A Statistical Analysis of the Length of Rainy Periods and their Quantity of Precipitation

by

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Csapadékos szakaszok hosszának és csapadékhozamának statisztikai elemzése. A dolgozat a budapesti és szegedi napi csapadékadatokból számított tartamvalószínűségeket vizsgálja. Megállapítja, hogy az egymásra következő észlelt csapadékos szakaszok hossza igen magas szignifikancia-szinten különbözik a véletlenszerűen egymásra következő csapadékos szakaszok hosszától. Meghatározza továbbá a tanulmány a csapadékos szakaszok hossza és csapadékhozama közötti összefüggésnek a lineáris függvénykapcsolattól való eltérését.

Statistische Analyse der Länge der Niederschlagsperioden und ihrer Niederschlagsmenge. Der Aufsatz untersucht die Wahrscheinlichkeiten der Dauer der Niederschlagsperioden, die aus den täglichen Niederschlagsangaben von Budapest und Szeged ausgerechnet wurden. Es wurde festgestellt, dass sich die Länge der nacheinanderfolgenden registrierten regnerischen Phasen von der Länge der zufällig nacheinanderfolgenden regnerischen Phasen in einem sehr hohem Signifikanzniveau unterscheidet. Die Arbeit bestimmt weiterhin die Abweichung des Zusammenhangs zwischen der Länge der Niederschlagsperioden und ihrer Niederschlagsmengen von dem linearen funktionellen Zusammenhang.

The paper examines the probability of durations calculated from the daily data of precipitation of Budapest and Szeged. It states that the length of the observed rainy periods differs from the length of the accidental rainy periods on a high level of significance. Besides the paper defines the deviation of functionality between the length of the rainy period and its amount of precipitation from the linear connection.

Data referring to the length and precipitation amount of rainy periods take a very important part among the pieces of climatological information for the distribution of rain. It is mainly the management of water supplies and agriculture that need such information.

A rainy period is defined as a sequence of rainy days. The definition allows the presence of more factors in bringing forth a rainy period since the amount of rain refers to 24 hours. So rainy periods cannot be considered homogenous from meteorological point of view. In the analysis of data these factors are neglected.

In this study the daily precipitation amounts of Budapest and Szeged are analysed for the 40 year long period of 1931—1970. Rainy day means the fulfilment of the R 0.1 mm condition where R represents one day's precipitation amount.

Tables 1—2 give the number of rainy periods of different length during the period in question. It can be stated that rainy periods of more than 5 days occur very rarely. The number of rainy periods shows a gradual decrease from May to June with a minimum in October, then a gradual increase follows. The number of rainy periods is less at Szeged than at Budapest.

Table 8 gives the probability of rainy days per month (p) and the complementer probability of days without rain (q). It holds for the whole year that the probability of rainy days is under 50 per cent, it is only the maximum of November when this value is 46 per cent at Budapest. The data of *Table 8* are used to calculate other probabilities for the characterization of the length of rainy periods, those which are valid on condition of independence. These are compared to real values.

In case of independence the probability (p) of rainy periods of a definite length (k) can be calculated with the

$$r(k) = Nq^2p^k \tag{1}$$

formula, where N is the number of data in the sequence i. e. the number of days in the period in question.

The frequency values $r(k)$ are contained in *Tables 3—4* comparing *Tables 1—2* and *3—4,* a great difference can be noticed between the number of rainy periods calculated on the basis of independence and those in reality: the number of short periods (1—3 days) is much greater on condition of independence than in reality; on the other hand the longer periods (4days or more) show an opposite deviation.

So rainy periods cannot be considered as a sequence of independent cases. The simplest mathematical-statistical proof of this can be given by counting the average length of the rainy periods on condition of independence. This value can be compared to the real lengths.

On condition of independence in a sequence of N data the possible number of periods in Npq, and from this the mean length of the periods (A) is:

$$A = \frac{1}{q} \tag{2}$$

The real mean lenght of the periods — calculated from the frequency of the event in question divided by the number of the periods — is a quantity A^*. In case of independence $A^*/A = 1$, while otherwise $A^*/A \neq 1$.

The hypothesis of independence of data neglected on condition that the value of the ratio A^*/A falls outside, a definite interval calculated

$$I = \frac{q}{q \pm d\sqrt{\frac{pq}{N}}} \tag{3}$$

where d is a factor of the wanted level of significance. In our study, $d=3$ with a level of significance 0.0027.

Table 5 gives the values of A^*, A and A^*/A with the limits of the interval of independence. The value of the ratio A^*/A is greater than 1 in every month and they fall outside the interval I. So the probability of independence in the length of the periods is 0.0027, this, being a very small value the idea of independence can be thrown away. The annual course of the value A^*/A is very characteristic. This is contained in *Fig. 1.* Maxima of both stations take place in October (this is the period of the least number of rainy seasons and that of the longest ones), the minima in summer. A weaker secondary maximum takes place in spring. This can be understood by the climatological fact that in autumn, at the beginning of winter and in spring the persistent precipitations of warm fronts dominate.

