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On some special limits of n-groups

JACEK MICHALSKI

1. Introduction

In [7], [8], [9] a systematic study of the category of n-groups has been started. The present paper follows the lines of these papers, especially [7]. We improve the results in [7] (Theorems 1 and 3) on the preservation of projective and inductive limits by the functors Φ and Ψ, respectively, under certain additional conditions on the diagram scheme in consideration. In the present paper we weaken the conditions that turned out in [7] to be sufficient for the preservation of limits so that they become necessary and sufficient. In this way the relation of Φ to projective limits and Ψ to inductive limits becomes clear.

2. Preliminaries

The terminology of this paper is the same as in [5]—[9], where we also discussed relevant notions. Recall briefly some of the most significant notions and notation introduced there.

We assume throughout the paper that $n=sk$ (allowing $k=1$). However, it is sensible (contrary to [7]) to make the assumption that $n>k$. For the case $n=k$ (i.e., $s=1$) some statements become trivial and others become false.

As in [10], n-groups will sometimes be called *polyadic groups,* especially when the arity of the operation is not crucial. Similarly, a sequence $a_1, \ldots, a_m$ of elements of an $(n+1)$-group G is called (following Post) a *polyad* (or shortly an *m-ad*). For convenience such sequences are denoted by $\langle a_1, \ldots, a_m\rangle$. To simplify the notation, in place of $\langle a_1, \ldots, a_{m-r}, \underbrace{b, \ldots, b}_{r}\rangle$ we shall write briefly $\langle a_1, \ldots, a_{m-r}, \overset{r}{b}\rangle$.

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Post has introduced an equivalence relation Θ on the set of all m-ads (for fixed m) of a given $(n+1)$-group (G,f). The relation Θ is defined as follows:

$$\langle a_1, \ldots, a_m\rangle \,\Theta\, \langle b_1, \ldots, b_m\rangle$$

if and only if for a certain $i=1, \ldots, n+1-m$, and for some elements $c_1, \ldots, c_{n+1-m} \in G$ we have the equality

$$f(c_1, \ldots, c_i, a_1, \ldots, a_m, c_{i+1}, \ldots, c_{n+1-m}) = f(c_1, \ldots, c_i, b_1, \ldots, b_m, c_{i+1}, \ldots, c_{n+1-m}).$$

One can prove (cf. [10]) that $\langle a_1, \ldots, a_m\rangle \Theta \langle b_1, \ldots, b_m\rangle$ implies that for every $i=1, \ldots, n+1-m$ and for every sequence $x_1, \ldots, x_{n+1-m} \in G$ the following equality holds:

$$f(x_1, \ldots, x_i, a_1, \ldots, a_m, x_{i+1}, \ldots, x_{n+1-m}) =$$

$$= f(x_1, \ldots, x_i, b_1, \ldots, b_m, x_{i+1}, \ldots, x_{n+1-m}).$$

The notion of polyads equivalent with respect to Θ will appear in Lemmas 10—15 and we will make use of the above mentioned theorem in the proof of Theorem 2.

In the paper we deal only with categorical properties of polyadic groups; however, in some proofs (especially in the proof of Theorem 2) we essentially turn to the inner view point, i.e., we consider polyadic groups as sets together with certain operations. This causes some inconsistency in notation. Usually we denote a polyadic group simply by one letter (say G), but whenever the group operation (say f) appears in an explicit form, we write (G,f). To avoid numerous repetitions, we assume that f and g always denote $(n+1)$-group and $(k+1)$-group operations, respectively, and we write (G,f) and (G,g) only to avoid a possible confusion.

The identity morphism is denoted by $e_A\colon A \to A$ or briefly by e, if it is not misleading.

For a $(k+1)$-semigroup (G,g) one can define a new $(sk+1)$-ary operation $g_{(s)}$ by

$$g_{(s)}(x_1, \ldots, x_{n+1}) =$$

$$= \underbrace{g(g(\ldots g(}_{s}g(x_1, \ldots, x_{k+1}), x_{k+2}, \ldots, x_{2k+1}), \ldots), x_{(s-1)k+2}, \ldots, x_{n+1}).$$

If (G,g) is a $(k+1)$-group, then the $(sk+1)$-group $(G,g_{(s)})$ is an $(n+1)$-group, too (cf. [2]). This $(n+1)$-group is said to be a *derived* $(n+1)$*-group of the* $(k+1)$*-group* (G,g) (cf. [2], [5]) and is denoted by $\Psi_s(G,g)$ or shortly by $\Psi_s(G)$ (cf. [3], [7]).

In this way one can obtain a forgetful functor $\Psi_s\colon \mathbf{Gr}_{k+1} \to \mathbf{Gr}_{n+1}$ (in this paper, as in [3], [7], $\mathbf{Gr}_n$ denotes the category of n-groups). The functor Ψ_s has a left adjoint. This is the functor Φ_s: $\mathbf{Gr}_{n+1} \to \mathbf{Gr}_{k+1}$ assigning to each $(n+1)$-group its free covering $(k+1)$-group (cf. [3], [5], [7]).

The notion of a free covering $(k+1)$-group of an $(n+1)$-group, introduced in [3] and investigated in [5], [7], is a generalization of the well-known notion of a free covering group which was introduced by Post in [10].

3. Some lemmas

This section is of auxiliary character. The facts presented can be treated as known and can be found in any basic course on category theory (e.g., [1], [11]) or easily inferred from statements given there. Most of these facts belong to the "folklore" of category theory, and therefore they are given without references. In this section we collect all the auxiliary categorical lemmas that be will applied in later sections.

As is known, each theorem of category theory can be given a dual formulation. To avoid repetitions, we do not formulate the dual versions to the given statements. When referring to the dual version of a lemma given in this section we indicate it by adding an asterisk to the number of the lemma.

In this paper the term functor always means a covariant functor. We use interchangeably the following terms: a small category and a diagram scheme, a functor from a small category and a diagram. The terms diagram scheme and diagram are used especially in dealing with limits. The symbol $\mathscr{D}$ always denotes a small category and the symbol F a functor from that category $\mathscr{D}$ (i.e., F denotes a diagram).

We assume in the lemmas (except Lemma 2) that the categories $\mathscr{K}$, $\mathscr{K}_1$ and $\mathscr{K}_2$ are complete with respect to projective limits of all diagrams, including the empty diagram scheme. As a consequence, these categories possess final objects. Since Lemma 2 is formulated for inductive limits, we assume in it of course the completeness of $\mathscr{K}$ with respect to inductive limits. This convention has to be understood so that the assumptions on the categories $\mathscr{K}$, $\mathscr{K}_1$ and $\mathscr{K}_2$ in the dual versions of the lemmas are also dual.

Lemma 1. *Let a faithful functor* Λ: $\mathscr{K}_1 \to \mathscr{K}_2$ *have the following property: if* $\Lambda(\gamma)=\Lambda(\beta)\delta$ *where* β: $B \to C$, γ: $A \to C$, δ: $\Lambda(A) \to \Lambda(B)$, *then the morphism* δ *is of the form* $\delta=\Lambda(\alpha)$ *for some* α: $A \to B$. *Then the functor* Λ *reflects projective limits.*

Proof. Let $[G; \{\alpha_D: G \to F(D)\}_{D\in\mathscr{D}}]$ and $[\Lambda(L); \{\Lambda(\pi_D): \Lambda(L) \to \Lambda F(D)\}_{D\in\mathscr{D}}]$ be the projective limits of F: $\mathscr{D} \to \mathscr{K}_1$ and ΛF: $\mathscr{D} \to \mathscr{K}_2$, respectively. From the faithfulness of Λ it follows that the family $\{\pi_D: L \to F(D)\}_{D\in\mathscr{D}}$ is compatible with

F. Therefore there exists a morphism δ such that $\alpha_D\delta=\pi_D$ for $D\in\mathscr{D}$. The family $\{\Lambda(\alpha_D)\colon \Lambda(G)\to\Lambda F(D)\}_{D\in\mathscr{D}}$ is compatible with ΛF, and so there exists a morphism $\eta\colon \Lambda(G)\to\Lambda(L)$ such that $\Lambda(\pi_D)\eta=\Lambda(\alpha_D)$. From the equalities $\Lambda(\pi_D)\eta\Lambda(\delta)=\Lambda(\alpha_D\delta)=\Lambda(\pi_D)$ for $D\in\mathscr{D}$ it follows that $\eta\Lambda(\delta)=e_{\Lambda(L)}$. Hence η is a retraction. On the other hand, from the assumption on Λ it follows that η is of the form $\eta=\Lambda(\mu)$ where $\mu\colon G\to L$. Thus $\Lambda(\alpha_D\delta\mu)=\Lambda(\pi_D)\eta=\Lambda(\alpha_D)$, which, by the faithfulness of Λ, implies that $\alpha_D\delta\mu=\alpha_D$ for $D\in\mathscr{D}$ and hence $\delta\mu=e_G$. Therefore μ is a co-retraction. The functor Λ, being faithful, preserves co-retractions and so $\eta=\Lambda(\mu)$ is a co-retraction. But η is also a retraction, and so η is an isomorphism. As is easy to check, Λ reflects isomorphisms, whence μ is an isomorphism (since $\eta=\Lambda(\mu)$). Therefore $[L;\ \{\pi_D\colon L\to F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of F, which is what was to be proved.

