Bibliographie

Approximation Theory, Tampa, Proceedings of the seminar held in Tampa, Florida, 1985—1986. Edited by E. B. Saff (Lecture Notes in Mathematics, 1287), VI+228 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

The Institute for Constructive Mathematics at the University of South Florida had its beginnings in 1985. The papers contained in these Proceedings of the Tampa Approximation Seminar prove the quality and variety of research activities conducted of the Institute and show the individual interest of the visitors to the Institute during the academic year 1985—1986.

Contents: P. R. Graves-Morris and J. M. Wilkins, A fast algorithm to solve Kalman's partial realisation problem for single input, multi-output systems; A. Knopfmacher and D. Lubinsky, Analogues of Freud's conjecture for Erdős type weights and related polynomial approximation problems; A. L. Levin and E. B. Saff, Some examples in approximation on the unit disk by reciprocals of polynomials; D. Lubinsky and E. B. Saff, Strong asymptotics for L_p extremal polynomials $(1<p\leqq\infty)$ associated with weights on $[-1, 1]$; L. S. Luo and J. Nuttall, Asymptotic behavior of the Christoffel function related to a certain unbounded set; H. N. Mhaskar, Some discrepancy theorems; J. Palagallo-Price and T. E. Price, Properties of projections obtained by averaging certain polynomial interpolants; L. Reichel, Boundary collocation in Fejér points for computing eigenvalues and eigenfunctions of the Laplacian; B. Shekhtman, On the geometry of real polynomials; H. Stahl, A note on a theorem by H. N. Mhaskar and E. B. Saff: "Where does the sup norm of a weighted polynomial live? (a generalization of incomplete polynomials)"; H. Stahl, Existence and uniqueness of rational interpolants with free and prescribed poles; J. Waldvogel, Zero-free disks in families of analytic functions.

J. Németh (Szeged)

N. H. Bingham—C. M. Goldie—J. L. Teugels, Regular Variation (Encyclopedia of Mathematics and its Applications, Vol. 27), XIX+491 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1987.

The publication of this book is a major mathematical event.

The theory of regularly varying functions was initiated by Jovan Karamata in 1930. A positive measurable function f defined on a half-line (a, ∞) with $a>0$ is called regularly varying (at ∞) of index $\varrho\in\mathbf{R}$ if $(*)$ $f(\lambda x)/f(x)\to\lambda^{\varrho}$, as $x\to\infty$, for each $\lambda>0$. (Measurability can usually be replaced by the Baire property for most of the basic results.) If R_ϱ denotes the class of all such functions then the functions in R_0 are called slowly varying, and for $f\in R_0$ we have $f(x)=x^{\varrho}l(x)$ with some $l\in R_0$. The notion of regular variation at zero rather than ∞, and then at any other point, is straightforward, every result at ∞ has a corresponding counterpart.

Karamata himself used his basic results on regular variation in Tauberian theorems and the theory was further developed by his Yugoslav School. As the authors write in their preface "The great potential of regular variation for probability theory and its applications was realised by William Feller, whose book [*An Introduction to Probability Theory and its Applications*, Vol. II, Wiley, New York, 1968 and 1971] did much to stimulate interest in the subject. Another major stimulus — again from a probabilistic viewpoint — was provided by Laurens de Haan in his 1970 thesis [*On Regular Variation and its Applications to the Weak Convergence of Sample Extremes*, Math. Centre Tract 32, Amsterdam], while Eugene Seneta gave a treatment of the basic theory of the subject in his monograph of 1976 [*Functions of Regular Variation*, Lecture Notes in Mathematics 506, Springer, Berlin]."

The first chapter is the essential Karamata theory (pp. 1—60). This is based on results like the uniform convergence theorem (stating that if $l \in R_0$ then the convergence in ($*$) above holds uniformly on each compact λ-set in $(0, \infty)$), the representation theorem (stating that if $l \in R_0$ then $l(x) = c(x) \exp\left\{\int_a^x b(t)\,dt/t\right\}$, $x \geqq a$, for some $a > 0$, where $c(x)$ is measurable and $c(x) \to c \in (0, \infty)$, $b(x) \to 0$ as $x \to \infty$), the characterisation theorem (stating that if for a positive measurable f relation ($*$) above holds for a λ-set of positive measure and with an unspecified limiting function $g(\lambda)$ on the right side, then it holds for all $\lambda > 0$ and necessarily $g(\lambda) = \lambda^\varrho$, $\lambda > 0$, for some $\varrho \in \mathbf{R}$) and the absolutely basic Karamata theorem, with many variants and refinements, stating very roughly that $f \in R_\varrho$ if and only if certain integral functions of f behave near ∞ as if $f(x)$ were constant times x^ϱ. There are many variants, versions or extensions of everything, monotone equivalents, asymptotic inverses and conjugates and various related notions and properties are discussed extremely intelligently together with special cases such as smooth variation and monotonicity with first applications as Karamata's Tauberian theorem for Laplace—Stieltjes transforms. Regularly varying sequences receive a separate discussion.

Chapter 2 (Further Karamata theory, pp. 61—126) is devoted to the investigation of the classes *ER* of extended regularly varying functions f and *OR* of *O*-regularly varying functions f (of positive measurable or Baire functions) for which $\lambda^d \leqq f_*(\lambda) \leqq f^*(\lambda) \leqq \lambda^c$, $1 \leqq \lambda < \infty$, for some c and d, and for which $0 < f_*(\lambda) \leqq f^*(\lambda) < \infty$, $1 \leqq \lambda < \infty$, respectively, where, as $x \to \infty$, $f_*(\lambda) = \lim\inf f(\lambda x)/f(x)$ and $f^*(\lambda) = \lim\sup f(\lambda x)/f(x)$, and to related classes. These are functions of bounded or positive increase or decrease, the classes $R_{-\infty}$ and R_∞, quasi-monotone and near-monotone functions, various subclasses of R_0, functions with Pólya peaks, Beurling slow variation, self-neglecting and self-controlled functions, to mention a few for those who know what these are or have the right sense of imagination.