Further on, the relation between the amounts of precipitation and the length of rainy periods is examined. The general tendency is that with the growth of the length of the period, the amount of precipitation grows as well. With the graphical presentation of the data for a month *(Fig. 2—3)* the increase seems to be not expressedly linear, but rather exponential. So now the equation of this exponential function is defined, then the difference between this and the linear growth. The exponential function is:

$$c_1 \cdot t^{\alpha} = y \tag{4}$$

where c_1 is the mean amount of precipitation per day in the month in question, t is the number of days and α is the exponent wanted.

The approaching values of α are calculated with the help of string method and contained in *Table 6*. This shows that the amount of precipitation is not in close connection with the length of the period. The value of α is greater than 1 nearly all over the year. Smaller values occur in February and June at Budapest and in November at Szeged. But in most cases there is only a small difference between α and 1, that the difference from linear values can be neglected. There are only two months at Budapest (August and November) and four at Szeged (March, July, August and December) when the difference is more than 1/10. So it is only August that refers to both places. *Table 6* shows that in case of 7 months, Szeged has the greatest values of

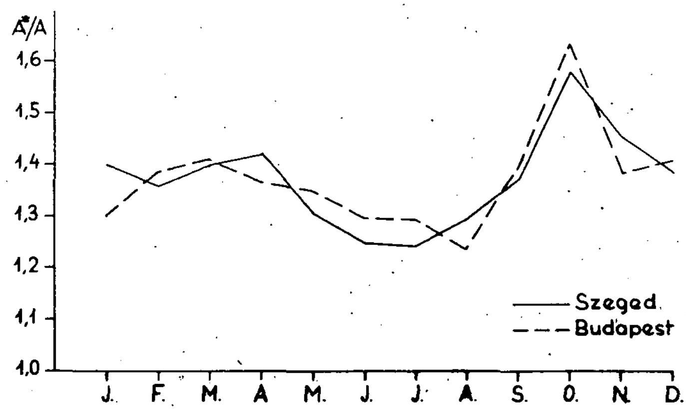

Fig. 1. The values of the quotient A^*/A
1. ábra. Az A^*/A értékei

α meaning that equally long rainy periods result in more precipitation at Szeged. The difference is more prominent in February and July and December, while in August it is nearly the same. As to its annual course, it decreases from December to January, then it increases in February and March. Then, after a short decrease, the summer maximum follows (in August at Budapest and in July in Szeged). Then again a decrease follows untill October at Szeged and November at Budapest.

The comparison of the two places from this point of view shows that the change is opposite in May, June, August and in the last three months of the year.

A general phenomenon is suspected to exist, when the nature of the change is the same referring to the previous month. To be sure about this the data of more stations have to be treated. (The greater value of α is probably caused by the greater moisture content of the air and the greater amount of convective precipitation).

For the sake of better comparison, the next rate was calculated:

$$Q = \frac{c_t}{c'_t}$$

This was calculated for each month and for both places. *Table* 7 shows the values. Based on the above facts, a new aspect of examination comes. The value of Q increases with the length of the period in February and June for Budapest and in November

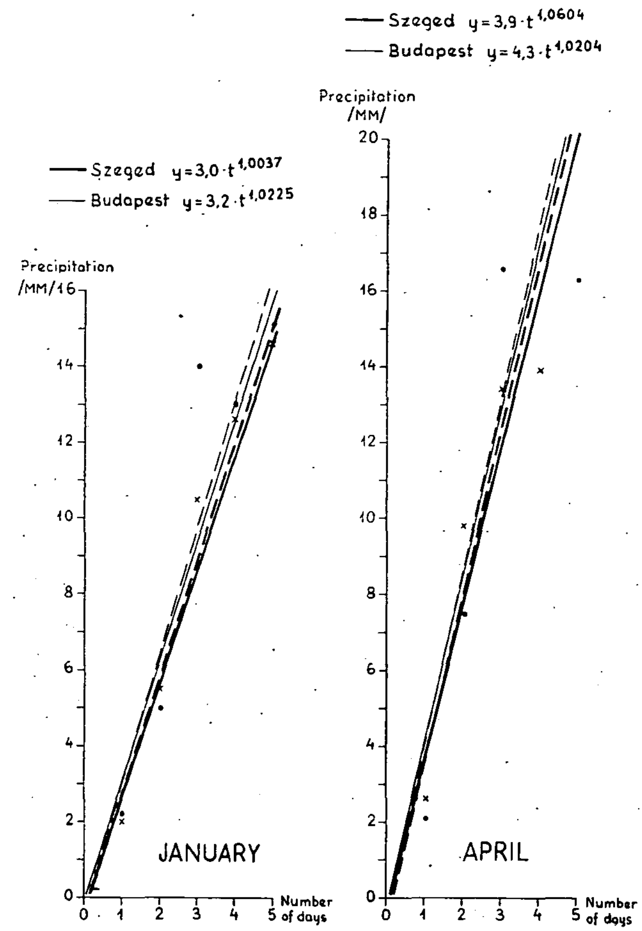

Fig. 2. The amount of precipitation of the periods in January and April
2. ábra. A periódusok csapadékhozama januárban és áprilisban

for Szeged since the exponent of the denominator is greater in these months than that of the numerator.

