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Bibliographie

Algebraic Logic and Universal Algebra in Computer Science, Edited by C. H. Bergman, R. D. Maddux and D. L. Pigozzi (Lecture Notes in Computer Science, 425), XI+292 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1990.

The conference "Algebraic Logic and Universal Algebra in Computer Science" was held in Ames, Iowa in June 1988. The aim of the conference was to bring together researchers from computer science and mathematicians working in universal algebra or algebraic logic. The LNCS volume contains the text of 6 invited papers and 10 contributed papers.

Two questions concerning finitely generated free algebras in a nontrivial variety of relation algebras are of particular interest in the paper *"Relatively Free Relation Algebras"* by H. Andréka, B. Jónsson and I. Németi. The first one is whether an n-generated free algebra contains as a subalgebra a free algebra on $n+1$ generators. The second question is if a free algebra on n generators can be nonfreely generated by some n element subset. The results on the first question are derived from general facts such as the congruence extension property and the existence of a nontrivial absolute retract. On the other hand, the results on the second question make use of arguments specific for relation algebras. It is shown that in general the two questions are completely independent of one another.

The informal paper *"The Value of Free Algebras"* by J. Berman exhibits through a series of examples how free algebras occur in computer science and how these free algebras are useful in solving problems in computer science. The examples include non-classical logics, one-pass algebras and data bases.

The paper *"Dynamic Algebras as a Well-Behaved Fragment of Relation Algebras"* by V. Pratt is devoted to the comparison of the merits of relation and dynamic algebras with converse and sometimes with star. Tarski proved in the 1940's that the equational theory of representable relation algebras is undecidable and not finitely based. On the other hand, the equational theory of dynamic algebras is both decidable and finitely based. Pratt attributes these advantages to the "maintenance of a suitable distance between the Boolean and monoidal sorts". One more argument justifying his opinion would be a proof that the equational theory of representable relation algebras with disjunction, relative product, converse, star and constants 0 and $1'$ is decidable, so that after dropping one part of the Boolean structure there results a decidable equational theory.

Conditional logic, studied in the paper *"The Implications in Conditional Logic"* by F. Guzmán, is a 3-valued logic which is a regular extension of Boolean logic. Because disjunction is not commutative, it is possible to define two kinds of implications. The main result is a complete equational axiomatization for these implications.

The contribution *"Other Logics for (Equational) Theories"* by G. C. Nelson consists of two parts. In the first part a complete proof system is described suitable for deriving all positive sentences that are logical consequences of a set of equational axioms. The proof system is extended to the case that the axioms are universal Horn sentences. Some computer science applications are mentioned. The second part is concerned with proving equations true in finite algebras. All facts and ideas exploited in the paper are well known in some form. The way how these facts are arranged accounts for the value of the paper.

Mal'cev algebras have been intended to serve as a variable free and signature independent formal treatment of (function) composition and term substitution in universal algebra. These algebras are the subject of the paper *"Mal'cev Algebras for Universal Algebra Terms"* by I. G. Rosenberg. After a definition of various Mal'cev algebras, it is shown how these algebras are related to varieties. Of course, the connection is the same as that between varieties and Lawvere theories.

The volume contains really good papers and covers a wide range. Everybody working in algebraic or logical aspects of computer science may find some papers of particular interest. The volume is dedicated to the memory of Evelyn M. Nelson.

Z. Ésik (Szeged)

Analysis III, Spaces of Differentiable Functions. Edited by S. M. Nikol'skiĭ (Encyclopaedia of Mathematical Sciences, 26), 221 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

In this volume the theory of differentiable functions in several variables is treated in detail. The book consists of two parts. Part I: Spaces of differentiable Functions of Several Variables and Imbedding Theorems (by L. D. Kudryavtsev and S. M. Nikol'skiĭ); Part II: Classes of Domains, Measures and Capacities in the Theory of Differentiable Functions (by V. G. Maz'ya). The aim of the authors of Part I is laid in the Introduction as follows: "... the authors undertake to give a presentation of the historical development of the theory of imbedding of function spaces, of the internal as well as the external motives which have stimulated it and of the current state of art in the field, in particular, what regards the methods employed today." The reader can be convicted that this aim is overfulfilled in many senses even if she or he only takes a look at the chapter headings, most significant ones of which are: Sobolev-Spaces; The Imbedding Theorems of Nikol'skiĭ; Sobolev—Liouville Spaces; Weighted Function Spaces; Orlicz and Orlicz—Sobolev Spaces. The main topic of Part II is justified by the author as follows: "An adequate description of the properties of function spaces has made it necessary to introduce new classes of domains of definition for the functions, or classes of measures entering in the norms. In this connection the universal importance of the notion of capacity of a set became manifest".

The principal questions treated in this Part are: The Influence of the Geometry of the Domain on the Properties of Sobolev Spaces; Inequalities for Potentials and Their Applications to the Theory of Spaces of Differentiable Functions; Imbedding Theorems for Spaces of Functions Satisfying Homogeneous Boundary Conditions.

In my opinion this book is very clearly and well written and it is warmly recommended both to researchers and to graduate students.

J. Németh (Szeged)

Béla Andrásfai, Graph Theory: Flows, Matrices, x+280 pages, Akadémiai Kiadó, Budapest, Hungary, 1991.

This book is the English translation (and revised version) of Béla Andrásfai's book Graph Theory: Flows, Matrices (in Hungarian, Akadémiai Kiadó, Budapest). The book includes various topics from graph theory and their applications to physical sciences, operation research and economics. The author also covers the algorithmic aspects of the topics discussed in the book.

The first chapter contains the basic results on connectivity, blocks and strongly connected digraphs. The second chapter includes results on bipartite graph matching, the Hungarian method, the max flow — min cut theorem and different flow problems. The final (third) chapter deals with some matrices related to graphs. Spectrum of graphs and planar graphs are also considered. The theory of linear electrical networks is discussed as an application of the matrix method.

At the end of each section there are several exercises with solutions (91 altogether). Solving these exercises gives a good practice for the methods.

Students and lecturers will enjoy this book. It can be also used as a textbook for classes in different fields where graph theoretical methods are used.

Péter Hajnal (Szeged)

D. K. Arrowsmith—C. M. Place, An Introduction to Dynamical Systems, VIII+423 pages, Cambridge University Press, Cambridge—New York—Port Chester—Melbourne—Sidney, 1990

In the classical sense, a dynamical system is a system of ordinary differential equations. The solutions of such a system defines a flow in a space. Similarly, if f is a diffeomorphism, then the iteration $x_{t+1}=f(x_t)$, where t is a natural number, also gives a dynamical system. Besides their great natural beauty, there are two reasons for studying these "discrete" dynamical systems: on the one hand there are tight connections between time-periodic vector fields and diffeomorphism problems; on the other hand, the same phenomena and problems of the qualitative theory of ordinary differential equations are present in their simplest form in the theory of discrete dynamical systems. In recent years there has been a marked increase of research interest in dynamical systems both continuous and discrete, and a number of good postgraduate texts have been published. The present book is specially aimed at the interface between undergraduate and postgraduate studies. The reader is assumed to be familiar with courses in analysis and linear algebra to second-year undergraduate standard.

