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Fourier—Stieltjes transforms of vector-valued measures on compact groups

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1. Introduction. In recent years, various studies have shown the growing importance of vector-valued measures as can be seen for instance from [1], [3], [4] and many others as well as the numerous references contained in them. To give just one specific example: the Fourier transforms of the distributions studied by BONNET [2] in generalizing the Bochner theorem to noncommutative Lie groups turn out to be vector-valued measures.

In the present paper, we study the Fourier—Stieltjes transforms of vector-valued measures defined on an infinite compact group. Let G be an infinite compact group with Σ as its dual object. We consider measures m on G with values in a Banach space E. Following ASSIAMOUA [1], we define the Fourier—Stieltjes transforms of such measures and obtain analogues of the results in § 28 of HEWITT and ROSS [6]. Among other results, we prove the celebrated Lebesgue theorem and the Parseval—Plancherel—Riesz—Fischer theorem.

2. Preliminaries

2.1. Definition. Let S be a locally compact Hausdorff space and $\mathcal{K}(S)$ the real (resp. complex) vector space of all continuous real (resp. complex) valued functions on S with compact supports. A *vector measure* on S with values in a real (resp. complex) normed linear space E is any linear mapping $m\colon \mathcal{K}(S)\to E$ with the following property: for every compact set $K\subset S$, there exists a positive constant a_K such that if $f\in\mathcal{K}(S)$ and supp $f\subset K$, then ([3], 2, no. 1)

$$\|m(f)\|_E \leqq a_k \sup\{|f(t)|\colon t\in K\}.$$

We note that if S is compact, then $\mathcal{K}(S)$ is equal to the vector space $\mathcal{C}(S, \mathbf{R})$ (resp. $\mathcal{C}(S, \mathbf{C})$) of all continuous functions on S into $\mathbf{R}$ (resp. $\mathbf{C}$) and a vector measure

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on S is a linear mapping $m\colon \mathcal{K}(S)\to E$ which is continuous in the uniform norm topology since in this case, there exists a constant $a=a_S$ such that

$$\|m(f)\|_E \leqq a\|f\|, \quad f\in\mathcal{K}(S),$$

where $\|f\|=\sup\{|f(t)|\colon t\in S\}$ is the uniform norm on $\mathscr{C}(S, \mathbf{R})$. If $m\colon \mathcal{K}(S)\to E$ is a vector measure, we shall write

$$m(f) = \int_S f(t)\,dm(t) \quad \text{or} \quad \int f\,dm.$$

2.2. Definition. An E-valued vector measure is said to be *dominated* if there exists a positive (real-valued) measure μ such that

$$\left\|\int f\,dm\right\|_E \leqq \int |f|\,d\mu, \quad f\in\mathcal{K}(S).$$

If m is dominated, then there exists a smallest positive measure $|m|$ called the *variation* or the *modulus* of m that dominates it.

A positive measure is said to be *bounded* if it is continuous in the uniform norm topology of $\mathcal{K}(S)$ and a dominated vector measure is said to be bounded if it is dominated by a bounded positive measure.

Thus every dominated vector measure on a compact space is bounded. (For these properties of vector measure and the general theory of vector integration, the reader is referred to [3] or [4].) We note also that if E is a Banach space and $S=G$ is a group, then the space $M^1(G, E)$ of all bounded E-valued measures on G is a Banach space with the norm

$$\|m\| = \int \chi_G\,d|m|,$$

where χ_G is the characteristic function of G.

3. The Fourier—Stieltjes transform. We shall now define the Fourier—Stieltjes transform of a vector-valued measure on a compact group G and obtain some of the properties of such transforms.

3.1. Definition. Let G be a compact infinite group and Σ its dual object. For each $\sigma\in\Sigma$, we choose once and for all, an element $U^{(\sigma)}$ in σ, denote its representation space by H_σ, fix a conjugation D_σ on H_σ and put $\overline{U}^{(\sigma)}=D_\sigma U^{(\sigma)}D_\sigma$, ([6], 27.28. C).