$$(c_t = c_1 \cdot t;\ c'_t = c_1 \cdot t^{\alpha} \text{ in the formula})$$

Table 7 shows that the change is opposite at the two places in October, November and February and December referring to the previous month. On the other hand there is not even 1/10 difference in the value of Q in any month of the year between Budapest and Szeged. Supposingly this is a general phenomenon, existing not only for these two places. This would mean that Q can be defined for any place from mean values with the help of c_1 through the calculation of c'_t. This is shown by *Fig. 2—3* where the distribution of monthly precipitation amounts among the periods

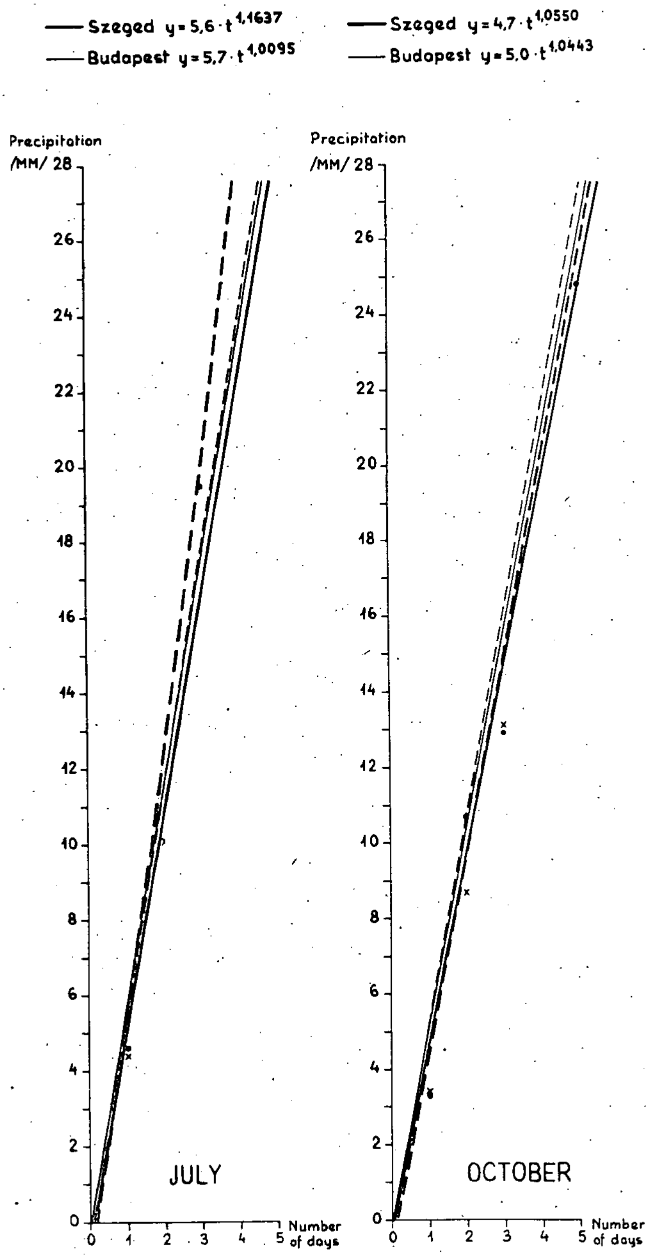

Fig. 3. The amount of precipitation of the periods in July and October
3. ábra. A periódusok csapadékhozama júliusban és októberben

