An Analysis of the Water Supplies of the Water System Danube — Tisza

by

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A Duna—Tisza vízrendszer vízjárásának analízise. A dolgozat a Duna és a Tisza vízhozamának és Magyarország csapadékmennyiségének változásait elemzi az 1915—1972 közötti adatok alapján. Trendszámítással megállapítják a szerzők, hogy az utóbbi hatvan évben csökkenő tendencia mutatkozik az évi közepes és évi maximális vízállások nagyságában és a csapadék mennyiségében. Harmonikus analízis segítségével több periódust mutatnak ki a vízállás és a csapadék adatsoraiban. Ezek közül a 13 éves periódus a legjellemzőbb.

Analyse des Regimes des Stromsystems Donau—Theiss. Der Aufsatz analysiert auf Grund der Angaben zwischen 1915—1972 den Wandel der Niederschlagsmengen Ungarns und die Wasserergiebigkeit der Donau und der Theiss. Mit Hilfe von Trendrechnung stellen die Verfasser fest, dass sich in den letzten sechzig Jahren in der Grösse der jährlichen mittleren, beziehungsweise in der Grösse der jährlichen maximalen Wasserstände und in der Menge des Niederschlags fallende Tendenz erweist. In den Agabenreihen des Wasserstandes und des Niederschlags wurde mit harmonischer Analyse mehrere Perioden ausgewiesen. Unter diesen Perioden ist die dreizehnjährige Periode am meisten charakteristisch.

The paper analyses the change of the water output of the Danube and Tisza and the amount of precipitation in Hungary, on the basis of the data between 1915—1972. It is established with trend calculation that in the last sixty years the height of the annual mean and annual maximum of water level and the amount of precipitation shows downward tendency. With the help of harmonic analysis many periods are revealed within the data of water-level and precipitation. Among them the most characteristic is the period of 13 years.

This study deals with the statistical analysis of the connection between water-level series and precipitation amount. Our goal is to answer the question what changes in water output and precipitation data can be traced in the last 60 years.

First we made some trend calculations, then tried to answer the question kind of periods can be noticed in the data by structural analysis of water-level series.

Data used are as follows: annual mean water-level for the years 1915—1972, for two stations on the Tisza (Vásárosnamény, Szeged), three on the Danube (Komárom, Budapest, Mohács) and one on the Rába (Árpás; annual maximal waterlevel for one station on the Danube (Budapest) and one on the Tisza (Szeged). No longer homogenous series of data was available, since old data cannot be taken homogenous because of riverbed controls, changes in the nul point of water-gauge and other human interventions.

Some precipitation series were treated as well. For the sake of comparability here also the 1915—1972 means were taken. The series were as follows: The mean precipitation amounts of Hungary for the civil year, the hidrological year (1th Oct.—30th Sept.), the summer half (1st April—30th Sept.) and the winter half (1st Oct—31st. March) of the year. The total series refer to the period 1870—1973.

Generally in time series some unbroken change can be traced. Our aim is to detect some tendency in the series available. To decide this we posses some methods for trend calculation. If the results are made as diagrams then they become more clear and easily comparable.

Since the time series equally contain random and regular values, the trend for a given period exists in a hidden form. For the sake of more simple evaluation we use first degree trends only. The results support the supposition that these series can be characterized by linear trends.

The computations themselves were done according to two different methods. Both treatments are based on the method of the smallest squares. First the essence of the analytical trend calculating method is described. The task is to define the equation of the straight the values of which and the real values of the series belonging to the same point of time differ only slightly. The values of the series are represented by the sign y, those of the trend line by y'. The equation of the trend is: $y=ax+b$. The requirement of the method of smallest squares is: $(y-y')^2=minimum$. The co-efficients can be defined from the following equations:

$$\begin{aligned} \sum y &= a\sum x+nb \\ \sum xy &= b\sum x+a\sum x^2 \end{aligned} \tag{1}$$

where n is the length of the time series, x is the sign for points of time, y represents the values belonging to the x values. From these equations the values of a and b are:

$$a = \frac{n\sum xy - \sum x\sum y}{n\sum x^2-(\sum x)^2} \qquad b = \frac{\sum y-a\sum x}{n} \tag{2}$$