As in [1], we define the *Fourier—Stieltjes transform* of a vector-valued measure $m\colon G\to E$ by

$$\hat{m}(\sigma)(\xi, \eta) = \int_G \langle \overline{U}_t^{(\sigma)}\xi, \eta\rangle\,dm(t), \quad (\xi, \eta)\in H_\sigma\times H_\sigma.$$

Let E be a Banach space. Then the mapping $(\xi, \eta)\to\hat{m}(\sigma)(\xi, \eta)$ from $H_\sigma\times H_\sigma$ into the space $\mathscr{S}(H_\sigma,\times H_\sigma, E)$ of the E-valued continuous sesquilinear mappings on

$H_\sigma \times H_\sigma$, equipped with the norm

$$\|\Phi(\sigma)\| = \sup\{\|\Phi(\sigma)(\xi,\eta)\|_E : \|\xi\|_{H_\sigma} \leqq 1, \|\eta\|_{H_\sigma} \leqq 1\}$$

is continuous ([1], 4.1).

Following HEWITT and ROSS [6], 28.24, we shall write

$$\mathscr{S}(\Sigma, E) = \prod_{\sigma\in\Sigma} \mathscr{S}(H_\sigma \times H_\sigma, E).$$

It is easy to see that, with addition and scalar multiplication defined coordinatewise, $\mathscr{S}(\Sigma, E)$ is a vector space. For $\Phi\in\mathscr{S}(\Sigma, E)$, we put

$$\|\Phi\|_\infty = \sup\{\|\Phi(\sigma)\| : \sigma\in\Sigma\}$$

and denote by $\mathscr{S}_\infty(\Sigma, E)$ the space $\{\Phi\in\mathscr{S}(\Sigma, E) : \|\Phi\|_\infty < \infty\}$. Also we denote by $\mathscr{S}_{00}(\Sigma, E)$ the space

$$\{\Phi\in\mathscr{S}_\infty(\Sigma, E) : \{\sigma\in\Sigma : \Phi(\sigma) \neq 0\} \text{ is finite}\}$$

and by $\mathscr{S}_0(\Sigma, E)$ the space

$$\{\Phi\in\mathscr{S}_\infty(\Sigma, E) : \text{for every } \varepsilon > 0,\ \{\sigma\in\Sigma : \|\Phi(\sigma)\| > \varepsilon\} \text{ is finite}\}.$$

The next theorem is an analogue of HEWITT and ROSS [6], 28.25.

3.2. Theorem.

(i) *The mapping* $\Phi \to \|\Phi\|_\infty$ *is a norm on* $\mathscr{S}_\infty(\Sigma, E)$ *and* $\mathscr{S}_\infty(\Sigma, E)$ *is a Banach space with respect to this norm.*

(ii) $\mathscr{S}_{00}(\Sigma, E)$ *is dense in* $\mathscr{S}_0(\Sigma, E)$.

Proof. (i) It is clear that $\Phi \to \|\Phi\|_\infty$ is a norm. Let $\{\Phi_n\}$ be a Cauchy sequence in $\mathscr{S}_\infty(\Sigma, E)$. Then for every $\sigma\in\Sigma$, $\{\Phi_n(\sigma)\}$ is a Cauchy sequence in $\mathscr{S}(H_\sigma \times H_\sigma, E)$. Since $\mathscr{S}(H_\sigma \times H_\sigma, E)$ is a Banach space, $\{\Phi_n(\sigma)\}$ converges to an element $\Phi(\sigma)$ in $\mathscr{S}(H_\sigma \times H_\sigma, E)$. An argument similar to [6], 28.25 shows that $\Phi = (\Phi(\sigma))$ belongs to $\mathscr{S}_\infty(\Sigma, E)$ and that $\{\Phi_n\}$ tends to Φ.

(ii) Let Φ be an element of $\mathscr{S}_0(\Sigma, E)$. For $n = 1, 2, \ldots,$ define the element Φ_n of $\mathscr{S}_{00}(\Sigma, E)$ by

$$\Phi_n(\sigma) = \begin{cases} \Phi(\sigma) & \text{if } \|\Phi(\sigma)\| \geqq 1/n, \\ 0 & \text{if } \|\Phi(\sigma)\| < 1/n. \end{cases}$$

Then plainly $\{\Phi_n\}$ converges to Φ in $\mathscr{S}_0(\Sigma, E)$.