The first chapter (Diffeomorphisms and flows) contains the basic definitions. In the second chapter (Local properties of flows and diffeomorphisms) the topological behaviour of diffeomorphisms and flows in the neighbourhood of an isolated fixed point is considered. The third chapter (Structural stability, hyperbolicity and homoclinic points) gives a description of the flows on two-dimensional manifolds, of the Anosov diffeomorphisms, and a very nice presentation of the horseshoe diffeomorphisms. The fourth and fifth chapters are devoted to the local bifurcations. The last Chapter 6 (Area-preserving maps and their perturbations) is directed at first-year postgraduate students. It contains current research topics arisen from the interaction of the theories of area-preserving and non-area-preserving maps.

The whole book is excellent, but its main value is its extensive set of exercises; more than 300 in all. They are companied by model solutions and hints to their construction.

We warmly recommend this book to both senior undergraduates and postgraduate students in mathematics, physics engineering, to the instructors and researchers interested in qualitative theory of nonlinear systems.

L. Hatvani (Szeged)

Bernard Aupetit, A Primer on Spectral Theory, (Universitext) x+193 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong—Barcelona, 1991.

The text is divided into seven chapters and an appendix. The first two chapters give a list of basic results in functional analysis without proofs and introduce the reader to the theory of operators on Banach spaces and Hilbert spaces. Some special types of operators are also examined. The third chapter introduces the notion of Banach algebras, gives some examples of commutative and non-commutative Banach algebras and develops the basic spectral theory of Banach algebras. In the fourth chapter the Gelfand representation theory of commutative Banach algebras and the representation theory of non-commutative Banach algebras are presented. The fifth chapter is devoted to some applications of subharmonicity. Here the spectral characterisations of commutative Banach algebras and finite-dimensional Banach algebras and the spectral characterizations of the radical are also discussed. The sixth chapter deals with special Banach algebras in which a continuous involution is given. Proving the basic theorem for the Gelfand representation of C^* algebras, as an application develops the spectral representation theory for selfadjoint and normal operators in a Hilbert space. The seventh chapter is an introduction to the theory of analytic multifunctions which has very important applications for instance to the distribution of spectral values in the plane. The appendix is essentially a list of results without proofs concerning subharmonic functions and functions of several complex variables. Each chapter ends with a collection of problems.

L. Gehér (Szeged)

Joseph A. Ball—Israel Gohberg——Leiba Rodman, Interpolation of rational matrix functions (Operator Theory: Advances and Applications, 45), XII+605 pages, Birkhäuser, Basel—Boston—Berlin, 1990.

The development over close to 100 years of the interpolation theory reached a considerable phase 40 years ago. Namely, since the early 1950's interpolation problems have been considered for matrix-valued functions, too.

A scalar interpolation problem admits often several generalizations for matrix case. For example: λ_0 can be a zero of the matrix-valued function $P(\lambda)$ in the sense that 1) $P(\lambda_0)$ is zero matrix, 2) $P(\lambda_0)\mathbf{x}=\mathbf{0}$, 3) $\mathbf{y}P(\lambda_0)=\mathbf{0}$, 4) $\mathbf{u}P(\lambda_0)\mathbf{v}=\mathbf{0}$ (with appropriate column or/and row vectors).

This book presents the interpolation theory for rational matrix functions. It would be difficult to list its content, it is much more informative — but not exhaustive — to say that classical results are generalized to this case. The presented theory admits applications to control and system theory; the last part of the book is devoted to such applications. In fact, the objects of this part are sensitivity minimization, model reduction and robust stabilization included their engineering motivations. An Appendix dealing with Sylvester, Lyapunov and Stein matrix-equations completes the main text, and more than two hundred items are listed as references.

The mean feature of this systematic and self-contained treatment is the realization approach. This is based on the fact that every proper rational matrix function can be expressed in the form

$$W(\lambda) = D+C(\lambda I-A)^{-1}B,$$

which allows to reduce the interpolation problems to problems in matrix theory.

This book certainly meets the interest of a great number of mathematicians and ingeneers as well as advanced students.

E. Durszt (Szeged)

Bifurcation and Chaos: Analysis, Algorithms, Applications, Edited by R. Seydel, F. W. Schneider, T. Küpper and H. Troger (International Series of Numerical Mathematics, 97), X+388 pages, Birkhäuser Verlag, Basel—Boston—Berlin, 1991.

This volume is the proceedings of a conference held in Würzburg, August 20—24, 1990. The main topics discussed in the papers are the following: symmetry, applications of manifolds, Takens—Bogdanov bifurcation, homoclinic orbits, oscillators, controllability, characterization of dynamical systems, general numerical procedures and specific algorithmic topics. The connection with applications is also strongly felt in many papers including chemical oscillations convection problems, climate modeling, economy, robot control, rolling motion of ships, motion of a moored pontoon, galvanostatic oscillation, excitable systems, dry friction, rotating shafts, an elastic model with continuous spectrum, rings under hydrostatic pressure, combustion, Turing structures, and a spinning satellite.

The volume gives the reader a good opportunity for getting an overview of the actual problems and results of the world of nonlinear phenomena.

L. Hatvani (Szeged)

Böhme, Analysis 1 (Anwendungsorientierte Mathematik, Funktionen, Differentialrechnung, 6. Auflage), XI+492 pages, Springer-Verlag, Berlin—Heidelberg—New York—London— Paris—Tokyo—Hong Kong—Barcelona, 1990.

The book essentially contains the material of the first semester. The discussion attaches great importance to applications. The text is divided into four parts. The first part is devoted to elementary functions of one real variable. The second part is a short glimpse into functions of one complex variable. The third part develops the differentiation of real function and gives the differentiation rules. The last part deals with the differentiation of functions of two real variables. To make easier the understanding lots of exercises are given, the solutions of which at the end of the book can be found.

L. Gehér (Szeged)

P. Concus—R. Finn—D. A. Hoffman, Geometric Analysis and Computer Graphics, Proceedings of a Workshop held May 23—25, 1988 (Mathematical Sciences Research Institute Publications, 17), IX+203 pages, 60 illustrations — 30 in full color, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong—Barcelona, 1991.

The unexpected title of this book comes from a workshop on differential geometry, calculus of variations, and computer graphics held at the Mathematical Sciences Research Institute in Berkley, May 23—25, 1988. Although nobody could imagine a meeting on such a divergent background in the past, now this book proves the successs. Reading the gathered papers in this book, it comes to light that scientific and technological frontiers being crossed with impressive speed and so the title gets a deeper meaning. Maybe this is the way of the future.

One reads about the multi-functions, Monge-Ampère equation, rendering algebraic surfaces, minimal surfaces, capillary surfaces, tories and so on from the papers by Almgren, Baldes, Wohlrab, Banchoff, Callahan, Concus, Finn, Sterling and others.

We recommend the book mainly to those who want to know the interaction between the two subjects in the title, but also to anyone interested in any of these subjects alone.

Á. Kurusa (Szeged)

M. Coornaert—T. Delzant—A. Papadopoulos, Géométrie et théorie des groupes (Lecture Notes in Mathematics, 1441), X+169 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

The main purpose of this book is to give a detailed treatment of the Gromov theory of hyperbolic groups. The material is based on the lectures which were held by the three authors at the University of Strasbourg. The text is divided into 12 chapters. In the first four chapters the basic concepts of Gromov product, hyperbolicity of metric spaces, Gromov boundary and hyperbolic groups are introduced, and hyperbolicity of the n-dimensional simply-connected Riemannian space with constant curvature -1 and more generally the hyperbolicity of simply connected Riemannian spaces the sectional curvatures of which is bounded from above by a strictly negative constant are investigated. In Chapter 5 for a given hyperbolic group a contractible locally finite and finite dimensional simplicial complex is constructed. Chapters 6, 7 examine linear isoperimetric inequalities in hyperbolic spaces and give isoperimetric characterisation of hyperbolic groups. Chapters 8, 9, 10 deal with approximations, isometries and quasi convexity. Chapter 11 is devoted to the investigation of the boundary of hyperbolic groups and the theory of automata.

