Bibliographie

M. A. Armstrong, Basic Topology, (Undegraduate Texts in Mathematics), XII+251 pages, Springer-Verlag, New York—Heidelberg—Berlin—Tokyo, 1983.

This is a topology book for undergraduates familiar with a first course in analysis, together with a knowledge of elementary group theory and linear algebra. The author shows several approaches in several branches of topology rather than too deep results in any articular area. The book deals with general topology, geometric and algebraic topology.

The clear geometric motivation is supplied by 132 figures. The author cares the delicate equilibrium of lengthy theories and applications, what helps the beginner reader.

The first chapter is an introductory one, the following three chapters contain a basic knowledge in general topology (compactness, connectedness, product spaces, glueing, topological groups). In the fifth chapter the fundamental group is introduced and is applied to prove the Brouwer fixed point theorem for a disc and the Jordan curve theorem. The following two chapters are devoted to triangulations, complexes, barycentric subdivision and simplicial approximation, the classification of closed surfaces. Two chapters deal with simplicial homology and its applications (degree of maps, the Euler-Poincaré formula, the Borsuk-Ulam theorem, the Lefschetz fixed point theorem, the invariance of dimension), but the author misses to give any systematic method for calculating homology groups, since the beginner may meet this trouble later as well. The last chapter is devoted to knots, an appendix on generators and relations is also included.

The book is highly recommended to undergraduate and first year graduate students. I have to emphasize its wide coverage, you have really an unusual introduction to topology. The previous edition of the book was published by McGraw-Hill (Great Britain) in 1979.

L. A. Székely (Szeged)

Differential Geometric Methods in Mathematical Physics, Proceedings, Clausthal, Germany, 1978. Edited by H. D. Doebner (Lecture Notes in Physics, 139), VIII+330 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This volume contains the text of 16 lectures presented at the conference "Differential Geometric Methods in Mathematical Physics" held at the Technical University of Clausthal, Germany, July 1978.

The lectures have been arranged into the following four groups according to the subject they are dealing with.

1. Quantization Methods and Special Quantum Systems: geometric quantization, vectorfield quantization, quantization of stochastic phase spaces, dynamics of magnetic monopoles, spectrum generating groups. 2. Gauge Theories: phase space of the classical Yang—Mills equation, nonlinear σ-models, gauging geometrodynamics, exceptional gauge groups. 3. Elliptic Operators, Spectral Theory and Applications: the Atiyah—Singer theorem applied to quantum-filed theory, spectral theory applied to phase transitions. 4. Geometric Methods and Global Analysis: systems of non-

Hausdorff spaces and non-Euclidean spaces, Weyl geometry, Lorentz manifolds, manifolds of embeddings.

The excellent papers communicated in this book are worth studying for mathematicians and physicists interested in any of the four mentioned research fields.

L. Gy. Fehér (Szeged)

Equadiff 82, Proceedings, Würzburg, 1982. Edited by H. W. Knobloch and K. Schmitt (Lecture Notes in Mathematics, 1017), XIII+666 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

The international conference EQUADIFF 82 held at the University of Würzburg during the week August 23 to August 28, 1982 was the fourth in a sequence of international conferences, with focus on the subject of differential equations, which were started in 1970 in Marseille.

What is the use of conferences? Besides to meet old friends and to get acquainted with new ones, it is to obtain a cross section of current research in the field. These proceedings consisting of 59 papers, give such a cross section also to those experts of the theory of differential equations not being able to participate the conference. It focussed on the branches of ordinary, functional and stochastic differential equations, partial differential equations of evolution type, and difference equations. Unfortunately, there is no room in such a review to list all the lectures, here are some topics special attention was paid to: infinite dimensional dynamical systems, semigroups of operators in Banach spaces, stability and bifurcation theory, Hamiltonian systems, functional differential equations with infinite delay, epidemic models, diffusion reaction model, numerical methods and applications in physics, engineering and biology.

The volume will give a very useful panoramic vision to every expert in the theory of differential equations and its applications.

L. Hatvani (Szeged)

P. Erdős, A. Hajnal, A. Máté, R. Rado, Combinatorial Set Theory: Partition Relations for Cardinals (Disquisitiones Mathematicae Hungaricae, 13 and Studies in Logic and the Foundations of Mathematics, Vol. 106), 347 pages, Jointly published by Akadémiai Kiadó, Budapest and North-Holland Publ. Co. Amstedam—New York—Oxford, 1984.

Starting from the familiar result known as Ramsey's theorem, a large portion of set theory, called the partition calculus, is developed in a considerable pace. The main interest of this calculus is in generalizing Ramsey's theorem for large cardinals. By now, the subject has achieved a stage when a systematic synthesis is possible (and desirable for further research). This book is devoted to such a systematic treatment. As the authors write: "we want to give a discussion of the ordinary partition relation for cardinals without the assumption of the generalized continuum hypothesis; we tried to make this latter as complete as possible".

