THE BILOCAL GENERALIZATION OF THE SCHRÖDINGER-GORDON EQUATION

By J. I. HORVÁTH

Department of Theoretical Physics, University of Szeged, Szeged, Hungary

In view of the experimental evidence of existence of families of the elementary particles the problem of a theoretical explanation of their mass spectra becomes a matter of urgency. Recently very interesting theories of the mass spectra based on YUKAWA's bilocal field theory were published by H. YUKAWA (1, 2,) and J. RAYSKI (3, 4). This fact gives the bilocal field theory a special interest.

This paper will indicate a natural geometrical basis for the bilocal theory eleborated originally by H. YUKAWA (5) in a quite abstract operator form and on the basis of this geometrization the generalized SCHRÖDINGER—GORDON equation of RAYSKI (3) will be deduced.

1. §. A CONNECTION BETWEEN THE BILOCAL AND THE GENERALIZED LINE ELEMENT SPACES

The world continuum in which the physical phenomenona take place, reppresented by the field, is a four dimensional space-time ensemble. This ensemble can be regarded as a four dimensional metrical space. The quantities which determine the state of the physical field are in the case of the usual local field theory ordinary space-time functions with a given law of transformation.

In YUKAWA's bilocal theory the quantities of the field depend on a point pair of the four dinemsional space-time continuum. Consequently, the natural geometrical basis for the bilocal field theory is the space of the point pairs. From this point of view YUKAWA's theory is a field theory of second kind (6) in the space of the point pairs and we will regarde the point pairs $(\underset{(1)}{x}, \underset{(2)}{x})$ of the world continuum as basic elements of the space $\mathfrak{B}$.

The point pair $(\underset{(1)}{x}, \underset{(2)}{x})$ is space-like, light-like and time-like resp. according to

$$(\underset{(1)}{x^\mu} - \underset{(2)}{x^\mu})(\underset{(1)}{x_\mu} - \underset{(2)}{x_\mu}) = (\underset{(1)}{x^0} - \underset{(2)}{x^0})^2 - (\underset{(1)}{x^1} - \underset{(2)}{x^1})^2 - (\underset{(1)}{x^2} - \underset{(2)}{x^2})^2 - (\underset{(1)}{x^3} - \underset{(2)}{x^3})^2 \lesseqgtr 0.$$

Since the physical phenomena which take place in space-like point pairs of the space-time continuum do not influence one another, we have to suppose that

$$(\underset{(1)}{x}, \underset{(2)}{x}) = (\underset{(2)}{x}, \underset{(1)}{x}) \qquad \textit{for space-like point pair.}$$

Now, we will introduce the so-called coordinates of YUKAWA by the definition

$$x^{\mu} = \frac{1}{2}(\underset{(1)}{x^{\mu}} + \underset{(2)}{x^{\mu}})\,; \; r^{\mu} = \underset{(1)}{x^{\mu}} - \underset{(2)}{x^{\mu}}. \tag{1, 1}$$

Instead of the point pairs $(\underset{(1)}{x}, \underset{(2)}{x})$ we can use the coordinates (x, r). Let the ensemble of the coordinates (x, r) be regarded as the basic elements of the $\mathfrak{Y}$ space.

Since in YUKAWA's theory a condition of normalization for the vectors r^{μ} is given by

$$r^{\mu} r_{\mu} = \lambda^2,$$

where λ is a constant, the four components of r^{μ} are not independent, consequently only one direction is determined by the second group of the coordinates of YUKAWA. But this means that the basic element of the $\mathfrak{Y}$ space and the basic element of a general line element space are equivalent.

The condition of normalization is negligible, if we introduce instead the YUKAWA's coordinates the coordinate

$$v^{\mu} = \varrho\, r^{\mu}$$

where ϱ is a positive factor and we suppose that the quantities of the physical field are homogeneous functions of the variable v^{μ} of zero degree. The space $\mathfrak{L}$ of the line elements (x, v) is then equaivalent with our previously defined spaces $\mathfrak{B}$ or $\mathfrak{Y}$.

The metrization of space $\mathfrak{L}$ and the basis of this kind of geometry was eleborated in a previous paper (7).

2. §. THE IDEA OF A CLASSICAL BILOCAL FIELD THEORY

The general idea of the physical field theory, given originally by M. FARADAY and J. C. MAXWELL, was that — instead of the idea of the point mechanics according to which the action of the forces is an action at distance — the interaction between two separate particles is transmitted by the physical field. This means that the changing of the state of the field in a point of the space-time world depends only on the changing of the state of the stress of the field in the immediate neighbourhood of the considered space-time point. If we take only into account the action of the state of the stress of the field in the infinitesimal neighbourhood of the considered space-time point we can deduce by the well known limiting process for the characterisation of the balance some field equations, which are based on the above consideration partial differential equations. The theory of the field is in this case an ordinary local field theory.