The second method is equivalent with the first. The equation of the regression line is done with the help of ortogonal polinoms. This method is somewhat more complicated than the first one but it can be used for trends of higher degree more easily if needed. This facility can be used mainly in case of computer treatment. The calculations done with either of the two methods had the same result. The results are presented in the following table:

The trends of annual water-level for the 58 years of the period 1915—1972 (cm/year)

Mean water-levels		Maximum water-levels	
Komárom	−0,323	Budapest	−0,732
Budapest	−1,083	Szeged	−0,105
Mohács	−1,352		
Vásárosnamény	−1,002		
Szeged	−0,41		
Árpás	−0,351		

Together with water-level series precipitation series are treated as well, regarding that water supplies are mainly governed by precipitation so it must not be neglected. Naturally a too rigorous connection is not expected, because other natural phenomena get role in the formation of water supplies. For most cases it holds that the change of the one causes a similar change in the other factor. We had a precipitation series of 103 years (1871—1973).

Trends of mean precipitation amounts in Hungary mm/year

	1915—1972	1871—1973
civil year	−0,872	−0,819
hydrological year	−0,238	−0,749
winter half year	−0,300	−0,204
summer half year	−0,061	−0,559

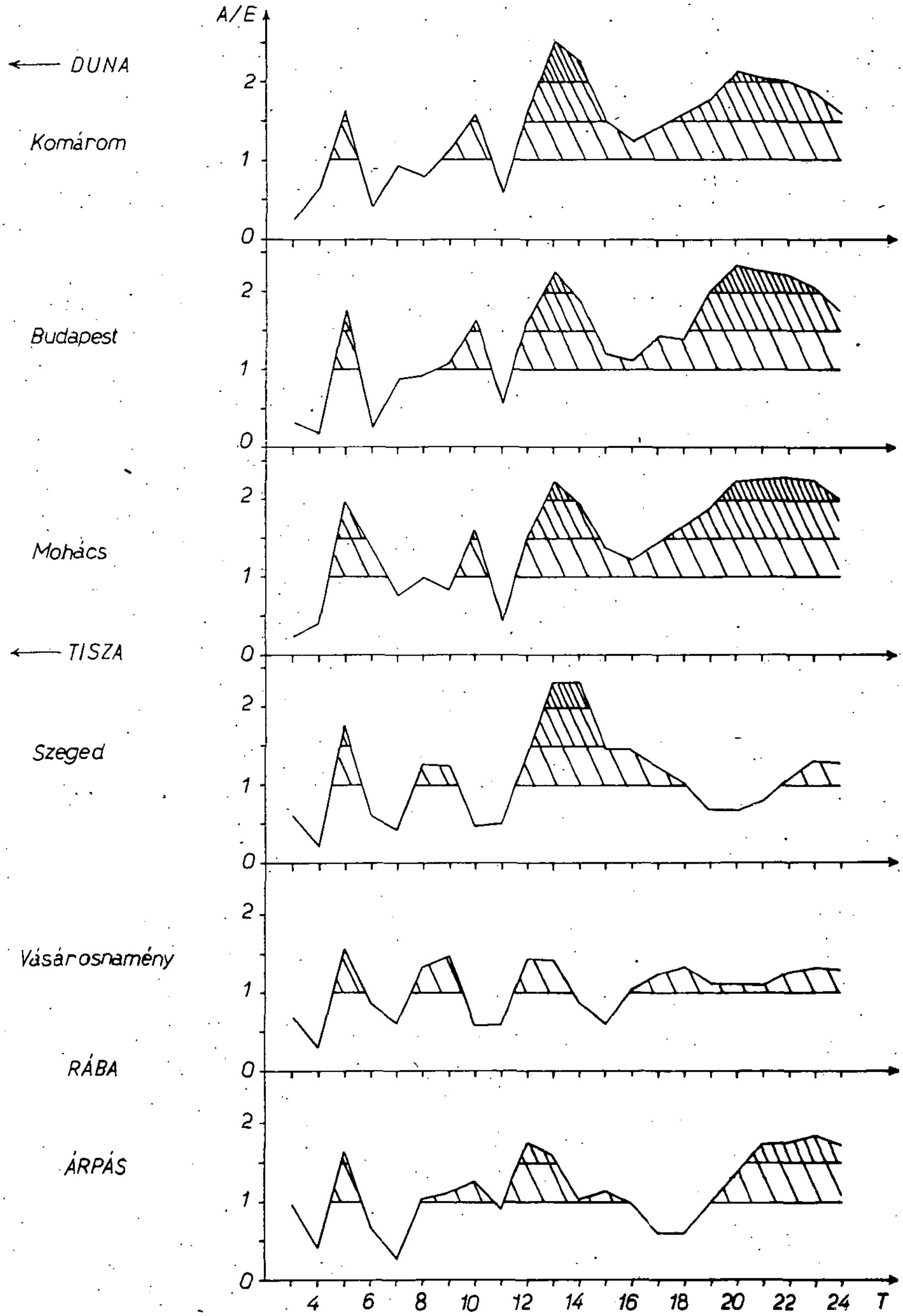

Fig. 1. The periods of mean annual water level
1. ábra. Az évi közepes vízállás periodogramjai

The results of the computations are presented on diagrams. *(Fig. 1.)* The curves represent the precipitation and watersupply series. $\bar{Y}$ is the mean of 58 years, Y' is the trend line. The scale of the vertical axis is 25 cm and 25 mm depending on whether water-level or precipitation is represented. The time series has a decreasing tendency both on the Danube and the Tisza. This negative trend is not continuous, but shows an oscillation around the mean. From the stations above the greatest trends occur with Budapest and Mohács, the smallest ones with Komárom and Árpás. The difference between the greatest and the smallest trend is only a bit greater than 1. The course of the data referring to the Danube is much the same with every station, while this does not hold for the ones on the Tisza. The cause of this is that the mean water supply of the Danube is greater and that the water supplies of the smaller rivers have a greater oscillation extreme values are more often observed. In the examination of periodicity the smoother results are considered realistic. In the case of Oscillating lines random ness has a greater part. Precipitation series also showed a negative trend. That of the civil year is greater and the trend of the summer half year may be taken 0. The negative trend of precipitation series agrees with the change of water supplies.