Taking logarithms, relation ($*$) above, with a general limiting function, can be written as $\varphi(\lambda x) - \varphi(x) \to h(\lambda)$. Chapter 3 (de Haan theory, pp. 127—192) provides the extended modern theory when the left side here is replaced by the ratio $(\varphi(\lambda x) - \varphi(x))/\psi(x)$, where ψ is some auxiliary function, with the corresponding *O*-, *o*-, *E*- and other versions or extensions.

Chapter 4 (Abelian and Tauberian theorems, pp. 193—258) and Chapter 5 (Mercerian theorems, pp. 259—283) together constitute a virtually complete and beautifully constructed account of that part of classical analysis which is defined by these names, obtained by full-force application of the results in the first three chapters, with many far-reaching extensions and complements. All integral transforms of convolution type and all matrix transforms receive detailed attention where some form or other of regular variation plays some rôle in the result. These five chapters form a completely integrated and unrivalled unit which will be difficult to surpass before the twenty-second century.

And now come two little pearls. The first is Chapter 6 (pp. 284—297) on applications to analytic number theory (partitions, the prime number theorem and the order of sums of multiplicative functions); while the second is Chapter 7 (pp. 298—325) with applications to complex analysis concerned mainly with the growth of entire functions.

The last Chapter 8 (Applications to probability theory, pp. 326—422) is a masterpiece in itself. It offers a fantastically rich field of applications of regular variation and here we restrict this review to listing section headings: tail-behaviour and transforms, infinite divisibility, stability and domains of attraction, further central limit theory, self-similarity, renewal theory, regenerative phenomena, relative stability, fluctuation theory, queues, occupation times, branching processes, extremes, records, maxima and sums.

Six short appendices (pp. 423—444) with indications of further fields of applications and technical necessities complete the main body of the text.

However, the remaining forty-seven pages are very important to the excellence of the book. There is a list of references of 645 different items, each one supplied with a list of all the page numbers where it is cited. Then there is an index of named theorems. This is followed by a seven-page comprehensive index of notation and a sixteen-page very detailed general index concludes this encyclopedia of regular variation. These, together with the extremely clever structuring of the material into chapters, sections and subsections, the page headings and the nine-page table of contents make the book very easily usable. This is just one sign of the authors' sense of scholarship. Throughout, all Serbian, Croatian, French, German, Hungarian, Russian or Scandinavian accent marks are proper and are at their own place. All second- or third-named authors have a separate entry in the bibliography with a reference to the first-named author. There are *no* misprints in this book. (The three trivial typos this reviewer found were probably left intentionally by the three authors: to satisfy reviewers who believe that perfect works are impossible.)

This is a perfect work of art in every sense of the word. The language is perfect, the taste is perfect, the typography is perfect and, above all, the mathematics is perfect. The amount of knowledge brought together and of the work that went into this book is truly fascinating. There should be dozens of mathematicians sweating on their problems at this late hour of the night, or early hour of the morning, all over the world who would only need to look up page x of it and exclaim 'heureka'. Many-many results are new: brand new or completely polished versions of older results, when, needless to say, the authors always give the original sources just as when they follow somebody else in the proof even if they greatly simplified and polished that proof. And they are never tired to do so, even when they give five different proofs for the uniform convergence theorem in Chapter 1. In a sense everything is new here: every word of the subject is redigested and the whole comprehensive theory and its many applications are unified and integrated. The writing style is very modest, the mathematical and general intellect shines through, each page ticks, it is sheer delight to read the book.

It is a classic right away. A book for all seasons.

Sándor Csörgő (Szeged)

H. G. Dales—W. H. Woodin, An Introduction to Independence for Analysts (London Mathematical Society Lecture Note Series, 115), XIII+241 pages, Cambridge University Press, Cambridge—New York—Melbourne, 1987.

Let X be an infinite compact space and let $C(X)$ be the Banach algebra of all continuous functions. A famous question, first discussed by Kaplansky in 1948, asks if every algebra norm on $C(X)$ is necessarily equivalent to the given uniform norm. In 1976, using the continuum hypothesis (CH) H. G. Dales and J. R. Esterle, independently of each other, showed that there are algebra norms on $C(X)$ which are not equivalent to the uniform norm. More surprisingly, also in 1976 R. M. Solovay and W. H. Woodin proved that the existence of such norms is independent of the basic axioms of set theory (ZFC).

As the authors write in the preface: "The purpose of this book is to explain what it means for a proposition to be independent of set theory, and to describe how independence results can be proved by the technique of forcing." A full proof of the independence of (*CH*) from (*ZFC*) is given and the first proof of the theorem of Solovay and Woodin, accessible not only to logicians but intelligible also to analysts, is provided here. The authors include a discussion of Martin's Axiom, "which can be used to establish independence results without the necessity of knowing any of the technicalities of forcing".

This book offers analysts a good possibility to get acquainted with the powerful technique of forcing and with its application in the resolution of a deep problem in analysis. It can be recommended also to students of set theory as an introductory work.

László Kérchy (Szeged)

Dependence in Probability and Statistics. A Survey of Recent Results (Oberwolfach, 1985). Edited by E. Eberlein and M. S. Taqqu (Progress in Probability and Statistics, Vol. 11), XI+473 pages, Birkhäuser, Boston—Basel—Stuttgart, 1986.

This is a fine collection of a large number of excellent survey papers and a smaller number of equally excellent research papers on various kinds of dependent random variables, concentrating mainly on limit theorems.

Section 2 is on various mixing conditions with papers by R. Bradley, M. Peligrad, W. Philipp, M. Denker, C. M. Goldie and G. J. Morrow and by N. H. Bingham, while Section 3 contains the papers by P. Gaenssler and E. Haeusler and by E. Eberlein on martingale types of dependence. Section 4 carries the articles by A. R. Dabrowski, E. Waymire and by R. H. Burton and E. Waymire on positive and Gibbs dependence, the papers by F. Avram and M. S. Taqqu and by R. A. Davis and S. Resnick on moving averages in independent variables belonging to the domain of attraction of a non-normal stable law constitute Section 5.

