Bibliographie

G. Alexits, Approximation Theory (Selected Papers), 298 pages, Akadémiai Kiadó, Budapest, 1983.

The volume is a collection of selected papers by George Alexits. It is my deep-seated conviction that this collection is of great value of mathematics. The thirthy-four articles included here cover a wide field of real analysis and show the characteristic mathematical style of Alexits, the admirably clear exposition of his profound mathematical ideas. More precisely the volume presents articles on approximation theory, the papers developing the theory of multiplicative function systems, and the recent items on function series. The earlier function-theoretic, set-theoretic and curve-theoretic papers of Alexits and his works on the history of mathematics have been left out together with those papers on the theory of function series, the results of which were incorporated in his monograph "Konvergenztheorie der Orthogonalreihen" (Akadémiai Kiadó, Budapest, 1960) published also in English and in Russian. The papers are reprinted in their original form, with the only exception being the English translation of an article originally published in Hungarian. In my view this article is one of the most significant papers of Alexits. In it he characterizes the Lipschitz class of order $\alpha=1$ by the order of approximation given by the Cesàro-means of the conjugate Fourier series. This paper was published in a Hungarian journal in 1941, presumably this was the reason that the result was reproved later in parts by A. Zygmund (1945) and M. Zamansky (1949). In addition to the papers, the volume contains a short description of the life and scientific activities of George Alexits and the full list of his scientific works. At the end of the book are some remarks and a list of errata. These remarks briefly discribe the effect of the presented papers and the further developments resulting from them, moreover they give references to later results, while the list of errors corrects some oversights and misprints in the originals.

The significance of Alexits' contributions to many areas of mathematics is nowadays well known. But, for the sake of correctness, it is necessary to mention in connection with the "Remarks" on p. 287 that the cited monograph of R. A. DeVore was not the first to give international recognition to the fact that Alexits proved already in 1941 both necessity and sufficiency of the characterization of the Lipschitz class $\alpha=1$ by (C, 1)-summation. The first monograph emphasizing this was that of P. L. Butzer and R. J. Nessel *Fourier Analysis and Approximation*, Birkhauser Verlag, Basel—Suttgart, 1971.

I am convinced that Professor Alexits had a wide international reputation by the time when his monograph on the convergence and summation problems of orthogonal series appeared in 1960 in three languages. This monograph has become one of the most cited works in the field of orthogonal series. Alexits was one of the most influential Hungarian mathematicians. He created a scientific school having numerous pupils in Hungary and all over the world.

Mathematicians working in approximation theory will surely find it very useful to have these selected papers of Alexits in one volume.

L. Leindler (Szeged)

V. I. Arnold, Geometrical Methods in the Theory of Ordinary Differential Equations (Grundlehren der Mathematischen Wissenschaften, 250), XI+334 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1983.

V. I. Arnold, Catastrophe Theory, 79 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1984.

The title of the original edition of the first book is „Дополнительные главы теории обыкновенных дифференциальных уравнений" (Supplementary Chapters to the Theory of Ordinary Differential Equations). The translator (or the editor of the translation) chose the new title rightly because it characterizes both the topics and the treatment of the book. However, it is worth recording the original title, which shows that the present book is the continuation and supplement of the author's excellent introductory text-book crowned success, and that the book consists of *almost independent chapters.*

The first two chapters deal with special equations (differential equations invariant under groups of symmetries, implicit equations, the stationary Schrödinger equation, second order differential equations, first order partial differential equations) and present classical results, that can be found in most monographs. Nevertheless, after having read these chapters the reader feels as if he had been just acquainted with these results because their deep mysteries have become clear and understandable, setting the facts in their true light.

Chapter 3 is devoted to structural stability. In the real world there always exist small perturbations, which cannot be taken into consideration in the mathematical models. It is clear, that only those properties of the model may be viewed as the properties of the real process which are not very sensitive to a small change in the model. The investigation of these properties led to the notion of structural stability.

The organization of the chapter is typical Arnold. First he gives the naive definition of structural stability and illuminates it by examples. Then he gathers together the necessary tools and gives the final precise definition of structural stability. The definition is followed by a detailed analysis of the one-dimensional case, which helps the reader to intensify the new notion. Then he presents a survey on the differential equation on the torus, hyperbolic theory and Anosov systems.

Chapter 4 is concerned with perturbation theory. In the theory of differential equations there are some equations of special form (e.g. linear equations) which admit an exact analytic solution or a complete qualitative description. Perturbation theory gives methods for the study of equations close to one with known properties. One of the most important sections of this theory is the averaging method that has been used among others in the celestial mechanics since the time of Lagrange and Laplace. "Nevertheless, the problem of strict justification of the averaging method is still far from being solved" — writes the author, and the reviewer can recommend this part of the book as an excellent comprehensive introduction to this interesting and actual topic.

In Chapter 5 the reader finds Poincaré's theory of normal form, which is a very useful device in many topics such as in bifurcation theory, to which Chapter 6 is devoted. In the models of the real world, in general, there are some parameters. It may happen that arbitrarily small variations of the parameters at fixed values cause essential change of the pictures of the solutions. This phenomenon is called bifurcation. The author studies bifurcations of phase portraits of dynamical systems in the neighbourhood of equilibrium positions and closed trajectories.

The subject-matter of the second book (or booklet) can also be considered as a chapter of the geometrical theory of dynamical systems. The origins of catastrophe theory lie in Whitney's theory of singularities of smooth mappings and the bifurcation theory of dynamical systems. Interpreting — not always mathematically — the results of these theories, catastrophe theory tries to provide a universal method for the study of all jump transitions, discontinuities and sudden qualitative changes. It has aroused a great controversy not only among specialists but also in the popular press. This booklet explains what catastrophe theory is about and why it arouses such a controversy.

While the first book is of advanced level, the second one can be recommended also "to readers having minimal mathematical background but the reader is assumed to have an inquiring mind".

L. Hatvani (Szeged)

Bernard Aupetit, Propriétés Spectrales des Algebres de Banach (Lecture Notes in Mathematics, 735) X+192, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

This nice book collects the results obtained till its publication in connection with the spectra of Banach algebra elements. It contains mainly the author's own results, but gives historical backgrounds and performs the classical results as well. The recent development of this area and the absence of a comprehensive work on this subject make this book very interesting and useful.

The author's interest started from two, apparently remote problems. These were: to generalize Newburgh's theorem on the continuity of the spectrum and to generalize the theorem of Hirschfeld and Želazko on the characterization of commutative algebras. Mixing in a suprising manner the methods of these areas, the author obtained a characterization of finite-dimensional algebras. The use of subharmonic functions and deep results of classical potential theory in functional analysis provides the essential new feature of his technique.

The text consists of five chapters. Continuity problems of the spectrum, characterizations of commutative, finite-dimensional, symmetric and C^* algebras, respectively, are systematically treated. Abundance of examples and counterexamples complete the discussions. Two appendices, one on Banach algebras and the other on potential theory, help the reader and make the text available for a wide audience. The book is recommended to everyone who is interested in this new field of functional analysis.

L. Kérchy (Szeged)

David Bleecker, Gauge Theory and Variational Principles (Global Analysis, Pure and Applied, Series A, No. 1), XX+179 pages, Addison—Wesley, London—Amsterdam—Don Mills—Ontario—Tokyo, 1981.

The present book is the first number of a new series on pure mathematics and applications of global analysis based on ideas of classical analysis and geometry. Series B will provide a collection of prerequisites for the reports of series A from the research frontiers.

The most successful models of the fundamental interactions of the matter as well as the most hopeful candidates for their unification are all gauge theories with local symmetries. The majority of developments of classical gauge field theory in the last 10 years is connected with the global aspects of the underlying fibre bundle theory. This book contains a detailed account of bundle theoretic foundations of gauge theory.

The author's point of view, that the particle fields are functions on the corresponding principal bundles, leads to very elegant formulation of the variational problems and Euler—Lagrange equations involved. This is done in Chapters 3—5 based on the geometric notations of the previous ones.

A short, clear explanation of the free Dirac's equation as Lagrange's equation for the Dirac spinor field on the spin bundle with Levi—Civita connection can be found in Chapter 6. In Chapter 7 a general framework is given for the unification of interactions, based on a construction to form a principal bundle with product group, a connection and Lagrangian on it from the principal bundles and their connections which are connected with the fields that are to be incorporated in a unified theory. The general scheme is applied to the Dirac electron field coupled to electromagnetic potential and to the original Yang—Mills nucleon model. In Chapter 8 the author treats the tensor calculus on a (pseudo-) Riemann manifold in the frame bundle picture. Chapter 9 is devoted to the unification of gravitation and Yang—Mills fields in the well-known Kaluza—Klein type way. The reality of the used canonical bundle metric is supported by calculation of its geodesics in Chapter 10, nicely intepreted as paths for the classical particle motion. Besides, Utiyama's theorem, the spontaneous symmetry breaking and the very basic notations of the characteristic classes in connection with the monopoles and instantons are treated within the additional topics of Chapter 10.

The book is very well organized, self-contained, concise and rigorous. In the preface and in the introduction to the chapters the intuitive ideas are also sketched by the author. It is highly recommended for everyone interested in gauge theory. Those working in the field as well as graduate students will find it useful without doubt.