Let a category $\mathscr{K}$ have an initial object U which satisfies an additional condition: for every object $X\in\mathscr{K}$ distinct from U we have $\operatorname{Mor}(X,U)=\emptyset$. It is worth adding that not every category with initial objects has initial objects with this property. For instance, this condition is not satisfied in $\mathbf{Gr}_n$ for $n=2$; however, for $n>2$ (and also in the category of sets) it is satisfied.

Consider a diagram $F\colon \mathscr{D}\to\mathscr{K}$. Let $\mathscr{D}_0$ be the full subcategory of $\mathscr{D}$ consisting of all objects D such that $F(D)\neq U$, and let F_0 be the restriction of F to $\mathscr{D}_0$. Then the following lemma is true.

Lemma 2. *$[L;\ \{\gamma_D\colon F(D)\to L\}_{D\in\mathscr{D}}]$ is the inductive limit of F if and only if $[L;\ \{\gamma_D\colon F_0(D)\to L\}_{D\in\mathscr{D}_0}]$ is the inductive limit of F_0.*

Proof. Let $[L;\ \{\gamma_D\colon F(D)\to L\}_{D\in\mathscr{D}}]$ be the inductive limit of F. The family $\{\gamma_D\colon F_0(D)\to L\}_{D\in\mathscr{D}_0}$ is compatible with F_0. Take an arbitrary family

$$\{\alpha_D\colon F_0(D)\to G\}_{D\in\mathscr{D}_0}$$

with $G\in\mathscr{K}$, which is compatible with F_0. That family can be extended to a family $\{\alpha_D\colon F(D)\to G\}_{D\in\mathscr{D}}$ by choosing as morphism $\alpha_D\colon F(D)\to G$ for $D\notin\mathscr{D}_0$, the only morphism from the initial object $F(D)$ (in the category $\mathscr{K}$) into the object G. It is easy to verify that the extended family of morphisms is compatible with F. Thus there exists a unique morphism $\delta\colon L\to G$ with $\delta\gamma_D=\alpha_D$ for $D\in\mathscr{D}$. Hence, in particular, $\delta\gamma_D=\alpha_D$ for $D\in\mathscr{D}_0$, which proves that $[L;\ \{\gamma_D\colon F_0(D)\to L\}_{D\in\mathscr{D}_0}]$ is the inductive limit of F_0.

Conversely, if $[L;\ \{\gamma_D\colon F_0(D)\to L\}_{D\in\mathscr{D}_0}]$ is the inductive limit of F_0, then the family $\{\gamma_D\}_{D\in\mathscr{D}_0}$ can be extended in a natural way to a family $\{\gamma_D\}_{D\in\mathscr{D}}$. So $[L;\ \{\gamma_D\}_{D\in\mathscr{D}}]$ is already the inductive limit of F. This completes the proof of Lemma 2.

In our further considerations the notions of discrete and connected categories prove to be very useful. A category $\mathscr{K}$ is said to be connected if for every pair of objects $X, Y \in \mathcal{K}$ there exists a finite sequence of objects $A_0, \ldots, A_m \in \mathcal{K}$ such that $A_0 = X$, $A_m = Y$ and $\mathrm{Mor}(A_i, A_{i+1}) \cup \mathrm{Mor}(A_{i+1}, A_i) \neq \emptyset$ for $i = 0, \ldots, m-1$. A full subcategory $\mathcal{K}'$ of a category $\mathcal{K}$ is said to be discrete if for any pair of distinct objects $X, Y \in \mathcal{K}'$ there exists no connected subcategory of $\mathcal{K}$ containing X and Y.

Using the Kuratowski—Zorn Lemma one can prove the following two lemmas.

Lemma 3. *For each object D of a small category $\mathcal{D}$, $\mathcal{D}$ has a maximal connected full subcategory $\mathcal{D}_D$, i.e., a connected subcategory $\mathcal{D}_D$ such that $D \in \mathcal{D}_D$ and for any pair of objects $A, B \in \mathcal{D}$ with $A \in \mathcal{D}_D$, $B \notin \mathcal{D}_D$ we have* $\mathrm{Mor}(A, B) \cup \mathrm{Mor}(B, A) = \emptyset$.

Lemma 4. *Every small category $\mathcal{D}$ has a maximal discrete full subcategory $\mathcal{D}_d$, i.e., a discrete subcategory $\mathcal{D}_d$ such that for each object $X \in \mathcal{D}$ there exists an object $A \in \mathcal{D}_d$ with $X \in \mathcal{D}_A$.*

Consider a diagram $F\colon \mathcal{D} \to \mathcal{K}$ with the following special property: every pair $\alpha, \beta \in \mathrm{Mor}(X, Y)$ with $X, Y \in \mathcal{D}$ satisfies the equality $F(\alpha) = F(\beta)$; furthermore, $F(\alpha)$ is an isomorphism. Let $\mathcal{D}_d$ be a maximal discrete full subcategory of $\mathcal{D}$ and let $F_d\colon \mathcal{D}_d \to \mathcal{K}$ be the restriction of F to the full subcategory $\mathcal{D}_d$. For such a diagram F we have the following lemma.

Lemma 5. *If $[L; \{\alpha_D\colon L \to F(D)\}_{D \in \mathcal{D}}]$ is the projective limit of F, then $[L; \{\alpha_D\colon L \to F_D(D)\}_{D \in \mathcal{D}_d}]$ is the projective limit of F_d.*

Proof. Let $[L; \{\alpha_D\}_{D \in \mathcal{D}}]$ be the projective limit of F. The family

$$\{\alpha_D\colon L \to F_d(D)\}_{D \in \mathcal{D}_d}$$

is compatible with F_d. Take any family $\{\beta_D\colon G \to F_d(D)\}_{D \in \mathcal{D}_d}$, where $D \in \mathcal{D}_d$, which is compatible with F_d. To show that this family can be extended to a family $\{\beta_D\colon G \to F(D)\}_{D \in \mathcal{D}}$ take an arbitrary object $X \in \mathcal{D}$. The definition of $\mathcal{D}_d$ implies the existence of an object $A \in \mathcal{D}_d$ with $X \in \mathcal{D}_A$. Then there exists a sequence of objects $A_0, \ldots, A_m \in \mathcal{D}$ such that $A_0 = A$, $A_m = X$ and $\mathrm{Mor}(A_i, A_{i+1}) \cup \mathrm{Mor}(A_{i+1}, A_i) \neq \emptyset$ for $i = 0, \ldots, m-1$. Let $\alpha_i \in \mathrm{Mor}(A_i, A_{i+1}) \cup \mathrm{Mor}(A_{i+1}, A_i)$ $(i = 0, \ldots, m-1)$. We define morphisms $\mu_i\colon F(A_i) \to F(A_{i+1})$ by putting $\mu_i = F(\alpha_i)$ for $\alpha_i \in \mathrm{Mor}(A_i, A_{i+1})$ and $\mu_i = F^{-1}(\alpha_i)$ for $\alpha_i \in \mathrm{Mor}(A_{i+1}, A_i)$. Let

$$\mu = \mu_{m-1}\mu_{m-2}\ldots\mu_0\colon F(A) \to F(X).$$

It is easy to check that the morphism μ does not depend on the choice of the objects connecting A to X. So we can define β_X as a composition of μ and β_A, i.e., $\beta_X = \mu\beta_A\colon G \to F(X)$. It is evident that the morphism β_X is uniquely determined, independently of the choice of the objects $A_0, \ldots, A_m$. In this way we get the family

$\{\beta_D: G\to F(D)\}_{D\in\mathscr{D}}$. From the construction of β_X it follows that this family is compatible with every diagram F_A (here F_A denotes the diagram F restricted to the subcategory $\mathscr{D}_A$) for $A\in\mathscr{D}_d$. Note that $\mathrm{Mor}(X; Y)=\emptyset$ whenever $X\in\mathscr{D}_A$, $Y\in\mathscr{D}_B$, $A, B\in\mathscr{D}_d$ and $A\neq B$. Hence it follows that the family $\{\beta_D\}_{D\in\mathscr{D}}$ is compatible with F. Then there exists a unique morphism $\delta: G\to L$ with $\alpha_D\delta=\beta_D$ for $D\in\mathscr{D}$. This shows that $[L; \{\alpha_D: L\to F_d(D)\}_{D\in\mathscr{D}_d}]$ is the projective limit of F, which is what was to be proved.