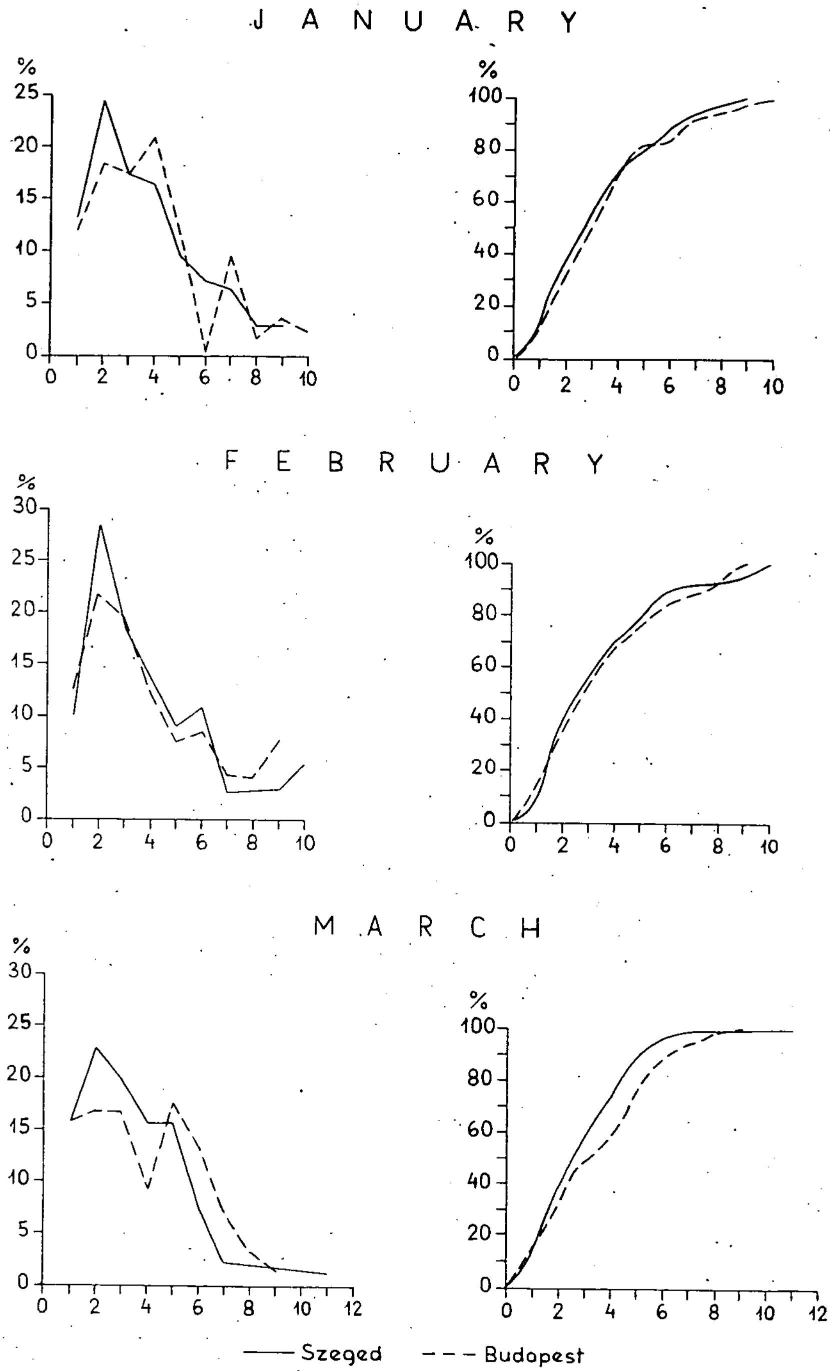

Fig. 4. The division of monthly precipitation between the periods in January, February and March

4. ábra. A havi csapadék megoszlása a periódusok között januárban, februárban és márciusban

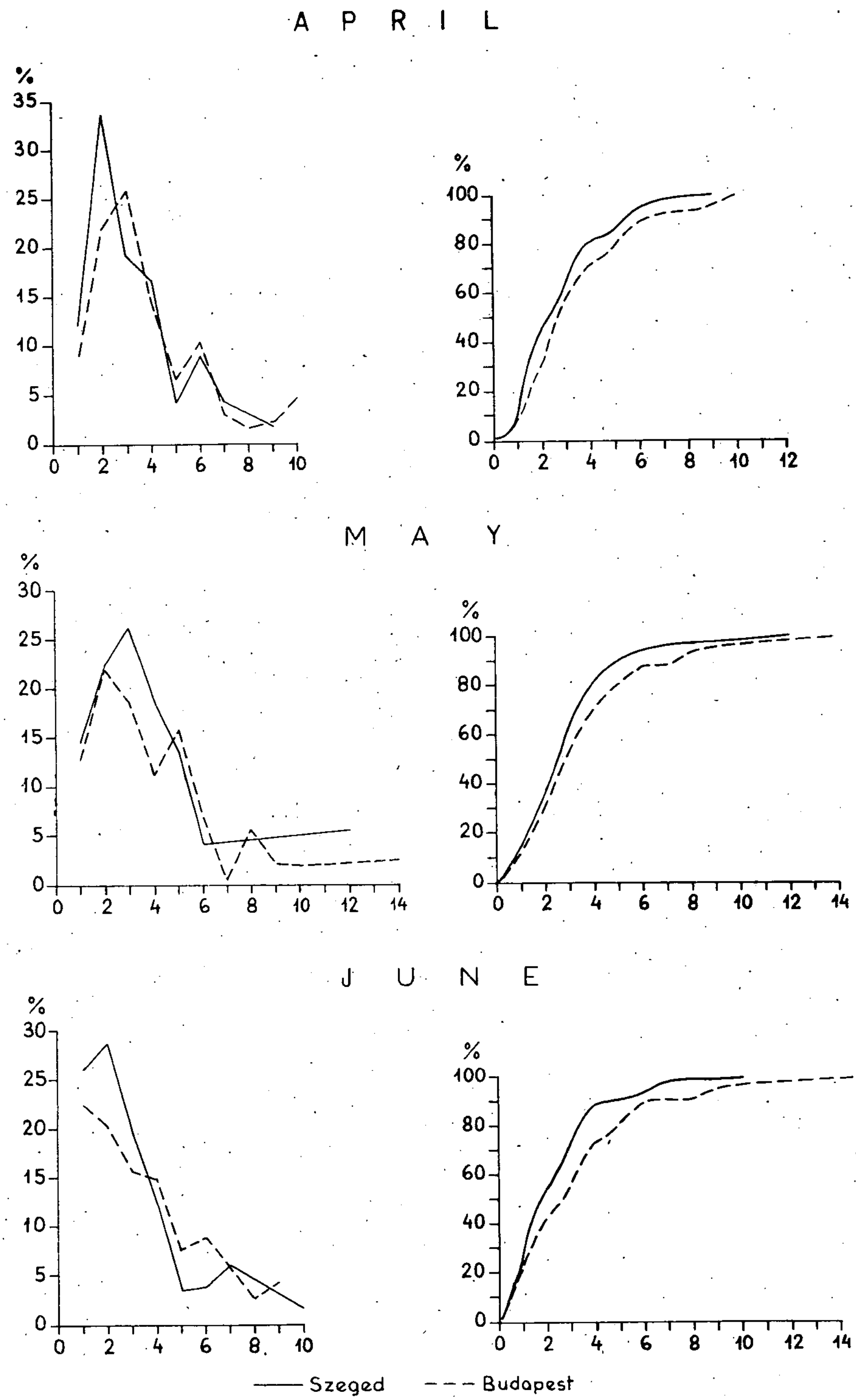

Fig. 5. The division of monthly precipitation between the periods in April, May and June
5. ábra. A havi csapadék megoszlása a periódusok között áprilisban, májusban és júni usban