L. Gy. Fehér (Szeged)

E. A. Coddington—H. S. V. de Snoo, Regular Boundary Value Problems Associated with Pairs of Ordinary Differential Expressions (Lecture Notes in Mathematics, 858), V+225 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This volume is devoted to the study of eigenvalue problems associated with pairs L, M of ordinary differential operators. The solutions f of $Lf=\lambda Mf$ subject to boundary conditions are considered. It is shown how these problems have a natural setting within the framework of subspaces in the direct sum of Hilbert spaces. A detailed discussion is worked out for the regular case, where the coefficients of the ordinary differential expressions L and M are sufficiently smooth and invertible functions on a closed bounded interval I, and M is positive in the sense that there exists a constant $c>0$ such that $(Mf,f)_2 \geqq c^2(f,f)_2$ for $f\in C_0^\infty(I)$. The key idea of the simultaneous diagonalization of two hermitian $n\times n$ matrices K, H, where $H>0$, is extended for the case where K, H are replaced by a pair of ordinary differential expressions L, M. The possible difficulties of the generalization are discussed in eleven chapters of this work. The authors say: "it is hoped that this detailed knowledge of the regular case will lead to a greater understanding of the more involved singular case".

The reader is assumed to have some familiarity with the main results proved in an earlier paper of the authors. We recommend these notes to everybody working in related fields of mathematics as well as to graduate students interested in the subject.

T. Krisztin (Szeged)

Combinatorial Mathematics X. Proceedings of the Conference held in Adelaide, Australia, August 23—27, 1982, edited by L. R. A. Casse (Lecture Notes in Mathematics, Vol. 1036), XI+419 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

These conference proceedings consist of seven invited papers and twenty-four contributed papers. According to the tradition of Australian conferences in combinatorial mathematics, a great part of the papers is concerned with finite geometries, Hadamard matrices, block designs and latin squares. Some papers investigate topics in combinatorial analysis, e.g. the Schröder—Etherington sequence, the solutions of $y^{(k)}(x)=y(x)$, and the method of generating combinatorial identities by stochastic processes.

The titles of invited papers are: C. C. Chen and N. Quimpo, Hamiltonian Cayley graphs of order pq; J. W. P. Hirschfeld, The Weil conjectures in finite geometry; D. A. Holton, Cycles in graphs;

A. D. Keedwell, Sequenceable groups, generalized complete mappings, neofields, and block designs; N. J. Pullmann, Unique coverings of graphs — A survey; D. Stinson, Room squares and subsquares; J. A. Thas, Geometries in finite projective spaces: recent results.

L. A. Székely (Szeged)

Complex Analysis and Spectral Theory (Seminar, Leningrad 1979/80), Edited by V. P. Havin and N. K. Nikol'skii (Lecture Notes in Mathematics, 864), IV+480 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This book may be considered as the third issue of selected works of the Seminar on Spectral Theory and Complex Analysis, organized by the Leningrad Branch of the Steklov Institute and the Leningrad University. It contains 9 papers written by the participants during the period 1979/80. The whole volume and most papers separately convincingly demonstrate how close the connection is between Spectral Theory and Complex Analysis both in their problems and methods.

The table of contents: 1. A. B. Aleksandrov, Essays on non Locally Convex Hardy Classes. — This paper contains a new approach to the problem of characterizing functions representable by Cauchy potential and at the same time, among others, gives a description of invariant subspaces of the shift operator. 2. E. M. Dyn'kin, The Rate of Polynomial Approximation in the Complex Domain. — This paper represents the classical Function Theory and provides a systematic exposition of the subject. 3. V. P. Havin, B. Jöricke, On a Class of Uniqueness Theorems for Convolutions. — This paper deals with a phenomenon of quasi-analicity exhibited by many operators commuting with translations. 4. S. V. Hruščev, S. A. Vinogradov, Free Interpolation in the Space of Uniformly Convergent Taylor Series. — The authors plan to publish in the future a survey of harmonic analysis in the space of the title and in the disc-algebra. The present paper collects some new results and some new approaches to the subject which appeared during their work. 5. S. V. Hruščev, N. K. Nikol'skii, B. S. Pavlov, Unconditional Basis of Exponentials and of Reproducing Kernels. — This nice paper contains a description of all subsets $\{\lambda_n\}_n$ of a half-plane $\{\lambda \in \mathbf{C}: \operatorname{Im} \lambda > \gamma\}$ such that the family $\{e^{i\lambda_n x}\}_n$ forms an unconditional basis in $L^2(I)$. (Here I is an interval of the real axis and the notion of unconditional basis is a slight generalization of the one of Riesz basis.) 6. S. V. Kisliakov, What is Needed for a 0-Absolutely Summing Operator to be Nuclear? — The results of this paper are concerned with the open problem: whether each continuous linear operator from the dual of the disc-algebra to a Hilbert space is 1-absolutely summing. 7. N. G. Makarov, V. I. Vasjunin, A Model for Noncontractions and Stability of the Continuous Spectrum. — The authors extend the Sz.-Nagy—Foiaş functional model from contractions to arbitrary bounded Hilbert space operators remaining in spaces with definite metrics and using auxiliary contractions. Applying this model they get nice results on the stability of the continuous spectrum in the case of "nearly unitary" operators. 8. N. A. Shirokov, Division and Multiplication by Inner Functions in Spaces of Analytic Functions Smooth up to the Boundary. — The results of this paper complete the list of basic classes X of "smooth analytic functions" with the property that for every function $f \in X$ and for every inner function I the relation $fI^{-1} \in X$ holds whenever fI^{-1} belongs to the Smirnov class. 9. A. L. Volberg, Thin and Thick Families of Rational Fractions. — A family of rational fractions $R_\Lambda = \{1/(z-\lambda): \lambda \in \Lambda\}$, where $\Lambda \subset \{z \in \mathbf{C}: \operatorname{Im} z > 0\}$, is called thick with respect to a Borel measure μ on the real line if R_Λ is dense in $L^2(\mu)$; R_Λ is called thin with respect to μ if e.g. the L^2-norms corresponding to μ and the Lebesgue measure are equivalent in the linear span of R_Λ. In this paper thick and thin families are described for measures with some properties.

L. Kérchy (Szeged)

Differential Equations Models (Edited by M. Braun, C. S. Coleman, D. A. Drew), XIX+380 pages;

Life Science Models (Edited by H. Marcus-Roberts, M. Thompson), XX+366 pages;
(Modules in Applied Mathematics, vol. 1, vol 4), Springer-Verlag, New York—Heidelberg—Berlin, 1983.

It is an old question even in the mathematical society "Why do people do mathematics?" There exist a great number of answers to this question from "We do mathematics because we enjoy doing mathematics" to "We do mathematics because it can be applied to the practice and other sciences". The first and last volume of the series "Modules in Applied Mathematics" convince us that good mathematics can be both enjoyable and applicable to the problems of the real world. These books show models which describe phenomena of nature or of the society and, simultaneously, they serve as a source of very interesting and very deep investigations in pure mathematics. For example, in population dynamics the co-existence of two interacting species is described by an autonomous system of two ordinary differential equations with polynomial right-hand sides. If the population shows periodical behaviour, then the system has a cycle as a trajectory. The following problem was posed by David Hilbert in 1900 and is still unsolved: what is the maximum number and position of the isolated cycles for a differential equations of this type?

Each chapter is concerned with a model. The construction of the chapters illustrates the steps of the method of the applied mathematics: the statement of the word problem; setting up to mathematical model; investigation of the model with the help of mathematical methods; the interpretation of the results.

The series has been written primarily for college teachers to be used in undergraduate programs. The independent chapters serve as the subject-matters of one-four lectures. Each chapter includes many exercises challenging the reader to further thinking, which are suitable to be posed for good students as well. Prerequisites for each chapter and suggestions for the teacher are provided.

The 23 chapters of the first volume are divided into six parts: I. Differential equations, models, and what to do with them; II. Growth and decay models: first order differential equations; III. Higher order linear models; IV. Traffic models; V. Interacting species: steady states of nonlinear systems; VI. Models leading to partial differential equations. Some of the most exciting problems: The Van Meegeren art forgeries; How long should a traffic light remain amber; Why the percentage of sharks caught in the Mediterranean Sea rose dramatically during World War I; The principle of competitive exclusion in population biology.

The fourth volume consists of three parts: I. Population models; II. Biomedicine: epidemics, genetics, and bioengineering; III. Ecology. The main mathematical devices used here are differential equations, probability theory, linear programming.

These excellent books will be very interesting and useful for both mathematicians interested in realistic applications of mathematics and those non-mathematicians wanting to know how modern mathematics is actually employed to solve relevant contemporary problems.

L. Hatvani (Szeged)

K. Donner, Extension of Positive Operators and Korovkin Theorems (Lecture Notes in Mathematics, 904) X+173 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This book deals with positive and norm-preserving extensions of linear operators in Banach lattices. Imbedding of Banach lattices into cones with infinitely big elements (i.e. $R\cup(+\infty)$ is used instead of R) and using the tensor product method, a useful new technique is obtained for solving the problems mentioned above. The results lead to a simple description of Korovkin systems in L^p.

The text is divided into eight sections. The reader is supposed to be familiar with some basic knowledge in Banach lattice theory.

László Gehér (Szeged)

Dynamical Systems and Turbulence, Warwick 1980, Edited by D. A. Rand and L. S. Young (Lecture Notes in Mathematics, 898), VI+390 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

The aim of the Organizing Committee was to bring together a wide variety of scientists from different backgrounds with a common interest in the problem of the dynamics of turbulence and related topics. The titles of some papers enumerated below show that this aim was fulfilled and so this volume is important and interesting for everyone who is interested in the general theory of dynamical systems.