Consider any small category $\mathscr{D}$. This category can be embedded in a small category $\mathscr{D}_e$ which is obtained by adding to $\mathscr{D}$ one (final) object E and a family of morphisms $\{\varepsilon_D: D\to E\}_{D\in\mathscr{D}}$, one morphism to each object $D\in\mathscr{D}_e$. The resulting category $\mathscr{D}_e$ is obviously connected. A diagram $F: \mathscr{D}\to\mathscr{K}$ can always be extended to $F_e: \mathscr{D}_e\to\mathscr{K}$ by defining $F_e(E)$ to be the final object in the category $\mathscr{K}$ and $F_e(\varepsilon_D)$ to be the morphism induced by the final object $F_e(E)$.

Lemma 6. *$[L; \{\pi_D: L\to F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of F if and only if $[L; \{\pi_D: L\to F_e(D)\}_{D\in\mathscr{D}_e}]$ (with $\pi_E: L\to F_e(E)$ the morphism induced by $F_e(E)$) is the projective limit of F_e.*

Lemma 7. *Let a functor $\Lambda: \mathscr{K}_1\to\mathscr{K}_2$ preserve projective limits of all diagrams of connected diagram schemes. If $[L; \{\pi_D: L\to F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $F: \mathscr{D}\to\mathscr{K}_1$ where $\mathscr{D}$ is any, not necessarily connected, diagram scheme, then $[\Lambda(L); \{\Lambda(\pi_D): \Lambda(L)\to\Lambda F_e(D)\}_{D\in\mathscr{D}_e}]$ is the projective limit of the extended diagram $\Lambda F_e: \mathscr{D}_e\to\mathscr{K}_2$.*

Proof. Let $[L; \{\pi_D\}_{D\in\mathscr{D}}]$ be the projective limit of F. According to Lemma 6, $[L; \{\pi_D: L\to F_e(D)\}_{D\in\mathscr{D}_e}]$ is the projective limit of $F_e: \mathscr{D}_e\to\mathscr{K}_1$. The category $\mathscr{D}_e$ is connected, whence $[\Lambda(L); \{\Lambda(\pi_D): \Lambda(L)\to\Lambda F_e(D)\}_{D\in\mathscr{D}_e}]$ is the projective limit of ΛF_e.

4. The relation of the functor Φ to projective limits

We devote this section to the study of the relation of Φ to projective limits. We start with a lemma.

Lemma 8. *If a composition of morphisms $\gamma: \Phi_s(A)\to\Phi_s(D)$ and $\Phi_s(\beta): \Phi_s(D)\to\Phi_s(B)$ with $A, B, D\in\mathrm{Gr}_{n+1}$ is of the form $\Phi_s(\beta)\gamma=\Phi_s(\alpha)$ for some $\alpha: A\to B$, then γ is also of the form $\gamma=\Phi_s(\delta)$ where $\delta: A\to D$.*

Proof. In view of Theorem 4 of [5] we have the equalities $\zeta_B\Phi_s(\alpha)=\zeta_A$ and $\zeta_B\Phi_s(\beta)=\zeta_D$. Then $\zeta_D\gamma=\zeta_B\Phi_s(\beta)\gamma=\zeta_B\Phi_s(\alpha)=\zeta_A$. Hence, by Theorem 4 of [5], there exists a morphism $\delta: A\to D$ such that $\Phi_s(\delta)=\gamma$, which is what was to be proved.

Note that from the faithfulness of Φ it follows that the morphism δ does not depend on the choice of β provided the morphism $\Phi_s(\beta)$ remains the same.

Proposition 1. *If* $[\Phi_s(L); \{\Phi_s(\pi_D)\colon \Phi_s(L)\to\Phi_s F(D)\}_{D\in\mathscr{D}}]$ *is the projective limit of* $\Phi_s F\colon \mathscr{D}\to\mathbf{Gr}_{k+1}$, *then* $[L; \{\pi_D\colon L\to F(D)\}_{D\in\mathscr{D}}]$ *is the projective limit of* $F\colon \mathscr{D}\to\mathbf{Gr}_{n+1}$.

Proof. From Lemma 8 it follows that Φ_s satisfies the assumption of Lemma 1. Then, by Lemma 1, Φ_s reflects projective limits, which is what was to be proved.

The theorem converse to Proposition 1 is not true in general. This was already indicated in [7], where an example was shown to demonstrate that Φ does not preserve the Cartesian product. On the other hand, in [7] a sufficient condition was given which, when imposed upon a diagram scheme $\mathscr{D}$, made Φ preserve the projective limits of diagrams $F\colon \mathscr{D}\to\mathbf{Gr}_{n+1}$.

Now we show that this condition fails to be necessary. Moreover, we characterize the categories $\mathscr{D}$ for which Φ preserves projective limits. Theorem 1 of [7] is a particular case of the theorem given below.

Theorem 1. *Let* $\mathscr{D}$ *be a nonempty diagram scheme. The functor* Φ_s *preserves the projective limits of all diagrams* $F\colon \mathscr{D}\to\mathbf{Gr}_{n+1}$ *if and only if* $\mathscr{D}$ *is connected.*

Proof. Let $\mathscr{D}$ be connected and let

$$[L; \{\pi_D\colon L\to F(D)\}_{D\in\mathscr{D}}] \text{ and } [L'; \{\gamma_D\colon L'\to\Phi_s F(D)\}_{D\in\mathscr{D}}]$$

be the projective limits of $F\colon \mathscr{D}\to\mathbf{Gr}_{n+1}$ and $\Phi_s F\colon \mathscr{D}\to\mathbf{Gr}_{k+1}$, respectively. The family $\{\Phi_s(\pi_D)\}_{D\in\mathscr{D}}$ is compatible with $\Phi_s F$, and so there exists a morphism $\mu\colon \Phi_s(L)\to L'$ with $\gamma_D\mu=\Phi_s(\pi_D)$ for $D\in\mathscr{D}$. Fix some (arbitrary) object $U\in\mathscr{D}$. Then $\gamma_U\mu=\Phi_s(\pi_U)$. By Corollary 6 of [5] the object $[L'; \{\gamma_D\}_{D\in\mathscr{D}}]$ (determined up to isomorphism) can be chosen in such a way that $L'=\Phi_s(G)$, $\gamma_U=\Phi_s(\eta_U)$, $\mu=\Phi_s(\delta)$, where $G\in\mathbf{Gr}_{n+1}$, $\eta_U\colon G\to F(U)$, $\delta\colon L\to G$. We show that every morphism γ_D is of the form $\gamma_D=\Phi_s(\eta_D)$ for an appropriately chosen $\eta_D\colon G\to F(D)$. To verify this, take an object $A\in\mathscr{D}$. The connectivity of $\mathscr{D}$ implies the existence of a finite sequence of objects $A_0,\ldots,A_l\in\mathscr{D}$ such that $A_0=U$, $A_l=A$, $\mathrm{Mor}(A_i,A_{i+1})\cup\mathrm{Mor}(A_{i+1},A_i)\neq\emptyset$ for $i=0,\ldots,l-1$. The morphisms η_{A_i} will be constructed by induction, step by step, starting with η_{A_1}. If $\mathrm{Mor}(U,A_1)\neq\emptyset$ (i.e., there exists a morphism $\alpha\colon U\to A_1$), we put $\eta_{A_1}=F(\alpha)\eta_U$. By the compatibility of the family $\{\gamma_D\}_{D\in\mathscr{D}}$ with $\Phi_s F$ and by the faithfulness of Φ_s it follows that $F(\alpha)\eta_U$ does not depend on the choice of α (note that the set $\mathrm{Mor}(F(U),F(A_1))$ may consist of a lot of morphisms!). So the morphism $\eta_{A_1}\colon G\to F(A_1)$ is well-defined. If, on the other hand, $\mathrm{Mor}(A_1,U)\neq\emptyset$ (i.e., there exists an $\alpha\colon A_1\to U$), then by Lemma 8 there exists a morphism $\eta_{A_1}\colon G\to F(A_1)$ which is well-defined (independently of the choice of α). In this way we get the morphism η_{A_1}. Further on, to obtain $\eta_{A_{i+1}}$ from η_{A_i} we proceed as in the first step, depending on which one of the sets $\mathrm{Mor}(A_i, A_{i+1})$ or $\mathrm{Mor}(A_{i+1}, A_i)$ is nonempty. After performing l such steps we obtain $\eta_A: G\to F(A)$. As is easy to verify the family $\{\eta_D\}_{D\in\mathscr{D}}$ is compatible with F. So there exists a morphism $\varrho: G\to L$ with $\pi_D\varrho=\eta_D$ for $D\in\mathscr{D}$ (since by assumption $[L; \{\pi_D\}_{D\in\mathscr{D}}]$ is the projective limit of F). The equalities $\pi_D\varrho\delta=\eta_D\delta=\pi_D$ hold for every $D\in\mathscr{D}$, whence $\varrho\delta=e_L$. Similarly, from the equalities $\gamma_D\Phi_s(\delta\varrho)=$ $=\Phi_s(\pi_D\varrho)=\Phi_s(\eta_D)$ it follows that $\Phi_s(\delta\varrho)=e_{L'}$, whence $\delta\varrho=e_G$. Then δ (thus also $\Phi_s(\delta)$) is an isomorphism, which proves that $[\Phi_s(L); \{\Phi_s(\pi_D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Phi_s F$.