The first two chapters, entitled Introduction and Preliminaries, respectively, provide the necessary backgrounds from classical set theory; for example, the basics on Zermelo—Fraenkel set theory, including axioms, Mostowski's Collapsing Lemma, equivalents for the Axiom of Choice, stationary sets, Fodor's and Solovay's theorems and the like, are considered briefly. Partition calculus proper begins in Chapter III, where Ramsey's result and its first important generalization, the Erdős—Dushnik—Miller theorem are proved. A separate section is included here in order to describe the main partition symbols used in the literature. In the next chapter (infinite) trees are treated in details, thus obtaining a powerful tool (the stepping-up lemma) for deriving positive ordinary partition relations. Chapter V is devoted to negative ordinary partition relations, while the next one develops important auxiliary results, called Canonization Lemmas, which are used later for establishing some positive partition relations for singular cardinals. Partition relations on large cardinals are investigated in Chapter VII, in particular a theorem due to Hanf and Tarski is proved. The next two chapters consider ordinary partition relations with "superscript" two and greater than two, respectively. Finally, the last two chapters give some applications of combinatorial methods, including Arhangel'skiĭ's result on the cardinality of the first countable compact Hausdorff space, the set-mappings theorems due to Fodor and Hajnal, the effect of using Suslin, Kurepa or Aronszajn trees in obtaining results without the (generalized) continuum hypothesis, and some positive results on the existence of (infinitary) Jónsson algebras.

The volume is clearly written; its complete understanding requires little familiarity with other branches of set theory and mathematics, only; and so it will certainly be a useful reading for anyone interested in infinite combinatorial methods.

P. Ecsedi—Tóth (Szeged)

Evaluating Mathematical Programming Techniques Proceedings, Boulder, Colorado, 1981. Edited by John M. Mulvey (Lecture Notes in Economics and Mathematical Systems, 199), XI+379 pages, Springer-Verlag Berlin, Heidelberg—New York.

This book contains approximately 30 lectures given at a twoday conference in Boulder. The main topic of this meeting was to consider how mathematical programming techniques ought to be evaluated.

The papers of the first two sections deal with several test problems and their computational experiments. The reader can find several comparisons of differently generated test problems for linear and non-linear problems. There are statistical reviews and methodological approaches as well. The next part of the book contains those examinations which consider computational comparisons for such integer programming and combinatorial optimization problems as the Euclidean travelling salesman — and the multidimensional knapsack problem. There is a description of an interesting algorithm, named SLIP, to choose the options from several algorithm factors. The next section is considered with identifying ideas that, perhaps, will guide future studies on comparisons of algorithms and codes. Three methods were selected for critical review: The Sandgreen—Ragsdell's, the Schittkowski's and the Miele—Gonzalez's studies. The remaining part of the book contains such approaches to software testing which use other disciplines, for example statistical methods.

These proceedings give a good overview of the recent research on this field of mathematics.

G. Galambos (Szeged)

E. Fried, Abstrakte Algebra. Eine elementare Einführung, IV+340 Seiten, Akadémiai Kiadó, Budapest, 1983.

Dieses Buch ist die Übersetzung des im Jahre 1972 erschienenen ungarischen Originals. Das Buch hat das Ziel, die Methoden darzulegen, die in der abstrakten Algebra auftreten. Gleichzeitig wird auch gezeigt, daß man diese Methoden zu Lösungen von Aufgaben welchen Typs verwenden kann. Dieses Ziel wird allerdings auf elementarem Weg erreicht. Die Kapitel beschäftigen sich mit Gruppen und Halbgruppen; Ringen, Körpern und Vektorräumen; Verbänden, Booleschen Algebren; universellen Algebren und Kategorien. Ein großer Vorzug des Buches ist, daß es die sich erhebenen algebraischen Begriffe durch interessante Beispiele klarstellt. Die Aufgaben sind außerordentlich nützlich und anschaulich. Dieses Buch populären Charakters ist für alle vorzuschlagen, die sich für die abstrakte Algebra interessieren.

L. Megyesi (Szeged)

James Glimm—Arthur Jaffe, Quantum Physics: A Functional Integral Point of View, XX+417 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

It is a basic "experimental fact" that physical problems of the most diverse origin can be dealt with be using common mathematical techniques. For classical physics the mathematical method of primary importance was provided by the theory of differential equations. The favourite candidates for this role in the case of modern physics are probability theory and analysis over function spaces. The authors, leading constructive field theorists, apply this fascinatingly uniform approach to three of the main branches of modern physics: quantum mechanics, statistical mechanics and quantum field theory.

The book consists of three parts of different style and intention. Part I is an introduction to the conceptual structure of quantum and statistical physics. Here the authors' purpose was to make the treatment of physics self-contained as far as it is possible. This is a big help for mathematicians but physicists and students also will find this survey useful. Among others there is a clear explanation of the famous Feynman—Kac formula here from the view-point of Wiener integrals. Part II is devoted to the main subject of the book: quantization of nonlinear fields. It gives a mathematically self-contained development of the theory of certain non-Gaussian measures on function spaces. Complete construction of boson fields with polynomial interaction in two spacetime dimensions has been presented. The compatibility of relativistic quantum mechanics and the constructed nonlinear quantum field theory is proved. This and other examples (all of them exist in two or three dimensions) answer the long standing question about mathematical implementability of quantization defined by renormalized perturbation theory in the positive. But it has been an open question up till now for the four dimensional case. Scattering theory, bound states, phase transitons and critical pints, the method of cluster expansion, reconstruction of quantum mechanics from path integrals form the theme of Part III. This part of the book is written at a more advanced level and provides an introduction to the literature.

The present book is highly recommended to all who are interested in the mathematical structure or applications of statistical and quantum physics.

L. Gy. Fehér (Szeged)

Victor Guillemin and Shlomo Sternberg, Symplectic techniques in physics, XI+468 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sidney, 1984.

This clearly written, excellent book contains extraordinarily wealthy material on pure symplectic geometry and on its extensive physical applications. Symplectic geometry has appeared as the modern version of the "old theory" of canonical transformations. Now it is one of the most active reserach areas where the frontiers of mathematics and physics are connected along questions of fundamental importance for both closely related sciences. Many of the results presented in the book were available only in journal articles so far and some of them are new. The authors' purpose is twofold: to provide an introduction to the subject and to present the central results from a modern point of view.