3.3 Lemma. *Every* $\Phi(\sigma)\in\mathscr{S}(H_\sigma \times H_\sigma, E)$ *is determined by the* d_σ^2 *elements* $a_{ij}^\sigma = \Phi(\sigma)(\xi_j, \xi_i)$ *of* E *where* d_σ *is the finite dimension of* H_σ *and* $(\xi_1, \xi_2, \ldots, \xi_{d_\sigma})$ *is an orthonormal basis of* H_σ. *More precisely, we have* $\Phi(\sigma) = \sum_{i,j=1}^{d_\sigma} d_\sigma a_{ij}^\sigma \hat{u}_{ij}^\sigma(\sigma)$ *where* $u_{ij}^\sigma(t) = \langle U_t^{(\sigma)}\xi_j, \xi_i\rangle$.

(Note that for a complex function u, $\hat{u}$ is the Fourier transform that is the Fourier—Stieltjes transform of the measure $u\lambda$, λ being the normalized Haar measure on G.)

Proof. We have

$$\Phi(\sigma)(\xi,\eta) = \sum_{i,j=1}^{d_\sigma} \alpha_j \bar{\beta}_i a_{ij}^\sigma$$

on putting

$$\xi = \sum_{j=1}^{d_\sigma} \alpha_j \xi_j \quad \text{and} \quad \eta = \sum_{i=1}^{d_\sigma} \beta_i \xi_i.$$

Now for a coordinate function $u_{ij}^\sigma\colon t \to \langle U_t^{(\sigma)}, \xi_j, \xi_i\rangle$, we have (by [6], 27.19)

$$\hat{u}_{ij}^\sigma(\sigma)(\xi,\eta) = \int_G \langle \bar{U}_t^{(\sigma)}\xi, \eta\rangle u_{ij}^\sigma(t)\, d\lambda(t) = \sum_{k,l} \int_G \alpha_l \bar{\beta}_k \bar{u}_{kl}^\sigma(t) u_{ij}^\sigma(t)\, d\lambda(t) = 1/d_\sigma \alpha_j \bar{\beta}_i.$$

Thus

$$\Phi(\sigma)(\xi,\eta) = \sum \alpha_j \bar{\beta}_i a_{ij}^\sigma = \sum d_\sigma \hat{u}_{ij}^\sigma(\sigma)(\xi,\eta) a_{ij}^\sigma.$$

Hence

$$\Phi(\sigma) = \sum_{i,j=1}^{d_\sigma} d_\sigma a_{ij}^{(\sigma)} \hat{u}_{ij}^\sigma(\sigma).$$

3.4. Definition. We shall write $\mathscr{S}_2(\Sigma, E)$ for the vector space

$$\Big\{\Phi \in \mathscr{S}(\Sigma, E)\colon \sum_{\sigma\in\Sigma} d_\sigma \sum_{i,j} \|\Phi(\sigma)(\xi_i, \xi_i)\|_E^2 < \infty\Big\}.$$

3.5. Lemma. *Suppose that E is a Hilbert space. Then the mapping*

$$(\Phi, \Psi) \to \langle \Phi, \Psi\rangle = \sum_{\sigma\in\Sigma} d_\sigma \sum_{i,j=1}^{d_\sigma} \langle \Phi(\sigma)(\xi_j, \xi_i), \Psi(\sigma)(\xi_j, \xi_i)\rangle$$

is an inner product on $\mathscr{S}_2(\Sigma, E)$.

Proof.

$$\sum\sum d_\sigma |\langle \Phi(\sigma)(\xi_j, \xi_i), \Psi(\sigma)(\xi_j, \xi_i)\rangle| \leqq \sum\sum d_\sigma^{1/2} \|\Phi(\sigma)(\xi_j, \xi_i)\|_E d_\sigma^{1/2} \|\Psi(\sigma)(\xi_j, \xi_i)\|_E \leqq$$
$$\leqq \sum\sum (d_\sigma \|\Phi(\sigma)(\xi_j, \xi_i)\|_2^2)^{1/2} \sum\sum (d_\sigma \|\Psi(\sigma)(\xi_j, \xi_i)\|^2)^{1/2} < \infty.$$

This shows that the mapping is well defined and the proof can be easily completed.