Conversely, let Φ_s preserve the projective limits of all diagrams $F: \mathscr{D}\to\mathbf{Gr}_{n+1}$ for a fixed category $\mathscr{D}$. Consider the functor $F: \mathscr{D}\to\mathbf{Gr}_{n+1}$ defined as follows: for $D\in\mathscr{D}$ let $F(D)$ be a one-element $(n+1)$-group and for $\alpha: X\to Y$ let $F(\alpha)$ be the unique morphism from $F(X)$ onto $F(Y)$. Let $[L; \{\pi_D: L\to F(D)\}_{D\in\mathscr{D}}]$ be the projective limit of F. Since all objects $F(D)$ for $D\in\mathscr{D}$ are final in $\mathbf{Gr}_{n+1}$, the object L is also a final object in $\mathbf{Gr}_{n+1}$, i.e. a one-element $(n+1)$-group. Thus $\Phi_s(L)=\mathfrak{C}_{s,k+1}$ (cf. [3], [7]). By assumption $[\Phi_s(L); \{\Phi_s(\pi_D):\Phi_s(L)\to\Phi_s F(D)\}_{D\in\mathscr{D}}]$ is the projective limit of $\Phi_s F$. However, we can see that for any $\alpha: X\to Y$ with $X, Y\in\mathscr{D}$, the morphism $\Phi_s F(\alpha)$ is the only isomorphism of the cyclic $(k+1)$-group $\Phi_s F(A)=\mathfrak{C}_{s,k+1}$ onto the cyclic $(k+1)$-group $\Phi_s F(B)=\mathfrak{C}_{s,k+1}$ with the property $\Phi_s F(\alpha)(0)=0$. Therefore, in view of Lemma 5, $[\Phi_s(L); \{\Phi_s(\pi_D): \Phi_s(L)\to\Phi_s F_d(D)\}_{D\in\mathscr{D}_d}]$ (where F_d is the restriction of F to $\mathscr{D}_d$) is the projective limit of $\Phi_s F_d: \mathscr{D}_d\to\mathbf{Gr}_{k+1}$. Since $\mathscr{D}_d$ is discrete, $[\Phi_s(L); \{\Phi_s(\pi_D)\}_{D\in\mathscr{D}_d}]$ is simply the Cartesian product of the family of $(k+1)$-groups $\{\Phi_s F(D)\}_{D\in\mathscr{D}_d}$, i.e., the Cartesian power of the cyclic $(k+1)$-group $\mathfrak{C}_{s,k+1}$. On the other hand, the $(k+1)$-group $\Phi_s(L)$ is the cyclic $(k+1)$-group $\mathfrak{C}_{s,k+1}$, whence the family $\{\Phi_s F(D)\}_{D\in\mathscr{D}_d}$ consists of one element, which means that $\mathscr{D}_d$ consists only of one object. Hence $\mathscr{D}$ is a connected category. This completes the proof of Theorem 1.

From Proposition 1 and Theorem 1 we immediately infer the following

Corollary 1. *Let $\mathscr{D}$ be a connected diagram scheme. Then* $[L; \{\pi_D: L\to F(D)\}_{D\in\mathscr{D}}]$ *is the projective limit of* $F: \mathscr{D}\to\mathbf{Gr}_{n+1}$ *if and only if*

$$[\Phi_s(L); \{\Phi_s(\pi_D): \Phi_s(L)\to\Phi_s F(D)\}_{D\in\mathscr{D}}]$$

is the projective limit of $\Phi_s F: \mathscr{D}\to\mathbf{Gr}_{k+1}$.

The question arises what are the free covering $(k+1)$-groups of projective limits of diagrams $F: \mathscr{D}\to\mathbf{Gr}_{n+1}$ in the case when the diagram scheme is not connected. Note that a partial answer was given in Lemma 7. In our case of the category of n-groups a more specific answer can be given.

Take an arbitrary diagram scheme $\mathcal{D}$ and a diagram $F\colon \mathcal{D}\to\mathbf{Gr}_{n+1}$. Let $\mathcal{D}_e$ and F_e have the same meaning as in Section 3. As is easy to see, $\Phi_s F(E)$ is nothing else than the cyclic $(k+1)$-group $\mathfrak{C}_{s,k+1}$ (cf. [3], [5]) and $\Phi_s F(\varepsilon_D)\colon \Phi_s F(D)\to\mathfrak{C}_{s,k+1}$ are simply the morphisms $\zeta_D\colon \Phi_s F(D)\to\mathfrak{C}_{s,k+1}$ (cf. [3], [5]). Every diagram $\Phi_s F$ can be extended to $\Phi_s F_e\colon \mathcal{D}_e\to\mathbf{Gr}_{k+1}$ by adding the object $\mathfrak{C}_{s,k+1}$ and the family of morphisms $\{\zeta_D\colon \Phi_s F(D)\to\mathfrak{C}_{s,k+1}\}_{D\in\mathcal{D}}$. Hence we obtain

Proposition 2. *If* $[L;\ \{\pi_D\colon L\to F(D)\}_{D\in\mathcal{D}}]$ *is the projective limit of* $F\colon \mathcal{D}\to\mathbf{Gr}_{n+1}$, *then* $[\Phi_s(L);\ \{\Phi_s(\pi_D)\colon \Phi_s(L)\to\Phi_s F_e(D)\}_{D\in\mathcal{D}_e}]$ *(where* $\Phi_s(\pi_E)=\zeta\colon \Phi_s(L)\to\mathfrak{C}_{s,k+1}$*) is the projective limit of the extended diagram* $\Phi_s F_e\colon \mathcal{D}_e\to\mathbf{Gr}_{k+1}$.

In this way free covering $(k+1)$-groups of projective limits are always projective limits, but perhaps of an extended diagram.

5. The relation of the functor Ψ to inductive limits

As in the dual case of Φ and projective limits, the functor Ψ reflects inductive limits. To show this fact, we need the following lemma.

Lemma 9. *If a composition of morphisms* $\Psi_s(\alpha)\colon \Psi_s(A)\to\Psi_s(D)$ *and* $\gamma\colon \Psi_s(D)\to\Psi_s(B)$ *with* $A, B, D\in\mathbf{Gr}_{k+1}$ *is of the form* $\gamma\Psi_s(\alpha)=\Psi_s(\beta)$ *for some* $\beta\colon A\to B$, *then* γ *is of the form* $\gamma=\Psi_s(\delta)$ *where* $\delta\colon D\to B$.

Proof. Take any element $c_0\in A$ and let d_0 be the skew element to c_0 in the $(k+1)$-group A. Let $d=\alpha(d_0)$, $c=\alpha(c_0)$. It is easy to check that d is skew to c in the $(k+1)$-group D. On the other hand, the element $\gamma(d)=\gamma\alpha(d_0)=\beta(d_0)$ is skew to $\gamma(c)=\gamma\alpha(c_0)=\beta(c_0)$ (since $\beta\colon A\to B$). Hence, by Corollary 3 of [6], γ is a homomorphism of $(k+1)$-groups, which is what was to be proved.

Proposition 3. *If* $[\Psi_s(L);\ \{\Psi_s(\gamma_D)\to\Psi_s(L)\}_{D\in\mathcal{D}}]$ *is the inductive limit of* $\Psi_s F\colon \mathcal{D}\to\mathbf{Gr}_{n+1}$, *then* $[L;\ \{\gamma_D\colon F(D)\to L\}_{D\in\mathcal{D}}]$ *is the inductive limit of* $F\colon \mathcal{D}\to\mathbf{Gr}_{k+1}$.

Proof. Lemma 9 shows that Ψ_s satisfies the assumption of Lemma 1*. Thus Ψ_s reflects inductive limits.

Theorem 1 describes the preservation of projective limits by Φ. Theorem 2 (dual to Theorem 1), formulated below, gives a condition characterizing those diagram schemes for which Ψ preserves inductive limits. The proof of Theorem 2 proceeds via complicated calculations. To stress the main idea of the proof a part of those calculations is presented in a sequence of five lemmas. All those lemmas have some common assumptions. To avoid repetition, we formulate these assumptions before starting the lemmas.

Given a diagram $F\colon \mathscr{D}\to \mathbf{Gr}_{k+1}$, let $[L'; \{\gamma_D\colon \Psi_s F(D)\to L'\}_{D\in\mathscr{D}}]$ be the inductive limit of $\Psi_s F$. As was mentioned in Section 2, we denote by g the $(k+1)$-group operation in all $(k+1)$-groups (i.e., $F(D)$, L, G), while by f the $(n+1)$-group operation in all $(n+1)$-groups (i.e., $\Psi_s F(D)$, $\Psi_s(L)$, $\Psi_s(G)$, L'). To avoid confusion we assume that the symbol $\bar{x}$ always denotes the element skew to x in the corresponding $(n+1)$-group (but not in a $(k+1)$-group). The equivalence of polyads is understood in the sense of [10].

