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On systems of N-variable weighted shifts

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1. Introduction. Let H be an infinite-dimensional Hilbert space with an orthonormal basis $\{e_i\}_{i=0}^{\infty}$, and let $\{w_i\}_{i=0}^{\infty}$ be a bounded sequence of complex numbers. An operator T on H defined by

$$Te_i = w_i e_{i+1}, \quad i = 0, 1, 2, \ldots$$

is called a (forward) unilateral weighted shift with the weight sequence $\{w_i\}_{i=0}^{\infty}$. JEWELL and LUBIN [2] have recently extended the theory of weighted shifts to the systems of N-variable weighted shifts. They concentrate in their paper mainly on exhibiting the interplay between such systems of operators and the analytic function theory of several variables on the lines of the survey article on (one-variable) shifts by SHIELDS [9]. The object of this note is to study the existence of cyclic vectors for the systems of N-variable backward and forward weighted shifts. We also obtain an application of these ideas to the theory of transitive operator algebras.

2. Notations and terminology. We shall follow mostly the definitions and notations given in [2]. To recapitulate, we denote by N an arbitrary but fixed positive integer, by $I=(i_1, i_2, \ldots, i_N)$ a multi-index of non-negative integers, and by $|I|$ the sum $i_1+i_2+\ldots+i_N$. E_k denotes the multi-index I having $i_k=\delta_{jk}$ for $j=1, 2, \ldots, N$, and O is the multi-index $(0, 0, \ldots, 0)$. We shall write $I\pm E_k$ for the multi-index $(i_1, \ldots, i_{k-1}, i_k\pm 1, i_{k+1}, \ldots, i_N)$ (where $i_k>0$ in $I-E_k$). If $T=\{T_1, T_2, \ldots, T_N\}$ is a system of N commuting operators on H, then, for a multi-index $I=(i_1, \ldots, i_N)$, T^I means the operator $T^I=T_1^{i_1}T_2^{i_2}\ldots T_N^{i_N}$. Let $\{e_I\colon I\geqq O\}$ be an orthonormal basis of H and let $\{w_{I,j}\colon j=1, 2, \ldots, N, I\geqq O\}$ be a bounded net of non-zero complex numbers. A system of N-variable backward weighted shifts with the weight net $\{w_{I,j}\}$ is defined as a family $S=\{S_1, \ldots, S_N\}$ of N operators on H such that $S_j e_I$ equals $w_{I,j}e_{I-E_j}$ if $i_j\neq 0$ and 0 otherwise, for all $j=1, 2, \ldots, N$.

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Similarly, a system $T=\{T_1, T_2, \ldots, T_N\}$ of N-variable forward weighted shifts with the weight net $\{w_{I,j}\}$ on H is defined by $T_j e_I = w_{I,j} e_{I+E_j}$, $1 \leqq j \leqq N$. We may and do assume $w_{I,j}$'s to be positive real numbers [2; Corollary 2]. We also assume that $w_{I,j}$'s satisfy either of the following two conditions:

(I) $$w_{I,j} w_{I-E_j,k} = w_{I,k} w_{I-E_k,j}, \quad \text{if} \quad i_j, i_k \neq 0,$$

(II) $$w_{I,j} w_{I+E_j,k} = w_{I,k} w_{I+E_k,j},$$

for all I, $1 \leqq k, j \leqq N$. As observed in [2], condition (I) (respectively condition (II)) implies that the operators S_j (respectively T_j) commute. We set $\beta(J, k, m)$ equal to $w_{J,k} w_{J-E_k,k} \ldots w_{J-(m-1)E_k,k}$ if $m \leqq j_k$, and 0 otherwise; $\alpha(J, k, m)$ equal to $w_{J,k} w_{J+E_k,k} \ldots w_{J+(m-1)E_k,k}$ if $m \neq 0$ and 1 otherwise, where $J=(j_1, j_2, \ldots, j_N)$, and $1 \leqq k \leqq N$. It is easy to see that, for $1 \leqq k \leqq N$, $S_k^m e_J$ is equal to $\beta(J, k, m) e_{J-mE_k}$ if $j_k \geqq m$ and 0 othewise, and $T_k^m e_J$ is equal to $\alpha(J, k, m) e_{J+mE_k}$.