Further on the examination ao periodicity of water-level series follows.

Periodicity means repetition, periodical process is encountered when there are regular periods during its course. Mathematically periodicity means that $f(x+a)=f(x)$ for every x in case of a functional connection $y=f(x)$ between two quantities. Here a is a constant. When a is the smallest number for which this requirement holds, it is called the period of the function.

In nature periodical processes are often encountered. Such are the change of days and nights and the dayly and yearly course of the different meteorological and climatological phenomena. There are hidden kinds of periodicity as well. In case of some processes the inclination for periodicity cannot be recognized because its close connection with its surroundings and the surrounding affects its course. In such cases random factors disturb periodicity. Such random factor is the affect of annual amount of precipitation in the water supply of a river.

For the demonstration of periodicity in a time series two methods are known:

— the method of autocorrelation

— harmonical analysis

The essence of the method of autocorrelation is that co-efficients and functions of the connection between two quantities are produced (or between the members of two series). The figure for correlation co-efficient:

$$R(\xi\eta) = \frac{M\{[\xi - M(\xi)][\eta - M(\eta)]\}}{D(\xi)\,D(\eta)} \tag{3}$$

where ξ and η are optional probability variables. It is used to characterize the connection between two or more probability variables. The value of R changes between 1 and -1 and the better it differs from 0 the closer is the connection (It its value is near to 0 the probability variable is considered independent.) The autocorrelation co-efficient is:

$$v_z = \frac{\sum_{i=1}^{n}(x_i-\bar{x})\,(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\,\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

It refers to a Z variable from the $z_1 \ldots z_m$ series to give the connection between members in d distance from each other. Here x_1 and x_m represent the first m member of the sample referring to z. $\bar{x}$ and $\bar{y}$ give the mathematical mean of the samples. The sequence of the autocorrelation co-efficients is called the autocorrelation function of the Z variable. In practice the sequence is given by a continuous curve. So the autocorrelation co-efficients between the mean water supplies of single years can be calculated in the case of any sections and also the autocorrelation function understood as the sequence of the first. The course of the autocorrelation function may demonstrate periodicity.