Lemma 10. *If for some objects $A, B\in\mathscr{D}$ we have $\operatorname{Mor}(A,B)\cup\operatorname{Mor}(B,A)\neq\emptyset$, then for an arbitrary element $a\in F(A)$ there exists an element $b\in F(B)$ such that the k-ads $\langle\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}\rangle$ and $\langle\gamma_B(g_{(s-1)}(\bar{b}, \overset{n-k}{b})), \overset{k-1}{\gamma_B(b)}\rangle$ are equivalent.*

Proof. Let $\alpha\colon A\to B$ and $b=F(\alpha)(a)$. Take elements $x_1, \dots, x_{n+1-k}\in L'$. Then

$$\begin{aligned}
f\big(x_1, \dots, x_{n+1-k}, \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}\big) &=\\
= f\big(\dots, x_{n+1-k}, \gamma_B\Psi_s F(\alpha)(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_B\Psi_s F(\alpha)(a)}\big) &=\\
= f\big(\dots, x_{n+1-k}, \gamma_B(F(\alpha)(g_{(s-1)}(\bar{a}, \overset{n-k}{a}))), \overset{k-1}{\gamma_B(F(\alpha)(a))}\big) &=\\
= f\big(\dots, x_{n+1-k}, \gamma_B(g_{(s-1)}(F(\alpha)(\bar{a}), \overset{n-k}{F(\alpha)(a)})), \overset{k-1}{\gamma_B(b)}\big) &=\\
= f\big(x_1, \dots, x_{n+1-k}, \gamma_B(g_{(s-1)}(\bar{b}, \overset{n-k}{b})), \overset{k-1}{\gamma_B(b)}\big).&
\end{aligned}$$

Next, let $\beta\colon B\to A$. Take an arbitrary element $b\in F(B)$. Then

$$\begin{aligned}
&f(x_1, \dots, x_{n+1-k}, \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}) =\\
&= f_{(2)}(\dots, x_{n+1-k}, \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_B(\bar{b}), \overset{n-1}{\gamma_B(b)}) =\\
&= f\big(\dots, x_{n+1-k}, f(\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A\Psi_s F(\beta)(\bar{b}), \overset{n-k}{\gamma_A\Psi_s F(\beta)(b)}), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(\dots, x_{n+1-k}, f(\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(F(\beta)(\bar{b})), \overset{n-k}{\gamma_A(F(\beta)(b))}), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(\dots, x_{n+1-k}, \gamma_A(g_{(s)}(g_{(s-1)}(\bar{a}, \overset{n-k}{a}), \overset{k-1}{a}, F(\beta)(\bar{b}), \overset{n-k}{F(\beta)(b)})), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(\dots, x_{n+1-k}, \gamma_A(g_{(s-1)}(g_{(s)}(\bar{a}, \overset{n-k}{a}, \overset{k-1}{a}, F(\beta)(\bar{b})), \overset{n-k}{F(\beta)(b)})), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(\dots, x_{n+1-k}, \gamma_A(g_{(s-1)}(f(\bar{a}, \overset{n-1}{a}, F(\beta)(\bar{b})), \overset{n-k}{F(\beta)(b)})), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(\dots, x_{n+1-k}, \gamma_A(F(\beta)(g_{(s-1)}(\bar{b}, \overset{n-k}{b}))), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(\dots, x_{n+1-k}, \gamma_A\Psi_s F(\beta)(g_{(s-1)}(\bar{b}, \overset{n-k}{b})), \overset{k-1}{\gamma_B(b)}\big) =\\
&= f\big(x_1, \dots, x_{n+1-k}, \gamma_B(g_{(s-1)}(\bar{b}, \overset{n-k}{b})), \overset{k-1}{\gamma_B(b)}\big).
\end{aligned}$$

Lemma 11. *If a category $\mathscr{D}$ is connected, then for every pair of objects $A, B\in\mathscr{D}$ and for any element $a\in F(A)$ there exists an element $b\in F(B)$ such that the k-ads $\langle\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}\rangle$ and $\langle\gamma_B(g_{(s-1)}(\bar{b}, \overset{n-k}{b})), \overset{k-1}{\gamma_B(b)}\rangle$ are equivalent.*

Proof. The category $\mathscr{D}$ is connected by assumption, so for any pair of objects $A, B\in\mathscr{D}$ there exists a sequence of objects $A_0, \dots, A_r\in\mathscr{D}$ such that $A_0=A$, $A_r=B$ and $\mathrm{Mor}(A_i, A_{i+1})\cup\mathrm{Mor}(A_{i+1}, A_i)\neq\emptyset$ for $i=0, \dots, r-1$. Applying Lemma 10 r times, we infer the equivalence of the polyads in question.

Lemma 12. *For any elements $a, y\in F(A)$ the $(k+1)$-ads*

$$\langle\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(y)\rangle \quad \textit{and} \quad \langle\gamma_A(y), \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}\rangle$$

are equivalent.

Proof. Let $x_1, \dots, x_{n-k}\in L'$. Then

$$\begin{aligned}
&f(x_1, \dots, x_{n-k}, \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(y)) = \\
&\quad = f_{(2)}(\dots, x_{n-k}, \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(y), \gamma_A(\bar{a}), \overset{n-1}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, f(\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(y), \gamma_A(\bar{a}), \overset{n-k-1}{\gamma_A(a)}), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(g_{(s)}(g_{(s-1)}(\bar{a}, \overset{n-k}{a}), \overset{k-1}{a}, y, \bar{a}, \overset{n-k-1}{a})), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(g_{(s-1)}(g_{(s)}(\bar{a}, \overset{n-k}{a}, \overset{k-1}{a}, y), \bar{a}, \overset{n-k-1}{a})), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(g_{(s-1)}(f(\bar{a}, \overset{n-1}{a}, y), \bar{a}, \overset{n-k-1}{a})), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(g_{(s-1)}(f(y, \bar{a}, \overset{n-1}{a}), \bar{a}, \overset{n-k-1}{a})), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(g_{(s-1)}(g_{(s)}(y, \bar{a}, \overset{n-1}{a}), \bar{a}, \overset{n-k-1}{a})), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(g_{(s)}(y, g_{(s-1)}(\bar{a}, \overset{n-k}{a}), \overset{k-1}{a}, \bar{a}, \overset{n-k-1}{a})), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, f(\gamma_A(y), \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(\bar{a}), \overset{n-1-k}{\gamma_A(a)}), \overset{k}{\gamma_A(a)}) = \\
&\quad = f(\dots, x_{n-k}, \gamma_A(y), f(\gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}, \gamma_A(\bar{a}), \overset{n-k-1}{\gamma_A(a)}, \gamma_A(a)), \overset{k-1}{\gamma_A(a)}) = \\
&\quad = f(x_1, \dots, x_{n-k}, \gamma_A(y), \gamma_A(g_{(s-1)}(\bar{a}, \overset{n-k}{a})), \overset{k-1}{\gamma_A(a)}).
\end{aligned}$$

Lemma 13. *If for some objects $A, B\in\mathscr{D}$ we have $\mathrm{Mor}(A, B)\cup\mathrm{Mor}(B, A)\neq\emptyset$, then for an arbitrary element $a\in F(A)$ there exists an element $b\in F(B)$ such that the $(n-k)$-ads $\langle\overset{s-1}{\gamma_A(a_s)}, \overset{(k-1)(s-1)}{\gamma_A(a)}\rangle$ and $\langle\overset{s-1}{\gamma_B(b_s)}, \overset{(k-1)(s-1)}{\gamma_B(b)}\rangle$ are equivalent (here a_s and*

b_s denote the skew elements to a and b in the $(k+1)$-groups $F(A)$ and $F(B)$, respectively).