There are two expository papers: D. Joseph: Lectures on bifurcation from periodic orbits; D. Schaeffer: General introduction to steady state bifurcation. Some of the contributed papers are: J. Guckenheimer: On a codimension two bifurcation; J. Hale: Stability and bifurcation in a parabolic equation; P. Holmes: Space- and time-periodic perturbations of the Sine-Gordon equation; I. P. Malta and J. Palis: Families of vector fields with finite modulus of stability; L. Markus: Controllability of multi-trajectories on Lie groups; W. de Melo, J. Palis and S. J. van Strien: Characterizing diffeomorphisms with modulus of stability one; S. J. van Strien: On the bifurcations creating horseshoes; F. Takens: Detecting strange attractors in turbulence.

L. Pintér (Szeged)

Emanuel Fischer, Intermediate Real Analysis, (Undergraduate Texts in Mathematics), XIV+770 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1983.

Today one finds a great number of books on introductory analysis, but sometimes the teacher cannot choose such a work from them which satisfies his students' background. The author — on the basis of his experience of many years — wrote a book for students who have completed a three-semester calculus course, possibly an introductory course in differential equations and one or two semesters of modern algebra. This determines the structure of the book and the spirit of the definitions and the proofs. Therefore, the author presents the material in "theorem — proof — theorem" fashion, interspersing definitions, examples and remarks.

The book is self-contained except for some theorems on finite sets.

At the end of Chapter XIV — having the title The Riemann Integral — we find Lebesgue's famous theorem: A function which is bounded on a bounded closed interval $[a, b]$ is Riemann-integrable if and only if the set of points in $[a, b]$ at which it is discontinuous has measure zero. We cited this theorem because in some sense it is characteristic for this book. The notions to understand this theorem are treated in the text, but the proof — which belongs to a next stage — is omitted. Nevertheless, the book concentrates on the specific and concrete by applying the theorems to obtain information about important functions of analysis.

Above all, this is a stylish book, well thought out and uses tested methods, which one could safely put into the hands of future users of mathematics. (There is an unexpected mistake in the Bibliography. Correctly the names of the authors of the world-famous book " Aufgaben und Lehrsätze aus der Analysis" are G. Pólya and G. Szegő.)

L. Pintér (Szeged)

G. B. Folland, Lectures on Partial Differential Equations (Tata Institute of Fundamental Research Lectures on Mathematics and Physics), VI+160 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1983.

This book consists of the notes for a course the author gave at the Tata Institute of Fundamental Research Centre in Bangalore in autumn 1981. The purpose of the course was the application of Fourier analysis (i.e. convolution operators as well as the Fourier transform itself) to partial differential equations. The book is divided into five chapters. In the first some basic results about convolutions and the Fourier transforms are given. In Chapter 2 the fundamental facts of partial differential operators with constant coefficients are studied. In the next one precise theory of L^2 differentiability is introduced to prove Hörmander's theorem on the hypoellipticity of constant coefficient differential operators. Chapter 4 comprises the basic theory of pseudo differential operators. The aim of the last chapter is to study how to measure the smoothing properties of pseudo differential operators of nonpositive order in terms of various important function spaces.

The reader is assumed to have familiarity with real analysis and to be acquainted with the basic facts about distributions. No specific knowledge of partial differential equations is assumed.

This book is directed to graduate students and mathematicians who are interested in the application of Fourier analysis.

T. Krisztin (Szeged)

F. Gécseg—M. Steinby, Tree Automata, 235 pages, Akadémiai Kiadó, Budapest, 1984.

The theory of tree automata is a relatively new field of theoretical computer science. More exactly, it is a new field of automata and formal language theory, though it has several aspects in common with flowchart theory, recursive program schemes, pattern recognition, theory of translations, mathematical logic, etc. The book of F. Gécseg and M. Steinby gives a systematic, mathematically rigorous summary of results on tree automata.

Every finite automaton, — more precisely, a finite-state recognizer — can be viewed as a finite universal algebra having unary operations only. This observation, though obvious, provides a way of generalization. Basically, a tree automaton is a finite universal algebra equipped with arbitrary finitary operations. However, problems investigated in the theory of tree automata essentially differ from that investigated in universal algebra. The introduction of tree automata as a new device was not only for the sake of generalizing automata theory. As explained in this book, the connection with context free grammars and languages, syntax directed translations, and other topics has been significant and vitally important.

The book consists of four chapters, a bibliography, and an index. Chapter I comprises an exposition on necessary universal algebra, lattice theory, finite automata and formal languages. Section 1 presents the terminology. Sections 2 and 3 recall some basic concepts of universal algebra, including terms, polynomials and free algebras. Section 4 deals with lattices, complete lattices, and a variant of Tarski's fixed-point theorem. Section 5 surveys finite-state recognizers and their relation to regular languages. Besides the various characterizations of regular languages, minimization and decidability results are also included. Section 6 is about Chomsky's hierarchy and, especially, context-free languages. Closure under operations, the pumping lemma, normal forms and decidability questions are treated. Section 7 reviews sequential machines. Almost all theorems on universal algebra and lattices appear with complete proofs. Automata and language theoretic proofs are mostly just outlined or omitted. Readers familiar with the topics of Chapter I may skim over it. Other readers will find enough material to understand the rest of the book, or, if needed, may consult the references given at the end of the chapter.

Chapter II is devoted to finite-state tree recognizers, i.e., tree automata without output. Section 1 explains the usage of the word tree for terms. Two kinds of tree recognizers are introduced in Section 2. Frontier-to-root recognizers read trees from the leaves toward the root, and root-to-frontier recognizers work in the opposite way. Both types have deterministic and nondeterministic versions. It is shown that all these recognizers accept the same class of tree languages — the so-called recognizable forests —, except for deterministic root-to-frontier recognizers. In Scetion 3 closure properties of recognizable forests are dealt with. Sections 4 and 5 give two different characterizations of recognizable forests through regular tree grammars and regular expressions. The latter is Kleene's theorem for recognizable forests. The minimization theory of deterministic frontier-to-root recognizers is developed in Section 6. Sections 7—9 provide four additional characterizations of recognizable forests: by means of congruences of the absolutely free term algebra, as fixed-points of forest equations, in terms of local forests, and by means of certain Medvedev-type operations. In Section 10 basic properties of recognizable forests are shown to be decidable. Section 11 treats deterministic root-to-frontier recognizers, their minimization, and characterizes forests accepted by these recognizers.

Chapter III provides a study of the connection of recognizable forests to context free grammars and languages. Section 1 exploites the yield function as a way of extracting a word from a tree and a language from a forest. In Section 2 the forest made up from the derivation trees of a context free grammar is shown to be recognizable. Hence, by the yield forming process, tree recognizers become acceptors for context free languages. Section 3 demonstrates some further properties of the yield function. The chapter ends with Section 4, where tree recognizers are used as acceptors for context free languages in an alternative way.

The last chapter, Chapter IV, treats tree automata with output, the so-called tree transducers. Two basic sorts of tree transducers are introduced in Section 1: frontier-to-root and root-to-frontier tree transducers. Many special cases and deterministic versions are investigated in the first two sections. These special cases give rise to the composition and decomposition theorems of tree transformations induced by tree transducers. This is the subject of Section 3. In Section 4, root-to-frontier tree transducers are generalized to transducers with regular look-ahead. Later this concept turns out to be a very useful tool in many ways. Section 6 provides a study of properties of surface forests, i.e. the images of regular forests under tree-transformations. Section 7 contains some auxiliary results in preparation for Section 8, where it is shown that an infinite hierarchy can be obtained by serial compositions of tree transformations. In the last section the equivalence problem of deterministic tree transducers is proven to be decidable.

Chapters II—IV also contain exercises and each of them ends with a historical and bibliographical overview reviewing some additional fields too. Applications of the theory are ignored, but interested readers may find enough orientation in the bibliographical notes.

The bibliography contains more than 250 entries. The index helps guide the reader in looking up notions and notations.

This well-written new book can be recommended as an important, systematic summary of the subject, as a reference book, and even for those who are familiar with some aspects of automata and formal language theory and want to increase their knowledge in this direction.

Zoltán Ésik (Szeged)

Geometric Dynamics. Proceedings, Rio de Janeiro, 1981. Edited by J. Palis Jr. (Lecture Notes in Mathematics, 1007), IX+827 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo 1983.

These are the Proceedings of an International Symposium on Dynamical Systems that took place at the Istituto de Matematica Pura e Aplicada, Rio de Janeiro, in July—August, 1981.

One hundred years before this conference H. Poincaré published his fundamental memoir "Sur les courbes définies par les équations différentielles", which was the origin of geometric or qualitative dynamics. Since that moment a great number of mathematicians have studied the properties of the trajectories of dynamical systems. New notions have come up, very interesting and deep problems have arisen.

The conference was participated by the most outstanding scholars in the West of this theory. They delivered 43 lectures on up-to-date topics. Some of them were: structural stability, entropy, local classification of vector fields, bifurcations, infinite dimensional dynamical systems (especially, functional differential equations), existence and nonexistence of periodic orbits, Lyapunov functions, Lyapunov exponents, strange attractors, random perturbations.

The Proceedings will be very useful for every scholar interested in the qualitative theory of differential equations.