In the first chapter they give a general survey of the mathematical and physical ideas involved in the development of symplectic geometry. Among them are the optical analogies of classical and quantum mehcanics, the problem of quantizations, particle motion in electromagnetic and gravity fields. The style is rather elementary but almost all the questions detailed further on with more sophisticated techniques are sketched here. Chapter II is devoted to the description of the key mathematical results about symplectic manifolds and homogeneous spaces, Hamiltonian group actions and their moment maps, foliations and reduction procedures. Applications concerning the problem of collective motions — collective Hamiltonians and an outline of the geometric quantization theory are given. The main points of the third chapter are the motion of particles in Yang—Mills and gravity fields, the principle of general covariance. A new local normal form description is presented for Hamiltonian actions of compact Lie groups. Chapter IV deals with the use of group theoretical methods in the investigation of complete integrability, with questions from the theory of solitop and the higher-order calculus of variations. The final part contains several standard results on Lie algebras ang highly non-standard ones about the deformation theory of Lie algebras and the associated symplectic homogeneous spaces. This is of central interest in the limiting process from a general physical theory to one of its special cases according to Bohr's principle of correspondence. The main results are taken from Coppersmith's unpublished thesis and generalize for example the widely known contraction of the Poincaré algebra to the Galilean one. Besides the mentioned themes many important physical examples and mathematical theorems are treated.

In conclusion this book is warmly recommended to everyone interested in symplectic geometry and its applications. It can be used as an up-to-date textbook for graduated students and will have durable significance for mathematicians and theoretical physicists.

L. Gy. Fehér (Szeged)

L. Henkin, J. D. Monk, A. Tarski, H Andréka, and I. Németi, Cylindric Set Algebras (Lectur, Notes in Mathematics, 883), VII+323 pages, Springer-Verlag, Berlin—Heidelberg—New York 1981.

Henkin, Monk and Tarski published a book in 1971, entitled Cylindric Algebras, Part I, which soon became a basic reference to algebraists and logicians. Their intention in writing a second part on the topic was clearly put in the title; and, indeed, they noted there that a preliminary chapter of the continuation was available in mimeographed copies. The first paper of the present volume is a revised and considerably extended version of that chapter, and also, is considered by the authors as a starting piece of a series of papers "which would form the bulk of Part II of their earlier work". The paper is (almost) self-contained and presents the basic definitions and properties of several different kinds of cylindric set algebras. In particular, applications of such general algebraic concepts as subalgebras, homomorphisms, direct and ultraproducts, relativization, reducts and the like are treated in details. The results obtained in this way are nice and deep.

The volume contains a second study due to Andréka and Németi. The first three authors write in the introduction (referring to their paper): "As their writing proceeded, they learned of the closely related results obtained by Andréka and Németi, and invited the latter to publish jointly with themselves". In the second paper "certain aspects of the theory are investigated more thoroughly; in particular, many results which are merely formulated in the first paper are provided with proofs in the second one".

Central to the discussion of Andréka and Németi are the so called regular generalized cylindric set algebras which play a role analogous to that of played by Boolean set algebras in the theory of general Boolean algebras.

These two long papers are extremely clearly written, contain several new, nice and deep results and so, no doubt, will become a basic reference for anyone interested in algebraization of logics. We warmly recommend to model theorists and algebraists to have this book on their bookshelf.

P. Ecsedi-Tóth (Szeged)

Paul van den Heuvel, The Stability of a Macroeconomic System with Quantity Constraints (Lecture Notes in Economics and Mathematical Systems, 211), VII+169 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1983.

One of the central questions of mathematical economics is the study of the existence, uniqueness and stability of equilibrium of economical systems. The classical walrasian equilibrium concept was investigated already in the 19^{th} century. This equilibrium is characterized by the equality of demand and supply for all goods by the assumption that the prices are fully flexible. In the keynesian economical models the prices are not completely flexible and the rigid prices cause inequalities between demand and supply. These inequalities are called quantity constraints in the mathematical model, and the corresponding equilibrium concept is called non-walrasian.

Barro—Grossman (1971) and Malinvaud (1977) defined a neokeynesian model where the economics consist of two sectors: consumption and production, and three commodities: labour, consumption good and money. This book is devoted to the detailed mathematical study of this economical model, the corresponding equilibrium concept and the related stability questions.

The book is written in a very clear style. The only prerequisite for its reading is some basic knowledge in convex analysis and ordinary differential equations. This book is of interest to specialists engaged in equilibrium theory of economical systems. Moreover it is warmly recommended to everyone who is willing to get acquainted with the background of mathematical economics.

Péter T. Nagy (Szeged)

Paul Kelly—Gordon Matthews, The Non-Euclidean, Hiperbolic Plane, Its Structure and Consistency (Universitext), XIII+333 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

The purpose of this book is to give an introduction to axiomatic Bolyai—Lobatchevsky plane geometry accessible to anyone with a good background in high school mathematics. The authors present a strong "metrical" axiom system which makes possible to derive the basic structure of hyperbolic plane geometry and of its euclidean models without difficulties concerning the order and congruence relations and their consequencies. They say "The development... is especially directed to college students who may become secondary teachers. For that reason, the treatment is designed to emphasize those aspects of hyperbolic plane geometry which contribute to the skills, knowledge, and insights to teach euclidean geometry with some mastery". Chapter I outlines the history of the "parallel-postulate" problem. Chapter II is a review of George Birkhoff' axiom system of absolute geometry. Chapter III gives a syntetic development of central theorems in hyperbolic plane geometry. Finally, a short introduction to "distance geometries" is given in an appendix.