3. Cyclic vectors for N-variable weighted shifts. We make the following definitions:

Definition 1. A net $\{w_{I,j}\}$ of (non-zero) positive real numbers is said to be monotonically decreasing if for all $I=(i_1, i_2, \ldots, i_N)$, $I'=(i_1', i_2', \ldots, i_N')$ with $|I| \geqq |I'|$, we have $w_{I,j} \leqq w_{I',j}$ for $1 \leqq j \leqq N$.

Definition 2. A vector x in H is said to be a cyclic vector for a system $A=\{A_1, A_2, \ldots, A_N\}$ of N commuting operators on H, if $H = \bigvee_I A^I x$ (i.e.: H is spanned by the vectors $A^I x$).

Theorem 1. *Let $S=\{S_1, S_2, \ldots, S_N\}$ be a system of N-variable backward weighted shifts with the weight-net $\{w_{I,j}\}$ which is monotonically decreasing and satisfies the condition* (I). *If $\{w_{I,j}\}_I$ is square-summable (i.e. $\sum_I w_{I,j}^2 < \infty$) for some j, $1 \leqq j \leqq N$, and $x = \sum_J x(J) e_J$ is any vector in H such that infinitely many elements of the set $\{x(J)\colon j_1 = j_2 = \ldots = j_N \geqq 0\}$ differ from zero, then x is a cyclic vector for S.*

Proof. For the sake of simplicity, we shall prove the theorem in case $N=2$. However, our method works also in the general case.

Assume that $\{w_{I,1}\}$ is square-summable. For given $\varepsilon > 0$, there exists an $I_0 = (i_1^0, i_2^0)$ such that

(1) $$\sum_{|J| \geqq |I_0|} w_{J,1}^2 < \varepsilon^2 (w_{(1,0),1})^2.$$

Choose $M=(m_1, m_2)$ with $m_1 \geqq i_1^0$, $m_2 \geqq i_2^0$, such that

(2) $$|x(M)| = \sup\{|x((j_1+i_1^0, j_2+i_2^0))| \colon j_1, j_2 \geqq 0\} > 0.$$

It is easy to verify, by using condition (I), that

$$w_{(1,m_2),1}\beta((0,m_2),2,m_2)=\beta((1,m_2),2,m_2)w_{(1,0),1}. \tag{3}$$

Now

$$S^M x=(S_2^{m_2}S_1^{m_1})\left(\sum x(J)e_J\right)=$$
$$=\sum x((j_1+m_1,j_2+m_2))\beta((j_1+m_1,j_2+m_2),1,m_1)\beta((j_1,j_2+m_2),2,m_2)e_J.$$

Hence, by making a successive use of (2), (3) and the fact that $\{w_{I,j}\}$ is monotonically decreasing, we have

$$\left\|\frac{S^M x}{x(M)\beta(M,1,m_1)\beta((0,m_2),2,m_2)}-e_O\right\|^2=$$
$$=\sum_{(j_1,j_2)\neq O}\left|\frac{x((j_1+m_1,j_2+m_2))}{x(M)}\right|^2\left(\frac{\beta((j_1+m_1,j_2+m_2),1,m_1)\beta((j_1,j_2+m_2),2,m_2)}{\beta((m_1,m_2),1,m_1)\beta((0,m_2),2,m_2)}\right)^2\leqq$$
$$\leqq\sum_{(j_1,j_2)\neq O}\left(\frac{\beta((j_1+m_1,j_2+m_2),1,m_1)\beta((j_1,j_2+m_2),2,m_2)}{\beta((m_1,m_2),1,m_1)\beta((0,m_2),2,m_2)}\right)^2=$$
$$=\sum_{(j_1,j_2)\neq O}\left(\frac{w_{(j_1+m_1,j_2+m_2),1}}{w_{(1,0),1}}\right)^2\times$$
$$\times\left(\frac{\beta((j_1+m_1-1,j_2+m_2),1,m_1-1)\beta((j_1,j_2+m_2),2,m_2)}{\beta((m_1,m_2),1,m_1-1)\beta((1,m_2),2,m_2)}\right)^2\leqq$$
$$\leqq\sum_{(j_1,j_2)\neq O}\left(\frac{w_{(j_1+m_1,j_2+m_2),1}}{w_{(1,0),1}}\right)^2<\varepsilon^2.$$