L. Hatvani (Szeged)

Geometric Techniques in Gauge Theories. Proceedings of the Fifth Scheveningen Conference on Differential Equations, The Netherlands, August 23—28, 1981. Edited by R. Martini and E. M. de Jager (Lecture Notes in Mathematics, 926), IX+219 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

The volume contains 10 lectures delivered at the conference on gauge theory, one of the important subjects of contemporary mathematics and physics. The first two papers give an introduction to the geometry of gauge field theory (R. Hermann: Fiber spaces, connections and Yang—Mills fields; Th. Friedrich: A geometric introduction to Yang—Mills equations). Four lectures were devoted to physical phenomena occurring in gauge field theory, the majority of which is based on global properties of the fibre bundle underlying the field equations. These lectures (F. A. Bais: Symmetry as a clue to the physics of elementary particles; Topological excitations in gauge theories; an introduction from the physical point of view; P. J. M. Bongaarts: Particles, fields and quantum theory; E. F. Corrigan: Monopole solitons) of informative character provide a common language for mathematicians and theoretical physicists. A Trautman's report — Yang—Mills theory and gravitation: A comparison — summarizes the analogies and differences between gauge theories of internal symmetries and Einstein's theory of general relativity. Two articles deal with the twistor method which is promising for solving nonlinear partial differential equations of mathematical physics (M. G. Eastwood: The twistor description of linear fields; R. S. Ward: Twistor techniques in gauge theories). Prolongation theory is the concern of the final paper (P. Molino: Simple pseudopotentials for the *KdV*-equation).

This well arranged book with single lectures very clearly written provides a comprehensive survey of classical gauge theory and can be warmly recommended for all students and research workers interested in the subject.

L. Gy. Fehér (Szeged)

Geometries and Groups, Proceedings, Berlin 1981. Edited by M. Aigner and D. Jungnickel (Lecture Notes in Mathematics, 893), X+250 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This volume contains five invited and 11 contributed papers presented at the colloquium in honour of Professor Hanfried Lenz held at the Freie Universität Berlin in May 1981. The invited survey lectures given by F. Buekenhout, J. Doyen, D. R. Hughes, U. Ott and K. Strambach are devoted to combinatorial and group theoretical aspects of geometry. The contributed papers deal with various problems of combinatorics and finite geometry.

Péter T. Nagy (Szeged)

Allan Gut—Klaus D. Schmidt, Amarts and Set Function Processes (Lecture Notes in Mathematics, 1042), 258 pages. Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

These lecture notes are based on a series of talks on real-valued asymptotic martingales (amarts) held at Uppsala University, Sweden. The main purpose of them is to introduce the reader to the theory of asymptotic martingales, on whose part the notes require the knowledge of classical martingale theory.

The book is divided into three parts. In the first part Allan Gut gives an introduction to amarts. This introduction contains, for example, the history and basic properties of amarts, convergence and stability theorems, and the Riesz decomposition. The much longer second part was written by Klaus D. Schmidt and it deals with amarts from a measures theoretical point of view. We list only the chapter headings here: Introduction, Real amarts, Amarts in a Banach space, Amarts in a Banach lattice, Further aspects of amart theory. The book ends with a rich bibliography. The bibliopgrahy contains papers which deal with or were inspired by amarts as well as some papers concerning further generalizations of martingales.

The book gives a good introduction to this field and the rich, up-to-date bibliography helps to find a way in the literature of amarts.

Lajos Horváth (Szeged)

A. Haraux, Nonlinear Evolution Equations—Global Behavior of Solutions (Lecture Notes in Mathematics, 841), IX+313 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

It is common in the modern theory of partial differential equations that the original equation is rewritten into an ordinary differential equation in an infinite-dimensional Banach space of functions as a state space. This allows the application of certain methods of topological dynamics and the theory of finite dimensional ordinary differential equations to partial differential equations. If the original equation is non-linear (e.g. the Schrödinger equation arising from non-linear optics) then the associated infinite-dimensional equation is non-linear as well. These lecture notes contain the basic material of the two semester seminar course on equations of this type given by the author at Brown University during the academic year 1979—80.

The study is centered on semi-linear, quasi-autonomous systems.

Chapter A, which is of preparatory character, deals with the uniqueness of the solutions of the Cauchy problem. Then the basic notions and facts of the theory of monotone operators are given, which is the main tool of investigation in the book.

Chapter B is concerned with the existence of periodic solutions to quasi-autonomous systems with especial regard to linear and dissipative cases.

Chapters C and D are the most original parts of the book. Concerning autonomous dissipative and quasi-autonomous dissipative periodic systems, the author gives theorems on the asymptotic behaviour of the solutions as $t\to\infty$.

The knowledge of elementary Banach space theory and the introductory chapters on Cauchy problem in nonlinear partial differential equations are prerequisites to read the book.

These lecture notes, containing several results not published previously in the literature, will be very useful and interesting for mathematicians dealing with the theory and applications of nonlinear partial differential equations.

L. Hatvani (Szeged)

Harmonic Maps, Proceedings, New Orleans 1980, edited by R. J. Knill, M. Kalka and H. C. J. Sealey (Lecture Notes in Mathematics 949), 158 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This volume contains papers contributed by participants of the N.S.F.—C.B.M.S. Regional Conference on Harmonic Maps at Tulane University in December 1980. The ten lectures given by James Eells and co-authored by Luc Lemaire at the conference are published separately in CBMS regional conference reports. The book gives a good survey on various topics connected with the theory of harmonic maps: singularities, deformation and stability theory, Cauchy—Riemann equations, Yang—Mills fields, foliations, and harmonic maps between classical spaces and surfaces.

Péter T. Nagy (Szeged)

Loo-Keng Hua, Selected Papers, Edited by H. Halberstam, XIV+889 pages, Springer-Verlag New York—Heidelberg—Berlin 1983.

Having edited recently some of Hua's books (*Introduction to Number Theory, Starting with the Unit Circle, Applications of Number Theory to Numerical Analysis,* the latter one written jointly with Wang Yuan) in English, it was just very timely to publish his Selected Papers. The Selected Papers consist of three main parts reflecting Hua's oeuvre in pure mathematics and a part classified miscellaneous, his biography and list of publications, and a sketch of his contributions to applied mathematics.

The first main part is, of course, number theory. It consists of 20 papers including his results on the estimation of exponential sums, on the generalized Waring's problem, on Goldbach's problem, on the Waring—Goldbach problem, on the Gauss circle problem, and on the number of partitions of a number into odd parts.

The second main part contains 18 papers on algebra and geometry, including Hua's results on the existence of pseudo-basis in p-groups, on semi-automorphisms of skew fields, on automorphisms of classical groups, and on the geometry of matrices.

The third main part is devoted to function theory in several variables (5 papers) in connection with partial differential equations and differential geometry.

We have to emphasize Hua's "offensive style" in solving mathematical problems what looms in his computations. Some of the present selected papers are the first English translations. This volume proves that those who know Loo-Keng Hua to be "only" number theorist are wrong.

L. A. Székely (Szeged)

Serge Lang, Undergraduate Analysis (Undergraduate Texts in Mathematics), 545 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1983.

This book is a revised and enlarged version of the author's "Analysis I", Addison—Wesley Publishing Company 1968. It is a logically self-contained first course in real analysis, "which presupposes the mathematical maturity acquired by students who ordinarily have had two years of calculus" (from the Foreword). The contents is as follows: Part 1: Review of Calculus (Sets and Mappings;

Real Numbers; Limits and Continuous Functions; Differentiation; Elementary Functions; The Elementary Real Integral); Part 2: Convergence (Normed Vector Spaces; Limits; Compactness; Series; The Integral in One Variable); Part 3: Applications of the Integral (Approximation with Convolutions; Fourier Series; Improper Integrals; The Fourier Integral); Part 4: Calculus in Vector Spaces (Functions on n-Space; Derivatives in Vector Spaces; Inverse Mapping Theroem; Ordinary Differential Equations); Part 5: Multiple Integration (Multiple Integrals; Differential Forms).

This survey shows how many topics are treated, more than in usual standard texts at this level. The emphasis is on the theoretical aspects, but the basic computational techniques are also demonstrated in detail. The central and deep concepts of analysis (convergence, limit, derivative, integral) are presented in a series of different forms, in ascending order of difficulty, and generality. There are many interesting technical and theoretical examples and problems, some easy, many hard; solutions to the problems are not included.

To conclude, this book is very well written and produced. Because of its flexible structure it is suitable for several advanced calculus and real analysis courses. It is not a book for the beginner, but it can be warmly recommended to all who want to learn the foundations of modern analysis.

Arnold Janz (Berlin)

Loren C. Larson, Problem-Solving Through Problems. Problem Books in Mathematics, XI + 344 pages with 104 illustrations, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1983.

This book is a volume of Springer-Verlag's new series Problem Books in Mathematics edited by P. Halmos. The reader can expect to find in this series collections of problems that have been discovered and gathered carefully together over years; interesting subjects not yet adequately treated elsewhere etc. As prototypes "Otto Dunkel Memorial Problem Book" and Pólya and Szegő's "Problems and Theorems in Analysis" are mentioned.

In another book of Pólya, in the world-famous "How to solve it" we find a chart of questions and answering some of them we have a good chance to obtain a solution of the problem. In answering the questions one of the crucial points is the knowledge of the various problem solving techniques. In this direction Larson's book will prove invaluable as a teaching aid. Chapter headings are: Heuristics; Two important principles: Induction and pigeonhole; Arithmetic; Algebra; Summation of series; Intermediate real analysis; Inequalities; Geometry.

One of the most interesting chapters is the first one, entitled Heuristics. The author focuses on the typically useful basic ideas such as: Search for a pattern; Draw a figure; Formulate an equivalent problem; Modify the problem; Choose effective notations; Exploit symmetry; Divide into cases; Consider extreme cases; Generalize. For example, in "Divide into cases" the problems can be divided into subproblems each of which can be handled separately in a case-by-case manner. The following three problems are solved: a) Prove that an angle inscribed in a circle is equal to one-half the central angle which subtends the same arc; b) A real valued function f, defined on the rational numbers, satisfies $f(x+y)=f(x)+f(y)$ for all rational x and y. Prove that $f(x)=f(1)x$ for all rational x; c) Prove that the area of a lattice triangle is equal to $I+(1/2)B-1$, where I and B denote respectively the number of interior and boundary lattice points of the triangle. Then some problems — from different branches of mathematics — for solution are listed and references to problems proposed in other chapters where this treated method may be useful. This is the structure of the other chapters too. At the end of the book one finds the sources of the more remarkable problems.