4. Properties of Fourier—Stieltjes transforms. Throughout this section, we adopt the following notation: if X is a subset of $M^1(G, E)$, we shall denote by $\hat{X}$ the set $\{\hat{u}\colon u \in X\}$. In the next two theorems we obtain analogues of Theorems 28.36 and 28.39 (i, ii) of [6], respectively.

4.1. Theorem. *The mapping $m \to \hat{m}$ from $M^1(G, E)$ into $\mathscr{S}_\infty(\Sigma, E)$ is linear, injective and continuous.*

Proof. That $m\to\hat{m}$ is linear is clear. We know that it is one-to-one by [1]; Lemma 4.1.5. Now,

$$\|\hat{m}(\sigma)\| = \sup\{\|\hat{m}(\sigma)(\xi,\eta)\|_E : \|\xi\|_{H_\sigma}\leqq 1 \text{ and } \|\eta\|_{H_\sigma}\leqq 1\} =$$
$$= \sup\left\{\left\|\int \langle \bar{U}_t^{(\sigma)}\xi,\eta\rangle\, dm(t)\right\|_E : \|\xi\|_{H_\sigma}\leqq 1,\ \|\eta\|_{H_\sigma}\leqq 1\right\} \leqq \int \chi_G\, d|m|,$$

since $\bar{U}_t^{(\sigma)}$ is unitary. Thus $\|\hat{m}(\sigma)\|\leqq\|m\|$, $\sigma\in\Sigma$ and $\|\hat{m}\|_\infty\leqq\|m\|$. Hence $\hat{m}\in\mathscr{S}_\infty(\Sigma,E)$ and the mapping is continuous.

4.2. Definition. Let $\mathscr{C}(G,E)$ denote complex Banach space of all continuous E-valued functions on G with pointwise operations and norm given by $\|f\|= =\sup\{\|f(t)\|_E : t\in G\}$. For $\sigma\in\Sigma$ and a fixed orthonormal basis $(\xi_1,\xi_2,\dots,\xi_{d_\sigma})$ in H_σ, $\mathscr{I}^\sigma(G)$ will denote the subspace of $\mathscr{C}(G,\mathbf{C})$ generated by the coordinate functions u_{ij}^σ. We set $\mathscr{I}^\sigma(G,E)=\{x\varphi : x\in E \text{ and } \varphi\in\mathscr{I}^\sigma(G)\}$ and define $\mathscr{I}(G,E)$ to be subspace of $\mathscr{C}(G,E)$ generated by the union $\bigcup_{\sigma\in\Sigma}\mathscr{I}^\sigma(G,E)$.

4.3. Theorem.

(i) *For each* $\sigma\in\Sigma$, *we have* $\widehat{\mathscr{I}^\sigma(G,E)}=\mathscr{S}(H_\sigma\times H_\sigma,E)$.

(ii) $\widehat{\mathscr{I}(G,E)}=\mathscr{S}_{00}(\Sigma,E)$.

Proof. (i) The result readily follows from Lemma 3.3 since

$$\Phi(\sigma)\in\mathscr{S}(H_\sigma\times H_\sigma,E)\Leftrightarrow$$
$$\Leftrightarrow a_{ij}^\sigma\text{'s in } E \text{ and } u_{ij}^\sigma\text{'s in } \mathscr{I}(G,\mathbf{C}) \text{ such that } \Phi(\sigma)=\sum d_\sigma a_{ij}^\sigma \hat{u}_{ij}^\sigma(\sigma)\Leftrightarrow$$
$$\Leftrightarrow \Phi(\sigma)\in\widehat{\mathscr{I}^\sigma(G,E)}.$$