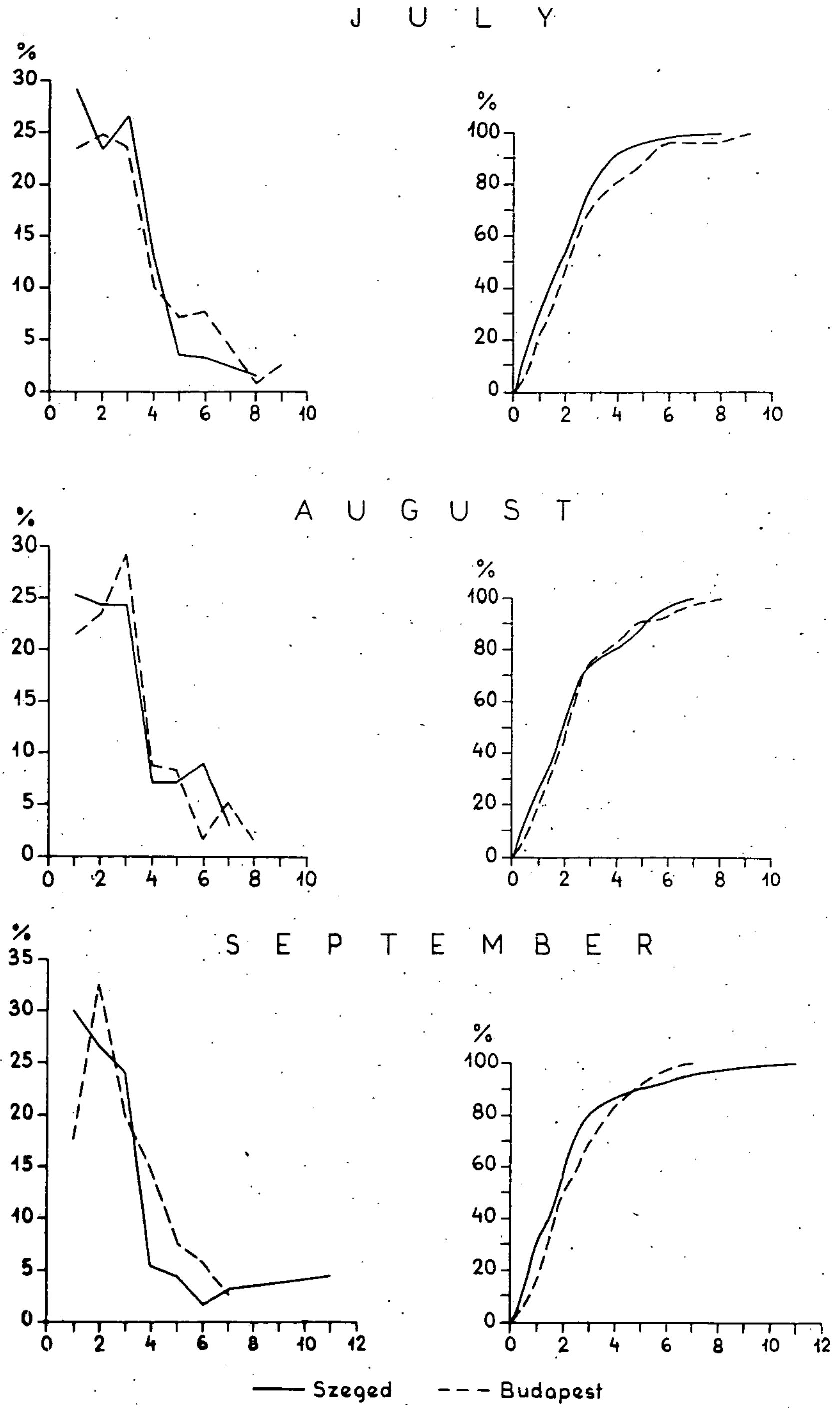

Fig. 6. The division of monthly precipitation between the periods in July, August and September
6. ábra. A havi csapadék megoszlása a periódusok között júliusban, augusztusban és szeptemberben