The style of the book is attractive, methods, problems and solutions are presented in a way which brings the printed page to life. No doubt, students and teachers will enjoy and use this book.

L. Pintér (Szeged)

George E. Martin, Transformation Geometry (Undergraduate Text in Mathematics) XII+237 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

The main purpose of this book is to describe the Euclidean plane geometry by the study of its transformation groups. The text starts with a short introduction (Chapter 1). In Chapter 2 the concept of transformation groups is defined. Chapters 3—4 deal with translations, halfturns and reflections, using the method of analytic geometry. In Chapter 5 it is shown that any congruence can be represented as a product of at most three reflections. Chapter 6 investigates congruence transformations which can be represented as products of two reflections; it turns out that these are the translations and the rotations. Chapters 7—8 introduce the concept of the congruences of even and of odd types and a complete classification of congruences is given. Chapter 9 gives the equations of congruence transformations. Chapters 10—12 describe the discrete congruence groups; the seven discrete groups having translations in only one direction (called "Frieze Groups") and the seventeen discrete groups having two independent translations (called "Wallpaper Groups"). The periodic tesselations can be obtained as an application. Chapter 13 is devoted to similarity transformations. Chapter 14 contains the classical theorems of elementary geometry. In Chapter 15 the affine transformations are defined, and their linear operator representations are given. Chapter 16 gives a short indication as to how the classification of congruences in three-space can be obtained. In Chapter 17 the Euler polyhedron theorem is proved, the regular polyhedrons are constructed and their symmetry groups are given.

The text requires only elementary geometric knowledge. The reader will surely enjoy the book.

László Gehér (Szeged)

Mathematical Models as a Tool for the Social Sciences, edited by B. J. West, V+120 pages, Gordon and Breach, New York—London—Paris, 1980.

This book is a collection of the talks of a seminar at the University of Rochester. The eight lectures present themselves as interesting examples of mathematical model building in economic and natural history (R. W. Fogel: Historiography and retrospective econometrics; A. Budgor and B. J. West: Natural forces and extreme events — the latter is on floods and droughts in the Nile River Valley), the psychology of learning, selection making and speculation (A. O. Dick: A mathematical model of serial memory; J. Keilson and B. J. West: A simple algorithm of contract acceptance; B. J. West: The psychology of speculation: a simple model), politics (W. Riker: A mathematical theory of political coalitions), inpopulation growth (J. H. B. Kemperman: Systems of mating — in which the problem is how stable population patterns are formed in large populations under given mating systems), and for economic income distribution (W. W. Badger: An entropyutility model for the size distribution of income).

"There is no one way, and indeed no best way, to construct a mathematical model of a natural or social system" as the editor writes in his introduction, but he believes "that any problem which may be well formulated verbally, may be well formulated mathematically". All of the above models are interesting and novel enough. If you don't believe in them, construct your own and confront it with the already existing ones. The book is a very good reading.

Sándor Csörgő and Lajos Horváth (Szeged)

Mathematical Programming. The State of the Art, Bonn 1982, edited by A. Bachem, M. Grötschel and B. Korte, VIII+655 pages with 30 figures. Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

This book consists of 21 state-of-the-art tutorials of the 23 having constituted the main frame of the XI. International Symposium on Mathematical Programming held at the University of Bonn in 1982. These survey papers written by leading experts can introduce everyone to the recent and most important results in several areas of mathematical programming. The book contains a brief review about the Fulkerson Prize and Dantzig Prize won in the year 1982. Since it seems to be unjust to mention some papers and to neglect other ones, no matter how long their list is, we give the author and the title of all the papers. We hope that the reader will forgive us upon seeing the list: E. L. Allgower and K. Georg, Predictor-corrector and simplicial methods for approximating fixed points and zero points of nonlinear mappings; L. J. Billera, Polyhedral theory and commutative algebra; G. B. Dantzig, Reminescences about the origins of linear programming; R. Fletcher, Penalty functions; R. L. Graham, Applications of the FKG inequality and its relatives; S.-Å. Gustafson and K. D. Kortanek, Semi-infinite programming and applications; M. Iri, Applications of matroid theory; E. L. Lawler, Recent results in the theory of machine scheduling; L. Lovász, Submodular functions and convexity; J. J. Morè, Recent developments in algorithms and software for trust region methods; M. J. D. Powell, Variable metric methods for constrained optimization; W. R. Pulleyblank, Polyhedral combinatorics; Stephen M. Robinson, Generalized equations; R. T. Rockafellar, Generalized subgradients in mathematical programming; J. Rosenmüller; Nondegeneracy problems in cooperative game theory, R. B. Schnabel, Conic methods for unconstrained minimization and tensor methods for nonlinear equations; A. Schrijver, Min-max results in combinatorial optimization; N. Z. Shor, Generalized gradient methods of nondifferentiable optimization employing space dilatation operations; S. Smale, The problem of the average speed of the simplex method; J. Stoer, Solution of large linear systems of equations by conjugate gradient type methods; R. J.-B. Wets, Stochastic programming: solution techniques and approximation schemes.

L. A. Székely (Szeged)

Measure Theory, Oberwolfach 1981, Proceedings of the Conference Held at Oberwolfach, Germany, June 21—27, 1981, edited by D. Kölzow and D. Maharam-Stone (Lecture Notes in Mathematics, 945), XV+431 pages. Springer-Verlag, Berlin—Heidelberg—New York, 1982.

These conference proceedings consist of 36 papers on several fields of measure theory such as general measure theory, descriptive set theory and measurable selections, lifting and disintegration, differentiation of measures and integrals, measure theory and functional analysis, non-scalar-valued measures, measures on linear spaces, stochastic processes and ergodic theory.

Although I must not list here all the titles of papers, I have to mention some of them. R. J. Gardner in his paper 'The Regularity of Borel Measures' gives a detailed survey on regularity assumptions of Borel measures with 15 pages of references. H.-U. Hess 'A Kuratowski Approach to Wiener Measure' exhibits a procedure that may be considered an alternative way of constructing Wiener measure. J. R. Choksi and V. S. Prasadin 'Ergodic Theory on Homogeneous Measure Algebras' continues previous efforts to generalize ergodic theory.

The book contains open research problems discussed in the problem session of the conference.

L. A. Székely (Szeged)

G. H. Moore, Zermelo's Axiom of Choice: Its Origins, Development and Influence (Studies in the History of Mathematics and Physical Sciences 8), XIV+410 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This book of four chapters is the first full-lenght history of the Axiom of Choice. David Hilbert wrote in 1926 that Zermelo's Axiom of Choice was the axiom "most attacked up to the present in the mathematical literature...". Later Abraham Fraenkel added to this that "the axiom of choice is probably the most interesting and, in spite of its late appearance, the most discussed axiom of mathematics, second only to Euklid's axiom of parallels which was introduced more than two thousand years ago".

In Chapter 1, The Prehistory of the Axiom of Choice, the author indicates four major stages through which the use of arbitrary choices passed on the way to Zermelo's explicit formulation of the Axiom of Choice. The first stage — choosing an unspecified element from a single set or arbitrary choice of an element from each of finitely many sets — can be found in Euklid's Elements (if not earlier). The second stage was when Gauss and others made infinite number of choices by stating a rule. In the third stage mathematicians made infinite number of choices but left the rule unstated. This was the case, e.g., when Cauchy demonstrated a version of the Intermediate Value Theorem in 1821. The fourth stage, where mathematicians made infinitely many arbitrary choices for which, consequently, the Axiom of Choice was essential, began in 1871 by a paper of Heine on real analysis. Heine's proof, borrowed from Cantor, implicitly used the Axiom to show that his definition of continuity implies the earlier one introduced by Cauchy and Weierstrass.

The boundary between finite and infinite, the various definitions of finiteness (by Bolzano, Dedekind and Pierce) and the connections among them are also discussed in this chapter, as well as several implicit uses of the Axiom by Cantor.

At the end of this chapter two equivalent statements to the Axiom, the Well-Ordering Principle and the Trichotomy of Cardinals are mentioned which were stated by Cantor before Zermelo formulated the Axiom of Choice.

Chapter 2, Zermelo and His Critics (1904—1908) is an exploration of the debate started when in 1904 Zermelo published his proof that every set can be well-ordered. The major questions were: "What methods were permissible in mathematics? Must such methods be constructive? If so, what constituted a construction? What did it mean to say that a mathematical object existed?" From 1905 to 1908 eminent mathematicians in England, France, Germany, Holland, Hungary, Italy, and the United States debated the validity of his demostration. Never in modern times have mathematicians argued so publicly and so vehemently over a proof.

In Chapter 3 we can read Zermelo's reply to his critics and his axiomatization of set theory and the counteropinions of Poincaré and Russel among others. Some equivalent statements to the Axiom of Choice are also discussed,

Chapter 4, The Warsaw School, Widening Applications, Models of Set Theory (1918—1940) deals with the wide-spread applications and the modern independence results.

There are an Epilogue: After Gödel, and two appendices. The first one consists of five letters on set theory (written by Baire, Borel and Hadamard), and the second is "Deductive Relations Concerning the Axiom of Choice".

While the author brings out aspects of a history that will fascinate mathematical researchers and philosophers, this book is warmly recommended to everybody interested in set theory, in the philosophy of mathematics and in historical questions.