(ii) Suppose that $f\in\mathscr{I}(G,E)$. Then f may be written $f=\sum_{i=1}^n \alpha_i f_{\sigma_i}$, $\alpha_i\in\mathbf{C}$, $\sigma_i\in\Sigma$ and $f_{\sigma_i}=\sum_{j=1}^n x_j u_j^\sigma$, $x_j\in E$, $u_{ji}^\sigma\in\mathscr{I}^{\sigma_i}(G,\mathbf{C})$. Thus

$$\hat{f}(\sigma)(\xi_1,\xi_m)=\sum_i \alpha_i \sum_j x_j \hat{u}_j^{\sigma_i}(\sigma)(\xi_1,\xi_m)\neq 0 \text{ only if } \sigma=\sigma_i,\ i=1,2,\dots,n.$$

Hence $\hat{f}\in\mathscr{S}_{00}(\Sigma,E)$.

Conversely, if $\Phi\in\mathscr{S}_{00}(\Sigma,E)$, then the set $P=\{\sigma\in\Sigma : \Phi(\sigma)\neq 0\}$ is finite. Moreover, each $\Phi(\sigma)=\sum_{i,j=1}^{d_\sigma} d_\sigma a_{ij}^\sigma \hat{u}_{ij}^\sigma(\sigma)$. Putting $f=\sum d_\sigma \sum_{i,j=1}^{d_\sigma} a_{ij}^\sigma u_{ij}^\sigma$, we get $\hat{f}=\Phi$ and so $\widehat{\mathscr{I}(G,E)}=\mathscr{S}_{00}(\Sigma,E)$.

4.4. Lemma. *The space* $\mathscr{I}(G,E)$ *is dense in* $\mathscr{C}(G,E)$.

Proof. We identify $\mathscr{I}(G,E)$ with $\mathscr{I}(G,\mathbf{C})\otimes_\varepsilon E$, the injective tensor product of $\mathscr{I}(G,\mathbf{C})$ and E, i.e. the tensor product carrying the norm

$$\Big\|\sum_{1\leqq i\leqq n} x_i\varphi_i\Big\|_\varepsilon=\Big\|\sum_{1\leqq i\leqq n}\varphi_i\otimes x_i\Big\|_\varepsilon=\sup\Big\{\Big|\sum_{1\leqq i\leqq n} u(x_i)v(\varphi_i)\Big| : \|u\|\leqq 1,\ \|v\|\leqq 1\Big\},$$

$u \in E'$, $v \in \mathscr{I}(G, \mathbf{C})'$ where E' and $\mathscr{I}(G, \mathbf{C})'$ are the topological duals of E and $\mathscr{I}(G, \mathbf{C})$, respectively ([7], 44.2 (3)). Since $\mathscr{I}(G, \mathbf{C})$ is dense in $\mathscr{C}(G, E)$, ([6], 27.39), it follows that $\mathscr{I}(G, E)$ is dense in $\mathscr{C}(G, E)$, because $\mathscr{C}(G, E)$ is norm isomorphic to $\mathscr{C}(G, \mathbf{C})\overline{\otimes}_\varepsilon E$, the completion of $\mathscr{C}(G, \mathbf{C})\otimes_\varepsilon E$, ([7], 44.7 (2)).

4.5. Theorem. *The space $\hat{L}_1(G, E)$ of the Fourier transforms of Haar-integrable functions f: $G \to E$ is dense in $\mathscr{S}_0(\Sigma, E)$.*

Proof. The space $\mathscr{I}(G, E)$ is dense in $L_1(G, E)$ because $\mathscr{I}(G, E)$ is dense in $\mathscr{C}(G, E)$ and $\mathscr{C}(G, E)$ is dense in $L_1(G, E)$ ([4], 7.16). Since $\mathscr{I}(\widehat{G, E}) = \mathscr{S}_{00}(\Sigma, E)$ is dense in $\mathscr{S}_0(\Sigma, E)$, $\hat{L}_1(G, E)$ which contains $\mathscr{I}(\widehat{G, E})$, is dense in $\mathscr{S}_0(\Sigma, E)$.

