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Erratum to "On a geometric problem concerning discs"

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As Professor Shlomo Reisner (Haifa) has kindly pointed out to us, the equality in our paper [1, lines 8—9 on page 327] is incorrect. Unfortunately, this computational error requires that — keeping the idea of the original argumentation unchanged — the second part of the verification of formula (1) in [1] should be rewritten.

In particular, lines 23—24 on p. 326 and lines 1—20 on p. 327 are to be replaced by the following:

"Given $w\in\mathbf{R}^n$ arbitrarily, there is a uniquely defined onto projection P_w: $\mathbf{R}^n\oplus\mathbf{R}\to\mathbf{R}^n$ with $\operatorname{Ker} P_w=\{\lambda(w,1)|\lambda\in\mathbf{R}\}$. Define

$$\Gamma(w)=\inf\{\gamma>0\,|\,\|(x,\gamma)\|=1,\ \|P_w(x,\gamma)\|\geqq 1+\delta_0\},$$
$$\Gamma_0=\inf\{\Gamma(w)\,|\,|w|\leqq 1\}.$$

By elementary compactness arguments, $\Gamma_0>0$. Similarily, there exists a constant $K>0$ such that $\|P_w(x,\gamma)\|\leqq K$ whenever $\|(x,\gamma)\|=1$, $|w|\leqq 1$.

Suppose now that $\alpha<\Gamma_0/K$ and choose $y\in\mathbf{R}^n$, $|y|\leqq\alpha$ arbitrarily.

For brevity, we write $P=P_{y/\alpha}$. By (c), there exist a (not necessarily uniquely determined) $x=x(y)\in\mathbf{R}^n$ and a (not necessarily uniquely determined) $c=c(y)\in\mathbf{R}$, $c>0$ such that

$$\|(x,\alpha c)\|=1 \quad\text{and}\quad \|P(x,\alpha c)\|\geqq 1+\delta_0. \tag{2}$$

Clearly we have $P(x,\alpha c)=(x-cy,0)$, so

$$K\geqq\|(x-cy,0)\|=|x-cy|\geqq 1+\delta_0. \tag{3}$$

Since $\alpha_c\geqq\Gamma_0$ and $\alpha<\Gamma_0/K$, there holds

$$c>K. \tag{4}$$

For $z=-\dfrac{x-cy}{|x-cy|}$ we have that $|z|=1$ and that

$$d_\alpha(z,y)=\|h_\alpha(z)-h_\alpha(y)\|=\left\|\left(-\frac{x-cy}{|x-cy|},0\right)-(y,\alpha)\right\|=$$
$$=\left\|-\frac{1}{c}(x,\alpha c)+\frac{c-|x-cy|}{c}\left(-\frac{x-cy}{|x-cy|},0\right)\right\|\leqq c^{-1}+c^{-1}\big|c-|x-cy|\big|.$$

In virtue of (3) and (4), we obtain that

$$c^{-1}+c^{-1}\big|c-|x-cy|\big| = c^{-1}+c^{-1}(c-|x-cy|) =$$
$$= 1-c^{-1}(|x-cy|-1) \leqq 1-c^{-1}\delta_0$$

so

$$(5) \qquad d_\alpha(z, y) < 1 \quad \text{provided that} \quad \alpha < \Gamma_0/K.$$

Since $|z|=1$, (1) is a trivial consequence of (5)".

Since the error in lines 8—9 on p. 327 has no effect on other parts of our original paper, it means that (to the best of our knowledge) all statements and examples of [1] are correct.

Further, lines 2—3 on p. 329 are to be replaced by the following "Given $\varepsilon > 0$ arbitrarily, (as a simple consequence of [2, Cor. 1]) there exist examples d with

$$\sup_{y\in B_X(0,1)} \inf_{x\in S_X(0,1)} d(y, x) < \varepsilon,"$$

We apologize for the mistakes.

References

[1] A. P. Bosznay and B. M. Garay, On a geometric problem concerning discs, *Acta Sci. Math.*, 52 (1988), 325—329.

[2] W. H. Cutler, Negligible subsets of infinite-dimensional Fréchet manifolds, *Proc. Amer. Math. Soc.*, 23 (1969), 668—675.

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