Lajos Klukovits (Szeged)

M. A. Naimark—A. I. Stern, Theory of Group Representations (Grundlehren der Mathematischen Wissenschaften, 246), IX+568 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This book is the second translation of the original edition of the book of M. A. Naimark, written in Russian, and in which M. A. Naimark describes his collaboration with A. I. Stern. The first translation was into French including a faithful transcription of misprints. The French translation already lists A. I. Stern as co-author. The book consists of 12 chapters. The text starts with a short algebraic foundation of representation theory. The next chapter summarizes the most important general results of the theory of representations of finite groups giving the representations of the symmetric group and of the group SL. Two chapters deal with topological groups, providing the general definition of a representation of a topological group, and especially the representation theory of compact groups in connection with the representations of the corresponding group algebra. In this part there are some mistakes that do not disturb the intelligibility of the text. Further chapters deal with the applications of the general theory of representations of compact groups. Two chapters investigate finite representations of the full linear group and of complex classical groups. The next one is devoted to covering spaces and simply connected groups. The last five chapters contain a detailed investigation of Lie groups and Lie algebras.

The reader is supposed to be familiar with linear algebra, elementary functional analysis and with the theory of analytic functions.

László Gehér (Szeged)

A. W. Naylor—G. R. Sell, Linear Operator Theory in Engineering and Science (Applied Mathematical Sciences, vol. 40), XV+624 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

As a lecturer, sometimes I have had to teach parts of mathematical analysis to scientists or students in chemistry, biology, medicine. So I know that this task can be more difficult than to lecture the same subject to mathematicians. How difficult could it be then to write a book for engineers and scientists on functional analysis, which is one of the most abstract fields of mathematical analysis? Thus having read the exciting title of this book I was very curious about answers to some questions: How to introduce the concepts of linear operator theory to the readers not bringing enough experiences from the classical chapters of mathematical analysis that make the definitions natural and understandable? Which concepts, results and methods and how deeply are they to be included into a mathematically rigorous book if it is known that the readers are interested mostly in the applications of functional analysis to their own sciences?

Fortunately, the authors resolve these conflicts excellently and find the balance between the different points of view. In order to illuminate the abstract concepts they give lots of examples and exercises. As far as it is possible they use the geometry and finite-dimensional analogies for the heuristic preparation of the subject-matter. For example, Chapter 6, concerned with the spectral analysis of linear operators, is divided into three parts. The first one is the geometric analysis of linear combinations of orthogonal projections giving a resolution of the identity in a Hilbert space. In the second part the spectrum of general bounded and unbounded linear operators is introduced and illuminated by examples. The chapter is concluded with the spectral theorem for compact normal operators in a Hilbert space and its applications (matched filter, the Karhunen—Loève expansion for discrete random processes, ε-capacity of a linear channel). It has been a very good decision to deal with the spectral theory of compact operators separately because it is relatively simple but demonstrates the distinction between the finite- and infinite-dimensional cases, which is the big jump in spectral theory.

We recommend this excellent text-book to every engineer, scientist and applied mathematician making the first steps in functional analysis.

L. Hatvani (Szeged)

Donald J. Newman, A Problem Seminar. Problem Books in Mathematics, VIII+113 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

This book contains some problems of D. J. Newman's problem seminar. The author says in the Preface: "There was once a bumper sticker that read "Remember the good old days when air was clean and sex was dirty?" Indeed, some of us are old enough to remember not only those good old days, but even the days when Math was fun (!), not the ponderous THEOREM, PROOF, THEOREM, PROOF, ..., but the whimsical, "I've got a good problem"."

This last sentence shows precisely what the reader can find on every page of this excellent book. The problems are interesting, natural, in general one cannot get away from them without having the solutions. This is not only the reviewer's personal impression but this was his experience after posing some problems of the text to his students.

The book consists of three parts: Problems, Hints and Solutions. Sometimes the solutions are not fully worked out, but the interested reader can fill the gaps. A great part of problems seems to be quite elementary, but in some cases the solution requires not only elementary notions. Therefore, the text forces the reader to do some more mathematics, to get acquainted with new notions. For illustration I tried to select a problem but I have so many favourites that I could not choose among them.

This problem seminar is warmly recommended to teachers, students and everyone who enjoy the fun and games of problem solving and have the opinion that asking and answering problems is what keeps a mathematician young in spirit.

L. Pintér (Szeged)

Ordinary Differential Equations and Operators, Proceedings, Dundee, 1982. Edited by W. N. Everitt and R. T. Lewis (Lecture Notes in Mathematics, 1032), XV+521 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

These are the Proceedings of the Symposium on Ordinary Differential Equations and Operators held in the Department of Mathematics at the University of Dundee, Scotland during the months of March, April, May, June and July 1982. They are dedicated to F. V. Atkinson by his many friends and colleagues in recognition of his mathematical contributions to the theory of differential equations.

The topics of the volume can be arranged in groups according to the many themes having been studied by F. V. Atkinson: boundary value problems, differential operators (Sturm—Liouville problems, spectral theory), second order oscillation theory, limit cycles, etc.

Some of the papers are surveys giving also the history of their topics, but the reader can find also articles including results not published before.

L. Hatvani (Szeged)

Ordinary and Partial Differential Equations. Proceedings, Dundee, Scotland 1980. Edited by W. N. Everitt and B. D. Sleeman (Lecture Notes in Mathematics, 846), XIV+384 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This volume contains lectures delivered at the sixth Conference on Ordinary and Partial Differential Equations held at the University of Dundee. As the name of the conference shows, the topics of lectures are taken from various branches of the theory of differential equations. To illustrate this assertion here are the titles of some lectures: Some unitarily equivalent differential operators with finite and infinite singularities; Nonlinear two-point boundary value problems; On the spectra of Schrödinger operators with a complex potential; Asymptotic distribution of eigenvalues of elliptic operators on unbounded domains; Some spectral gap results; Some topics in nonlinear wave propagation; Oscillation properties of weakly nonlinear differential equations; Norm inequalities for derivatives; Fixed point theorems; A bound for solutions of a fourth order dynamical system;

Convergence of solutions of infinite delay differential equations with an underlying space of continuous functions; Symmetry and bifurcation from multiple eigenvalues; Variational methods and almost solvability of semilinear equations.

The book is warmly recommended to everybody who works in differential equations and perhaps it will stimulate other readers to make research in this field.

L. Pintér (Szeged)

Ordinary and Partial Differential Equations, Proceedings, Dundee, Scotland, 1982. Edited by W. N. Everitt and B. D. Sleeman (Lecture Notes in Mathematics, 964), XVIII+726 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

These Proceedings include the lectures delivered at the seventh Conference on Ordinary and Partial Differential Equations which was held at the University of Dundee, Scotland, March 29—April 2, 1982.

Unfortunately, there is no room in this review to present the complete list of the 60 lectures, which shows a very wide spectrum. Some of the key words and phrases: boundary value problems, eigenvalue problems, eigenfunction expansions, oscillations, bifurcations, differential equations with delay, integrodifferential equations, stochastic functional differential equations, scattering theory, generalized Schrödinger operators, partial differential equations of infinite order, control theory, astronomy, thermodynamics.

Like the Proceedings of the earlier Dundee Conferences, this volume, which is dedicated to the University of Dundee on the occasion of its centenary celebrations, gives a good flavour of the actual problems of the theory of differential equations.

L. Hatvani (Szeged)

Radical Banach Algebras and Automatic Continuity (Proceedings, Long Beach 1981), Edited by J. M. Bachar, W. G. Bade, P. C. Curtis Jr., H. G. Dales and M. P. Thomas (Lecture Notes in Mathematics, 975), VII+470 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1983.

This collection contains 30 papers, the contributions to the conference indicated in the title, held at the California State University between July 13—17, 1981.

The editors write: "The basic problem of automatic continuity theory is to give algebraic conditions which ensure that a linear operator between, say, two Banach spaces is necessarily continuous. This problem is of particular interest in the case of a homomorphism between two Banach algebras. Other automatic continuity questions arise in the study of derivations from Banach algebras to suitable modules and in the study of translation invariant functionals on function spaces. There is a fundamental connection between questions of automatic continuity and the structure of radical algebras. ... The purpose of the conference was to present recent developments in these two areas and to explore the connections between them."

The volume is divided into five sections. Section I deals with the general theory of commutative radical Banach algebras and contains (together with a paper of F. Zouakia) two lengthy papers by J. Esterle. The first one gives a classification of these algebras, while the second one is devoted to the question of whether or not such algebras must contain non-trivial closed ideals. This latter problem is related to the invariant subspace problem for Banach spaces.

Papers in Section II (by H. G. Dales, Y. Domar, W. G. Bade, K. B. Laursen, M. P. Thomas, S. Grabiner, G. R. Allan, G. A. Willis, N. Gronbaek and G. F. Bachelis) are concerned with radical convolution algebras on $\mathbf{R}^+$ and $\mathbf{Z}^+$. The central problem here is to determine for which radical weights, ω, every closed ideal of $L^1(\omega)$ is a standard ideal, that is, an ideal consisting of those functions with support in an interval $[\alpha, \infty)$.

Section III contains papers by B. Aupetit, R. J. Loy, P. C. Curtis Jr., J. C. Tripp, P. G. Dixon, E. Albrecht, M. Neumann, H. G. Dales and G. A. Willis, and is devoted to the automatic continuity of homomorphisms (between semisimple, nonsemisimple, local and C^* algebras) and derivations.

The automatic continuity of (mostly translation invariant) linear functionals on Banach algebras is discussed in Section IV, which includes papers by G. H. Meisters, R. J. Loy and H. G. Dales.