The harmonical analysis means the determination of the constants of the following equation:

$$y = A \cdot \sin\left(\frac{2\pi}{T} \cdot x + U\right)$$

where A is the amplitude, T is the period, x is the time, U is the phase angle.

Further it must be decided how to evaluate the period got, regarding that random periods may be present too. These have no real physical basis. For these the ratio A/E is used where A is the amplitude, E is the expectation

$$E = \sigma \sqrt{\frac{\pi}{N}}$$

(where σ is the standard deviation, N is the number of the members of the series). Standard deviation is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{N}}$$

where $\bar{x}$ is the mean of x_i values. If the amplitude is expressed in ratio of the expectation the ratio A/E shows the P probability of having the amplitude resulted from the randomness of data in the following way:

$A/E=k=0.5$	$P=0.8217$
1.0	0.4559
1.5	0.1708
2.0	0.0432
2.5	0.0074
3.0	0.0009

If the value of A/E is great enough the probability of random arrangement of data is slight. For such values the reality of the period can be taken real. Generally $A/E>2$ is acceptable but here the values $A/E>1.5$ and $A/E>1$ were considered as well.

The examination of periodicity in water supplies of the river-system Danube-Tisza the method of harmonical analysis was adopted. The method of autocorrelation requires less computation than that of autocorrelation and it can be used for the examination of periodicity but it does not give any information about the temporal positions of the periods but only about their lenght . This disadvantage is overcome with the harmonical analysis together the advantage that the basic series can be reconstructed from the characteristic waves. Even it can be used for extrapolation. This forecast must be handled however with cautiousness.