This shows that $e_O\in L=\bigvee_I S^I x$. Now for each $I=(i_1,i_2)$, we see that

$$y_I=S^I x-x(I)\beta(I,1,i_1)\beta((0,i_2),2,i_2)e_O$$

is in L. Using this fact and making computations such as given above, we show that $e_{(1,0)}$ is in L.

Let $N=(n_1,n_2)$ be such that

$$|x(N+E_1)|=\sup\{|x((j_1,j_2))|:\ j_1\geqq n_1+1,\ j_2\geqq n_2\}>0.$$

Proceeding as above,

$$\left\|\frac{y_N}{x(N+E_1)\beta(N+E_1,1,n_1)\beta((1,n_2),2,n_2)}-e_{(1,0)}\right\|^2=$$
$$=\sum_{(j_1,j_2)\neq O,(1,0)}\left|\frac{x((j_1+n_1,j_2+n_2))}{x((n_1+1,n_2))}\right|^2\times$$
$$\times\left(\frac{\beta((j_1+n_1,j_2+n_2),1,n_1)\beta((j_1,j_2+n_2),2,n_2)}{\beta((n_1+1,n_2),1,n_1)\beta((1,n_2),2,n_2)}\right)^2\leqq$$

$$\leqq \sum_{(j_1,j_2)\neq O,(1,0)} \left(\frac{\beta((j_1+n_1, j_2+n_2), 1, n_1)\beta((j_1, j_2+n_2), 2, n_2)}{\beta((n_1+1, n_2), 1, n_1)\beta((1, n_2), 2, n_2)} \right)^2 =$$

$$= \sum_{(j_1,j_2)\neq O,(1,0)} \left(\frac{w_{(j_1+n_1, j_2+n_2),1}}{w_{(2,0),1}} \right)^2 \times$$

$$\times \left(\frac{\beta((j_1+n_1-1, j_2+n_2)1, n_1-1)\beta((j_1, j_2+n_2), 2, n_2)}{\beta((n_1+1, n_2), 1, n_1-1)\beta((2, n_2), 2, n_2)} \right)^2 \leqq$$

$$\leqq \frac{1}{w^2_{(2,0),1}} \sum_{(j_1,j_2)\neq O,(1,0)} w^2_{(j_1+n_1, j_2+n_2),1} \to 0 \quad \text{as} \quad |N| \to \infty.$$

This implies that $e_{(1,0)} \in L$. Similarily, we can show that $e_{(0,1)} \in L$. Continuing this process, we see that $e_I \in L$ for all $I \geqq 0$, and hence x is a cyclic vector for S.

Theorem 2. *Let $T = \{T_1, T_2, \ldots, T_N\}$ be a system of N-variable forward weighted shifts with the weight-net $\{w_{I,j}\}$ which is monotonically decreasing and satisfies condition* (II). *If $\{w_{I,j}\}_I$ is square-summable for some j, $1 \leqq j \leqq N$, then any vector $x = \sum_J x(J) e_J$ in H with $x(O) \neq 0$ is a cyclic vector for T.*