Finally Section V contains a list of open problems, some well known and others posed at the conference.

L. Kérchy (Szeged)

D. M. Sandford, Using Sophisticated Methods in Resolution Theorem Proving (Lecture Notes in Computer Science, 90), VI+239 pages. Springer-Verlag, Berlin—Heidelberg—New York, 1980.

The motto of the volume "There are no solved problems; there are only problems that are more or less solved" indicates quite well the author's intention when choosing an area of research, the development of which — after a promising decade — has come to a sudden standstill. The author is right; the book convinces the reader that there remains a large room for further thinking on open problems in the theory of theorem proving, whose solutions can point ahead.

The main topic of the volume is a certain refinement of the familiar resolution principle, called Hereditary Lock Resolution (HLR, for short). HLR is an amalgamation of a modification of Boyer's Lock Resolution rule and an extension of the Model Strategy due to Luckham. The basic properties of HLR are presented in Chapter 2. Chapter 3 is devoted to completeness problems; in fact it is proved that HLR are a sound and complete inference rule. The last chapter deals with a general theory of model specification techniques. The results obtained are employed to show the flexibility and sophistication of models in pragmatic environments.

The book is not self-contained. Actually, its complete understanding requires a considerable amount of brackground knowledge in the "classical" theory of theorem proving. Accordingly, this volume can be useful for experts and graduate students.

P. Ecsedi-Tóth (Szeged)

Ryuzo Sato-Takayuki Nōno, Invariance Principles and the Structure of Technology (Lecture Notes in Economics and Mathematical Systems 212), 94 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

This book is devoted to the study of the mathematical models of production theory in the period of technical progress. The production process can be described by an input-output function and the technical change can be considered as a 1-parameter transformation group acting on the manifold of input variables. Thus it is very natural to use the methods of Lie transformation groups in this theory.

The main results of this monograph are connected with invariance principles of production processes. The possible input-output functions are classified and the classical production functions are characterized by means of invariance properties.

Péter T. Nagy (Szeged)

J.-P. Serre, Linear Representations of Finite Groups (Graduate Texts in Mathematics, 42) Springer-Verlag, New York—Heidelberg—Berlin 1977, X+170 pages.

This book consists of three parts. One of them deals with the general theory and two are devoted to special questions of representation theory. The first part introduces the basic concepts of representation of finite groups, and describes the correspondence between representations and characters.

The proofs are elegant and as elementary as possible. A short indication shows how the preceding results carry over to compact groups. The general theory is applied for some known classical groups. The second part investigates degrees of representations and integrality properties of characters, induced representations, theorems of Artin and Brauer and their applications, rationality questions. The third part contains an introduction to the Brauer theory using the language of abelian categories. Several applications to the Artin representations are given. At the end of the text a short Appendix can be found on the definition of Artinian rings, the Grothendieck group, projective modules and discrete valuations.

László Gehér (Szeged)

J. Sesiano, Books IV to VII of Diophantus' Arithmetica in the Arabic translation attributed to Qustā ibn Lūqā, (Sources in the History of Mathematical and Physical Sciences 3), XII+502 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1982.

According to our present knowledge the Greek mathematician Diophantus of Alexandria (lived probably between 150 B. C. and A. D. 350, but it seems fairly probable that he flourished about A. D. 250) wrote at least two treatises: one of them dealing with problems in indeterminate equations and systems of equations, the Arithmetica, and another, a smaller tract, on polygonal numbers. Both are only partially extant today. We can read from the introduction of the Arithmetica that it originally consisted of thirteen Books. But only six of these have survived until now in Greek, and they have been edited and translated several times. The remaining seven were considered irretrievably lost until 1973, when Gerald Toomer learned of existence of a manuscript in A. Gulchīn-i Ma'ānī's just-published catalogue of the mathematical manuscripts in the Mashhad Shrine Library. This manuscript, a codex, consists of four other, hiterto unknown Books in an Arabic translation which, since it is attributed to Qustā ibn Lūqā, must have been made around or after the middle of the ninth century.

This book, which has five parts, is based on the author's 1975 Ph. D. thesis at Brown University. Major changes, however, are found in the mathematical commentaries. The discussion of Greek and Arabic interpolations is entirely new, as is the reconstruction of the history of the Arithmetica from Diophantine to Arabic times.

In Part One the first chapter deals with historical questions: the authenticity of the Arabic Books, the placement of the Arabic Books among the presently known Books of the Arithmetica, Diophantus in islamic and Bysantine times. This analysis leads to the conclusion that the four Arabic Books are the IV—VII books of the Arithmetica. Three Greek Books precede the Arabic four and the other Greek Books follow them.

In Part Two we can find the English translation of the Arabic Books. Part Three, the largest one, contains the author's detailed mathematical commentaries on the material. Part Four is the complete Arabic text of the manuscript. Part Five is an extensive Arabic index.

There is an Appendix under the title Conspectus of the Problems of the Arithmetica.

"Readers — mathematicians and non-mathematicians alike — will gain new perspectives on the techniques of Greek algebra and will learn of the fate and modifications of a scientific classic in the time between its classical origin and its medieval Arabic translation."

Lajos Klukovits (Szeged)

D. J. Shoesmith—T. J. Smiley, Multiple-Conclusion Logic, pp. IX+396. Cambridge University Press, Cambridge—London—New York—Melbourne, 1978.

The volume is a systematic study of multiple-conclusion proofs which can have, as opposed to traditional proof theory, more than one conclusions, say $B_1, \ldots, B_n$. These are to be understood as the "field within which the truth must lie", provided, of course, that the premisses $A_1, \ldots, A_m$ are accepted. The subject goes back to the works of G. Gentzen, R. Carnap and W. Kneale.

The book is divided into four parts. In Part I the familiar logical notions are generalized for multiple-conclusion proof rules and the connections between conventional and multiple-conclusion logics are investigated. In particular, adequateness (completeness) of several multiple-conclusion proof rules is proved. Part II treats graph proofs. This concept has been introduced to give an explicit tool for describing interdependencies among the components of an argument; quoting the authors: "it is not enough that each of its component steps is valid in isolation: they must also relate to one another properly". Graph proofs enable one to visualize arguments (independently from any particular axiom system) and hence to investigate the connection between the "form of arguments" (i.e. their graphs) and the semantical notion of validity.

In the rest of the volume the authors apply the techniques developed in the first two parts. In Part III, a thorough study of a particular many-valued multiple-conclusion inference system can be found. It is proved, for example, that every finite-valued multiple-conclusion propositional calculus is finitely axiomatizable. The last part of the book is devoted to investigate how "natural deduction" can be replaced by direct multiple-conclusion proofs. In particular, cut-elimination-like theorems are proved for classical predicate and for intuitionistic propositional calculi.

The book is clearly written and easily comprehensible. It can be useful for proof theorists on expert and graduate levels.

P. Ecsedi-Tóth (Szeged)

Ya. G. Sinai, Theory of Phase Transitions: Rigorous Results. VIII+150 pages, Akadémiai Kiadó, Budapest and Pergamon Press, Oxford, 1982.

The concept of limit Gibbs distributions (LGD) is relatively new, it was introduced in 1968 by Dobrushin, Lanford and Ruelle. Their construction made possible the rigorous development of the theory of phase transition in a probabilistic language. However, the special mathematical structures related to statistical physics involve highly non-standard methods.

Sinai's outstanding book gives a systematic survey of the results obtained using the concept of LGD. A great deal of these results is due to the author himself and his school (Chapters II and IV).

The book is well constructed, each chapter is almost selfcontained. The presentation is clear, the author always finds the appropriate level of generality. Both mathematicians and physicists — if they are inclined to deal with statistical physics directly and seriously — can grasp the major problems of the theory of phase transitions and the necessary information to try to solve them.

Chapter I has an introductory character, the author defines the notion of LGD and elucidates it by the most important examples related to lattice systems (e.g. Ising model, Heisenberg's continuous spin model, Yang—Mills model). The existence of the LGD is proved for general lattice systems and for the lattice model of quantum field theory.

In Chapter II the existence of phase diagram for small $(r-1)$-parameter perturbations of a periodic Hamiltonian having r ground states is proved. The result is due to Sinai and Pirogov; the proof is based on a far-reaching generalization of the contour method proposed by Peierls for proving the existence of long range order in the Ising-model at low temperature.

In Chapter III continuous spin systems are considered. By the Dobrushin—Shlosman theorem there is no continuous symmetry breakdown in the two-dimensional Heisenberg model. On the other hand, in models of three or more dimensions at low temperature, as Fröhlich, Simon and Spencer have proved, a spontaneous breakdown of continuous symmetry is present.

Chapter IV is devoted to the exact mathematical foundation of the renormalization group method — due to Bleher and Sinai — in the theory of second-order phase transitions. Dyson's hierarchical model is studied in detail; this model is an instructive example, where all interesting phenomena arise. The most intriguing problem is to find non-Gaussian invariant distributions under the action of the renormalization group. A special kind of bifurcation theory is developed for solving the above problem.

The subject of this book is presented "in statu nascendi"; the deep mathematical tools treated by the author were further developed — a great deal even by the Moscow school of mathematical physics — since the book has been written.

András Krámli (Budapest)

Statistics and Probability, Proceedings of the 3rd Pannonian Symposium on Mathematical Statistics, Visegrád, Hungary, 13—18 September, 1982, edited by J. Mogyoródi, I. Vincze and W. Wertz, X+415 pages, Akadémiai Kiadó, Budapest and D. Reidel Publishing Company, Dordrecht—Boston—Lancaster, 1984.