Proof. Let $\alpha\colon A\to B$ and $b=F(\alpha)(a)$ (hence also $b_s=F(\alpha)(a_s)$). Take elements $x_1,\dots,x_{k+1}\in L'$. Then

$$f(x_1,\dots,x_{k+1},\overset{s-1}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)})=f(\dots,x_{k+1},\overset{s-1}{\gamma_B\Psi_sF(\alpha)(a_s)},\overset{(k-1)(s-1)}{\gamma_B\Psi_sF(\alpha)(a)})=$$

$$=f(\dots,x_{k+1},\overset{s-1}{\gamma_B(F(\alpha)(a_s))},\overset{(k-1)(s-1)}{\gamma_B(F(\alpha)(a))})=f(x_1,\dots,x_{k+1},\overset{s-1}{\gamma_B(b_s)},\overset{(k-1)(s-1)}{\gamma_B(b)}).$$

Next, let $\beta\colon B\to A$. Take an arbitrary element $b\in F(B)$. Let b_s be the skew element to b in the $(k+1)$-group $F(B)$. Then

$$f(x_1,\dots,x_{k+1},\overset{s-1}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)})=$$

$$=f(\dots,x_{k+1},\gamma_A(g_{(s)}(\overset{s-1}{F(\beta)(b_s)},\overset{(k-1)(s-1)}{F(\beta)(b)},$$
$$F(\beta)(b_s),\overset{k-1}{F(\beta)(b)},a_s)),\overset{s-2}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)})=$$

$$=f(\dots,x_{k+1},f(\overset{s-1}{\gamma_A(F(\beta)(b_s))},\overset{(k-1)(s-1)}{\gamma_A(F(\beta)(b))},$$
$$\gamma_A(F(\beta)(b_s)),\overset{k-1}{\gamma_A(F(\beta)(b))},\gamma_A(a_s)),\overset{s-2}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)})=$$

$$=f(\dots,x_{k+1},f(\overset{s-1}{\gamma_A\Psi_sF(\beta)(b_s)},\overset{(k-1)(s-1)}{\gamma_A\Psi_sF(\beta)(b)},$$
$$\gamma_A\Psi_sF(b_s),\overset{k-1}{\gamma_A\Psi_sF(b)},\gamma_A(a_s)),\overset{s-2}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)})=$$

$$=f(\dots,x_{k+1},\overset{s-1}{\gamma_B(b_s)},\overset{(k-1)(s-1)-1}{\gamma_B(b)},f(\gamma_A(F(\beta)(b)),$$
$$\gamma_A(F(\beta)(b_s)),\overset{k-1}{\gamma_A(F(\beta)(b))},\gamma_A(a_s),\overset{s-2}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)}))=$$

$$=f(\dots,x_{k+1},\overset{s-1}{\gamma_B(b_s)},\overset{(k-1)(s-1)-1}{\gamma_B(b)},\gamma_A(g_{(s)}(F(\beta)(b),$$
$$F(\beta)(b_s),\overset{k-1}{F(\beta)(b)},\overset{s-1}{a_s},\overset{(k-1)(s-1)}{a})))=$$

$$=f(\dots,x_{k+1},\overset{s-1}{\gamma_B(b_s)},\overset{(k-1)(s-1)-1}{\gamma_B(b)},\gamma_A(F(\beta)(b)))=$$

$$=f(x_1,\dots,x_{k+1},\overset{s-1}{\gamma_B(b_s)},\overset{(k-1)(s-1)}{\gamma_B(b)}).$$

Lemma 14. *If a category $\mathscr{D}$ is connected, then for every pair of objects $A,B\in\mathscr{D}$ and for an arbitrary element $a\in F(A)$ there exists an element $b\in F(B)$ such that the $(n-k)$-ads $\langle\overset{s-1}{\gamma_A(a_s)},\overset{(k-1)(s-1)}{\gamma_A(a)}\rangle$ and $\langle\overset{s-1}{\gamma_B(b_s)},\overset{(k-1)(s-1)}{\gamma_B(b)}\rangle$ are equivalent (here a_s and b_s denote the skew element to a and b in the $(k+1)$-groups $F(A)$ and $F(B)$, respectively).*

Proof. The proof of this lemma is analogous to that of Lemma 11.

Lemma 15. *If a, b are any elements of an object $F(D)$ with $D\in\mathcal{D}$, then the k-ads $\langle\gamma_D(g_{(s-1)}(\bar a, \overset{n-k}{a})), \overset{k-1}{\gamma_D(a)}\rangle$ and $\langle\gamma_D(g_{(s-1)}(\bar b, \overset{n-k}{b})), \overset{k-1}{\gamma_D(b)}\rangle$ are equivalent.*

Proof. Let $x_1, \ldots, x_{n+1-k}\in L'$. Then

$$f(x_1, \ldots, x_{n+1-k}, \gamma_D(g_{(s-1)}(\bar a, \overset{n-k}{a})), \overset{k-1}{\gamma_D(a)}) =$$

$$= f_{(2)}(\ldots, x_{n+1-k}, \gamma_D(g_{(s-1)}(\bar a, \overset{n-k}{a})), \overset{k-1}{\gamma_D(a)}, \overset{n-1}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(\ldots, x_{n+1-k}, f(\gamma_D(g_{(s-1)}(\bar a, \overset{n-k}{a})), \overset{k-1}{\gamma_D(a)}, \overset{n+1-k}{\gamma_D(b)}), \overset{k-2}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(\ldots, x_{n+1-k}, \gamma_D(g_{(s)}(g_{(s-1)}(\bar a, \overset{n-k}{a}), \overset{k-1}{a}, \overset{n+1-k}{b})), \overset{k-2}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(\ldots, x_{n+1-k}, \gamma_D(g_{(s-1)}(f(\bar a, \overset{n-1}{a}, b), \overset{n-k}{b})), \overset{k-2}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(\ldots, x_{n+1-k}, \gamma_D(g_{(s-1)}(f(\bar b, \overset{n-1}{b}, b), \overset{n-k}{b})), \overset{k-2}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(\ldots, x_{n+1-k}, \gamma_D(g_{(s)}(g_{(s-1)}(\bar b, \overset{n-k}{b}), \overset{k-1}{b}, \overset{n+1-k}{b})), \overset{k-2}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(\ldots, x_{n+1-k}, f(\gamma_D(g_{(s-1)}(\bar b, \overset{n-k}{b})), \overset{k-1}{\gamma_D(b)}, \overset{n+1-k}{\gamma_D(b)}), \overset{k-2}{\gamma_D(b)}, \gamma_D(\bar b)) =$$

$$= f(x_1, \ldots, x_{n+1-k}, \gamma_D(g_{(s-1)}(\bar b, \overset{n-k}{b})), \overset{k-1}{\gamma_D(b)}).$$

Theorem 2. *Given a diagram scheme $\mathcal{D}$, assume that $\mathcal{D}$ is nonempty or $k>1$. The functor Ψ_s preserves the inductive limits of all diagrams $F\colon \mathcal{D}\to\mathbf{Gr}_{k+1}$ if and only if the full subcategory $\mathcal{D}_0$ of $\mathcal{D}$, which consists of those objects D for which $\Psi_s F(D)$ is not an initial object in $\mathbf{Gr}_{n+1}$, is connected.*

Proof. Assume that the nonempty category $\mathcal{D}_0$ is connected. Let

$$[L; \{\sigma_D\colon F(D)\to L\}_{D\in\mathcal{D}}] \quad\text{and}\quad [L'; \{\gamma_D\colon \Psi_s F(D)\to L'\}_{D\in\mathcal{D}}]$$

be the inductive limits of $F\colon \mathcal{D}\to\mathbf{Gr}_{k+1}$ and $\Psi_s F\colon \mathcal{D}\to\mathbf{Gr}_{n+1}$, respectively. Note that for $k=1$ the diagram scheme $\mathcal{D}_0$ is equal to $\mathcal{D}$ (since $\Psi_n F(D)$ is not an empty $(n+1)$-group). On the other hand, for $k>1$ the full subcategory of $\mathcal{D}$ consisting of those objects for which $F(D)$ (but not $\Psi_s F(D)$ as in the definition of $\mathcal{D}_0$) is a nonempty $(k+1)$-group, equals $\mathcal{D}_0$ (since $F(D)$ is nonempty iff $\Psi_s F(D)$ is nonempty). In view of Lemma 2, $[L; \{\sigma_D\colon F_0(D)\to L\}_{D\in\mathcal{D}_0}]$ and $[L'; \{\gamma_D\colon \Psi_s F_0(D)\to L'\}_{D\in\mathcal{D}_0}]$ (where F_0 is the restriction of F to $\mathcal{D}_0$) are also the inductive limits of F_0 and $\Psi_s F_0$.

Take an arbitrary (but fixed) object $C \in \mathscr{D}_0$ and choose an element $c_0 \in C$. Let $c_s \in C$ be the skew element to c_0 in the $(k+1)$-group $F(C)$. We prove that the element $d = \gamma_C(c_s)$ is an s-skew element to the element $c = \gamma_C(c_0)$ in the $(n+1)$-group L'. Indeed, for any element $x \in L'$ we have

$$f(\overset{s}{d}, \overset{(k-1)s}{c}, x) = f(\overset{s}{\gamma_C(c_s)}, \overset{(k-1)s}{\gamma_C(c_0)}, f(\gamma_C(\bar{c}_0), \overset{n-1}{\gamma_C(c_0)}, x)) =$$

$$= f(f(\overset{s}{\gamma_C(c_s)}, \overset{(k-1)s}{\gamma_C(c_0)}, \gamma_C(\bar{c}_0)), \overset{n-1}{\gamma_C(c_0)}, x) =$$

$$= f(\gamma_C(g_{(s)}(\overset{s}{c_s}, \overset{(k-1)s}{c_0}, \bar{c}_0)), \overset{n-1}{\gamma_C(c_0)}, x) = f(\gamma_C(\bar{c}_0), \overset{n-1}{\gamma_C(c_0)}, x) = x,$$

which shows that the elements d and c satisfy condition 1° of the definition of an s-skew element (cf. [6]).