Advances in dependent extreme value theory are sketched in the papers of G. O'Brien, J. Hüsler, and W. Vervaat in Section 6. Finally, Section 1 is on the recent hot topic of long-range dependence with papers by T. C. Sun and H. C. Ho, L. Giraitis and D. Surgailis, M. S. Taqqu and J. Levy, M. Maejima, N. Kôno, H. Dehling, the last paper being here a bibliographical guide by M. S. Taqqu to some 286 items. The preface of the two editors provides an intelligent guide to the collection itself, which will probably be indispensable for anyone with dependencies.

Sándor Csörgő (Szeged)

Luc Devroye, A Course in Density Estimation (Progress in Probability and Statistics, Vol. 14), XIX+183 pages, Birkhäuser, Boston—Basel—Stuttgart, 1987.

This seems like a most enjoyable book on density estimation using the L_1 criterion. The larger part of it appears as a lighter edition of the author's research monograph with L. Győrfi, *Nonparametric Density Estimation: The L_1 View,* Wiley, New York, 1985. It is based on the notes of a course the author has taught at Stanford University in 1986. Indeed, it is a first-class textbook for a graduate course with many examples, figures and exercises. The author should indeed be commended for having made the results of a very fresh and sophisticated research available for a wide public in just a little more than no time at all. In comparison to the earlier research monograph, however, some new material is also found in the present book. (Indeed, the opposite would have been very much uncharacteristic for the author.) These are chapters on robustness, minimum distance estimation, estimation of monotone densities, and on relative stability. The book can be enthusiastically recommended to every statistician: students, instructors, research workers, and the layman for that matter.

Sándor Csörgő (Szeged)

Differential Geometry, Calculus of Variations, and their Applications, Edited by G. M. Rassias and Th. M. Rassias (Lecture Notes in Pure and Applied Mathematics, Vol. 100), XIII+521 pages, Marcel Dekker, Inc., New York—Basel, 1985.

This book contains a series of papers dedicated to the memory of Leonhard Euler (1707—1783) on 200th anniversary of his death. His discoveries and significant contributions were devoted to every area of the mathematical sciences that existed in his day: calculus of variations; differential geometry of surfaces; the geometric origins of topology and combinatorics; particle, rigid body and celestial mechanics etc. The pure and applied aspects of mathematics and mechanics were not separated yet in that time. Lagrange, Laplace and Gauss were influenced directly by Euler's work, thus his activity belongs to the foundaments of the modern science. The papers in this volume are written by the authorities of the fields: dynamical systems, differential topology and geometry, calculus of variations, differential equations, control theory, and history and philosophy of sciences.

Péter T. Nagy (Szeged)

H. Edelsbrunner, Algorithms in Combinatorial Geometry, ETACS Monographs on Theoretica Computer Science, XV+423 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Computational geometry is a rapidly expanding part of mathematics today and several books have been published on this topic. This is not "just another book" but certainly one of the best ones. The theory emerged as the unification of computational technics and results of combinatorial geometry, and this book follows this line. The author's aim was "to demonstrate that computational and combinatorial investigations in geometry are doomed to profit from each other". According to this intention the book is divided into three parts.

The first part is devoted to the combinatorial geometry. It contains the fundamental geometric structures (arrangements of hyperplanes, configurations of points, convex polytopes, Voronoi diagrams), the main combinatorial tools and basic results of the complexity of families of cells (the Euler relation, the Dehn—Sommerville equations, an asymptotic version of the upper bound theorem).

The second part contains the computational methods, the organization of data structures of arrangements and the most important geometric algorithms (construction of convex hulls, linear programming, point location search).

The third part presents applications of the first two parts proving that the combination of these two fields results a really fruitful method.

Each chapter contains a problem section including exercises as well as research problems and the chapters end with a complete and updated bibliographical notes.

The book can be useful to specialists as a reference book but it is also recommended to everybody interested in the present advances in computational geometry.

J. Kincses (Szeged)

F. Forgó, Nonconvex Programming, 188 pages, Akadémiai Kiadó, Budapest, 1988.

Nonconvex programming deals with the class of mathematical programming problems in which a local maximum-point is not necessarily a global maximum-point. This book is devoted to provide a survey of the basic research directions of the nonconvex programming except its two major fields, the integer programming and the global optimization which are treated in a number of excellent monographs.

The book consists of ten chapters. The first three ones comprise such topics and techniques as optimality conditions, nonlinear duality, convex and concave envelopes of functions, direct and implicit enumerations, branch and bound method, different cuts which will be used in subsequent chapters.

Chapter 4 deals with the problem of maximizing a quasi-convex function over a polytope. Cutting-plane methods based on different kinds of cuts such as convexity, polaroid and shallow cuts are given for solving the problem in question. For the case of convex objective function the method of Falk and Hoffmann is presented. This part ends with the treatment of the Tuy—Zwart method.

Chapter 5 studies the problem of maximizing a linear objective function with convex inequality constraints, and an indirect cutting-plane algorithm is discussed.

The general case is studied in Chapter 6, where a continuous function is to be maximized over a compact subset of the n-dimensional Euclidean space. For solving it a branch and bound algorithm developed by Horst is presented, then some bounding techniques are discussed. Finally, the special case of separable objective function is investigated.

Chapter 7 is devoted to the nonconvex quadratic programming problems. Such methods are presented which more or less utilize the quadratic nature of the objective function.

A special nonconvex problem, the fixed charge problem, and some methods of solution are investigated in Chapter 8.

Chapter 9 deals with techniques for converting constrained problems to unconstrained ones and gives an explicit formula for the optimal solution of a nonconvex programming problem in terms of a multiple integral.

Finally, Chapter 10 contains a partition algorithm to decompose the nonconvex programming problem.

The book is well-written. The material is well-organized, the proofs are clear, a subject index helps the reader. It may be recommended to mathematicians, operation researchers, and computer scientists.

B. Imreh (Szeged)

S. Gallot—D. Hulin—J. Lafontaine, Riemannian Geometry (Universitext), XII+248 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

It is a great fun to read this book. The authors had found the ideal rate of abstractions and examples. When a new definition or theorem occurs the reader will meet a detailed recurrent study of the most important examples of Riemannian geometry like spheres, tori, projective spaces, etc. At the same time, throughout the book there are several exercises (the solutions of most of them are given at the end of the book) to help to understand the text.