The thirty-six papers included in this volume move on a very wide scale. This, of course, is no surprise if the major organizing principle of a conference is geographical. The authors are: G. Baróti, M. Bolla—G. Tusnády, E. Csáki, S. Csörgő—H. D. Keller, P. Deheuvels, I. Fazekas, L. Gerencsér, T. Gerstenkorn—T. Jarzebska, B. Gyires, L. Horváth, J. Hurt, P. Kosik—K. Sarkadi, A. Kováts, A. Krámli—D. Szász, M. Krutina, L. Lakatos, A. Lesanovsky, E. Lukacs, P. Lukács, Gy. Michaletzky, J. Mogyoródi, T. F. Mári, H. Neudecker—T. Wansbeek, H. Niederreiter, J. Pintér, W. Polasek, L. Rutkowski, F. Schipp, A. Somogyi, C. Stepniak, G. J. Székely, A. Vetier, I. Vincze, A. Wakolbinger—G. Eder, M. T. Weselowska—Janczarek and A. Zempléni. A subject index helps orientation.

Sándor Csörgő (Szeged)

Studies in Pure Mathematics. To the Memory of Paul Turán. Edited by P. Erdős, L. Alpár, G. Halász and A. Sárközy, 773 pages, Akadémiai Kiadó, Budapest and Birkhäusler Verlag, Basel—Boston—Stuttgart, 1983.

The volume, dedicated to the memory of Paul Turán includes 66 papers of 88 invited authors from 16 countries of the world. The subjects of the papers are in most cases near to Turán's researches, in many cases problems of Turán are solved or the works were initiated by his earlier results. Nearly half of the papers deal with number theory what was his favourite topic during his very successful mathematical activity.

The wide scope of topics which found place in this volume — number theory, theory of functions of a complex variable, approximation theory, Fourier series, differential equations, combinatorics, statistical group theory — reflects Turán's universality and his large influence in mathematics. His pioneering contribution to many branches of mathematics can never be forgotten. This volume gives also an impression of his endeavour of searching for new paths, since various flourishing fields represented here, as, e.g., his main achievement, the power sum method (to which topic he devoted two books already, the third appears in 1984 at J. Wiley Interscience Tracts Series under the title "On a new method in the analysis and its application"), furthermore extremal graph theory, probabilistic number theory, statistical group theory owe their birth or/and their main developments to ideas of Turán. The high level of the works has been ensured by the authors whose list is the following: H. L. Abbot, M. Ajtai, L. Alpár, J. M. Anderson, R. Askey, C. Belna, B. Bollobás, W. G. Brown, L. Carleson, F. R. K. Chung, J. Clunie, Á. Császár, J. Dénes, E. Dobrowolski, Á. Elbert, P. D. T. A. Elliott, P. Erdős, W. H. J. Fuchs, D. Gaier, T. Ganelius, R. L. Graham, K. Győry, G. Halász, F. Harary, B. Harris, I. Havas, W. K. Hayman, E. Heppner, E. Hlawka, L. Iliev, K.-H. Indlekofer, Mourad E.—H. Ismail, H. Jager, M. Jutila, J.—P. Kahane, I. Kátai, Y. Katznelson, K. H. Kim, B. Kjellberg, G. Kolesnik, J. Komlós, W. Lawton, L. Lorch, G. G. Lorentz, L. Lovász, A. Meir, Z. Miller, H. L. Montgomery, Y. Motohashi, W. Narkiewicz, D. J. Newman, H. Niederreiter, P. P. Pálfy, Z. Z. Papp, R. Pierre, J. Pintz, G. Piranian, Ch. Pommerenke, N. Purzitsky, Q. I. Rahman, F. W. Roush, I. Z. Ruzsa, H. Sachs, A. Sárközy, A. Schinzel, W. M. Schmidt, I. J. Schoenberg, W. Schwarz, S. M. Shah, A. B. Shidlovsky, H. Siebert, M. Simonovits, G. Somorjai, V. T. Sós, J. Spencer, C. L. Stewart, M. Stiebitz, E. G.. Straus, J. Surányi, H. P. F. Swinnerton-Dyer, M. Szalay, E. Szemerédi, P. Szüsz, R. Tijdeman, R. C. Vaughan, P. Vértesi, M. Waldschmidt, K. Wiertelak.

J. Pintz (Budapest)

The Mathematics and Physics of Disordered Media: Percolation, Random Walk, Modeling, and Simulation, Proceedings of a Workshop held at the IMA, University of Minnesota, Minneapolis, February 13—19, 1983, edited by B. D. Hughes and B. W. Ninham (Lecture Notes in Mathematics 1035), VIII+431 pages, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1983.

It is most appropriate to cite a few sentences from the charter of the Workshop: "One of the fundamental questions of the 1980's facing both mathematicians and scientists is the mathematical characterisation of disorder.... The last decade has seen the beginnings of a unity of methods and approaches in statistical mechanics, transport in amorphous and disordered materials, properties of heterogeneous polymers and composite materials, turbulent flow, phase nucleation, and interfacial science. All have an underlying structure characterised in some sense by chaos, self-avoiding irregular walks, percolation, and fractals. Some real progress has been made in understanding random walks and percolation processes on the one hand, and through mean field or effective medium approximation and simulation of liquids and porous media on the other. The subject is directly connected with the statistics of extreme events and important pragmatic areas like fracture of solids, comminution of particulate materials, and flow through porous media."

In this extremely carefully compiled workshop volume very well-known theoreticians and applied scientists present their views of the foundations of disordered media. Following a long introductory paper in two parts (by B. D. Hughes on random discrete models and by P. Prager on diffusions in disordered media), two papers emphasize the important role of stable distributions in various physical phenomena, nine papers discuss various aspects (theoretical and applied) of percolation theory, and the five further papers deal with probabilistic models of fluids, permeability, diffusion, waves and crack growth.

Among various other kind of specialists, this volume is certainly a must for the applied probabilist.

Sándor Csörgő (Szeged)

Twistor Geometry and Non-Linear Systems (Proceedings, Primorsko, 1980), edited by H. D' Doebner and T. D. Palev, (Lecture Notes in Mathematics, 970), V+216 pages, Springer-Verlag Berlin—Heidelberg—New York, 1982.

This book contains the review lectures given at the 4th Bulgarian Summer School on Mathematical Problems of Quantum Field Theory held in Primorsko, Bulgaria, in Septermber 1980. The list of the papers is as follows.

I. Twistor Geometry: 1. S. G. Gindikin; Integral geometry and twistors. — This is a new approach to twistor geometry using the methods of Gelfand' integral geometry. 2. Yu. I. Manin; Gauge fields and cohomology of analytic sheaves. — This gives a deep analysis of holomorphic Yang—Mills fields, the vacuum Yang—Mills equations and the full system of Yang—Mills—Dirac equations in the language of holomorphic vector bundles over analytic spaces. 3. Z. Perjés; Introduction to twistor particle theory. 4. N. J. Hitchin; Complex manifolds and Einstein's equations. — This is a generalization of Penrose's twistor theory based on the geometry of rational curves in complex manifolds.

II. Non-Linear Systems: 1. A. A. Kirillov; Infinite dimensional Lie groups: their orbits, invariants and representations. The geometry of moments. 2. A. S. Schwartz; A few remarks on the construction of solutions of non-linear equations. 3. A. K. Pogrebkov—M. C. Polivanov; Some topics in the theory of singular solutions of non-linear equations. 4. V. K. Melnikov; Symmetries and conservation laws of dynamical systems. — The infinite dimensional symmetry group and several infinite series of conservation laws are found for a nonlinear evolution equation. 5. M. A. Semonov—Tianshansky; Group-theoretical aspects of completely integrable systems. — This paper treats several applications of the so-called orbit method in representation theory. 6. A. V. Mikhailov; Relativistically invariant models of the field theory integrable by the inverse scattering method. 7. P. A. Nikolov—I. T. Todorov; Space-time versus phase space approach to relativistic particle dynamics.

The book gives a good account of the present stage of the subject. We recommend it to everybody working in related fields of mathematics or mathematical physics.

Péter T. Nagy (Szeged)

Frank W. Warner, Foundations of Differentiable Manifolds and Lie Groups, (Graduate Texts in Mathematics; 94) VI+271 pages, Springer-Verlag, New York—Berlin—Heidelberg—Tokyo, 1983.

This Springer edition is a reproduction of the book originally published by Scott, Foresman and Co. in 1971. It is a very clear, detailed and carefully developed graduate-level textbook of analysis on manifolds. The reader must be familiar with the material by a good undergraduate course in algebra and analysis, some knowledge of point set topology, covering spaces and fundamental groups is also assumed. Chapters 1, 2 and 4 treat the fundamental methods of calculus on manifolds. These include differentiable manifolds, tangent vectors, submanifolds, implicit function theorems, vector fields, distributions and the Frobenius theorem, differential forms, integration, Stokes' theorem and the de Rham cohomology. Chapter 3 is devoted to the foundations of Lie group theory, including the relationship between Lie groups and Lie algebras, adjoint representation, properties of classical groups, the closed subgroup theorem and homogeneous spaces. The subject of Chapter 5 is the proof of a strong form of de Rham theorem. An axiomatic treatment of sheaf cohomology theory is given. The canonical isomorphism of all classical cohomology theories on manifolds is proved. In Chapter 6 the Hodge theorem and a complete description of the local theory of elliptic operators is presented, using Fourier series as the basic tool.

A lot of exercises are included, which constitute an integral part of the text. Some of them are routine, but in some cases they contain major theorems. Hints are provided for difficult exercises.

The book may be recommended to students and research workers interested in manifold theory.

Péter T. Nagy (Szeged)