Next, take elements $x_1, \ldots, x_{n+1-k} \in L'$ and fix $i = 1, \ldots, n+1-k$. Then

$$f(x_1, \ldots, x_i, d, \overset{k-1}{c}, x_{i+1}, \ldots, x_{n+1-k}) =$$

$$= f(\ldots, x_i, \gamma_C(c_s), \overset{k-1}{\gamma_C(c_0)}, f(\gamma_C(\bar{c}_0), \overset{n-1}{\gamma_C(c_0)}, x_{i+1}), x_{i+2}, \ldots) =$$

$$= f(\ldots, x_i, f(\gamma_C(c_s), \overset{k-1}{\gamma_C(c_0)}, \gamma_C(\bar{c}_0), \overset{n-k}{\gamma_C(c_0)}), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$= f(\ldots, x_i, \gamma_C(g_{(s)}(c_s, \overset{k-1}{c_0}, \bar{c}_0, \overset{n-k}{c_0})), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$= f(\ldots, x_i, \gamma_C(g_{(s)}(\overset{k-1}{c_0}, c_s, \bar{c}_0, \overset{n-k}{c_0})), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$= f(\ldots, x_i, f(\overset{k-1}{\gamma_C(c_0)}, \gamma_C(c_s), \gamma_C(\bar{c}_0), \overset{n-k}{\gamma_C(c_0)}), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$= f(\ldots, x_i, \overset{k-1}{c}, d, f(\gamma_C(\bar{c}_0), \overset{n-k}{\gamma_C(c_0)}, \overset{k-1}{\gamma_C(c_0)}, x_{i+1}), x_{i+2}, \ldots) =$$

$$= f(\ldots, x_i, \overset{k-1}{c}, d, x_{i+1}, \ldots, x_{n+1-k}).$$

Moreover, by the definition of the $(n+1)$-group L' (as an inductive limit of $(n+1)$-groups) it follows that the elements of L' are generated by the set $\bigcup_{D \in \mathscr{D}} \gamma_D(F(D))$. Hence in particular $x_i = f_{(\cdot)}(\gamma_{D_1}(y_1), \ldots, \gamma_{D_r}(y_r))$, where $r \equiv 1 \pmod{n}$, $y_j \in F(D_j)$ for $j = 1, \ldots, r$, and $x_1 = f_{(\cdot)}(\gamma_{A_1}(z_1), \ldots, \gamma_{A_t}(z_t))$, where $t \equiv 1 \pmod{n}$, $z_j \in F(A_j)$ for $j = 1, \ldots, t$.

To explain the sequence of transformations, we will write the numbers of the lemmas we refer to, below the sign of equality. The elements chosen according to Lemmas 10 and 11 will be denoted by d_i in the $(k+1)$-groups $F(D_i)$ and by a_i in the

$(k+1)$-groups $F(A_i)$. Then

$$f(x_1, \ldots, x_i, d, \overset{k-1}{c}, x_{i+1}, \ldots, x_{n+1-k}) =$$

$$= f(\ldots, x_i, \gamma_C(c_s), \overset{k-1}{\gamma_C(c_0)}, f(\gamma_C(\bar{c}_0), \overset{n-1}{\gamma_C(c_0)}, x_{i+1}), x_{i+2}, \ldots) =$$

$$= f(\ldots, x_i, f(\gamma_C(c_s), \overset{k-1}{\gamma_C(c_0)}, \gamma_C(\bar{c}_0), \overset{n-k}{\gamma_C(c_0)}), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$= f(\ldots, x_{i-1}, f_{(\cdot)}(\gamma_{D_1}(y_1), \ldots, \gamma_{D_r}(y_r)), \gamma_C(g_{(s)}(c_s, \overset{k-1}{c_0}, \bar{c}_0, \overset{n-k}{c_0})), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$= f_{(\cdot)}(\ldots, x_{i-1}, \gamma_{D_1}(y_1), \ldots, \gamma_{D_r}(y_r), \gamma_C(g_{(s-1)}(\bar{c}_0, \overset{n-k}{c_0})), \overset{k-1}{\gamma_C(c_0)}, x_{i+1}, \ldots) =$$

$$\underset{(11)}{=} f_{(\cdot)}(\ldots, x_{i-1}, \gamma_{D_1}(y_1), \ldots, \gamma_{D_{r-1}}(y_{r-1}), \gamma_{D_r}(y_r), \gamma_{D_r}(g_{(s-1)}(\bar{d}_r, \overset{n-k}{d_r})), \overset{k-1}{\gamma_{D_r}(d_r)}, x_{i+1}, \ldots) =$$

$$\underset{(12)}{=} f_{(\cdot)}(\ldots, x_{i-1}, \gamma_{D_1}(y_1), \ldots, \gamma_{D_{r-1}}(y_{r-1}), \gamma_{D_r}(g_{(s-1)}(\bar{d}_r, \overset{n-k}{d_r})), \overset{k-1}{\gamma_{D_r}(d_r)}, \gamma_{D_r}(y_r), x_{i+1}, \ldots) =$$

$$\underset{(11)}{=} f_{(\cdot)}(\ldots, x_{i-1}, \gamma_{D_1}(y_1), \ldots, \gamma_{D_{r-1}}(y_{r-1}), \gamma_{D_{r-1}}(g_{(s-1)}(\bar{d}_{r-1}, \overset{n-k}{d_{r-1}})),$$
$$\overset{k-1}{\gamma_{D_{r-1}}(d_{r-1})}, \gamma_{D_r}(y_r), x_{i+1}, \ldots) =$$

$$\underset{(12)}{=} f_{(\cdot)}(\ldots, x_{i-1}, \gamma_{D_1}(y_1), \ldots, \gamma_{D_{r-2}}(y_{r-2}), \gamma_{D_{r-1}}(g_{(s-1)}(\bar{d}_{r-1}, \overset{n-k}{d_{r-1}})),$$
$$\overset{k-1}{\gamma_{D_{r-1}}(d_{r-1})}, \gamma_{D_{r-1}}(y_{r-1}), \gamma_{D_r}(y_r), x_{i+1}, \ldots) = \ldots =$$

$$= f_{(\cdot)}(\gamma_{A_1}(g_{(s-1)}(\bar{a}_1, \overset{n-k}{a_1})), \overset{k-1}{\gamma_{A_1}(a_1)}, \gamma_{A_1}(z_1), \ldots, \gamma_{A_t}(z_t), x_2, \ldots) =$$

$$\underset{(11),(15)}{=} f(\gamma_C(g_{(s-1)}(\bar{c}_0, \overset{n-k}{c_0})), \overset{k-1}{\gamma_C(c_0)}, x_1, \ldots) =$$

$$= f(\gamma_C(g_{(s)}(c_s, \overset{k-1}{c_0}, \bar{c}_0, \overset{n-k}{c_0})), \overset{k-1}{\gamma_C(c_0)}, x_1, \ldots) =$$

$$= f(f(\gamma_C(c_s), \overset{k-1}{\gamma_C(c_0)}, \gamma_C(\bar{c}_0), \overset{n-k}{\gamma_C(c_0)}), \overset{k-1}{\gamma_C(c_0)}, x_1, \ldots) = f(d, \overset{k-1}{c}, x_1, \ldots),$$

which proves that the elements d and c satisfy condition 2° of the definition of an s-skew element (cf. [6]). Therefore the element d is s-skew to c in the $(n+1)$-group L'. Thus, by Proposition 1 of [6] (cf. also Theorem 5 of [5]), the $(n+1)$-group (L', f) is derived from some $(k+1)$-group (G, g), i.e., $\Psi_s(G) = L'$. Furthermore, the $(k+1)$-group operation g in G is given by

$$g(x_1, \ldots, x_{k+1}) = f(x_1, \ldots, x_{k+1}, \overset{s-1}{d}, \overset{(k-1)(s-1)}{c}).$$