The excellent textbook is warmly recommended to students and to everyone who is interested in teaching of geometry.

Péter T. Nagy (Szeged)

D. V. Lindley—W. F. Scott, New Cambridge Elementary Statistical Tables, 80 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1984.

The concept of what constitutes a familiar or elementary statistical procedure has changed in the last 30 years and the rapid progress of pocket calculators made some statistical tables unnecessary. The reason for the publication of this set of tables is to replace the Cambridge Elementary Statistical Tables (Lindely and Miller, 1953). As the authors explain in the Preface they wanted to provide a convenient set of tables for the teaching and study of statistics in schools and university.

The binomial, Poisson, normal, χ^2 and t distributions have been fully tabulated and the book contains the percentage points of these distributions as well. The percentage points of the Behrens', F, Spearman's S, Kendall's K, Wilcoxon's signed — rank and Mann — Whitney distributions and the upper percentage points of the one — and two — sample Kolmogorov—Smirnov, Friedman's and Kruskal—Wallis statistics are also given. There are also tables of sums of squares of normal scores, random sampling numbers, random normal deviates and expected values of normal order statistics. The gerat experience of the Cambridge University Press guarantees that there are not misprints in the tables.

Lajos Horváth (Szeged)

J. Macki—A. Strauss, Introduction to Optimal Control Theory (Undergraduate Texts in Mathematics), XIII+165 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

Perhaps one of the most characteristic feature of this book can be gathered already from the sentences of the first chapter: "In control theory, one is interested in governing the state of a system by using controls. The best way to understand these three concepts is through examples." Then we find examples: A national economy, Water storage and supply and the example which is used throughout the monograph: The rocket car. Because of the generality of the concepts of systems, state and control the authors could just as well have chosen examples from biology, space flight and other fields. It is worth enumerating some of the exercises of Chapter 1.: A model for the optimal harvesting of fish; A model for the control of epidemics; The moon landing problem; Neoclassical economic growth model.

The book is written in a nice style, emphasizing motivation and explanation, avoiding the ponderous "definition — axiom — theorem — proof" approach.

In proving theorems the authors often just prove a relatively simple case. The general case is omitted or is given in the appendices. At the end of the chapters one finds several notes, references and examples.

The book is in some sense self-contained, the prerequisities are only advanced calculus (including Lebesgue integration), basic course in ordinary differential equation and linear algebra.

Chapter headings are: Introduction and motivation; Controllability (with an appendix containing the proof of the bang-bang principle); Linear autonomous time-optimal control problems; Existence theorems for optimal control problems; Necessary conditions for optimal controls — the Pontryagin Maximum principle; Appendix to the previous chapter: a proof of the Pontryagin Maximum principle.

Summarizing, this excellent concise introduction is useful not only for advanced undergraduates in mathematics, but also for economists, engineers and applied scientists because the authors find the ideal balance between the theory and application of mathematics.

L. Pintér (Szeged)

Measure Theory and its Applications, Proceedings of a Conference held at Sherbrooke, Québec, Canada, June 7—18, 1982. Edited by J. M. Belley, J. Dubois and P. Morales (Lecture Notes in Mathematics, 1033), XV+317 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

This volume contains 28 papers on several branches of measure theory. The topics are ergodic theory, vector measures, measure theory and topology. Choquet, G., Representation integrale, Convexes et cones convexes non localement compacts, Formes lineaires positives et mesures; Oxtoby, J., Transitive points in a family of minimal sets; Hida, T. and Streit, L., White noise analysis and its application to Feynman integral.

Seven papers are in French. The conference proceedings are completed with a selection of open problems of the problem section. In order to call your attention to the contained open problems, I quote a beautiful problem of P. Erdős (cited by D. Mauldin in the book).

Let K be a compact subset of $\mathbf{R}^2$, with Lebesgue measure $\lambda(K)>0$. Does there exist a point x such that $\{\|y-x\|: y\in K\}$ contains an open interval?

L. A. Székely (Szeged)

Angelo B. Mingarelli, Volterra—Stieltjes Integral Equations and Generalized Ordinary Differential Expressions (Lecture Notes in Mathematics, 989), XIV+317 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

The author's aim is to present a qualitative and spectral theory of Volterra—Stieltjes integral equations giving special interest to applications in ordinary differential equations as well as in difference equations.

In Chapter 1. the author studies the extension of the classical Sturmian results — comparison and separation theorems — on Stieltjes integro—differential equations. In Chapter 2. oscillation and non-oscillation theorems are given on Volterra—Stieltjes integral equations with applications to second order differential and difference equations. In Chapter 3. the author uses a natural extension of a method introduced by I. S. Kac. This method shows that the integral equation investigated before can be thought of as defining generalized differential operators. Here is treated the famous Weyl classification (limit point, limit circle) of singular generalized differential operators.

In the final two chapters one finds Sturm—Liouville difference and differential equations with an indefinite weight-function, and the spectral theory of generalized differential operators.

The appendices containing a part of the necessary mathematical background make the book in some sense self-contained.

This thoughtful work on a vividly developing field is warmly recommended for everybody interested in differential and integral equations.

L. Pintér (Szeged)

Model Theory of Algebra and Arithmetic (Proceedings, Karpacz, Poland, 1979). Edited by L Pacholski, J. Wierzejewski, and A. J. Wilkie (Lecture Notes in Mathematics, 834), VI+410 pages Springer-Verlag, Berlin—Heidelberg—New York, 1980.