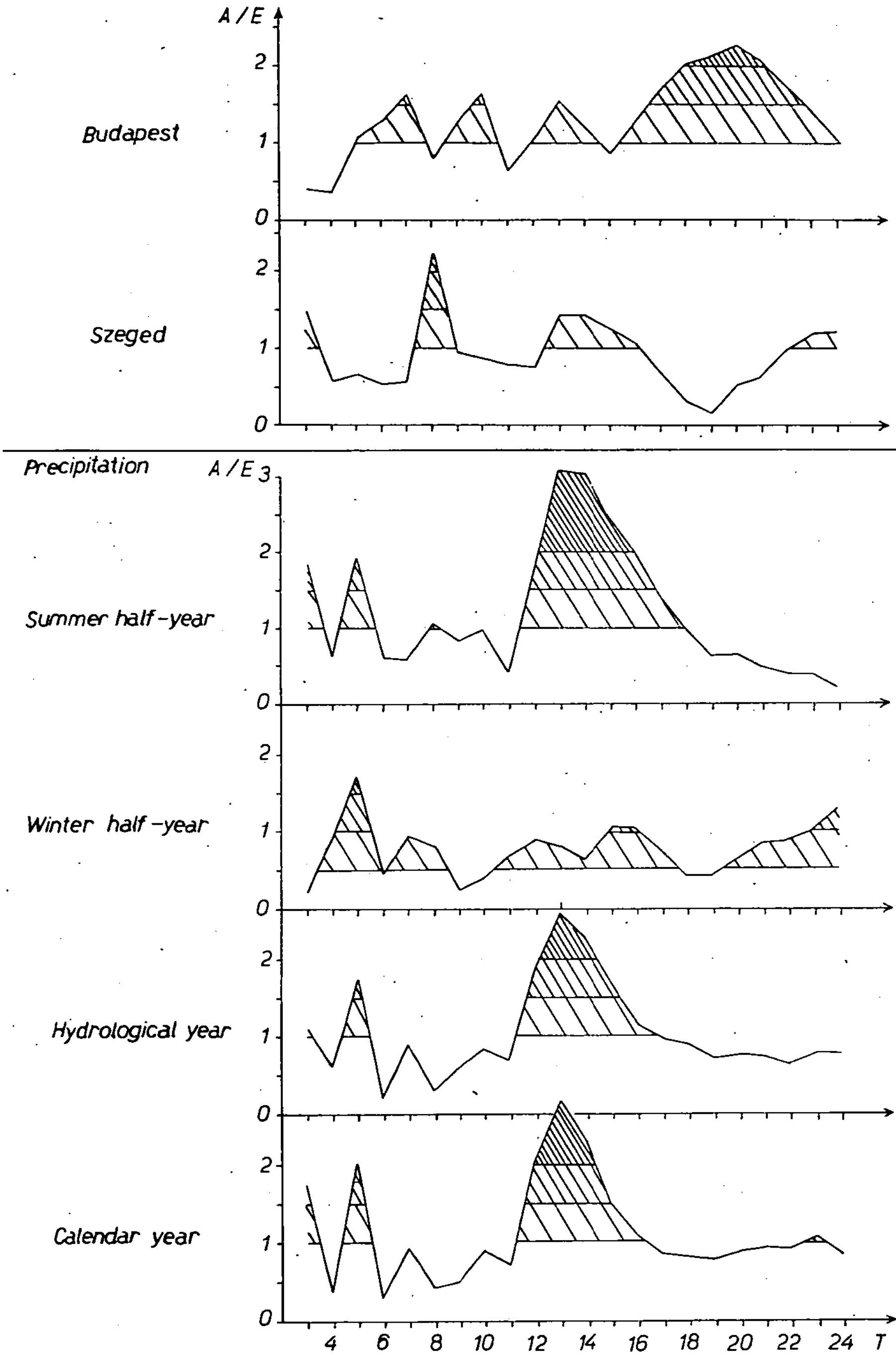

Fig. 2. The periods of yearly maximum of water level and the periods of precipitation

2. ábra. Az évi maximális vízállás valamint a nyári félév, a téli félév, a hidrológiai év és a naptári év csapadékának periodogramjai

In the present examination $T=3-24$ periods were supposed. The results are presented in *Tables I—III* and in *Fig. 1—2.* It can be stated that real period in water-level can be shown for 13 and 20—22 years. This result can be reached even with autocorrelation method. Good accordance can be seen for the stations on the Danube, not so good for the Tisza ones in periodicity. The difference can be explained by the fact that the Danube has no affluents in the section examined, while the Tisza does. These (Bodrog, Sajó, Zagyva, Körösök, Maros) come from territories with different amounts of precipitation. A slight but definite period is the 5 year one. The maximum water-levels have a quite different curve: the Danube has a period of 18—21 years, the Tisza one of 8 years. This result is not unexpected considering that maximum water-levels are random events.

The period in precipitation amount of Hungary for 13—15 years is very striking. Only the winter half year is excluded. This period coincides greatly with that in water supplies (13 years). It also can be stated that the curve of water supplies is not affected by the temporal position of the most precipitation.

Table I

The Trends of Annual Mean Water-Level
The Constant Values of the Following Equation:

$$y = A \sin\left(\frac{2\pi}{T}x + U\right)$$

T	Komárom U	Komárom A	Komárom A/E	Budapest U	Budapest A	Budapest A/E	Mohács U	Mohács A	Mohács A/E
1	327,5	2,6	0,2	332,6	4,4	0,3	346,6	4,1	0,2
2	30,2	3,8	0,4	24,4	2,7	0,2	34,8	6,7	0,4
3	58,0	18,0	1,7	55,0	22,7	1,8	52,6	32,3	1,9
4	194,3	4,6	0,4	210,3	3,3	0,3	172,7	2,4	0,1
5	342,4	9,9	0,9	331,2	11,6	0,9	347,6	12,4	0,8
6	16,0	8,5	0,8	12,8	12,2	1,0	24,5	16,5	1,0
7	198,8	12,2	1,2	198,3	12,4	1,1	212,0	14,1	0,9
8	45,6	17,0	1,6	43,4	21,0	1,6	49,9	26,0	1,6
	208,5	6,1	0,6	217,2	7,4	0,6	212,1	7,3	0,4
	30,6	17,1	1,6	27,2	21,2	1,6	35,7	25,2	1,5
3	115,6	26,5	2,5	10,9	29,0	2,2	113,6	36,8	2,2
4	173,6	23,8	2,3	169,1	24,2	1,9	173,6	32,0	1,9
15	221,0	15,8	1,5	222,1	15,7	1,2	231,7	22,5	1,4
16	244,1	13,3	1,3	248,1	14,6	1,1	263,7	20,0	1,2
17	269,2	15,1	1,4	274,2	18,7	1,4	287,0	23,4	1,4
18	293,7	16,8	1,6	301,5	22,1	1,7	311,5	27,1	1,6
19	319,9	19,1	1,8	326,9	25,7	2,0	333,5	31,0	1,9
20	340,5	22,6	2,1	345,7	30,1	2,3	349,6	36,5	2,2
21	11,0	21,6	2,1	13,1	29,4	2,3	16,1	37,1	2,3
22	36,4	21,0	2,0	35,2	28,6	2,2	36,6	37,6	2,3
23	58,6	19,7	1,9	54,3	26,6	2,1	55,6	36,8	2,2
24	77,2	16,8	1,6	69,1	22,4	1,7	71,5	32,9	2,0