The book is divided into five chapters. The first one is a quick introduction to differential manifolds. The next two chapters contain the basics of Riemannian geometry until Myer's and

Milnor's theorems. Chapter IV deals with analysis on manifolds and Chapter V is about Riemannian submanifolds.

Summing up, this is a modern, well built and useful book, and we warmly recommend it to all who need a good introduction to Riemannian geometry.

Árpád Kurusa (Szeged)

M. Göckeler—T. Schücker, Differential geometry, gauge theories, and gravity (Cambridge monographs on Mathematical physics), XII+230 pages, Cambridge University Press, New York—New Rochelle—Melbourne—Sydney, 1987.

This book is an introduction to those concepts of differential geometry which are fundamental for applications in elementary particle theory and general relativity. On the mathematical side, the only prerequisites are linear algebra and real analysis. The physical part of the book is essentially self-connected, but it is useful if the reader is already motivated by some knowledge of Yang—Mills theory, general relativity, and the Dirac equation.

The first three chapters contain an elementary account of differential forms in $\mathbf{R}^n$. This machinery is used then to reinterpret and rewrite basic quantities and equations of Yang—Mills theory and general relativity in geometric, coordinate-free terms. Next, the reader is acquainted with the notion and some applications of the Lie derivative. This is followed by chapters providing the rudiments of manifolds and Lie groups.

In Chapter 9 the authors present an introduction to fiber bundles and connections on them. This is a topic of growing importance in applications. The following chapter illustrates the theory on the examples of the Dirac monopole, the 't Hooft—Polyakov monopole, Yang—Mills and gravitational instantons.

Chapter 11 treates the algebraic (Clifford algebra, spinor representations) and analytic (Dirac operator, spin structures) aspects of the Dirac equation. The concept of Kähler fermions is also touched upon here. The next chapter is devoted to a subject of more advanced character, to the algebraic approach to anomalies. The final sections contain some background material on anomalous graphs.

This book is intended for graduate students in theoretical physics in the first place. The reviewer warmly recommends it also to everybody else searching for a well written, elementary introduction to modern differential geometry with emphasis on applications in particle theory and relativity.

László Fehér (Szeged)

Lj. T. Grujic—A. A. Martynyuk—M. Ribbens-Pavella, Large Scale Systems Stability under Structural and Singular Perturbations (Lecture Notes in Control and Information Sciences, 92), XVI+366 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Almost one hundred years ago, 1892, A. M. Lyapunov founded the mathematical stability theory in his famous doctoral dissertation. Previously stability concepts had been used only for mechanical systems. He has not only formulated the abstract definitions of stability concepts for arbitrary differential systems but established methods of investigation of these properties. One of them, the so-called direct method is suitable for finding conditions of stability properties via the system state differential equation without use of its solutions. This method has been proved to be extremely useful not only in mechanics but in many fields of the applications of differential equations such as control theory, reaction kinetics, population dynamics, biology and so on.

These lecture notes, which are a revised and completed version of their original Russian edition, are devoted to a recently developed branch of stability theory, to large scale systems stability. It is also based upon Lyapunov's direct method.

The first two chapters give an up-to-date survey on the state of Lyapunov's direct method. It was an excellent idea to start some sections with citations of the original definitions of stability concepts and fundamental theorems from Lyapunov's work. The reader can follow the arch of the one hundred years' development realizing that Lyapunov's original theorems are important and actual even today. Chapter I entitled Outline of the Lyapunov Stability Theory in General gives new versions of the definitions of stability concepts and theorems involving the earlier generalizations. The absolute stability is also treated.

Chapter II (Comparison Systems) contains the theory and application of the comparison method with scalar, vector and matrix functions. (The theory of comparison matrix functions initiated by A. A. Martynyuk was available earlier only in papers.)

The second part of the book is devoted to large-scale systems. The main idea here is to decompose the whole system into interconnected subsystems and then to find an aggregation form of the system yielding conditions under which the desired property of the original system can be deduced from the same properties of its interconnected subsystems and from qualitative properties of their interactions.

Chapter V (Large-Scale Power Systems Stability), which is essentially revised and completed in comparison with the original Russian edition, gives a good example for the process of mathematical modelling from the introduction of the physical problem, through the mathematical formulation and treatment until the interpretation of the results.

This book — which should be found on the book shelf of every mathematician, engineer, and any other user of mathematics interested in stability theory — is worthy of celebrating the oncoming hundredth anniversary of the publication of Lyapunov's fundamental work.

L. Hatvani (Szeged)

Jack Carl Kiefer, Introduction to Statistical Inference, Edited by G. Lorden (Springer Texts in Statistics), VIII+334 pages, 60 illustrations, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

This book is unique and is best in its kind. It gives a systematic development of decision-theoretic statistics, and it does this as a first course in mathematical statistics. It is based upon lecture notes of the late Professor Kiefer, one of the great masters of the subject to be compared only to Neyman and Wald, on whose work he builds here. So this is a posthumus book and it would have been a very great loss to the whole international statistical community if these notes had remained only in the privileged possession of those individuals who were fortunate enough to be around Cornell where Kiefer has developed them. It is a gift to all of us. The editor and the publisher should be thanked for making it available.

The first three short chapters (Introduction, Specification of a statistical problem, Classifications of statistical problems; pp. 1—30) introduce the basic decision-theoretic notions such as decision or procedure, loss function, operating characteristic, risk function and admissibility. Chapter 4 (Some criteria for choosing a procedure; pp. 31—80) explains the Bayes, the minimax and the unbiasedness criteria, and gives the essentials on randomized procedures and the methods of maximum likelihood and moments. Following the important Chapter 5 (Linear unbiased estimation; pp. 81—136) concentrating on the general linear model, least squares, orthogonalization and the Gauss—Markov Theorem, the whole Chapter 6 (pp. 137—157) is devoted to sufficiency.

The criteria of completeness, unbiasedness, sufficiency, invariance and asymptotic efficiency are discussed at length in Chapter 7 (pp. 158—245) in the context of point estimation, where more on minimax procedures and maximum likelihood are naturally found. Chapter 8 (pp. 246—286) is on hypothesis testing with less than twenty pages on "common normal theory tests", and the main body of the book concludes with Chapter 9 (pp. 287—311) on confidence intervals. Three short appendices, a list of fifteen references and an index complete the volume.