The volume consists mostly of papers presented by invited lecturers at the Conference on Applications of Logic to Algebra and Arithmetic held at Bierutowice—Karpacz in Poland, September 1—7 1979. Some invited papers not presented personally and a few contributed ones are included, too. The 21 titles of the book are as follows: J. Becker, J. Denef and L. Lipshitz, Further remarks on the elementary theory of formal power series rings; C. Berline, Elimination of quantifiers for nonsemi-simple rings of characteristic p; M. Boffa, A. Macintyre and F. Point, The quantifier elimination problem for rings without nilpotent elements and for semi-simple rings; E. Bouscaren, Existentially closed modules: types and prime models; G. Cherlin, Rings of continuous functions: decision problems; P. Clote, Weak partition relations, finite games, and independence results in Peano arithmetic; F. Delon, Hensel fields in equal characteristic $p>0$; M. A. Dickmann, On polynomials over real closed rings; J. Duret, Les corps faiblement algébriquement clos non séparablement clos ont la propriété d'indépendance; U. Felgner, Horn-Theories of abelian groups; P. Hájek and P. Pudlák, Two orderings of the class of all countable models of Peano arithmetic; A. Macintyre, Ramsey quantifiers in arithmetic; K. L. Manders, Computational complexity of decision problems in elementary number theory; K. McKenna, Some diophantine Nullstellensätze; G. Mills, A tree analysis of unprovable combinatorial statements; J. B. Paris, A hierarchy of cuts in models of arithmetic; C. Smorynski and J. Stavi, Cofinal extension preserves recursive saturation; L. von Den Dries, Some model theory and number theory for models of weak systems of arithmetic; A. J. Wilkie, Applications of complexity theory to Σ_0-definability problems in arithmetic; G. Wilmers, Minimally saturated models; B. I. Zilber, Totally categorial theories: structural properties and the non-finite axiomatizability.

The book gives a good overview on the present state of the arts (of course, at the date of edition) and so it is recommended to experts as well as to graduate students intrested in the subject.

P. Ecsedi—Tóth (Szeged)

E. E. Moise, Introductory Problem Courses in Analysis and Topology (Universitext), VII+94 pages, Springer-Verlag, New York—Heidelberg—Belin, 1982.

This book consists of two parts: Analysis and Topology. In each chapter there are given definitions and theorems guaranteed to be true. The first job for the reader is to prove the proposed theorems. In the problems stated after one finds true and false propositions the reader's task is to give the right answer. This requires to focus his/her attention on the heart of the matter which is not easy for a student, but the result is an unusually rapid progress.

Let us mention some examples: After defining continuous and Lipschitzian functions we have: "Theorem (?). Every continuous function is Lipshitzian. Theorem (?). If f is a continuous function $[a, b]\to\mathbf{R}$, then f is Lipschitzian. Theorem (?). Every Lipschitzian function is continuous." The following example is taken from the second part (Topology): „A set $M\subset\mathbf{R}$ has the Heine—Borel property if for each collection G of open intervals covering M, there is a finite subcollection G' of G which also covers M. Theorem (?). If M has the Heine—Borel property, then M is closed and bounded."

These problem courses are useful for the major part of students because as the author says:

"Some have supposed that problem courses are advantageous only for students of real brilliancy, but my own experience over many years indicates the contrary. The time that is 'wasted' while students grope their way makes the pace of a problem course very slow. (It often happens that a whole hour is spent analyzing a 'proof' which turns out to be quite worthless.) This means that a competent student is able to keep track, and finds at the end that he understands the course completely. This is a valuable experience, and for many students it is new."

L. Pintér (Szeged)

Roe Nottrot, Optimal Processes on Manifolds; an Application of Stokes' Theorem (Lecture Notes in Mathematics, 963), VI+124 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

The optimal control theory developed extremely rapidly during the last twenty years. The central topic of investigations was the generalizations of Pontryagin's "maximum principle" which has to be satisfied by an "optimal process". This book gives a unified treatment of variational theory of optimal processes described by a set of ordinary or partial differential equations. The general maximum principle is proved by an application of Stokes' theorem.

Chapter I contains a brief introduction to alternating differential forms, integration, Stokes' theorem on a smooth manifold. Chapter II is devoted to the proof of the maximum principle. Chapter III—V deal with applications of the general theory to processes described by ordinary, first and second order partial differential equations.

This book is highly recommended to all who are interested in control theory.

Péter T. Nagy (Szeged)

David R. Owen, A First Course in the Mathematical Foundations of Thermodynamics (Undergraduate Texts in Mathematics), XVII+178 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

There are two approaches to the foundations of thermodynamics. One of them, beeing in close contact with statistical physics, postulates the existence of the state functions energy and entropy. The other, more traditional treatment, having its roots in the works of Carnot, Joule and Clausius, is based on the concepts of work, heat and temperature.

This book is a modern and a mathematically precise version of the traditional approach, operating of course on more general and more abstract notions. The truth of the classical theory has not been questioned since the end of the last century, nevertheless when reading a standard textbook on thermodynanics, often a kind of dissatisfaction may rise in the reader, wanting a more coherent expansion of the subject. The feeling of frustration is resolved by D. R. Owen. Unsophisticated mathematics and appropriate amount of reference to physical content make this book interesting and useful. It will be certainly appreciated by everyone, who is familiar with the empirical facts of heat theory, but who wishes to see the strict logical order of precisely defined quantities, fundamental assumptions and exact theorems of thermodynamics. After an overview of the properties of homogeneous fluid bodies, the concept of a system with perfect accessibility and the general notion of a thermodynamical system, containing both the states and the possible processes of them, are introduced. They make possible to state the first and second laws, and to prove the existence of energy and entropy in a simple manner.