T	Vásárosnamény			Szeged			Árpás		
	U	A	A/E	U	A	A/E	U	A	A/E
3	302,0	10,1	0,1	248,4	14,9	0,9	80,7	6,2	1,0
4	281,7	4,2	0,3	9,6	8,2	0,3	179,8	2,5	0,4
5	56,9	22,5	1,6	56,3	42,8	1,7	59,7	10,1	1,6
6	83,2	12,5	0,9	81,1	15,0	0,6	342,2	4,1	0,7
7	216,2	8,7	0,6	232,2	11,1	0,4	44,1	1,5	0,2
8	26,4	19,6	1,4	27,4	31,3	1,3	345,5	6,3	1,0
9	162,3	21,4	1,5	147,6	30,5	1,2	334,7	7,0	1,1
10	306,2	8,5	0,6	20,0	11,6	0,5	86,6	7,8	1,3
11	261,0	8,6	0,6	202,9	12,6	0,5	315,2	5,5	0,9
12	337,5	20,8	1,4	352,5	34,1	1,4	50,7	10,9	1,8
13	54,2	20,5	1,4	80,4	57,0	2,3	132,5	9,8	1,6
14	127,1	12,5	0,9	146,2	57,7	2,3	218,6	6,4	1,0
15	248,3	8,7	0,6	211,6	42,3	1,7	305,4	6,9	1,1
16	300,9	14,9	1,0	268,2	36,8	1,5	348,0	6,1	1,0
17	334,4	17,8	1,2	313,9	31,0	1,2	19,9	3,7	0,6
18	0,7	19,2	1,3	351,2	25,5	1,0	338,1	3,7	0,7
19	12,9	16,2	1,1	11,4	17,1	0,7	348,4	6,1	0,9
20	8,0	15,9	1,1	2,2	17,1	0,7	5,3	8,5	1,4
21	17,1	16,2	1,1	14,4	20,1	0,8	21,6	10,6	1,7
22	25,8	18,3	1,3	28,8	27,0	1,1	40,3	10,8	1,8
23	32,8	19,2	1,3	45,9	32,6	1,3	60,4	11,4	1,9
24	36,6	18,7	1,3	57,4	32,0	1,3	79,1	10,6	1,7

Table II

The Trends of Annual Maximum Water-Level
The Constant Values of the Following Equation