L. Gehér (Szeged)

C. Corduneanu: Integral equations and applications, IX+366 pages, Cambridge University Press, Cambridge—New York—Port Chester—Melbourne—Sidney, 1991.

Since it is so classical subject of the analysis it is a very natural question for a nonspecialist mathematician that "What new about the integral equations can a book write?". The book of Corduneanu gives a very striking answer. I recommend to all to read the excellent "Introduction" of the book. It is very well written, and contains not only a detailed description of the book's contents, but also some interesting historical considerations as well as some important notes of the author about the built up of the theory.

I think the author successfully reached his aim to write the book for three purposes. It is good for graduate textbook and for reference book as well as for young researchers to become acquainted with this field.

The book is based on the integral and the abstract Volterra equation as a unified starting point. It deals with the Fredholm theory of the linear integral equations with the Hammerstein equations and some of their generalizations to the Banach spaces. Applications of these integral equations are discussed in the last chapter. A very valuable part of the book is its big list of references, that contains more than 500 entries.

I recommend this very well written book to everybody who get in touch with the integral equations even in teaching, learning or in research.

Á. Kurusa (Szeged)

CSL'89, Edited by E. Börger, H. Kleine Büning and M. M. Richter (Lecture Notes in Computer Science, 440), VI+437 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1990.

These are the proceedings of the 3rd Workshop on Computer Science Logic held in Kaiserslautern, Germany, in October 1989. Altogether 45 talks were presented at the workshop, 28 of which have been collected in the volume. The authors of the papers are: *K. Ambos-Spies and D. Yang;*

G. Antoniou and V. Sperscheider; E. Börger; D. Cantone, V. Cutello and A. Policriti; E. Dahlhaus; B. I. Dahn; H. Decker and L. Cavedon; M. Droste and R. Göbel; A. Goerdt; E. Grädel; Y. Gurevich and L. S. Moss; J. Krajicek and P. Pudlák; H. Leiss; A. Leitsch; C. Meinel; D. Mey; D. Mundici; H.-J. Ohlbach; M. Parigot; A. Pasztor and I. Sain; W. Penczek; L. Priese and D. Note; E. Speckenmeyer and R. Kemp; R. F. Stärk (two papers), *O. Stepankova and P. Stepanek; H. Volger; E. Wette.*

The volume can be recommended to those interested in logical aspects of theoretical computer science.

Z. Ésik (Szeged)

Effective Methods in Algebraic Geometry (Progress in Mathematics, 94), Edited by Teo Mora and Carlo Traverso, XIV+500 pages, Birkhäuser, Boston—Basel—Berlin, 1991.

The development of computers has made it possible to complete calculations which previously were not feasible, thus the formulation of effective methods is now an important part of many areas of mathematics.

This book contains the proceedings of the symposium "MEGA—90 — Effective Methods in Algebraic Geometry", Castiglioncello, April 17—21, 1991. Two main areas were addressed at the symposium, that of effective methods and complexity issues in algebraic geometry and related areas (such as commutative algebra and algebraic number theory) and the use of algebraic geometry in algebraic computing. The book contains 33 papers, treating the resolution of singularities, codes and elliptic curves, algebraic differential equations, membership problems and other topics in algebraic geometry and algebra.

The book is recommended to those interested in the algorithmic aspects of algebraic geometry at graduate level and beyond.

G. Megyesi (Szeged)

A. Simovici—Peter A. Fejer—Peter Dan, Mathematical Foundations of Computer Science, Vol. 1, X+425 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1990.

The volume presents basic discrete mathematics relevant to computer science courses. The five chapters collected in the first volume are *Elementary set theory, Relations and functions, Partially ordered sets, Induction* and *Enumerability and diagonalization.* The computer science orientation can be witnessed by a thorough treatment of induction and diagonalization and topics such as databases, complete partially ordered sets, grammars, primitive recursive and partial recursive functions. The book is written in a rigorous style. New concepts are usually introduced through a series of examples and a number of applications are given for most theorems. In addition, each chapter contains a large number of exercises. Many of them are related to various fields of computer science or provide background information. Care is taken that general results are preceeded by a treatment of some particular instances. All these make the volume available for a large audience including undergraduate students. The second volume will cover topics of logical nature.

Z. Ésik (Szeged)

C. A. Floudas—P. M. Pardalos, A Collection of Test Problems for Constrained Global Optimization Algorithms (Lecture Notes in Computer Science, 455), XIV+180 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

Global optimization has been extensively studied in recent years, and numerous new theoretical, algorithmic and computational results have been achieved. In spite of these contributions, there has been still a lack of nonconvex test problems for comparing constrained global optimization algorithms.

The book of the authors contains a systematic collection of over 50 test problems for evaluating and testing constrained global optimization methods. For each test problem, the problem formulation, data problem statistics (like number of variables, linear and nonlinear constraints) and global or best known solutions are given. The test problems collected reflect a wide range of practical applications: e.g. distillation column sequencing, pooling, blending, heat exchanger network synthesis, reactor-separator-recycle system design etc.
An extensive bibliography of more than 250 references completes the book. The volume can be recommended to those working in the field of nonlinear constrained optimization and to engineers who want to test the numerical effectivity, efficiency and reliability of optimization algorithms.

T. Csendes (Szeged)

Functional-Analytic Methods for Partial Differential Equations, Proceedings of a Conference and a Symposium held in Tokyo, July 3—9, 1989. Edited by H. Fujita, T. Ikebe, and S. T. Kuroda, (Lecture Notes in Mathematics, 1450) VII+251 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

An "International Conference on Functional Analysis and its Application in Honor of professor Tosio Kato" was held on July 3 through 6, 1989 at University of Tokyo, which was followed by a "Symposium on Spectral and Scattering Theory" held on July 7 through 9 at Gakushuin University. In these meetings the study of Schrödinger operators and functional analytic study of nonlinear PDEs were the major subjects. The connection with applications is also strongly discussed in many papers.

L. Hatvani (Szeged)

I. M. Gel'fand—E. G. Glagoleva—E. E. Shnol, Functions and Graphs, IX+105 pages, Birkhäuser, Boston—Basel—Berlin, 1990.

The book is dealing with transferring of formulae and data into geometrical form by sketching the graphs of several functions without calculus.

It is very important to show the way how to "see" functions, formulae and how to observe the ways in which these functions change. To see simultaneously the formula of a given function and its geometrical representation and to draw the graph of a function is very useful not only in studying mathematics but in studying any subject, because the graphs are widely used not only in mathematics but in economy, medicine, engineering, physics, biology, business and so on.

The chapter headings of the book are: Examples; The Linear Function; The Function $y=|x|$; The Quadratic Trinomial; The Linear Fractional Function; Power Functions; Rational Function; Problems for Independent Solution; Answers and Hints to Problems and Exercises.

The book is very useful for high school teachers helping them in presenting basic mathematics in a clear and simple form.

J. Németh (Szeged)

B. Golubov—A. Efimov—V. Skvortsov, Walsh Series and Transforms. Theory and Applications (Mathematics and its Applications, 64), XIII+368 pages, Kluwer Academic Publishers, Dordrecht—Boston—London, 1991.