In the course of development the role of the ideal gas — the classical object of the theory — is sometimes felt to be exaggerated, but the reader is compensated at the end of the book by the treatment of elastic filaments and viscous bodies, usually falling out of the scope of traditional applications.

M. G. Benedict (Szeged)

L. C. Piccinini, G. Stampacchia, G. Vidossich, Ordinary Differential Equations in R^n. Problems and Methods (Applied Mathematical Sciences, 39), XII+385 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1984.

The authors write in the Preface: "Our text ... aims to give a simple and rapid introduction to the various themes, problems, and methods of the theory of ordinary differential equations. The book has been conceived in such a way so that even the reader who has merely had a first course in calculus may be able to study it and to obtain a panoramic vision of the theory". The book answers this not easy purpose in an excellent way.

The purpose is not easy because just the basic theory of differential equation requires knowledges not included in the first courses in calculus. One of the advantages of the book making it a very good introductory lecture notes is the way as it acquaints the reader with these things. We can find short (but completed with proofs where it is necessary) surveys on these topics in bodies of the chapters where they are applied (e.g. review of metric spaces, review of Banach spaces, elements of linear algebra, the topological degree). At the end of the sections exercises are proposed illustrating the results and giving possible applications and complementary material.

The book is not only a good text-book but also a very useful monograph for the experts in differential equations. Reading the book they can get acquainted with new aspects of the basic theory. The most original chapter is the third one, titled Existence and Uniqueness for the Cauchy problem under the condition of continuity. It gives a "panoramic vision" of the results obtained by the Italian school in this field (the Peano-phenomenon, G-convergence, ...). Separate chapter is devoted to boundary value problems, which is written in a modern way but so that it is understandable for beginners, too. The chapters are concluded by bibliographical notes which serve as a guide for further studies.

L. Hatvani (Szeged)

Probability Measures on Groups VII. Proceedings, Oberwolfach, 24—30 April 1983. Edited by H. Heyer (Lecture Notes in Mathematics, 1064), X+588 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The main aim of this series of meetings on Probability Measures on Groups is to cover the recent advances in this field of probability theory and to present new results. This volume contains four survey articles written by H. Heyer, A. Janssen, R. Schott, M. E. Walter and 31 research papers. The authors have raised also some open problems and possible extensions of the presented material. The editor of this collection has classified the contributions under the following topics: (i) probability measures on locally compact groups (decomposition, infinite divisibility), (ii) random walks on groups and homogeneous spaces (reccurence, polynomial growth, dichotomy theorems), (iii) Markov processes on hypergroups (transience, Lévy—Khinchin formulae, central limit theorems), (iv) Non-commutative probability theory (subadditive ergodic theorems, Gaussian functionals), (v) Random matrices and operators (law of large numbers, random Schrödinger operators, characteristic exponents).

Lajos Horváth (Szeged)

W. T. Reid, Sturmian Theory for Ordinary Differential Equations, XII+559 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

On the first page one finds: "Dedicated to Dr. Hyman J. Ettlinger. Inspiring teacher, who introduced the author as a graduate student to the wonderful world of differential equations."

In this book the reader is introduced by the author into one of the most interesting branches of ordinary differential equations in such a way, that he can hardly get rid of attractive problems.

The famous 1836 paper of Sturm, dealing with oscillation and comparison theorems for linear homogeneous second order ordinary differential equations, is one of the most important starting points of the investigations of the qualitative theory of solutions of differential equations.

The basic works of M. Bocher, D. Hilbert, G. D. Birkhoff, G. A. Bliss, M. Morse are fundamental for the development of the theory.

"The prime purpose of the present monograph is the presentation of a historical and comprehensive survey of the Sturmian theory for self-adjoint differential systems, and for this purpose the classical Sturmian theory is but an important special instance."

The organization of the chapters seems to be ideal to the reviewer. Every chapter contains a body of material which presents concepts and methods central for the investigated theme. This is followed by a section with more detailed comments and references to pertinent literature and — frequently — up to date problems. Finally — the most interesting for many readers — we have a section on Topics and Exercises. The well chosen problems characterizing the author's interest, make the typical feature of the book.

The titles of chapters are: Historical prologue; Sturmian theory for real linear homogeneous second order ordinary differential equations on a compact interval; Self-adjoint boundary problems associated with second order linear differential equations; Oscillation theory on a non-compact interval; Sturmian theory for differential systems; Self-adjoint boundary problems; A class of definite boundary problems; Generalizations of Sturmian theory.

This book is warmly recommended to everyone who is interested in differential equations, and also to other mathematicians — working in various branches of mathematics — who after having read this book will have a survey of this theme and perhaps will find pleasure in making research in this field.

L. Pintér (Szeged)

Séminaire de Probabilités XVIII, 1982/83. Edited by J. Azéma and M. Yor (Lecture Notes in Mathematics, 1059), IV+518 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

This 18[th] of the now Paris-based Seminaires de Probabilités consists of original research papers in diverse areas of the theory of stochastic processes. The presented 33 papers provide up-to-date overviews of the research activities of the French school of probability. We do not have space to list the titles of the papers published in this volume but we hope that the following list of authors will help to form an opinion of the content of this book. The authors of these proceedings: M. T. Barlow, E. Perkins, R. F. Bass, L. C. G. Rogers, F. B. Kninght, W. S. Kendall, R. F. Gundy, F. Bronner, J. Jacod, M. Liao, H. Rost, C. Stricker, W. A. Zheng, P. A. Meyer, S. W. He, A. Erhard, D. Bakry, C. S. Chou, J. Ruiz de Chavez, J. G. Wang, L. Schwartz, M. Talagrand, Ph. Novelis, F. Russo, R. C. Dalang, J. Neveu, R. Azencott and M. Emery.