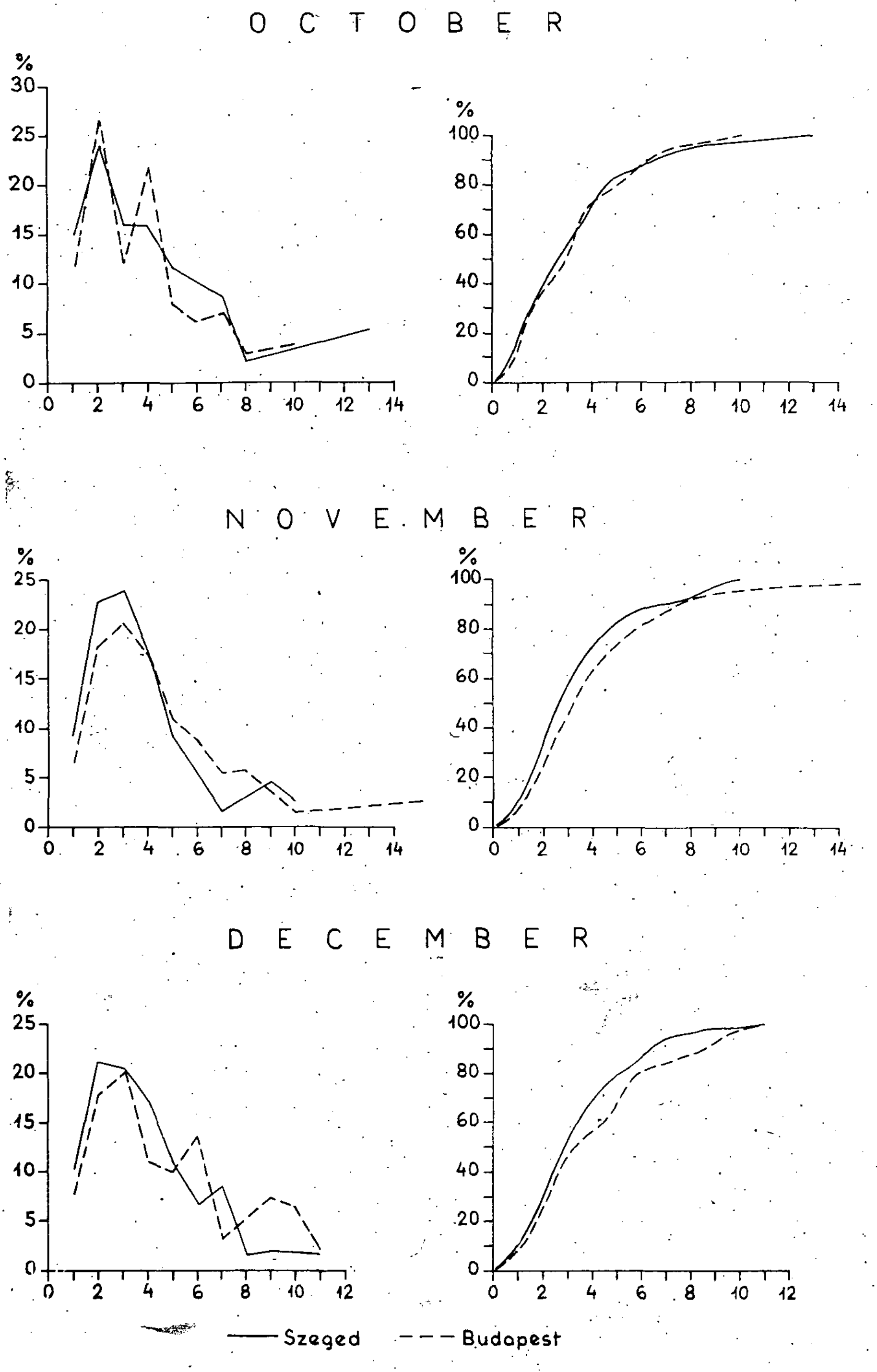

Fig. 7. The division of monthly precipitation between the periods in October, November and December

7. ábra. A havi csapadék megoszlása a periódusok között októberben, novemberben és decemberben is reflected in per cents. So it can be said that the greatest part of monthly mean precipitation amounts comes from rainy periods shorter than four days in every month. Considering the amount of precipitation for every period it can be seen that the difference from the equalizing curve is less than 3 mm in each period. In case of longer periods the differences are greater. So we are able to define the precipitation amount of periods of 2—3 days by calculation from the mean value of one day's amount quite exactly, but estimations are possible to be given for longer periods as well. (It is possible only if the value of Q is nearly the same for the whole country, and so the treatment of the data of more stations is needed.)

The reliability of this computation is to some extent decreased by the standard deviation of real data. Their periodicity can be examined, in order to make uncertainty decrease, for different amounts of precipitation.

Fig. 4—7 shows the distribution of monthly mean precipitation amounts among periods of different length. Figures on the left show how many per cents of the monthly precipitation amounts are given by the different periods, while those on the right show how many per cents of the monthly precipitation are given by periods not longer than a given value.

It is seen that more than 30 per cent of the monthly total is given only in two cases by a period of fixed length. Even the number of periods giving more than 25 per cent is small. So most of the monthly amounts of precipitation is given by periods of 10—25 per cent.

Regarding the whole year a change in favour of the short periods can be noticed in the summer months.

Every diagram shows the longest period giving more than 10 per cent of the monthly precipitation, but the 5 days' periods are generally of less importance.

Comparing the two stations it can be stated that in the case of Szeged with a more arid climate the greatest part of the monthly precipitation amount is given by shorter (2—3—4 days) periods. In the case of Budapest the longer periods have the greater importance. This shown by the diagrams on the right, since the curves of Szeged data are above the Budapest ones, they move off each other until the periods of 2—3—4 days, then move closer. Less difference can be noticed in January, February, October and August.

Table 1.