This book is the translation of the work published in Russian in 1987. This volume is a very good and useful introduction to Walsh—Fourier analysis with applications of the theory. Chapters 1 and 2 give the definitions of Walsh system and examine the basic properties of Walsh—Fourier series. Chapters 3—5 deal with the uniqueness of representation of functions by Walsh series, summation of Walsh series by the method of arithmetic means and convergence in L^p of Walsh—Fourier series. The main topic of Chapter 6 is the theory of generalized multiplicative transforms. In Chapters 7 and 8 Walsh series with monotone decreasing coefficients and lacunary subsystems of the Walsh system are considered.

Chapter 9 is dealing with divergence, almost everywhere convergence of Walsh—Fourier series of L^2 functions. Chapter 10 is devoted to the question of approximation by Walsh and Haar polynomials.

The last chapters (11 and 12) contain the methods for applying the Walsh system and its generalizations to digital information processing, to construct special computational devices to digital filtering, and to digital holograms. The appendices at the end of the book contain background information relating to more advanced material (group theory, measure theory, the Lebesgue integral, functional analysis). The appendices are followed by commentary including some remarks of historical nature and information about the latest developments in the area.

The book ends with a very rich, valuable "References" containing more than 150 items (30 of them are books). The volume is clearly and very well written. It will certainly be very useful book for engineers, technical specialists, graduate students of applied mathematics, and for everybody interested in Fourier analysis and its application.

J. Németh (Szeged)

The Grothendieck Festschrift, A collection of Articles Written in Honor of the 60th Birthday of Alexander Grothendieck (Progress in Mathematics, 86—88), Edited by P. Cartier, L. Illusie, N. M. Katz, G. Laumon, Y. Manin and K. A. Ribet, 3 volumes, Volume I XX+498 pages, Volume II VIII+563 pages, Volume III VII+495 pages, Birkhäuser, Boston—Basel—Berlin, 1991.

This book contains 35 papers by leading mathematicians from around the world. Most of the contributions are on various areas of algebraic geometry, but there are also several on algebraic number theory, topology and other areas of geometry. This variety of topics reflects the vast area on which Grothendieck worked and the book is a worthy tribute for his 60th birthday.

The diversity of the topics in this book, and also its price, mean that this book is less suitable for the individual, but it would be a good addition to any mathematical library.

G. Megyesi (Szeged)

Martin C. Gutzwiller, Chaos in Classical and Quantum Mechanics (Interdisciplinary Applied Mathematics, 1), xiv+432 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

Simple elementary and deterministic mechanical systems can have very complicated motions. Their behavior is exceedingly sensitive to the precise starting conditions and they do not follow simple, regular and predictable patterns, but run along a seemingly random, yet well-defined, trajectory. The name for this phenomenon is chaos.

"Chaos ... will challenge many of our assumptions about the typical behavior of dynamical systems. Since mechanics underlies our view of nature, we will probably have to modify some of our ideas concerning the harmony and beauty of the universe. As a first step, we will have to study entirely different basic examples in order to re-form our intuition. We must become familiar with certain novel specimens of simple mechanical systems based on chaotic rather than regular behavior."

This book offers a collection of instructive examples, which are chaotic, yet simple enough to be understood thoroughly. The central theme is the connection between classical and quantum mechanics: classical chaos should be the limit of quantum chaos as Planck's quantum becomes small. The style of the book is informal. The arguments based on elementary rather than algebraic manipulations. In order to gain a better perspective on the more important results, the historical and cultural background is mentioned and related disciplines are connected. The comments on the motivation behind certain results and on possible future developments provide the reader with a new perspective and prepare her/him to attack new problems.

Reading the book requires a knowledge of both classical and quantum mechanics at the level of beginning graduate students. This excellent book will certainly appeal to people working on this very active area of physics and its closest relatives: mathematics, astronomy and chemistry.

T. Krisztin (Szeged)

Werner Heise—Pasquale Quattrocchi, Informations- und Codierungstheorie, (Studienreihe Informatik), XII+392 pages, Springer-Verlag, Berlin—Heidelberg—New York—Paris—Tokyo, 1989.

The aim of the German and Italian authors is to provide a tutorial for those who are interested in the mathematical theory of communication. According to the preface of the first edition, the assumed readers are students majored in informatics. However, the authors present the applied mathematical background, so no specific knowledge is required to understand the book.

In the first part we can get an introduction to the theory of message transmission. The first two chapters introduce the basic concepts, such as code, source, channel, and give the definition of some special classes of channel. Chapter 3 summarizes the classical results of information theory, Chapters 4 and 5 discuss source and channel encoding.

The second part of the book is devoted to the theory of error detecting and correcting codes. After discussing the best known combinatorial bounds for these codes in Chapter 6, in Chapter 7 the authors present those algebraic concepts and theorems which are necessary to understand the theory of linear codes. The detailed description of these codes can be found in Chapter 8. The sections of this chapter deal with the basic concepts of linear codes (such as generator and parity check matrix, syndrome decoding, etc.), the modifications which preserve the linearity and the Reed—Muller codes. A separate chapter (Chapter 9) is devoted to the special class of linear codes, the cyclic codes, paying particular attention to the *BCH* and quadratic residue (*QR*) codes.

A lot of example help to understand the theoretical material of this clearly written book, besides funny pictures make the reading more enjoyable. We can warmly recommend this work both to students and teachers.

T. Gaizer (Szeged)

J. H. Hubbard—B. H. West, Differential Equations. A Dynamical Systems Approach, Part I: Ordinary Differential Equations, (Texts in Applied Mathematics, 5), XIX+348 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong—Barcelona, 1991.

This is an introductory textbook, which essentially differs from the traditional courses on differential equations. According to the fact that most of the differential equations do not admit solutions which can be written in elementary terms, it takes the view that a differential equation defines functions, and the object of the theory is to understand the behaviour of these functions. To this end it uses numerical and qualitative methods. While numerical methods approximate a single solution as closely as one wishes qualitative methods involve graphing the field of slopes, which enables one to draw approximate solutions following the slopes, and to study these solutions all at once. These method, are companied with a software, MacMath, which brings the notions to life and yields the majority of the 144 illustrations.

Not only the approach is new but the basic terminology as well. The authors introduce the terms "fence" "funnel" and "antifunnel". A fence is a curve on the (t, x) plane that channels the solutions in the direction of the slope field. A lower fence pushes solutions up, an upper fence pushes solutions down. A set bounded above by an upper fence and below by a lower fence is called a funnel. A set bounded above by a lower fence and below by an upper fence is called an antifunnel. It is interesting that these concepts give simple, noniterative proofs of the important theorems, e.g. the Sturm comparison theorem.

The book is ended by a chapter on iteration, which is also unusual in a text on differential equations. The reason of the appearance is that the iteration is another type of dynamical systems playing an important role in the theory of continuous dynamical systems generated by differential equations.

This excellent book will be very useful for instructors and students of undergraduate courses in differential equations and their applications.

L. Hatvani (Szeged)

J. E. Humpreys, Reflection Groups and Coxeter Groups, (Cambridge studies in advanced mathematics, 29), XII+204 pages, Cambridge University Press, Cambridge—New York—Port Chester—Melbourne—Sydney, 1990.

This is an easy-to-follow introductory graduate text on the theory of Coxeter groups.

The book consists of two parts. Part I describes the classical examples of Coxeter groups and provides the motivation for Part II, which is devoted to the general study of Coxeter groups. The first two chapters introduce the basic notions, such as for example roots, Coxeter graphs and Coxeter systems of generators, on the example of finite reflection groups, and give the classification of such groups. The next chapter describes in detail the theory of polynomial invariants of finite reflection groups. In particular, it presents the interesting relationships between the properties of the Coxeter elements and the orders of the fundamental invariants. In this part special attention is payed to the important examples of finite reflection groups provided by the Weyl groups of semisimple Lie algebras and to the related affine Weyl groups.