What is so special in such a book? *It is the lucidity of the mind and, as a result, the simplicity* of the language. Every sentence has a clear meaning (and this in itself would be sufficient to make the book unique) and Kiefer always means something. *Every* single-minded direction gets its share from him, sometimes in rather sharp terms, thus those who decide to cite Kiefer against something for their own benefit should be careful enough to leaf one or two before they do so. This is the work of a thinker. With the possible exception of Charles Stein alone, every living statistician will find something interesting or new in this book. And, at the same time, this is a textbook of introductory statistics for (good) students with minimal mathematical background but with a necessary maturity, seriousness and interest. This is achieved by a very large amount of examples and homework problems with fascinating notes and suggestions from the author. Instructors with the necessary characteristics just listed for students will want to have a copy of the book, independently of the nature of the statistics course they teach.

Sándor Csörgő (Szeged)

A. Kertész, Lectures on artinian Rings, Edited by R. Wiegandt, 427 pages, Akadémiai Kiadó, Budapest, 1987.

The text is a substantially extended and completed translation of the original German edition "Vorlesungen über artinsche Ringe" of the late A. Kertész. The present edition realizes the ideas and intentions of A. Kertész left behind in his notes.

Rather than being a comprehensive account of the theory of artinian rings, this book provides a well-written elementary text on ring theory centered on the basic theorems on artinian rings. Moreover, its scope is considerably wider than the title suggests and the main topic is developed within the framework of those modern generalizations which resulted from the systematic use of the artinian approach.

The book consists of fifteen chapters from which only nine have been treated in the German edition. The first four chapters are developments of the general theory of rings and modules, and requires practically no previous knowledge of that topic. This introduction to rings, modules, prime and Jacobson radical is carried out with care in an almost leisure manner.

The following part of the book deals with artinian rings and with generalizations without assuming the existence of unit element. For artinian rings it presents the classical theorems on semi-simple, primary and simple rings as well as on projective and injective modules. There we also find the general theory of rings of linear transformations, Jacobson's Density theory, the Wedderburn—Artin structure theorem, and Maschke's theorem. Steinfeld's theory on quasi-ideals is developed and used in giving ideal-theoretical characterization of semi-simple rings. The Litoff—Anh theorem on local matrix rings and Vámos' theorem on characterizing artinian modules by finitely embedded modules are also included, which have not been treated in the German edition. A full account of the additive structure for artinian rings is given including the fundamental theory of Fuchs and Szele.

The last six chapters of the book were written by A. Betsch, A. Widinger, and R. Wiegandt. During the last decades many new branches of ring theory have been developed and several important results have been proved involving artinian rings and modules. Thus it has become highly desirable to supplement the text with important topics such as Goldie's theory on rings of quotients, quasi-Frobenius rings, and Connell's theorem on artinian group rings. This part includes also a general decomposition theorem on strictly artinian rings, and investigations of linearly compact rings. In the study of rings with minimum condition on principal right ideals, the splitting theorem due to Ayoub and Huynh is also treated.

For better understanding, each chapter ends by a set of exercises with hints for solutions.

Writing this book the contributors and the editor made an excellent job, and the extended English version enlivens the reputation of the original German edition.

N. V. Loi (Budapest)

Serge Lang, Calculus of Several Variables, Third Edition (Undergraduate Texts in Mathematics), XII+503+A91+I4 pages, Springer-Verlag, New York—Berlin—Heidelberg—London—Paris—Tokyo, 1987.

Sometimes one pays less attention to the functions of several variables than to the functions of one variable. Once a famous mathematician told that: "In several variables everything goes just as in one variable." It was true for him, but the teachers know that in general this is not true for the students. We have several problems teaching this theme.

This book was previously published in 1973 and 1979, and therefore it is widely known. In a self-contained presentation it covers all essential topics in the calculus of several variables. Having read this book the reader will be familiar e.g. with the mathematics of mechanics.

Perhaps the best way to characterize the method of the book is to sketch the discussion of two, slightly embarrassing problems. The paragraph on inverse mappings contains three examples after the definition, then the inverse mapping theorem comes: Let $F\colon U\to R^n$ be a C^1-map. Let P be a point of U. If the Jacobian determinant $\Delta_F(P)$ is not equal to 0, then F is locally C^1-invertible at P. The proof of this theorem is beyond the scope of this book. Then we have three examples again. The next paragraph contains ten proposed exercises, the answers can be found at the end of the book. In the paragraph on implicit functions, after a short introduction the implicit function is stated in the form: Let U be open in R^2 and let $f\colon U\to R$ be a C^1-function. Let (a, b) be a point of U, and let $f(a, b)=c$. Assume that $D_2f(a, b)\neq 0$. Then there exists an implicit function $y=\varphi(x)$ which is C^1 in some interval containing a, and such that $\varphi(a)=b$. Before the proof we can find four examples. The next paragraph consists of various interesting exercises.

In connection with mathematical analysis in the former century it has been said that while Berlin found Göttingen lacking in rigour, Göttingen found Berlin lacking in ideas. These standpoints are problematical today as well. In my opinion "this book is between Berlin and Göttingen", it has a proper level in rigour and in ideas.

L. Pintér (Szeged)

Ricardo Mañé, Ergodic Theory and Differentiable Dynamics (Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, Band 8), XII+317 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Modelling systems, especially in mechanics, one often comes to a measure space and a measurable map on it such that the measure is invariant with respect to the map. The theme of ergodic theory is the dynamic behavior of such measure-preserving maps. The first theorem of the theory was proved by Poincaré. His celebrated recurrence theorem says that if the evolution of a system is described by a vector field whose divergence vanishes identically, then the system returns infinitely often to configurations arbitrarily close to the initial one, except for a set of initial configurations with zero Lebesgue measure, i.e. except for a set which can be neglected from the probabilistic point of view.

Around the turn of the century the work of Boltzmann and Gibbs on statistical mechanics raised the following mathematical problem: Given a measure-preserving map T of a space $(X, \mathcal{A}, \mu)$ and an integrable function $f\colon X \to R$, find conditions under which the limit

$$\lim_{n\to\infty} \frac{f(x)+f(Tx)+\ldots+f(T^{n-1}(x))}{n}$$

exists and is constant almost everywhere. Birkhoff proved that for any T and f the limit exists almost everywhere, and a necessary and sufficient condition for its value to be constant almost everywhere is that there exists no set $A \in \mathcal{A}$ such that $0<\mu(A)<1$ and $T^{-1}(A)=A$. Maps which satisfy this condition are called *ergodic*.