Proof. Again, we prove the theorem for $N=2$. Let the net $\{w_{I,1}\}_I$ be square-summable. It can be easily seen from condition (II) that

$$\alpha(O, 2, i_2) w_{(1,i_2),1} = w_{O,1} \alpha((1, 0), 2, i_2). \tag{4}$$

Now

$$T^I x = (T_1^{i_1} T_2^{i_2})(\sum x(J) e_J) =$$

$$= \sum x((j_1, j_2)) \alpha((j_1, j_2), 2, i_2) \alpha((j_1, j_2+i_2), 1, i_1) e_{(j_1+i_1, j_2+i_2)},$$

and hence, using successively (4) and the fact that $\{w_{I,j}\}$ is monotonically decreasing, we have

$$\left\| \frac{T^I x}{x(O)\alpha(O, 2, i_2)\alpha((0, i_2), 1, i_1)} - e_I \right\|^2 =$$

$$= \sum_{(j_1,j_2)\neq O} \left| \frac{x((j_1, j_2))}{x(O)} \right|^2 \left(\frac{\alpha((j_1, j_2), 2, i_2)\alpha((j_1, j_2+i_2), 1, i_1)}{\alpha(O, 2, i_2)\alpha((0, i_2), 1, i_1)} \right)^2 =$$

$$= \left(\frac{w_{I,1}}{w_{O,1}} \right)^2 \sum_{(j_1,j_2)\neq O} \left| \frac{x((j_1, j_2))}{x(O)} \right|^2 \left(\frac{\alpha((j_1, j_2), 2, i_2)\alpha((j_1, j_2+i_2), 1, i_1)}{\alpha((1, 0), 2, i_2)\alpha((1, i_2), 1, i_1)} \right)^2 \leqq$$

$$\leqq \left(\frac{w_{I,1}}{w_{O,1}} \right)^2 \sum_{(j_1,j_2)\neq O} \left| \frac{x((j_1, j_2))}{x(O)} \right|^2 \leqq \left(\frac{\|x\|}{|x(O)| w_{O,1}} \right)^2 w^2_{I,1}.$$

Since $\{w_{I,1}\}_I$ is square-summable and $\{e_I\}_I$ is an orthonormal basis of H, using the fact tha tthe vectors $\{T^I x\}_I$ are linearly independent, it follows by the Paley—Wiener Theorem [8] that x is a cyclic vector for T.

4. A partial solution of the transitive algebra problem. Let $\mathfrak{A}$ denote a weakly closed subalgebra of $B(H)$, the Banach algebra of all bounded operators on H. We shall write Lat $\mathfrak{A}$ for the lattice of all invariant subspaces of $\mathfrak{A}$, and shall call $\mathfrak{A}$ transitive if Lat $\mathfrak{A}=\{\{0\}, H\}$. It is easy to see that $B(H)$ is a transitive algebra. Whether there exist transitive algebras other than $B(H)$ is an open question, known as the transitive algebra problem, raised by KADISON [3] first in 1955. Since then, although the problem remains unsolved in general, a number of partial solutions of the problem have been obtained by a number of mathematicians, based mainly on the first partial solution given by ARVESON [1]. LOMONOSOV [4], however, used different techniques to obtain a partial solution of far reaching consequences. For an elegant account of these results we refer to RADJAVI and ROSENTHAL [7], see also PEARCY and SHIELDS [6]. We prove here the following theorem which generalizes a result due to NORDGREN, RADJAVI and ROSENTHAL [5]:

Theorem 3. *If a transitive algebra $\mathfrak{A}$ contains a system of N-variable backward weighted shifts S with the monotonically decreasing weight-net $\{w_{I,j}\}$ and $\{w_{I,j}\}_I$ is square-summable for some j, $1\leqq j\leqq N$, then $\mathfrak{A}=B(H)$.*

In order to prove the theorem we shall need the following lemma which is a generalization of Corollary 1 of [5; p. 176]. Let us denote by $B^{(n)}$ the direct sum of n copies of an operator B on H.

Lemma. *If $\mathfrak{A}$ is a transitive algebra containing operators A and B on H such that*
(i) *every common eigenspace of A and B is one-dimensional,*
(ii) *for each $n>0$, every non-zero common invariant subspace of $A^{(n)}$ and $B^{(n)}$ contains a common eigenvector,*
then $\mathfrak{A}$ is $B(H)$.