The first chapter in Part II develops the theory of Coxeter groups in general. For example, the geometric representation of general Coxeter groups and the properties of the Bruhat ordering are among the topics treated here. The following chapter deals with special cases: finite, affine, crystallographic and hyperbolic Coxeter groups. Then the author gives an introduction to the theory of Hecke algebras associated to Coxeter groups. The last chapter provides the reader with a guide to additional topics related to the subject-matter treated in the book, which can be further studied by using the extensive bibliography.

The book is clearly written and is self-contained. It can be profitably used by everybody interested in the general theory of Coxeter groups and its applications, or in the special Coxeter groups featuring so prominently in Lie theory.

László Fehér (Szeged)

A. E. Ingham, The Distribution of Prime Numbers (Cambridge Mathematical Library), XVII+114 pages, Cambridge University Press, Cambridge—New York—Port Chester—Melbourne—Sidney, 1990.

This book was first published in 1932. Number theory is a very strange part of mathematics. One of my professors told us that number theory is beautiful and good for nothing. But this was more than forty years ago. Nowadays number theory plays a more and more important role in real applications, too. The "fairly tales" become reality.

One of the most interesting parts of number theory deals with prime numbers. The subject of this book is the discussion of the theory of distribution of prime numbers in the series of natural numbers. After an introduction which contains the history of the problem and elementary facts too, the discussion depends on the theory of zeta-function. Chapter headings are: Foreword, Preface, Introduction, Elementary theorems, The prime number theorem, Further theory of (s), Applications, Explicit formulae, Irregularities of distribution.

An important part of this book is the Foreword written by R. C. Vaughan containing up to date results, comments and references.

This work is warmly recommended to teachers and students as well.

Finally we cite two interesting things. The last two sentences of the author's preface are: "The proof-sheets have been read by Prof. H. Bohr and Proof. J. E. Littlewood and also by Prof. G. H. Hardy, Dr. A. Zygmund ..., To Prof. N. Wiener I am indebted for some valuable comments ...". What a list of names!

One of the first reviews of the book from Zentralblatt für Mathematik (1933) was written by F. Bohnenblust.

A sentence from the review: Von vielen Sätzen werden verschiedene Beweisvarianten manchmal vollständig ausgeführt, manchmal nur skizziert, so dass der Leser-neben einer durchsichtigen systematischen Darstellung — eine klare Übersicht über die inneren Zusammenhänge der Theorie gewinnen kann.

L. Pintér (Szeged)

Bernd Jähne, Digitale Bildverarbeitung, XII+331 pages, 144 pictures, Springer-Verlag, Berlin—Heidelberg—New York—Paris—Tokyo, 1989.

To analyze and understand pictures is a simple task for us, humans, but — at least for the first sight — hardly tractable for computers. Even the most obvious operations: storing pictures, making simple corrections or detecting some simple patterns can require strong hardware background and involved algorithms.

Jähne's book provides a good overview of the present state of image processing. The outline of the book was the author's two-semester course had been held at University of Heidelberg. In spite of this the book is far more than a tutorial, it can also be a helpful guide for those ingeneers and researchers who want to use image processing in their work.

The chapters of the book cover the phases of image processing. The first two chapters contain some general introduction and concentrate on the problem of making and digitalizing pictures. The next two chapters are devoted to the mathematical background applied in image processing: Chapter 3 to the unitary transformations, Chapter 4 to the basic concepts of one and two variable statistics. In Chapters 5 to 10 we can read about the techniques that are used for modifying and analyzing digitalized pictures: for example, filtering and clustering procedures are discussed here. Chapter 11 is devoted to an area which has a lot of use e.g. in medical applications: the problem of reconstructing a picture from its projections. The last four chapter focus on analyzing and processing a sequence of pictures that were taken of moving objects or by moving camera.

The importance of Fourier transformation in image processing is emphasized by the fact that besides the *DFT* algorithm is described in Chapter 3, in Appendix A the author provides a summary of the one and two dimensional Fourier transformation. Finally, Appendix B contains a complete description of a *PC*-based digital image processing system.

The book is well illustrated with experimental results: 144 pictures help to demonstrate the effects of the studied procedures. We can warmly recommend this work both to those who just wish to get familiar with image processing and to those who want to apply it in the practice.

T. Gaizer (Szeged)

N. Korneichuk, Exact Constants in Approximation Theory (Encyclopedia of Mathematics and its Applications, 38), XII+452 pages, Cambridge University Press, Cambridge—New York—Port Chester—Malbourne—Sidney, 1991.

This book is very useful for non-specialists as a self-contained introduction to the important and widely applied area of approximation theory that is dealing with exact constants and for experts as a rich reference book to this topic (28 monographs, 17 books and more than 300 articles are cited in the References). The results are concerning extremal problems in approximation theory and are tightly related to numerical analysis and optimization.

Chapter 1 (Best approximation and duality in extremal problems) and Chapter 3 (Comparison theorems and inequalities for the norms of functions and their derivatives) contain the deep theorems of analysis and function theory on which the exact constant results are based. Chapter 2 (Polynomials and spline functions as approximating tools) gives an introduction to polynomial and spline approximation. Chapters 4 to 7 (Polynomial approximation of classes of functions with bounded rth derivative in L_p; Spline approximation of classes of functions with a bounded rth derivative; Exact constants in Jackson inequalities; Approximation of classes of functions determined by modulus of continuity) are devoted to approximation by polynomials (trigonometric or algebraic) and by polynomial splines. Chapter 8 (N-widths of functional classes and closely related extremal problems) deals with n-widths and generalizes some of the ideas of the earlier chapters.

Each chapter ends with valuable commentary and exercises.

The former contains references to the authors and their works related to the results included in the chapter in question and the latter contains in many cases the extensions of the corresponding results.

Since many of the results collected in this book have not been gathered together in book form before, this excellently written book of high level is warmly recommended to everybody who searches, teaches or applies the approximation theory.

J. Németh (Szeged)

Logic and Computer Science, Edited by P. Odifreddi (Lecture Notes in Mathematics, 1429), V+162 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1990.

This volume contains the lecture notes of the C.I.M.E. meeting on Logic and Computer Science held in June 1988 in Monteatini, Italy.

Table of Contents:

S. Homer, *The Isomorphism Conjecture and its Generalizations*

A. Nerode, *qome Lectures on Intuitionistic Logic*

R. A. Platek, *Making Computers Safe for the World. An Introduction to Proofs of Programs.* Part I

G. E. Sacks, *Prolog Programming*

A. Scedrov, *A Guide to Polymorphic Types*

It has been conjectured by L. Berman and J. Hartmanis that all *NP*-complete problems are are polynomial time isomorphic. This conjecture and its generalizations are discussed in the paper by S. Homer. The second paper, written by A. Nerode, is an exposition of one part of the undergraduate course on Intuitionistic Logic at Cornell. It focuses on Kripke's frame semantics for intuitionistic predicate logic (without function symbols) and on the correctness and completeness of a variant of Hughes and Cresswell's, or Fitting's prefixed tableaux. The paper by R. A. Platek develops flowchart semantics and Floyd's inductive assertion method on the basis of inductive definability. The fourth paper is written in a rather technical style. It fails to explain the aim and scope of PROLOG programming and its scientific level is well below the level of the other contributions. The last paper provides a highlight of (second order) polymorphic lambda calculus and the semantics of polymorphism. Complete proofs of the confluence theorem and the strong minimalization theorem are given.