Lajos Horváth (Szeged)

K. T. Smith, Primer of Modern Analysis (Directions for Knowing All Dark Things, Rhind Papyrus, 1800 B. C.), (Undergraduate Texts in Mathematics), XV+446 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

Seeing a new introduction to analysis, the reader is to look for those topics the author can show from a new point of view, topics, which are unusual on the level of introduction. You will find these topics, so Smith's book is worth reading.

The author has dealt with the crucial points of analysis from the notion of limit up to research level. The book consists of three parts. The first one is a usual first course in calculus, the rest of the book is of greater importance. The second part contains metric spaces, algebra and geometry in $\mathbf{R}^n$, the calculus in $\mathbf{R}^n$ and surfaces. It is a fruitful approach to treat algebraic and geometric structures of $\mathbf{R}^n$ in a textbook of analysis, since just the connection of several structures belongs to the "set of dark things" of a part of students.

The third part of the book is devoted to some advanced topics of analysis, e.g. Lebesgue measure (included Sard's theorem on regular values), differentiability of regular Borel measures and of Lipschitz functions a.e. surface area, degree of maps (through degree of C^∞ maps by the approach of Milnor), and extensions of differentiable functions.

It is impossible to write a book introducing to all chapters of analysis. The present book omits e.g. complex analysis and Fourier analysis. I think the author is right, a too long book would terrify the possible readers.

I recommend the book to under graduates and graduates as a reference. It is the second edition of the original book published by Bodgen and Quigley, 1971, with substantial revisions.

L. A. Székely (Szeged)

J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, IX+609 pages, Springer-Verlag, New York—Berlin—Heidelberg, 1980.

Since the publication of its original German edition this book has found very wide acceptance. The table of contents reads as follows.

1. Error Analysis 2. Interpolation 3. Topics in Integration 4. Systems of Linear Equations 5. Finding Zeros and Minimum Points by Iterative Methods 6. Eigenvalue Problems 7. Ordinary Differential Equations 8. Iterative Methods for the Solution of Large Systems of Linear Equations. Some Further Methods.

This introduction contains all standard topics and much more. To name some extras, the sections on minimization problems, extrapolation methods and for example, the multiple shooting method are not considered in all customary introductory courses.

The presentation of the material meets the highest mathematical standards. Most of the results including theorems about convergence rates and error bounds appear with full proofs. Special attention is paid to the practical implementation and the comparison of different methods. Numerous algorithmic descriptions are more or less formally provided in ALGOL 60. Each chapter ends with a list of references covering a significant amount of research articles and textbooks published between 1960 and 1974. Some new items up to 1978 are contained, too.

Nearly 200 exercises referring to interesting generalizations and additional results help the reader in the deeper understanding of the ideas presented in the main text. The readability of the book has considerably been increased by illuminating figures, comprehensive informal descriptions and fully worked out numerical examples.

Because of these outstanding features we can warmly recommend the book to all students of mathematics or computer science at the advanced undergraduate or graduate level. Even the more experienced lecturer can utilize some useful ideas about teaching this topic.

J. Virágh (Szeged)

K. R. Stromberg, An Introduction to Classical Real Analysis, IX+575 pages, Wadsworth International Group, Belmont, California, 1981.

This volume is the outgrowth of lectures held by the author during the past twenty years". The emphasis is on "twenty years". This characterizes the whole work, which seems to be a masterpiece among the various books discussing classical analysis.

Chapter headings are: Preliminaries; Numbers; Sequences and series; Limits and continuity; Differentiation; The elementary transcendental functions; Integration; Infinite series and infinite products; Trigonometric series.

This is real analysis in the sense that we do not have the theory of analytic functions but complex numbers and complex functions appear throughout the book. This text contains the preparatory topics which are necessary for learning complex and abstract analysis. The book is recommended first of all for advanced undergraduate and beginning graduate students, but also an experienced teacher will find something new in every chapter: a natural introduction of a notion, a simpler proof of a well-known theorem etc. Here in Szeged the sixth chapter is especially interesting, containing the elementary development of the theory of the Lebesgue integral due to F. Riesz (who spent his most fruitful years at Szeged University).

The excellent, attractive examples and exercises mobilize the reader's activity, sometimes one cannot get away from them without having the solution. The author says in the Preface: "I spent at least three times as much effort in preparing the exercises as I did on the main text itself".

Having read the book one comprehends a former reviewer's opinion (taken from a prepublication review): "This is the book I wish that I had written."

L. Pintér (Szeged)

J. Szép—F. Forgó, Einführung in die Spieltheorie XXIX+292 Seiten, Akadémiai Kiadó, Budapest, 1983.

Dieses Buch ist die deutsche Übersetzung der im Jahre 1974 erschienenen ungarischen Ausgabe.

Das Buch gewhärt einen Überblick über die Ergebnisse der Spieltheorie. Die ersten Kapitel enthalten die ganz allgemeinen Begriffe und Sätze der Spieltheorie. Das Buch beschäftigt sich am ausführlichsten mit dem 2-Personen-Spiel — das ist der am besten verwendbare Zweig der Spieltheorie, aber es behandelt auch einige spezielle n-Personen-Spiel und wirft mehrere neue Probleme auf.