$$y = A \sin\left(\frac{2\pi}{T}x + U\right)$$

T	Budapest			Szeged		
	U	A	A/E	U	A	A/E
3	186,5	8,9	0,4	276,5	52,5	1,5
4	346,1	8,1	0,4	13,2	20,6	0,6
5	132,2	23,8	1,1	6,5	23,7	0,7
6	269,8	28,7	1,3	89,0	19,1	0,6
7	317,8	36,2	1,6	268,2	20,2	0,6
8	16,8	17,6	0,8	38,3	77,5	2,2
9	241,2	28,6	1,3	141,3	33,9	1,0
10	51,9	37,1	1,7	120,3	31,6	0,9
11	241,6	14,4	0,6	225,1	27,5	0,8
12	30,2	23,6	1,1	22,8	26,3	0,8
13	121,2	34,9	1,6	120,5	49,4	1,4
14	176,0	27,4	1,2	190,0	49,4	1,4
15	191,0	19,7	0,9	264,3	44,0	1,3
16	204,7	28,7	1,3	313,8	38,2	1,1
17	233,9	38,4	1,7	351,8	24,7	0,7
18	264,1	45,9	2,1	34,0	11,7	0,3
19	292,9	48,1	2,2	276,9	5,6	0,2
20	317,3	51,4	2,3	301,8	18,9	0,5
21	339,0	46,0	2,1	331,6	22,5	0,7
22	2,4	38,2	1,7	4,9	34,0	1,0
23	23,4	31,6	1,4	26,8	40,5	1,2
24	22,3	23,5	1,1	45,1	42,5	1,2

Table III
Precipitation
The Constant Values of the Following Equation:

$$y = A \sin\left(\frac{2\pi}{T}x + U\right)$$

T	Summer Half Year			Winter Half Year		
	U	A	A/E	U	A	A/E
3	53,1	31,1	1,8	208,5	3,1	0,2
4	327,1	11,0	0,6	80,1	14,5	0,9
5	115,6	32,5	1,9	27,6	26,6	1,7
6	219,4	10,7	0,6	54,3	6,9	0,5
7	318,2	10,2	0,6	12,4	14,8	1,0
8	5,8	18,0	1,1	166,7	12,9	0,8
9	243,1	14,1	0,8	325,8	4,2	0,3
10	73,4	17,1	1,0	121,2	6,2	0,4
11	307,0	7,1	0,4	291,1	10,6	0,7
12	59,9	31,4	1,9	33,4	14,8	1,0
13	124,2	52,1	3,1	137,0	12,8	0,8
14	181,2	51,4	3,0	250,7	9,7	0,6
15	231,2	40,7	2,4	328,2	16,3	1,1
16	273,3	33,5	2,0	22,5	16,0	1,0

T	Summer Half Year			Winter Half Year		
	U	A	A/E	U	A	A/E
17	315,7	23,7	1,4	60,0	11,6	0,8
18	353,5	17,3	1,0	46,2	6,7	0,4
19	27,7	11,0	0,7	29,2	6,7	0,4
20	82,8	11,6	0,7	25,5	10,3	0,7
21	101,0	8,2	0,5	40,0	13,1	0,9
22	132,6	6,9	0,4	58,2	13,5	0,9
23	137,9	7,0	0,4	71,4	15,9	1,0
24	193,3	3,9	0,2	90,2	20,1	1,3

T	Hydrological Year			Calendar Year		
	U	A	A/E	U	A	A/E
3	55,7	28,3	1,1	38,0	40,4	1,8
4	36,4	14,8	0,3	28,0	8,1	0,4
5	77,2	42,7	1,3	103,5	46,2	2,0
6	196,0	4,4	0,2	183,4	8,9	0,4
7	349,2	22,5	0,9	26,1	21,5	0,9
8	41,5	7,2	0,3	14,0	10,0	0,4
9	259,1	15,9	0,6	254,7	11,1	0,5
10	85,6	21,8	0,9	83,1	21,7	1,0
11	297,5	17,6	0,7	317,4	16,4	0,7
12	51,5	45,1	1,8	60,6	48,2	2,1
13	126,8	64,6	2,6	129,5	64,3	2,8
14	190,6	55,6	2,3	191,3	52,6	2,3
15	254,0	41,3	1,7	249,0	33,2	1,5
16	301,2	32,7	1,3	298,6	24,5	1,1
17	343,7	24,1	1,0	341,7	19,5	0,9
18	9,4	22,2	0,9	9,1	18,9	0,8
19	28,3	17,8	0,7	31,8	18,4	0,8
20	56,0	19,2	0,8	50,0	20,6	0,9
21	62,9	18,5	0,8	65,8	21,8	1,0
22	81,7	16,8	0,7	84,3	21,8	1,0
23	90,3	19,8	0,8	96,8	25,1	1,1
24	101,4	19,6	0,8	108,2	20,1	0,9

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