Z. Ésik (Szeged)

Mathematical Foundations of Programming Semantics, Edited by M. Main, A. Melton and M. Mislove (Lecture Notes in Computer Science, 442), VI+pages, Springer-Verlag, Berlin—Heidelberg—New York, 1990.

The volume contains the papers presented at the Fifth International Conference on the Mathematical Foundations of Programming Semantics held at Tulane University, New Orleans, Louisiana' from March 29 to April 1, 1989. The contributions address concurrency, domain theory, type theory and lambda calculus, categorial semantics and program correctness. The authors of the papers are: *S. Abramsky; L. Cardelli and J. C. Mitchell; E. W. Stark; G. M. Reed; J. Davies and S. Schneider; A. W. Roscoe and G. Barrett; G. Barrett; F. Pfenning and C. Paulin-Mohring; K. Malmkjaer; M. G. Main and D. L. Black; A. Stoughton; L. S. Moss and S. R. Thatte; L. Aceto and M. Henessey; P. Panangaden and J. R. Russell; J. M. E. Hyland, E. P. Robinson and G. Rosolini; E. L. Gunter; R. Jagadeesan; A. Pasztor; A. J. Power; H. Jifeng and C. A. R. Hoare; J. W. Gray.*

The volume can be recommended to those interested in recent research in semantics.

Z. Ésik (Szeged)

Mappings of Operator Algebras. Proceedings of the Japan—U.S. Joint Seminar, University of Pennsylvania, 1988. Edited by Huzihiro Araki and Richard V. Kadison (Progress in Mathematics, 84), X+307 pages, Birkhäuser, Boston—Basel—Berlin, 1991.

This volume is dedicated to Professor Shôichirô Sakai and it is the proceedings of the fourth Japan — U.S. Joint Seminar on Operator Algebras held in honor of his 60th birthday. The content is (of course) adequate to this occasion. Index theory is a frequented topic in the papers, and — among others — derivation of operator algebras, actions of groups on C^*-algebras and completely bounded mappings are discussed. The content is connected also with quantum physics and ergogic theory.

The 19 articles of several length form a really good proceedings. The reader can find some new results, but mainly expositions of recent results and effective methods. Open problems and conjectures presented with hints stimulate to make attempt at solving some of them. Thus, this book provides a useful reference for researchers and graduate students working in the field of operator algebras.

E. Durszt (Szeged)

Mircea Martin—Mihai Putinar, Lectures on Hyponormal Operators, (Operator Theory, 39), 304 pages, Birkhäuser Verlag, Basel—Boston—Berlin, 1989.

The Hilbert space operator T is called hyponormal if its selfcommutator T^*T-TT^* is a positive operator. An important subclass of hyponormal operators is formed by the subnormal operators, which are restrictions of normal operators to invariant subspaces. The significant progress having reached in the study of subnormal operators up to 1981 was summarized in a monograph by J. Conway. There are known however hyponormal operators which are not subnormal, even more such operators naturally arise in many applications, e.g. in the theory of singular integral operators.

This book collects the various results achieved in the study of hyponormal operators in the last decades, including the basic inequalities, the invariant subspace theorems, the functional models and the role of the principal function. A number of examples and exercises make the treatment more colourful.

This volume can be recommended to graduate students as an introduction to this rapidly developing, fruitful field of mathematics. At the same time it will surely serve as an indispensable reference for the specialists.

L. Kérchy (Szeged)

Jean Mawhin—Michel Willem, Critical Point Theory and Hamiltonian Systems (Applied Mathematical Sciences, 74), xiv+277 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1989.

The development of a general theory of periodic solutions of Hamiltonian systems is a fundamental step in understanding the structure of their solution set. The main difficulty in applying the naive idea of finding the periodic solutions of a general Hamiltonian system through the critical points of its Hamiltonian action on a suitable space of periodic functions lies in the fact that this action is unbounded from below and from above. Therefore, the direct method of the calculus of variations (which deals with absolute minima) cannot be applied in a straightforward way and more sophisticated approaches like minimax methods and dual least action principles have to be used.

The aim of this interesting survey is to initiate the reader to the fundamental techniques of critical point theory which have been used recently in the framework of periodic solutions of Hamiltonian systems. The main subjects are the dual least action principle developed by Clarke and Ekeland, minimax approaches such as the Lusternik—Schnirelman theory and the mountain pass theorem of Ambrosetti and Rabinowitz, the Morse theory and some local and global aspects of the theory of nondegenerate critical manifolds. Various important problems concerning Hamiltonian systems are considered as applications of the techniques.

The book consists of ten chapters. The titles of the chapters are the following: the direct method of the calculus of variation, the Fenchel transform and duality, minimization of the dual action, minimax theorems for indefinite functionals, a Borsuk—Ulam theorem and the index theories, Lusternik—Schnirelman theory and multiple periodic solutions with fixed energy, Morse—Ekeland index and multiple periodic solutions with fixed energy, Morse theory, application of Morse theory to second order systems, nondegenerate critical manifolds. Some exercises are provided at the end of each chapter and a very extensive bibliography is presented.

The excellent style of the presentation of the book may help to make critical point theory more popular among people working and trained in ordinary differential equations.

T. Krisztin (Szeged)

Nonlinear Analysis and Applications. Edited by V. Lakshmikantham (Lecture Notes in Pure and Applied Mathematics, 109), XIX+649 pages, Marcel Dekker, Inc., New York and Basel, 1987.

The 7th International Conference on Nonlinear Analysis and Applications held at the University of Texas at Arlington, July 28—August 1, 1986 was in some sense a festive occasion because the main organizer, the moving spirit of these conferences V. Lakshmikantham became sixty years old. In this volume one finds the proceedings of this conference. Nowadays nonlinear analysis is a very broad part of mathematics both in theory and applications. In this book you have more than eighty papers. To enumerate the various problems is a nearly impossible task and to cite only a few of the talks could be misleading. (To enumerate all the titles is too long.) But let us emphasize an important feature of these talks. Everyone knows that sometimes (perhaps fairly often) the talks on conferences after the first five minutes are interesting for a few specialist only. We get some results but not ideas and important problems. Im my opinion in this collection the reader will find relatively many well-written, inspiring paper. Perhaps this is the best recommendation.

L. Pintér (Szeged)

Wlodzimierz Odyniec—Grzegorz Lewicki, Minimal Projections in Banach Spaces, (Lecture Notes in Mathematics, 1449), VIII+168 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

It is a well-known problem to find the best approximation of an element x in a Banach space X with elements of a subspace D of X. If the subspace D is complemented in X, i.e. if there exists a projection of X onto D, then it is of special interest to find a projection onto D (provided that there exists) the norm of which is the distance $\varrho(x, D)$ between x and D; such projections are called minimal projections. (The relationship between the two above mentioned problems is not apparent.) The text consists of four chapters. The first chapter is devoted to the problem of uniqueness of minimal projections. The second chapter deals with the connection between the problem of uniqueness of a minimal projection for subspaces with finite codimension of infinite dimensional spaces, and certain linear programming problems in n-dimensional euklidean spaces. In Chapter 3 lots of Kolmogorov type characterizations of minimal projections are presented. Chapter 4 studies isometries of a Banach space onto itself and gives characterisations of Hilbert spaces in the class of uniformly smooth strictly normed Banach spaces with the aid of minimal projections.