It can be very difficult to decide whether or not a map occurring in statistical mechanics is ergodic. For example, Gibbs initiated the study of billiards as models for a perfect gas. In a billiard spheres move with constant velocity within a bounded region colliding with one another and with the boundary in a perfectly elastic way. In the thirties Birkhoff gave an abstract formulation of the problem, but only in the sixties, starting with Sinai's work, were any billiard proved to be ergodic. The first example of a convex ergodic billiard was given by Bunimovich in 1974, but no examples of ergodic billiards with convex C^∞ boundary are known.

The book is an excellent survey on the ergodic theory of differentiable dynamical systems.

Chapter 0 summarizes the basic definitions and theorems of measure theory. This is a quick review, but the reader can find results with proofs on derivatives with respect to sequences of partitions, which cannot be found in standard references. Chapter 1 entitled Measure-Preserving Maps starts with a brilliant introduction outlining the basic problems of the ergodic theory, then presents the main kinds of dynamical systems around which ergodic theory has developed. Chapter II (Ergodicity) contains the classical concepts and results including Birkhoff's Theorem, Kolmogorov—Arnold—Moser Theorem, Gaussian and Markov Shifts. Chapter III (Expanding Maps and Anosov Diffeomorphisms) and Chapter IV (Entropy) are devoted to contemporary ergodic theory. A good part of the information is contained in the great number of exercises which give the reader the opportunity of working actively and individually in the field.

The book can be highly recommended either as an introduction or as a monograph for mathematicians and physicists.

L. Hatvani (Szeged)

Bernard Maskit, Kleinian Groups (Grundlehren der mathematischen Wissenschaften, 287), XIII+326 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

The fractional linear transformation group $PSL(2, \mathbf{C})$ with complex coefficients on the extended complex plane $\mathbf{C}\cup\{\infty\}$ has fundamental importance in theoretical and applied mathematics. This group is isomorphic to the orientation preserving conformal transformation group of the euclidean plane, to the isometry group of the hyperbolic space and to the rotation group of the pseudo-euclidean space-time and contains as subgroup the isometry groups of the euclidean and non-euclidean planes. The discrete subgroups of $PSL(2, \mathbf{C})$ were investigated already by Felix Klein in the relation with the space-form problem of the hyperbolic geometry. This theory had a large development in the last century and has many applications in complex analysis for the investigation of automorphic functions in topology, differential equations and number theory.

The new aspects of this theory are connected with the geometry and topology of 3-manifolds. The fundamental results of W. P. Thurston and his active school show that one could analyse discrete subgroups of $PSL(2, \mathbf{C})$ using 3-dimensional hyperbolic geometry.

The present book is an introduction to the theory of Kleinian groups which are subgroups of $PSL(2, \mathbf{C})$ acting freely and discontinuously at some point $Z \in \mathbf{C} \cup (\infty)$. The methods of hyperbolic geometry are used consequently in the treatment. The book is designed for using as a textbook for a one year advanced graduate course in Kleinian groups. The first three chapters give an introduction to the basic notions and results concerning fractional linear transformations, discontinuous groups acting on the plane and the theory of covering spaces. Chapters IV—VII contain the explanation of the general theory and can be used as foundation of Thurston's work, too. Chapter VIII is a collection of examples of Kleinian groups with diverse properties. The last two chapters give a study of special groups and discusse their structure theory.

The chapters are followed by a set of exercises which are quite uneven in terms of difficulty and also by notes giving a brief historical outline of the theory.

The reader is assumed to be familiar in group theory, topology, analytical and differential geometry of hyperbolic spaces. The book is highly recommended to everyone interested in the related fields of mathematics.

Péter T. Nagy (Szeged)

Non-Linear Equations in Classical and Quantum Field Theory, Proceedings, Meudon and Paris VI, France 1983/84. Edited by N. Sanchez (Lecture Notes in Physics, 226), VIII+400 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1985.

Field Theory, Quantum Gravity and Strings, Proceedings, Meudon and Paris VI, France 1984/85. Edited by H. J. de Vega and N. Sanchez (Lecture Notes in Physics, 246), VI+381 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.

Field Theory, Quantum Gravity and Strings II, Proceedings, Meudon and Paris VI, France 1985/86. Edited by H. J. de Vega and N. Sanchez (Lecture Notes in Physics, 280), VI+245 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

These three volumes contain the lectures delivered at the series of seminars on current developments in mathematical physics held alternately at DAPHE-Observatoire de Meudon and LPTHE-Universite Pierre and Marie Curie (Paris). The series of seminars started in October 1983 and these volumes account for the lectures (60 altogether) read up to October 1986. The lectures delivered by outstanding experts together provide the reader with a comprehensive review of recent advances and trends in mathematical physics. The following list of key-words can give only a taste of the variety of topics covered in this collection.

The central themes of the first volume are integrable non-linear theories and methods to solve them. Among the key-words are: Lax pairs, Bäcklund transformations, Yang—Baxter and Kac—Moody algebras. The models reviewed include self-dual Yang—Mills fields, Bogomolny—Prasad—Sommerfield monopoles and sigma models.

A number of lectures in the second volume of this set are devoted to the superstring attempt of unification of interactions and to the related topic of conformally invariant two dimensional models. Other reviews treate Kaluza—Klein theories, quantum cosmology and stochastic quantization. Exact solvability is amongst the key-words of most frequent occurrence here too.

In the third volume the reader finds lectures on string theory, quantum gravity, integrable systems, soliton dynamics, twistor theory, dynamical symmetries and critical phenomena.

This collection of stimulating and comprehensive reviews encourages further the interaction between different fields of theoretical physics and mathematics. It should have a place on the shelves of every theoretical physics and mathematics library.