If we will now neglect the mentioned limiting process, we can assume that the state of the field in the considered space-time point will be directly influenced by the phenomena taking place in the space-time point of a sphere with a λ radius which surrounds the considered point. Apparently this assumption represents the general idea of the bilical field theory. It is clear that according to this supposition the idea of the action at distance is recalled in the inside of the sphere with the radius λ, but it does not matter because the aspect of the field theory remains macroscopically — between elementary particles — unchanged.

Undoubtedly, the general idea of the bilocal field theories means a radical change of the field theoretical aspect, but the bilocal field theories have the advantage over the local field theories that they are free from the well known divergencies of the local field theories.

3. §. THE GENERALISED SCHÖDINGER-GORDON EQUATION OF THE BILOCAL FIELDS

We shall deal in the following with the skalar bilocal field theory. In this case the field is characterised by a skalar bilocal function $\psi = \psi(x, r)$ and we suppose that

$$r^{\mu} r_{\mu} = \lambda^2 \ll 1.$$

Let the LAGRANGE-function of the field be given in the form

$$\mathfrak{L} = \mathfrak{L}\left(\psi, \frac{\partial \psi}{\partial x^{\mu}}, \frac{\partial \psi}{\partial r^{\mu}}\right),$$

where the character of transformation of the function $\mathfrak{L}$ is a skalar density. The field equations can be deduced from the variational principle:

$$\delta_{\psi} \mathfrak{J} = \delta_{\psi} \iint \mathfrak{L} \, d^4 x \, d^4 r = 0.$$

Index ψ refers to the fact that in integral $\mathfrak{J}$ the function ψ should be varied. The variation of ψ has to be zero on the limit of the domain of integration.

It can be proved by infinitesimal transformations (8) that the bilocability of the function ψ does not make any difficulty and the EULER-LAGRANGE equation of the above variational principle is

$$\frac{\partial \mathfrak{L}}{\partial \psi} - \frac{\partial}{\partial x^{\varrho}} \frac{\partial \mathfrak{L}}{\partial \dfrac{\partial \psi}{\partial x^{\varrho}}} - \frac{\partial}{\partial r^{\varrho}} \frac{\partial \mathfrak{L}}{\partial \dfrac{\partial \psi}{\partial r^{\varrho}}} = 0.$$

Finally, let us suppose that our LAGRANGE function is quadratic in ψ and its derivatives and consequently has the general form

$$\mathfrak{L} \stackrel{\text{def}}{=} \frac{1}{2} \left\{ A^{\mu\nu} \frac{\partial \psi}{\partial x^{\mu}} \frac{\partial \psi}{\partial x^{\nu}} + B^{\mu\nu} \frac{\partial \psi}{\partial x^{\mu}} \frac{\partial \psi}{\partial r^{\nu}} + C^{\mu\nu} \frac{\partial \psi}{\partial r^{\mu}} \frac{\partial \psi}{\partial r^{\nu}} + D^{\mu} \frac{\partial \psi}{\partial x^{\mu}} \psi + \right.$$

$$\left. + E^{\mu} \frac{\partial \psi}{\partial r^{\mu}} \psi + F \psi^2 \right\}, \qquad (3,2)$$

where the constant coeffitiens fullfill the symmetry relations:

$$A^{\mu\nu} = A^{\nu\mu},\ B^{\mu\nu} = B^{\nu\mu},\ C^{\mu\nu} = C^{\nu\mu}.$$

Then our field equations have the explicit form

$$A^{\mu\nu} \frac{\partial^2 \psi}{\partial x^\mu \partial x^\nu} + B^{\mu\nu} \frac{\partial^2 \psi}{\partial x^\mu \partial r^\nu} + C^{\mu\nu} \frac{\partial^2 \psi}{\partial r^\mu \partial r^\nu} - F\psi = 0. \qquad (3, 3)$$

In the case of

$$F = \varkappa^2,\ A^{\mu\mu} = B^{\mu\mu} = C^{\mu\mu} = 1 \text{ and } A^{\mu\nu} = B^{\mu\nu} = C^{\mu\nu} = 0 \text{ for } \mu \neq \nu$$

our equations (3,3) and RAYSKI's generalised SCHRÖDINGER—GORDON equations are identical.

References

1. *Yukawa, H.: Phys. Rev.* **91**. 415 (1953).
2. *Yukawa, H.: Phys. Rev.* **91**. 416 (1953.
3. *Rayski, J.: Acta Phys. Polonica* **13**. 77 (1953).
4. *Rayski, J.: Acta Phys. Polonica,* **14**. 107, 149 (1955).
5. *Yukawa, H.: Phys. Rev.* **77**. 219 (1949).
6. *Horváth, J. I. und Moór, A.: Zs. f. Phys.* **131**. 544 (1952)
7. *Horváth, J. I. und Moór, A.: Proc. Kon. Nederl. Acad. Wetensch.,* (A) **58**. 421, 762 (1955).
8. *Horváth, J. I.: Magyar Fizikai Folyóirat,* **4**. 567 (1955).