4.6. Corollary. *If $f \in L_1(G, E)$, then the set $\{\sigma \in \Sigma: \hat{f}(\sigma) \neq 0\}$ is countable.*

4.7. Lemma. *Let $L_2(G, E)$ denote the Banach space of the Haar-square integrable functions on G into E. If $f \in L_2(G, E)$, then*

$$f = \sum_\sigma \sum_{i,j} d_\sigma \hat{f}(\sigma)(\xi_j, \xi_i) u_{ij}^\sigma.$$

Proof. If $f = xh$, $x \in E$ and $h \in L_2(G, \mathbf{C})$, then

$$f = \sum_{\sigma \in \Sigma} d_\sigma \sum_{i,j=1}^{d_\sigma} \left(\int xh(t) \bar{u}_{ij}^\sigma(t)\, d\lambda(t)\right) u_{ij}^\sigma$$

(use [6], 27.40 for h). Hence $f = \sum_\sigma d_\sigma \sum_{i,j=1} \left(\int f(t) \bar{u}_{ij}^\sigma(t)\, d\lambda(t)\right) u_{ij}^\sigma$. Since $L_2(G, \mathbf{C}) \otimes E$ is dense in $L_2(G, E)$ it is clear that the last equality holds for $f \in L_2(G, E)$. Now,

$$\int f(t) \bar{u}_{ij}^\sigma(t)\, d\lambda(t) = \int \langle \bar{U}_t^{(\sigma)} \xi_j, \xi_i \rangle f(t)\, d\lambda(t) = \hat{f}(\sigma)(\xi_j, \xi_i).$$

Hence $f = \sum_\sigma d_\sigma \sum_{i,j} \hat{f}(\sigma)(\xi_j, \xi_i) u_{ij}^\sigma$.

Finally, we obtain the analogue of [6], 28.43.

4.8. Theorem. *Assume that E is a Hilbert space. Then the mapping $f \to \hat{f}$ is an isometry from $L_2(G, E)$ onto $\mathscr{S}_2(\Sigma, E)$ and so $\mathscr{S}_2(\Sigma, E)$ is a Hilbert space.*

Proof. If E is a Hilbert space, than $L_2(G, E)$ is a Hilbert space so that $f \in L_2(G, E)$ if and only if

$$\|f\|_2^2 = \langle \sum_\sigma \sum_{i,j} d_\sigma a_{ij}^\sigma u_{ij}^\sigma, \quad \sum_\sigma \sum_{i,j} d_\sigma a_{ij}^\sigma u_{ij}^\sigma \rangle,$$

where $a_{ij}^\sigma = \hat{f}(\sigma)(\xi_j, \xi_i)$, $1 \leq i,\ j \leq d_\sigma$. Hence

$$\|f\|_2^2 = \sum_\sigma \sum_{i,j} d_\sigma^2 \|a_{ij}^\sigma\|_E^2 \|u_{ij}^\sigma\|_2^2 = \sum_\sigma \sum_{i,j} d_\sigma \|\hat{f}(\sigma)(\xi_j, \xi_i)\|_2^2,$$

since $d_\sigma \|u^\sigma_{ij}\|_2^2 = 1$ ([6], 27.40). Thus $\hat{f} \in \mathcal{S}_2(\Sigma, E)$ and

$$\|\hat{f}\|_2^2 = \sum_\sigma \sum_{i,j} d_\sigma \|\hat{f}(\sigma)(\xi_j, \xi_i)\|_E^2 = \|f\|_2^2.$$

Conversely, let $\Phi \in \mathcal{S}_2(\Sigma, E)$. Then $\sum_\sigma \sum_{i,j} d_\sigma \|\Phi(\sigma)(\xi_j, \xi_i)\|_E^2 < \infty$ and hence the set $\{\Phi(\sigma)(\xi_j, \xi_i) \neq 0\}$ is countable, say $\{a_k\}_{k \in \mathbf{N}}$. Put $f_n = \sum_{k=1}^n d_{\sigma_n} a_k u_k$, where u_k replaces u^σ_{ij} whenever $a^\sigma_{ij} = a_k$ is different from zero. Then the functions f_n form a Cauchy sequence in $L_2(G, E)$ whose limit f satisfies $\hat{f} = \Phi$ and the proof is complete.

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