The Number of Periods Occured in Budapest
(1931—1970)

Length of the Period (Day)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Other
January	*109*	*62*	*21*	*27*	*13*	*1*	*4*	*1*	*1*	*1*						
	109	124	63	108	65	6	28	8	9	10						
February	*84*	*53*	*33*	*17*	*7*	*6*	*2*	*2*	*2*							
	84	106	99	68	35	36	14	16	18							
March	*99*	*51*	*23*	*14*	*11*	*6*	*2*	*2*	*1*							
	99	102	69	56	55	36	14	16	9							
April	*94*	*60*	*31*	*12*	*8*	*5*	*2*	*1*	*1*	*1*						
	94	120	93	48	40	30	14	8	9	10						
May	*103*	*61*	*30*	*19*	*12*	*6*	*1*	*3*	*1*	*1*				*1*		
	103	122	90	76	60	36	7	24	9	10				14		
June	*110*	*53*	*33*	*15*	*9*	*6*		*1*	*2*						*1*	
	110	106	99	60	45	36		8	18						15	
July	*118*	*51*	*28*	*10*	*6*	*3*		*1*	*1*							
	118	102	84	40	30	18		8	9							
August	*123*	*56*	*27*	*7*	*5*	*1*	*2*	*1*								
	123	112	81	28	25	6	14	8								
September	*87*	*47*	*18*	*9*	*5*	*2*	*1*									
	87	94	54	36	25	12	7									
October	*67*	*46*	*18*	*15*	*6*	*5*	*2*	*2*		*1*						
	67	92	54	60	30	30	14	16		10						
November	*83*	*58*	*34*	*25*	*8*	*8*	*4*	*4*		*1*						(20) 1
	83	116	102	100	40	48	28	32		10						20
December	*87*	*62*	*29*	*16*	*10*	*13*	*3*		*3*	*2*	*1*					
	87	124	87	64	50	78	21		27	20	11					

The number of the wanted length periods
The number of the days being in the wanted length and wanted number periods

Table 2

The Number of Periods Occured in Szeged
(1931—1970)

Length of the Period (Day)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Other
January	*93*	*61*	*23*	*18*	*9*	*3*	*3*	*1*	*1*							
	93	122	69	72	45	18	21	8	9							
February	*85*	*56*	*24*	*15*	*8*	*6*	*3*		*1*	*2*						
	85	112	72	60	40	36	21		9	10						
March	*93*	*56*	*21*	*18*	*10*	*3*	*2*				*1*					
	93	112	63	72	50	18	14				11					
April	*79*	*58*	*24*	*20*	*3*	*8*	*2*		*1*							
	79	116	72	80	15	48	14		9							
May	*92*	*66*	*30*	*22*	*9*	*3*						*1*				
	92	132	90	88	45	18						12				
June	*113*	*64*	*24*	*14*	*3*	*4*	*2*			*1*						
	113	128	72	56	15	24	14			10						
July	*120*	*42*	*18*	*8*	*2*	*3*		*1*								
	120	84	54	48	10	18		8								
August	111	36	21	5	3	4	1									
	111	72	63	20	15	24	7									
September	*90*	*36*	*20*	*6*	*2*	*1*	*1*				*1*					
	90	72	60	24	10	6	7				11					
Oktober	*69*	*43*	*19*	*11*	*7*		*3*	*1*					*1*			
	69	86	57	44	35		21	8					13			
November	*74*	*54*	*27*	*22*	*12*	*5*	*1*	*1*	*2*	*1*						
	74	108	81	88	60	30	7	8	18	10						
December	*93*	*55*	*25*	*16*	*10*	*7*	*3*	*1*	*2*		*1*					
	93	110	75	64	50	42	21	8	18		11					

The number of the wanted length periods
The number of the days being in the wanted length and wanted number periods

Table 3

The Monthly Values of $r(k)$ in Budapest

Length of the Period (Day)	J	F	M	A	M	J	J	A	S	O	N	D
1	186	171	193	185	184	181	192	192	177	190	169	180
2	76	68	67	105	77	73	60	60	45	55	78	79
3	31	27	24	25	33	29	19	19	10	16	36	35
4	12	11	8	9	14	12	6	6	3	5	17	15
5	5	4	3	4	6	5	2	2	1	1	8	7
6	2	2	1	1	2	2	1	1	(0,17)	(0,39)	4	3
7	1	1	(0,35)	(0,4)	1	1	(0,17)	(0,17)	(0,04)	(0,11)	2	1
8	(0,36)	(0,28)	(0,12)	(0,17)	(0,4)	(0,29)	(0,05)	(0,05)	(0,01)	(0,04)	1	1
9	(0,14)	(0,11)	(0,04)	(0,07)	(0,17)	(0,12)	(0,01)	(0,01)	(0,002)	(0,009)	(0,34)	(0,25)