L. Gehér (Szeged)

Operator Algebras, Unitary Representations, Enveloping Algebras, and Invariant Theory (Actes du colloque en l'honneur de Jacques Dixmier), Edited by A. Connes, M. Duflo, A. Joseph and R. Rentschler (Progress in Math., 92), XVI+579 pages, Birkhäuser Verlag, Boston—Basel—Berlin, 1990.

This volume is the proceedings of the Colloquium held in Paris in 1989, celebrating the 65th anniversary of Professor Jacques Dixmier. The expository and research articles presented by the 22 invited speakers cover the four great areas of research, listed in the title, where Jacques Dixmier achieved significant progress. The first chapter deals with "C^*-algebras" and contains papers by E. Stormer, M. Takesaki and D. Voiculescu. The second chapter is devoted to "Lie groups and Lie algebras" and includes papers by L. Pukanszky, A. A. Kirillov, B. Kostant, D. Kazhdan, M. Kashiwara—T. Tanisaki, V. Lakshmibai, P. Littelmann—C. Procesi, W. Rossmann, R. K. Brylinski, D. Barbasch, D. A. Vorgan, Jr., W. M. McGovern, J. Bernstein, J.-E. Björk—E. K. Ekström, T. Levasseur, C. De Concini—V. G. Kac. Finally papers by M. Brian—C. Procesi, V. L. Popov and H. Kraft constitute the third "Invariant Theory" chapter.

This book can be recommended first of all to the specialist, but beyond this any interested reader will find it useful who wants to get an insight into these areas of mathematics inspired by Jacques Dixmier.

L. Kérchy (Szeged)

Paradoxa Klassische und neue Überraschungen aus Wahrscheinlichkeitsrechnung und mathematischer Statistik, 240 pages, Akadémiai Kiadó, Budapest, 1990.

This book illustrates excellently that probability theory is not only a chapter of measure theory.

The book has a historical framework. Chapter 1 is devoted to the oldest and most classical paradoxes of probability theory connected to problems of chance like card-playing, lottery, horse kickings and misprints ... to name a few.

Chapter 2 presents paradoxes in mathematical statistics. The explanations of these paradoxes help the reader to see through statistical absurdities and understand the useful and essential conclusions of statistics.

In Chapter 3 the reader can find paradoxes of random processes. Most of these paradoxes arose in the second half of the last century when the results of classical deterministic mechanics proved to be insufficient in different fundamental branches of science.

Chapter 4 — the most interesting for specialists of probability theory — presents paradoxes in the foundations of probability theory. These paradoxes are closely related to the development of Kolmogorov's fundamental theory.

Each paradox is discussed in five parts: the history, formulation, explanation of the paradox, remarks and references. Each chapter finishes with quickes. These are not discussed in detail, not because they are of less importance or interest, but because they do not fit into the main line of the book.

The book is recommended to probability specialists and nonspecialists as well.

L. Viharos (Szeged)

H.-O. Peitgen—E. Maletsky—H. Jürgens—T. Perciante—D. Saupe—L. Yunker, Fractals for Classroom: Strategic Activities Volume One, XII+128 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong—Barcelona, 1991.

The subject of fractals is nowadays a rapidly increasing area of the mathematics. At the same time it is one of the most suitable territory of recent mathematics to introduce in a classroom, because also its most abstract theories keep the freshness of the basic experiences. A story of a young lady, who determined the dimensions of fractals generated by Pascal's triangle, in the foreword of Benoit Mandelbrot justifies also this establishment.

As an introduction of the fractals, this book tries to drive students along a sequence of experiments. The activities need the reader to construct, count, compute and measure. The fractal theory seems from this point of view an experimental science like physics, and this makes easier to understand the underlying mathematical principles for the students. I think this approach can be made complete in teaching thanks to the modern computers. While these experiments are very interesting they make always good opportunity for the authors to call the student's attention to the most interesting experiences. The concept the book is built on is the self-similarity, the chaos game and complexity.

It is worth noting, that all the sheets of the book are perforated. This makes possible to use the sheets as separated exercise-forms. Other interest of the book is the enclosed slide package. This contains nine very good quality slide about fractal images.

In sum, we warmly recommend this book to the teachers, who want to bring mathematics out of past history for their students, to the students, who want to know in the visual sense the most color and beauty geometric structures of mathematics and want to discover new exciting territories. I am sure that fractals, and also this publication, open new ways in teaching and learning mathematics.

Á. Kurusa (Szeged)

A. M. Perelomov, Integrable Systems of Classical Mechanics and Lie Algebras, Volume I, X+307 pages, Birkhäuser Verlag, Basel—Boston—Berlin, 1990.

This book gives a systematic, up-to-date account of the rapidly developing theory of integrable) classical mechanical systems with finitely many degrees of freedom. The reader is assumed to be familiar with the fundamentals of classical mechanics, the theory of differentiable manifolds and Lie groups, but apart from these prerequisities the vook is self-contained.

The study of such well-known integrable systems as for example the motion of a point mass in a central potential or various special cases of the motion of a rigid body about a fixed point, played an important role both in the development of the mathematical formalism of classical mechanics and in its applications in describing physical phenomena. However, until quite recently only a rather small number of nontrivial examples of integrable systems was known. During the last twenty years or so the situation changed dramatically, the complete integrability of a large number of mechanical systems has been proven, mainly by applying the isospectral deformation (inverse scattering) method to classical mechanics. Practically all know integrable systems are related to Lie algebras in some way or other, for example quite a few of them live on coadjoint orbits of Lie groups, or can be obtained by reduction of some sort from some higher dimensional system with a large, manifest symmetry group, which underlies the integrability.

The main subject of the present volume is the isospectral deformation method and its combination with various Lie algebraic techniques. The author also gives an exposition of the classical methods and results of the theory of integrable Hamiltonian systems. The book contains a detailed survey of important classes of the known integrable systems and a good bibliography as well, and thus it can serve as a standard reference on its subject.

The first chapter contains a clear presentation of the general theory, including the isospectral deformation method, the description of Hamiltonian systems with symmetry, and the closely related questions of symmetry reduction and the so called projection method. Chapter 2 deals with the simplest, classic examples of integrable systems. Chapter 3 offers a survey of many-body problems of generalized Calogero—Moser type. The subject of Chapter 4 is the non-periodic Toda lattice and its various generalizations, described here both from the point of view of coadjoint orbits and Lax pairs and also as reductions of the geodesic motion on certain symmetric spaces. The last chapter deals with additional questions of many-body problems. Throughout the book, various aspects of the theory of semisimple Lie algebras are used, and the basic facts of this theory are summarized in an appendix. There are also three further appendices, e.g. one on symmetric spaces.

The book is clearly written and is very redable. It will be useful to students and lectures in theoretical physics and mathematics as well as for researchers on related areas.

László Fehér (Szeged)

M. H. Protter—C. B. Morrey, A First Course Calculus in Real Analysis (Undergraduate Texts in Mathematics), XVIII+534 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo—Hong Kong—Barcelona, 1991.

In this second edition of the successful book there are a lot of change and improvements. Many new problems and noticable clarifications of many proofs are added improving the readability of the book.

Since the first course in real analysis follows the elementary calculus where the emphasis is on problem-solving and the development of manipulative skills, the book like this has to show the students that the higher mathematics is not simply manipulative but the rigorous proofs have great importance from the point of view of advanced mathematics. This problem has been excellently solved in this book since precise proofs of the theorem are given but the way leading from the intuitive ideas to the end of the proofs is not exhausting. At the same time the book makes clear to the students that the proofs of the basic statements are necessary to their further study in mathematics.