L. Megyesi (Szeged)

Theory and Applications of Singular Perturbations. Proceedings. Edited by W. Eckhaus and E. M. de Jager (Lecture Notes in Mathematics, 942), V+363 pages, Springer-Verlag, Berlin—Heildeberg—New York, 1982.

This volume contains 22 papers delivered at the conference on "Theory and Applications of Singular Perturbations" held at Oberwolfach, August 16—22, 1981. The papers deal with recent specialized research in the theory of singular perturbations which is an important area of the theory of ordinary and partial differential equations. Roughly speaking a differential equation is singularly perturbed if the differential operator in the equation is multiplied by a small parameter.

The first part of these notes consists of primarily pure analytic considerations on free boundary problems, nonselfadjoint and nonlinear elliptic eigenvalue problems, coercive singular perturbations singular-singularly perturbed equations, etc. Classical, functional, nonstandard mathematical techniques and numerical methods are applied.

The second part is devoted to the applications concerning two-dimensinal viscous flows, incompressible flows at high Reynolds number, swirling flow between rotating coaxial disks, wave pattern, combustion theory the physics of ionized gases, Kramers' diffusion problem and the kinetic theory of enzymes.

T. Krisztin (Szeged)

B. L. van der Waerden, Geometry and Algebra in Ancient Civilizations, X+233 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

Until quite recently, in almost all books and papers on the early history of mathematics, we could read that the history of mathematics begins with the Babylonian and Egyptian arithmetic, algebra and geometry. However, this picture has been changed by three recent discoveries.

The first of them was made by A. Seidenberg, who studied the altar constructions and other ritual applications of mathematics in the Indian Śulvasūtras. In these relatively ancient texts squares are constructed equal in area to a given rectangle. In the constructions the "Theorem of Pythagoras" was used similarly to that of Euclid.

Secondly, the author compared the ancient Chinese collection "Nine Chapters of the Arithmetical Art" with Babylonian collections of mathematical problems and found so many similarities that the existence of a common origin seemed to him unavoidable. According to his views the "Theorem od Pythagoras" must have played a central role in this source.

The third discovery was made by A. Thom and A. S. Thom. They found that in the construction of megalithic monuments in Southern England and Scotland "Pythagorean Triangels", right-angled triangles whose sides are integral multiples of a fundamental unit of length, have been used.

Combining these three discoveries the author has ventured a tentative reconstruction of a mathematical science which must have existed in the Neolitic Age, between 3000 and 2500 B.C., and spread from Central Europe to the British Islands, to the Near and Far East.

In Chapters 1 and 2 we can read the author's ideas on this ancient science analyzing written sources, archeological evidences, and comparing Chinese and Babylonian mathematical texts as well. One of the fundamental evidences of his hypothesis on the common origin is that we not only meet "Pythagorean Triples" like (3, 4, 5) in Babylonian, Indian, Chinese and Greek texts, but their methods of calculating such triples are very similar, and there are equivalent methods as well.

In the subsequent chapters there is a very interescting comparative analysis of some fields of the Greek, Babylonian, Indian and Chinese mathematics and astronomy.

In Chapters 3, 4 and 6 the several traces of the pre-Babylonian geometry and algebra, which can be discerned in the work of Euclid and Diophantos and in popular Greek mathematics are discussed. In this discussion the four newly found books of Diophantos' Arithmetica (the Books IV to VII) are concerned, too.

In Chapter 5 the different methods in solving linear Diphantine equations and Pell's equations due to the Indian and Chinese mathematicians are compared with the Greek science. We can conclude here to the fact that the Euclidean Algorithm played a central role in their solutions.

In Chapter 7 the author points out that the work of the excellent Chinese geometer Liu Hui (third century A. D.) and some mathematical passages in the works of the great Indian astronomer Āryabhata (sixth century A. D.) are influenced by the work of Greek geometers and astronomers like Archimedes and Apollonios.

This very interesting, informative and enjoyable book — the first volume of the author's "History of Algebra" — is highly recommended to anyone, who is interested in the ancient history of mathematics, especially in the origin of mathematics. We hope that the further volume (or volumes) will appear soon.

Lajos Klukovits (Szeged)

M. J. Wygodski, Höhere Mathematik griffbereit, XI+832 Seiten, Akademie-Verlag, Berlin, 1982.

Dieses Buch ist die (verarbeitete, erweiterte) deutsche Übersetzung des russischen Originals. Über das Ziel des Buches vermittelt das Vorwort folgendes:

"Das Buch ... hat eine zweifache Bestimmung. Erstens übermittelt das Buch Auskünfte über sachgemäße Fragen: Was ist ein Vektorprodukt? Wie bestimmt man die Fläche eines Drehkörpers? Wie entwickelt man eine Funktion in eine trigonometrische Reihe? usw. Die entsprechenden Definitionen, Theoreme, Regeln und Formeln, begleitet von Beispielen und Hinweisen, findet man schnell.

Zweitens ist das Buch für eine systematische Lektüre bestimmt. Es beansprucht nicht die Rolle eines Lehrbuches. Beweise werden daher nur in Ausnahmefällen vollständig gegeben. Jedoch kann das Buch als Hilfsmittel für eine erste Auseinandersetzung mit dem Gegenstand dienen".

Der überwiegende Teil des Buchinhalts fällt auf das Gebiet der Geometrie und der mathematischen Analysis.

L. Megyesi (Szeged)