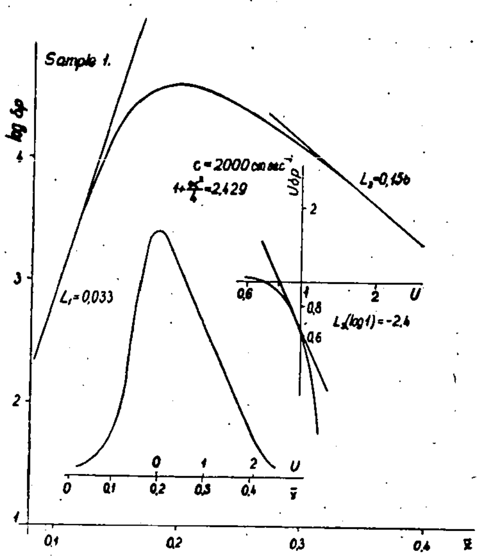

Fig. 8. The determination of the three slopes needed in calculations of D_0, τ and S, sample 1

The determination of D_0', τ and L is evidently seen on Figs. 5—8. *The oscillogram curves*, in the middle of the figures, *were plotted* logarithmically on the one hand *versus* $\bar{x} = ct$, *for the determination of* D_0, τ, etc. [Eq. (13)], then, on the other hand, *against* $\log U = \log t - \log \tau$, *which rendered possible the determination of* S [Eq. (16)]. The mobilities (μ_p) were calculated by using the EINSTEIN relation.

The present investigations show that the solution of the continuity equation in the case of the flying spot method may be succesfully extended for surface recombination velocities differing from zero, giving thus a method for the simultaneous determination all the characteristic parameters occurring in continuity equation (1) and boundary condition (2), and, as dates of the tables show, this determination involves a rather little uncertainty, the deviations from the mean values did not amount in average 30 *per cent.*

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ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТА ДИФФУЗИИ, ВРЕМЕНИ ЖИЗНИ, А ТАКЖЕ СКОРОСТИ ПОВЕРХНОСТНОЙ РЕКОМБИНАЦИИ В ПОЛУПРОВОДНИКАХ, ПОСРЕДСТВОМ ДВИЖУЩЕГОСЯ СВЕТОВОГО ПУЧКА

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В статье содержится подробное обсуждение решения уравнения непрерывности в случае генерации носителей тока движущимся световым пучком. Простая форма решения пригодна к применению её к экспериментальной работе и к одновременному определению коеффициента диффузии, времени жизни и скорости поверхностной рекомбинации. Пригодность метода обоснована измерениями над Ge-кристаллами.