Table 4

The Monthly Values of $r(k)$ in Szeged

Length of the Period (Day)	J	F	M	A	M	J	J	A	S	O	N	D
1	183	163	183	183	180	183	177	175	164	178	173	173
2	66	65	64	66	70	66	46	44	38	48	69	69
3	24	26	22	24	27	24	12	10	9	13	28	28
4	9	10	8	9	11	9	3	3	2	4	11	11
5	3	4	3	3	4	3	1	1	(0,5)	1	4	4
6	1	2	1	1	2	1	(0,2)	(0,17)	(0,11)	(0,26)	2	2
7	(0,5)	1	(0,4)	(0,5)	1	(0,5)	(0,05)	(0,04)	(0,02)	(0,07)	1	1
8	(0,12)	(0,2)	(0,1)	(0,12)	(0,2)	(0,12)	(0,01)	(0,01)	(0,006)	(0,02)	(0,28)	(0,28)
9	(0,05)	(0,1)	(0,05)	(0,05)	(0,1)	(0,05)	(0,003)	(0,003)	(0,001)	(0,006)	(0,13)	(0,13)

Table 5

The Values of A; A^*; A^*/A and the Limits of the Interval of Independence (I)

Budapest	J	F	M	A	M	J	J	A	S	O	N	D
A	1,694	1,666	1,538	1,588	1,724	1,666	1,450	1,450	1,333	1,408	1,852	1,785
A^*	2,209	2,311	2,172	2,168	2,326	2,161	1,876	1,788	1,864	2,302	2,563	2,518
A^*/A	1,303	1,387	1,411	1,366	1,349	1,293	1,294	1,234	1,398	1,635	1,384	1,410
I	0,9354	0,9337	0,9563	0,9391	0,9353	0,9354	0,9473	0,9473	0,9537	0,9497	0,9255	0,9313
	−1,074	−1,076	−1,065	−1,069	−1,076	−1,074	−1,059	−1,059	−1,052	−1,056	−1,084	−1,082

Szeged	J	F	M	A	M	J	JL	AU	S	O	N	D
A	1,563	1,666	1,538	1,563	1,640	1,538	1,352	1,333	1,299	1,370	1,666	1,666
A^*	2,156	2,275	2,122	2,221	2,139	1,920	1,681	1,724	1,783	2,162	2,432	2,310
A^*/A	1,400	1,366	1,400	1,421	1,305	1,248	1,244	1,293	1,374	1,579	1,459	1,387
I	0,9290	0,9322	0,9456	0,9391	0,9361	0,9822	0,9541	0,9532	0,9550	0,9506	0,9342	0,9396
	−1,082	−1,078	−1,067	−1,069	−1,074	−1,069	−1,053	−1,052	−1,050	−1,054	−1,076	−1,069

Table 6

The Values of c_1 and α

Budapest	J	F	M	A	M	J	J	A	S	O	N	D
$c_1\,\alpha$ (mm)	3,2	3,8	3,6	4,3	5,4	6,3	5,7	5,4	4,9	5,0	5,2	3,8
α	1,0225	0,9995	1,0427	1,0204	1,0922	0,9834	0,0095	1,1672	1,0599	1,0443	1,1391	1,0405

Szeged	J	F	M	A	M	J	J	A	S	O	N	D
c_1 (mm)	3,0	3,1	3,2	3,9	4,9	5,8	5,6	6,1	5,6	4,7	4,5	3,4
α	1,0037	1,0361	1,1107	1,0604	1,0401	1,0122	1,1637	1,1553	1,0250	1,0550	0,9730	1,1074

Table 7

The Values of Q

Budapest

Length of the Period (Day)	J	F	M	A	M	J	J	A	S	O	N	D
1	1	1	1	1	1	1	1	1	1	1	1	1
2	0,9867	1,0005	0,9727	0,9867	1	1,0142	0,9995	0,8889	0,9592	0,9728	0,9078	0,9727
3	0,9785	1,0006	0,9569	0,9785	0,9061	1,0227	0,9990	0,8297	0,9363	0,9569	0,8619	0,9569
4	0,9725	1,0007	0,9459	0,9725	0,8826	1,0277	0,9985	0,7899	0,9202	0,9459	0,8302	0,9459
5	0,9682	1,0008	0,9376	0,9683	0,8651	1,0326	0,9984	0,7607	0,9081	0,9376	0,7982	0,9376

Szeged

Length of the Period (Day)	J	F	M	A	M	J	J	A	S	O	N	D
1	1	1	1	1	1	1	1	1	1	1	1	1
2	0,9995	0,9727	0,9267	0,9592	0,9728	0,9936	0,8949	0,8948	0,9828	0,9624	1,0188	0,9268
3	0,9990	0,9569	0,8863	0,9363	0,9569	0,9891	0,8389	0,8389	0,9734	0,9416	1,0302	0,8863
4	0,9985	0,9458	0,8585	0,9202	0,9459	0,9864	0,8011	0,8011	0,9655	0,9266	1,0384	0,8585
5	0,9984	0,9376	0,8377	0,9081	0,9376	0,9841	0,7730	0,7730	0,9604	0,9151	1,0443	0,8376

Table 8

The Values of p and q in the Year (%)

		J	F	M	A	M	J	J	A	S	O	N	D
Budapest	p	41	40	35	37	42	40	31	31	25	29	46	44
	q	59	60	65	63	58	60	69	69	75	71	54	54
Szeged	p	36	40	35	36	39	36	26	25	23	27	40	40
	q	64	60	65	64	61	64	74	75	77	73	60	60

Reference

[1] *Péczely, G.:* Tartamvalószínűségek vizsgálata. (The Examination of the Probability of Durations) Időjárás. 61, № 4, pp. 241—244. 1957.