László Fehér (Szeged)

Nonlinear Semigroups, Partial Differential Equations and Attractors, Proceedings, Washington, D.C., 1985. Edited by T. L. Gill and W. W. Zachary (Lecture Notes in Mathematics, 1248), IX+185 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

Reading the classic textbooks and monographs in partial differential equations nowadays one can realize in surprise that everything was linear at that time. During the last three decades it has been pointed out that nonlinear structures are of interest, e.g. that the chaotic behaviour of some nonlinearities offers new explanations for some misterious phenomena. The methods of nonlinear theory have been developing so fast, and so many books have appeared on them that now one must think: everything is nonlinear.

These lecture notes are the proceedings of the symposium on the topics involved in the title held at Howard University in Washington, D. C. on August 5—8, 1985. In the reviewer's opinion, all the articles are of such interest and importance that each of them has to be cited: Joel D. Avrin, Convergence Properties of Strongly-Damped Semilinear Wave Equations; S. A. Belbas, Numerical Solution of Certain Nonlinear Parabolic PDE; Melvyn S. Berger, The Explicit Solution of Nonlinear ODE's and PDE's; Whei-Ching C. Chan and Shui-Nee Chow, Uniform Boundedness and Generalized Inverses in Liapunov—Schmidt Method for Subharmonics; Hans Engler, Existence of Radially Symmetric Solutions of Strongly Damped Wave Equations; H. Engler, F. Neubrander, and J. Sandefur, Strongly Damped Semilinear Second Order Equations; Lawrence C. Evans, Nonlinear Semigroup Theory and Viscosity Solutions of Hamilton—Jacobi PDE; Jerome A. Goldstein, Evolution Equations with Nonlinear Boundary Conditions; Jack K. Habe, Asymptotically Smooth Semigroups and Applications; John Mallet—Paret and George R. Sell, The Principle of Spatial Averaging and Inertial Manifolds for Reaction Diffusion Equations; Robert H. Martin, Jr., Applications of Semigroup Theory to Reaction-Diffusion Systems; Jeffrey Rauch and Michael C. Reed, Ultra Singularities in Nonlinear Waves; M. C. Reed and J. J. Blum, A Reaction-Hyperbolic System in Physiology; Eric Shechter, Compact Perturbations of Linear M-Dissipative Operators Which Lack Gihman's Property; Thomas I. Seidman, Two Compactness Lemmas; Andrew Vogt, The Riccati Equation: When Nonlinearity Reduces to Linearity.

L. Hatvani (Szeged)

Numerical Analysis, Proceedings of the Fourth IIMAS Workshop held at Guanajnato, Mexico, July 23—27, 1984. Edited by J. P. Hennart (Lecture Notes in Mathematics, 1230), X+234 pages, Springer-Verlag, Berlin—Heidelberg, 1986.

This volume contains 18 selected items (mainly of the invited lecturers) from the 29 papers delivered at the Fourth Workshop on Numerical Analysis hosted by the National University of Mexico.

The program of the workshop was centered on the following main areas: optimization problems, the solution of systems of both linear and nonlinear equations, and the numerical aspects of differential equations. Most of the papers deal with special problems/methods of these fields. Moreover, many practical hints and experimental results are provided, too.

The authors' motivations vary from practical problems, e.g. the planning of semiconductor devices and the stability of capillary waves, to 'pure' (numerical) mathematics such as the derivation of new Runge—Kutta formulae and convergence results on the secant methods in Hilbert space.

Let us quote some titles just to give a taste of the book:

Goldfarb: Efficient primal algorithm for strictly convex quadratic programs; Falk and Richter: Remarks on a continuous finite element scheme for hyperbolic equations; Elman and Streit: Polynomial iteration for nonsymmetric indefinite linear systems.

Although a part of the contributions is available in a more polished form in journal this book may be a valuable guide for the specialists working in these subfields to the directions of current interest.

J. Virágh (Szeged)

Tadao Oda, Convex Bodies and Algebraic Geometry; An Introduction to the Theory of Toric Varieties (Ergebnisse der Mathematik and ihrer Grenzgebiete, 3. Folge, Band 15), VII+212 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

The beginners learning algebraic geometry usually have difficulties with the lot of new and abstract notions familiarity of which is necessary to understand the theory. The purpose of this book is to give an introduction to algebraic geometry, especially to the theory of toric varieties, using the language of the visuable convex geometry. The author writes in the introduction: "For this reason, we chose to construct toric varieties as complex analytic spaces, so that they can be understood more easily without much prior knowledge of algebraic geometry. Not only can some of the important complex analytic properties of these spaces be translated into easily visualized elementary geometry of convex figures, but many interesting examples of complex analytic spaces can be easily constructed by means of this theory." Chapter 1 is devoted to the basic notions and facts about toric varieties. Chapter 2 contains results on the cohomology of compact toric varieties and the imbedding theory into projective spaces. Chapter 3 contains a study of the automorphism group using holomorphic differential forms. Chapter 4 deals with applications of the theory to the investigation of singularities. In Appendix the basic results of convex geometry are collected without proofs.

Péter T. Nagy (Szeged)

Nicolae H. Pavel, Nonlinear Evolution Operators and Semigroups. Application to Partial Differential Equations (Lecture Notes in Mathematics, 1260), VI+285 pages, Springer-Verlag,) Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

In the last two decades a very successful new branch has appeared in the theory of differential equations. It has been pointed out that semigroups and evolution equations techniques can be widely used to solve problems related to partial differential equations and functional differential equations. This allows these equations to be treated as suitable ordinary differential equations in infinite dimensional Banach spaces.

The book presents some of the fundamental results and recent research on nonlinear evolution operators and semigroups and their applications. Most of the results involved were available earlier only in papers.

The first chapter is devoted to the construction and main properties of nonlinear evolution operator associated with nonautonomous differential inclusions. Chapter 2 is concerned with nonlinear semigroups generated by dissipative operators (Crandall—Liggett Theory). The most interesting chapter, the third one, shows how to apply the abstract results of the theory to unify the treatments of several types of partial differential equations arising in physics and biology (equation of long water waves of small amplitude, Porous Medium Equation, the heat equation, Schrödinger Equation, Semilinear Schrödinger Equation, and so on).