The main topics of the book are: The Real Number System; Basic Properties of Functions on R^1; Elementary Theory of Differentiation and Integration in R^1 and in R^N; Infinite Sequences and Infinite Series; Fourier Series; Functions of Bounded Variation; Riemann—Stieltjes integral; Implicit Function Theory; Approximation Theorems; Vector Field Theory.

The great number of exercises (more than one thousand) help the students in understanding the material.

The book is excellently written, it is recommended to all instructors and students who want to teach or to learn first course in real analysis using an outstanding textbook.

J. Németh (Szeged)

László Rédei, Endliche p-Gruppen (Finite p-Groups), 304 pages, Akadémiai Kiadó, Budapest, 1989.

The large number and variety of finite p-groups have caused the demand for a classification or at least for the creation of a comprehensive theory. Successful efforts in this direction were made only within special (although sufficiently wide) families, such as regular p-groups and p-groups of maximal class. All these (and other) treatments mainly happen by means of the subgroups, as well as throughout in (finite) group theory.

In his last work Rédei had given a new classification method based on handling the group elements themselves. The initial idea is very simple and natural: Using a (strictly) normal series of the group with cyclic factors, one can choose for each factor an element to cover. The chosen elements form a special generating set (a *basis*) such that any group element possesses a presentation as the product of powers of the generators in fixed order, determined by the order of the factors in the series. With the aid of a basis one easily obtains a set of defining relations for the group. However, this set of relations heavily depends on the particular choice of basis and that makes the classification problem so difficult.

Since powers in finite p-groups are more conveniently thought of having p-adic integers in the exponents, Chapter 1 is completely devoted to the ring of p-adic integers $\mathscr{I}_p$ and the p-adic number field $\mathscr{K}_p$. Paragraphs 1—14 provide a practical and elementary introduction into the structure of these objects, and give the very basic terms of p-adic analysis. In paragraph 15 a new concept of generalized sum is defined, with a p-adic integer as the number of terms.

In Chapter 2 the general theory is developed; the starting point is the following.

Let $G=N_0\supset N_1\supset\ldots\supset N_l=1$ be a normal series for G and $a_1, \ldots, a_l$ a corresponding basis such that $N_{i-1}=\langle a_i, a_{j+1}, a\rangle$ holds for $i=1, \ldots, l$. If q_j denotes the order of a_j mod N_j then the group is defined by the following relations:

$$a_j^{q_j} = \prod_{k=j+1}^{l} a_k^{r_{jk}} \quad (1 \leq j \leq l), \qquad a_j^{a_i} = \prod_{k=j}^{l} a_k^{s_{ijk}} \quad (1 \leq i \leq j \leq l).$$

Let

$$\mathscr{D} = \{q_1, q_2, \ldots;\ r_{12}, r_{13}, \ldots;\ s_{122}, s_{123}, \ldots\},$$

be considered as a set of symbols, and for any integer l the *l-th segment* $\mathscr{D}_l$ denotes the subset of $\mathscr{D}$ with all elements, whose indices do not exceed l. Thus a presentation $\mathscr{D}_l(G)$ of a group G by means of an l element basis can be thought of as a place (a special value) of $\mathscr{D}_l$. Places generally do not lead to desired (p-)groups, therefore the text goes on with determining the conditions for that. All these conditions are of the form that the structure constants be zeros of some continuous p-adic function.

(It is worth mentioning that *not* all the functions in question are polynomials.)

The first step towards a classification is the concept of *natural classes;* a group belongs to the k-th natural class $\mathscr{C}_k$ iff k is the minimal length of its bases. Then natural classes split into several *parameter classes* in the following way: With given n, l and continuous p-adic functions

$$q_k \ (1 \leq k \leq l), \quad r_{jk} \ (1 \leq j < k \leq l), \quad s_{ijk} \ (1 \leq i < j \leq k \leq l)$$

whose variables are $t_1, \ldots, t_n$, let $\mathscr{T}_1, \ldots, \mathscr{T}_n$ be at least two element subsets of $\mathscr{I}_p$. Suppose that the values of the above functions at all $t_h \in \mathscr{T}_h$ ($h=1, \ldots, n$) result in presentations of pairwise nonisomorphic groups in $\mathscr{C}_l$; then the set of these groups is called a *parameter class of degree n.* To find parameter classes (for fixed l and n) is not generally easy, and a partition of $\mathscr{C}_l$ into parameter classes of degree n is far from being uniquely determined. The aim is always to get $\mathscr{C}_l$ as the disjoint union of minimal (possibly finite) number of parameter classes of lowest degree. Concerning this problem, the following conjecture is set: *$\mathscr{C}_l$ can be splitted into the disjoint union of finitely many parameter classes of degree* $\leqq \binom{l+2}{3}$.

Going through general theory one will recognize the difficulty of application to concrete classes. However, Chapter 3 is an evidence to the fact that classifying certain classes is not hopeless: Rédei succeeded in dividing $\mathscr{C}_2$ into 3 (for odd p) or 9 (for $p=2$) parameter classes of degree at most 4 *(6.1 Satz)*. Even in this simplest case a good deal of extra calculation was needed to get the final result, which does not give rise to much optimism concerning $\mathscr{C}_l$ in general. On the other hand, the theory yields the following beautiful "classical type" theorem:

For any group G in $\mathscr{C}_2$ (i.e., for any metacyclic p-group) $|G|$, $\exp(G)$, $|G/G'|$, $\exp(G/G')|$, $|Z(G)|$, $\exp(Z(G))$, $|\{x^p\colon x\in G\}|$ *and* $|\{x\in G\colon x^p=e\}|$ *form a complete system of invariants.*

It seems to be obvious that similar theorems will not occur very often. Within the scope of Rédei's new method, instead, there must be more possibilities for anyone not waiting for an easy success.

Péter Z. Hermann (Budapest)

Arto Salomaa, Public-Key Cryptography (EATCS Monographs in Theoretical Computer Science, 23), X+245 pages, with 18 figures, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo—Hong Kong—Barcelona, 1990.

Cryptography, secret writing, is probably as old as writing. This old activity, i.e., to send secret messages, has become the object of scientific research only recently. It is partly due to the need to guarantee the security of data bases, but the military aims are also important.

The first chapter is an outline of the classical two-way cryptography. All the other chapters are devoted to the public-key systems.

In the "classical" cryptosystems both keys, the encrypting and the decrypting keys, are supposed to be secret, in the public-key systems the encrypting keys can be published (like a telephon directory), but the decrypting keys are secret.

In Chapter 1 several classical systems are considered and analysed in the cryptoanalist's (a person, who wants to decypher the secret message without knowing the key) point of view.

In the subsequent five chapters we can read a systematic treatment of the public-key systems (the main point is the RSA system). These systems appeared in the middle of the 70's only. The security of this type of systems are based on results of the complexity theory. The fundamental idea of them is closely related with the following: given an argument value x, it is easy to compute the function value $f(x)$, whereas it is intractable (in the sense of the complexity theory) to compute x from $f(x)$.

To read this very interesting book the knowledge of the basic notions and results of the complexity theory (e.g. time complexity, Turing machine, the classes P and NP, etc.) and some results from the classical number theory (e.g. congruences, Euler's theorem, quadratic residues, etc.) are also supposed to be known. To help the reader in these fields there are two tutorials as appendices.

Lajos Klukovits (Szeged)