By Corollary 2 of [6] the element d is skew to c in that $(k+1)$-group G. Let $x_1, \ldots, x_{k+1} \in F(D)$ for any $D \in \mathscr{D}$. Then

$$g(\gamma_D(x_1), \ldots, \gamma_D(x_{k+1})) = f(\gamma_D(x_1), \ldots, \gamma_D(x_{k+1}), \overset{s-1}{\gamma_C(c_s)}, \overset{(k-1)(s-1)}{\gamma_C(c_0)}) =$$

$$\underset{(14)}{=} f(\gamma_D(x_1), \ldots, \gamma_D(x_{k+1}), \overset{s-1}{\gamma_D(d_s)}, \overset{(k-1)(s-1)}{\gamma_D(d_0)}) = \gamma_D(g(x_1, \ldots, x_{k+1})),$$

where d_0 is some element of $F(D)$ and d_s is the skew element to d_0 in this $(k+1)$-group $F(D)$. This shows that γ_D is of the form $\gamma_D = \Psi_s(\beta_D)$ with $\beta_D\colon F(D) \to G$ for $D \in \mathscr{D}$. The faithfulness of Ψ_s implies the compatibility of the family $\{\beta_D\colon F_0(D) \to G\}_{D \in \mathscr{D}_0}$ with F_0. So there exists a unique morphism $\delta\colon L \to G$ such that $\delta\sigma_D = \beta_D$ for $D \in \mathscr{D}_0$. The family $\{\Psi_s(\sigma_D)\}_{D \in \mathscr{D}}$ is compatible with $\Psi_s F$, which implies the existence of a unique morphism $\omega\colon \Psi_s(G) \to \Psi_s(L)$ with $\omega\gamma_D = \Psi_s(\sigma_D)$ for $D \in \mathscr{D}$ (since $L' = \Psi_s(G)$ is the inductive limit of $\Psi_s F$). Then $\Psi_s(\delta)\omega\gamma_D = \Psi_s(\delta\sigma_D) = \Psi_s(\beta_D) = \gamma_D$ for $D \in \mathscr{D}_0$, which shows that $\Psi_s(\delta)\omega = e_{L'}$. Hence $\Psi_s(\delta)$ is an epimorphism, and so δ is an epimorphism, too. It is easy to verify that the element $\omega(d)$ is skew to $\omega(c)$ in the $(k+1)$-group L. As was proved above, d is skew to c in the $(k+1)$-group G. Therefore by Corollary 3 of [6] ω is of the form $\omega = \Psi_s(\nu)$ where $\nu\colon G \to L$. Hence $\Psi_s(\nu\delta\sigma_D) = \omega\Psi_s(\beta_D) = \Psi_s(\sigma_D)$ for $D \in \mathscr{D}_0$. By the faithfulness of Ψ_s we obtain $\nu\delta\sigma_D = \sigma_D$ for $D \in \mathscr{D}_0$. Then $\nu\delta = e_L$, whence δ is a monomorphism. The morphism $\Psi_s(\delta)$, being an epimorphism and a monomorphism, is an isomorphism. Therefore

$$[\Psi_s(L);\ \{\Psi_s(\sigma_D)\colon \Psi_s F_0(D) \to \Psi_s(L)\}_{D \in \mathscr{D}_0}]$$

is the inductive limit of $\Psi_s F_0$, and so by Lemma 2

$$[\Psi_s(L);\ \{\Psi_s(\sigma_D)\colon \Psi_s F(D) \to \Psi_s(L)\}_{D \in \mathscr{D}}]$$

is the inductive limit of $\Psi_s F$. The functor Ψ_s preserves the inductive limit of F.

Conversely, let Ψ_s preserve the inductive limits of all diagrams $F\colon \mathscr{D} \to \mathbf{Gr}_{k+1}$ where $\mathscr{D}$ is nonempty. Consider the functor $F\colon \mathscr{D} \to \mathbf{Gr}_{k+1}$ defined as follows: for $D \in \mathscr{D}$ the object $F(D)$ is a one-element $(k+1)$-group and for $\alpha\colon X \to Y$ the morphism $F(\alpha)$ is the (unique) isomorphism of $F(X)$ onto $F(Y)$. By the definition of F it follows that in this case $\mathscr{D}_0 = \mathscr{D}$. Let $[L; \{\gamma_D\colon F(D) \to L\}_{D \in \mathscr{D}}]$ be the inductive limit of F. By assumption, Ψ_s preserves inductive limits, therefore

$$[\Psi_s(L);\ \{\Psi_s(\gamma_D)\colon \Psi_s F(D) \to \Psi_s(L)\}_{D \in \mathscr{D}}]$$

is the inductive limit of $\Psi_s F$. Note that for any morphism $\alpha\colon X \to Y$ with $X, Y \in \mathscr{D}$, the morphism $\Psi_s F(\alpha)$ is the (unique) isomorphism of the one-element $(n+1)$-group $\Psi_s F(X)$ onto the one-element $(n+1)$-group $\Psi_s F(Y)$. Therefore, in view of Lemma 5*, $[\Psi_s(L);\ \{\Psi_s(\gamma_D)\colon \Psi_s F_d(D) \to \Psi_s(L)\}_{D \in \mathscr{D}_d}]$ (where F_d is the restriction of F to $\mathscr{D}_d$) is the inductive limit of $\Psi_s F_d$. The category $\mathscr{D}_d$ is discrete, and hence $[\Psi_s(L);\ \{\Psi_s(\gamma_D)\}_{D \in \mathscr{D}_d}]$ is simply the free product of the family of $(n+1)$-groups

$\{\Psi_s F_d(D)\}_{D\in\mathscr{D}_d}$. According to Theorem 3 of [8] the free product of at least two nonempty $(n+1)$-groups is not an $(n+1)$-group derived from a $(k+1)$-group; so the family of $(n+1)$-groups $\{\Psi_s F_d(D)\}_{D\in\mathscr{D}_d}$ is a one-element family (since $\Psi_s(L)$ is obviously derived from the $(k+1)$-group L). Thus $\mathscr{D}_d$ consists of one object only, whence $\mathscr{D}$ is a connected category.

If $\mathscr{D}$ is an empty category and $k>1$, then L (as the inductive limit of the empty diagram F) is the empty $(k+1)$-group. Hence $\Psi_s(L)$ is the inductive limit of $\Psi_s F$. The empty category is obviously connected. This completes the proof of Theorem 2.

Corollary 2. *Let $\mathscr{D}$ be a nonempty connected diagram scheme. Then $[L; \{\sigma_D\colon F(D)\to L\}_{D\in\mathscr{D}}]$ is the inductive limit of $F\colon \mathscr{D}\to \mathbf{Gr}_{k+1}$ if and only if $[\Psi_s(L); \{\Psi_s(\sigma_D)\colon \Psi_s F(D)\to\Psi_s(L)\}_{D\in\mathscr{D}}]$ is the inductive limit of $\Psi_s F\colon \mathscr{D}\to\mathbf{Gr}_{n+1}$.*

Note that in the case $k=1$ we always have $\mathscr{D}_0=\mathscr{D}$ (since $(n+1)$-groups derived from groups are always nonempty). But for $k>1$ the $(n+1)$-group derived from the empty $(k+1)$-group is empty. That case has to be excluded. This is the reason for considering the category $\mathscr{D}_0$ instead of $\mathscr{D}$. This, however, is only a minor restriction since, as mentioned in Lemma 2, in considering inductive limits of $(n+1)$-groups the empty $(n+1)$-group is inessential.

As in the dual case of Φ (Section 4), the question arises what are the $(n+1)$-groups derived from the inductive limits of diagrams $F\colon \mathscr{D}\to\mathbf{Gr}_{k+1}$ in the case when $\mathscr{D}$ is not connected. As in the case of Φ, a partial answer is offered by Lemma 7* for $k=1$, but here too (i.e. in the case of $\mathbf{Gr}_2$) more details can be given.

Take any diagram scheme $\mathscr{D}$ and a diagram $F\colon \mathscr{D}\to\mathbf{Gr}_2$. Let $\mathscr{D}_i$ denote the category obtained from $\mathscr{D}$ by adding an initial object I and F_i the functor F extended to that category $\mathscr{D}_i$. The object $F_i(I)$ is obviously a trivial (i.e., one-element) group. For $D\in\mathscr{D}$ let $\mu_D\colon F_i(I)\to F_i(D)$ denote the embedding of the trivial group into any group $F(D)$. Every $(n+1)$-group $\Psi_s F_i(D)$, being derived from a group, contains an invariant element of order one (cf. [2], [10]). The embedding of that element (treated as a one-element group) is just the morphism $\Psi_n(\mu_D)\colon \Psi_n F_i(I)\to\Psi_n F_i(D)$. Thus every diagram $\Psi_n F$ can be extended to $\Psi_n F_i$ by adding the one-element $(n+1)$-group $\Psi_n F_i(I)$ and the family of morphisms $\{\Psi_n(\mu_D)\colon \Psi_n F_i(I)\to\Psi_n F_i(D)\}_{D\in\mathscr{D}}$. Hence we obtain

Proposition 4. *If $[L; \{\gamma_D\colon F(D)\to L\}_{D\in\mathscr{D}}]$ is the inductive limit of $F\colon\mathscr{D}\to\mathbf{Gr}_2$, then $[\Psi_n(L); \{\Psi_n(\gamma_D)\colon \Psi_n F_i(D)\to\Psi_n(L)\}_{D\in\mathscr{D}_i}]$ is the inductive limit of the extended diagram $\Psi_n F_i\colon \mathscr{D}\to\mathbf{Gr}_{n+1}$.*

In particular, for the case when $\mathscr{D}$ is a discrete category we get

Corollary 3. *An $(n+1)$-group derived from a free product of groups is the free product of $(n+1)$-groups with an amalgamated one-element sub-$(n+1)$-group.*

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UL. DAWIDA 1A
50-527 WROCŁAW, POLAND