L. Hatvani (Szeged)

Probability Theory and Mathematical Statistics, Proceedings of the 5th Pannonian Symposium on Mathematical Statistics, Visegrád, Hungary, 20—24 May, 1985. Edited by W. Grossmann, J. Mogyoródi, I. Vincze and W. Wertz, XIII+457 pages, Akadémiai Kiadó, Budapest and D. Reidel Publishing Company, Dordrecht, 1988.

The proceedings of the 3rd and 4th Symposia have been reviewed in these *Acta*, 47 (1984), page 513 and 51 (1987), page 283.

Just as the financial support shrinks as the symposium moves over to Hungarian territory from the Austrian, the number of participants decreases. Accordingly, the proceedings reduce from two volumes to one. Fortunately, however, the level of the quality achieved by the proceedings of the 4th Symposium has been maintained.

Part A (pages1—234) containes the papers on various probability problems by G. Baróti, N. L. Bassily, E. Csáki and A. Földes, G. Elek and K. Grill, I. Fazekas, S. Fridli and F. Schipp, J. Galambos and I. Kátai, B. Gyires, I. Gyöngy, M. Janžura, I. Kalmár, A. Kováts, L. Lakatos, E. G. Martins and D. D. Pestena, T. F. Móri, T. Nemetz and J. Ureczky, P. M. Peruničić, D. Plachky, T. Pogány, G. J. Székely, I. Vincze, and by A. Zempléni. Part B then consists of the papers on diverse statistical topics and applications written by J. Anděl, G. Apoyan and Yu. Kotojants, J. Hurt, P. Kosik and K. Sarkadi, A. Pázmán, Z. Prášková, L. Rüschendorf, A. K. Md. E. Saleh and P. K. Sen, L. Szeidl, G. Terdik, R. Thrum, J. Tóth, S. Veres (2 papers), J. A. Víšek, L. Vostrikova, P. Volf, and by W. Wefelmeyer.

Those who liked the "Pannonian" flavour in the preceding Proceedings will want to savour it in this volume as well.

Sándor Csörgő (Szeged)

Probability Theory and Mathematical Statistics, Proceedings of the Fifth Japan—USSR Symposium, Kyoto, Japan, July 8—14, 1986. Edited by S. Watanabe and Yu. V. Prohorov (Lecture, Notes in Mathematics, 1299), VIII+589 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1988.

The volume contains 61 papers, 20 of which are by authors from the USSR, 2 by visitors in Japan from Czechoslovakia and France, and the remaining 39 articles are written by Japanese authors, 2 with co-authors from France and the USA. There are 2 papers describing the work of G. Maruyama who deceased three days before the symposium, 5 papers which could be classified as belonging to statistical theory, and the great majority of the rest is in probability theory with a few contributions representing related fields such as probabilistic number theory, ergodic theory or information theory. A number of the papers are expository in nature, most of them are proper research articles on a wide variety of different topics.

Sándor Csörgő (Szeged)

Maurice Roseau, Vibrations in Mechanical Systems. Analytical Methods and Applications, XIV+515 pages, Springer-Verlag, Berlin—Heidelberg—New York—London—Paris—Tokyo, 1987.

The vibrations and prediction of their effects are of great importance in construction of machines and devices. The change in time of the mechanical variables is governed by ordinary or partial differential equations. Based on the analysis of differential equations various theories have been developed such as linearized or non-linearized, and very often of an asymptotic nature. These theories deal with the conditions of stability and resonance and the coupling of modes in non-linear systems. In this book such methods are developed which deal with free and induced vibrations in discrete or continuous mechanical structures.

In each of the twelve chapters the reader can find well selected illustrations to the theories and methods. Numerous important examples, known and original, are discussed in a complex and thoroughful way. To show the variety of the subject covered in this book it is enough to cite some items of the contents: Forced Vibrations, Vibrations in Lattices, Gyroscopic Coupling, Stability of Linear Systems, The Stability of Operation of Non-Conservative Mechanical Systems, Flexible Vibrations of Beams, Longitudinal and Torsional Vibrations of Bars, Vibrations of Elastic Solids and of Plane Elastic Plates, Vibrations in Periodic Media, Model Analysis, Synchronisation Theory, Stability of a Column Under Compression, The Method of Amplitude Variation, Rotating Machinery, Non-Linear Waves and Solitons.

This volume is a translation of the French original published in 1984. Several chapters have been taught to graduate students at the Pierre and Marie Curie University in Paris.

The book is useful for mathematicians dealing with applications of differential equations and is recommended to students and researchers interested in mechanics and mechanical engineering.

I. K. Gyémánt (Szeged)

Kennan T. Smith, Power Series from a Computational Point of View (Universitext), VIII + 132 pages, Springer-Verlag, New York—Berlin—Heidelberg, 1987.

The author summarizes his aims as follows:

"The purpose of this book is to explain the use of power series in performing calculations, such as approximating definite integrals or solutions to differential equations. This focus may seem narrow but, in fact, such computations require the understanding and use of many of the important theorems of elementary analytic function theory, ... These computations provide an effective motivation for learning the theorems, and a sound basis for understanding them."

In the refree's opinion, the title could be paraphrased such as "Power Series from the Point of View of Complex Analysis" because most of the material is hard-core mathematics. The chapter headings are the following: Taylor polynomials, Sequences and Series, Power Series and Complex Differentiability, Local Analytic Functions, Analytic continuations. So the minimal prerequisite would be an introductory analysis course. In this case, however, some elementary parts of the second chapter could be omitted. For the convenience of the reader the theorems, definitions and formulae are numbered and cross-referenced throughout the text. However, references such as "According to the next section, this is Taylor's formula for $\log(1+x)$ centered at a, but this is not needed" (pp. 25), or "Referring to the picture in Section 4, use Definition 1.7 and Theorems 2.4 and 2.8 to show that ..." (pp. 84) are rather awkward. The book ends with a useful index of notions.

After each chapter a set of selected problems can be found. They belong mainly to the two categories "prove the following theorem" or "compute the definite integral/Taylor polynomial of the following function". There are only a few scattered indications of computer practice one of them, e.g. "Write the FORTRAN program to compute..." (pp. 26).

Finally, I miss the links with Numerical Analysis from the book. (The rare exceptions are the trapezoid and the Simpson's rule mentioned in some problems.)

J. Virágh (Szeged)