In the proof of the theorem we consider again only the case $N=2$. Thus we assume that $\mathfrak{A}$ contains the family $S=\{S_1, S_2\}$. Assume that $\{w_{I,1}\}_I$ is square-summable. In the light of above Lemma, it is sufficient to show that S_1 and S_2 satisfy conditions (i) and (ii).

Let L be a common eigenspace of S_1 and S_2. Then $L=\{x\in H\colon S_1x=\lambda x$ and $S_2x=\mu x\}$ for some complex numbers λ, μ. For any vector $x=\sum x(J)e_J$ in L, we have

$$\sum_{j_1\neq 0} x((j_1,j_2))w_{(j_1,j_2),1}e_{(j_1-1,j_2)} = \lambda\left(\sum x((j_1,j_2))e_{(j_1,j_2)}\right),$$

$$\sum_{j_2\neq 0} x((j_1,j_2))w_{(j_1,j_2),2}e_{(j_1,j_2-1)} = \mu\left(\sum x((j_1,j_2))e_{(j_1,j_2)}\right),$$

which implies that

$$\lambda x((j_1,j_2)) = x((j_1+1,j_2))w_{(j_1+1,j_2),1}, \quad \mu x((j_1,j_2)) = x((j_1,j_2+1))w_{(j_1,j_2+1),2}$$

if $j_1, j_2 \geqq 0$. This leads to

$$x((j_1, j_2)) = \lambda^{j_1}\mu^{j_2}[\beta((j_1, j_2), 1, j_1)\beta((0, j_2), 2, j_2)]^{-1}x((0, 0))$$

for $(j_1, j_2) \neq O$. Thus $x = x((0, 0))y$, where

$$y = \sum \lambda^{j_1}\mu^{j_2}[\beta((j_1, j_2), 1, j_1)\beta((0, j_2), 2, j_2)]^{-1}e_{(j_1, j_2)}$$

is a fixed vector in H, and hence L is one-dimensional.

Next, let M be a non-zero common invariant subspace of $S_1^{(n)}$ and $S_2^{(n)}$, for $n>0$. Let $x_1 \oplus x_2 \oplus \ldots \oplus x_n$ be any non-zero element of M, where $x_k = \sum x(k, J)e_J$, $1 \leqq k \leqq n$. If for each k the net $\{x(k, J)\}_J$ has only finitely many non-zero elements, then the common invariant subspace M_1 of $S_1^{(n)}$ and $S_2^{(n)}$ generated by $x_1 \oplus x_2 \oplus \ldots \oplus x_n$ is finite-dimensional. Therefore, M_1, and consequently M, has a common eigenvector of $S_1^{(n)}$ and $S_2^{(n)}$. We therefore assume that for every $I = (i_1, i_2)$, we have

$$\max\{|x(k, (j_1 + i_1, j_2 + i_2))|:\ j_1, j_2 \geqq 0,\ 1 \leqq k \leqq n\} > 0.$$

Let $I = (i_1, i_2)$ be fixed. Then there is a multi-index $R = R(I) = (r_1, r_2)$, $r_1 \geqq i_1$, $r_2 \geqq i_2$ and a number $s = s(I)$ between 1 and n such that

$$|x(s, R)| = \max\{|x(k, (j_1 + i_1, j_2 + i_2))|:\ j_1, j_2 > 0,\ 1 \leqq k \leqq n\}. \tag{5}$$

Now $(S^{(n)})^R\left(\bigoplus_{k=1}^{n} x_k\right) = \bigoplus_{k=1}^{n} S_2^{r_2}S_1^{r_1}x_k$ gives us that

$$(S^{(n)})^R(x_1 \oplus x_2 \oplus \ldots \oplus x_n)[x(s, R)\beta(R, 2, r_2)\beta((r_1, 0), 1, r_1)]^{-1} =$$

$$= \left[(x(s, R))^{-1} \bigoplus_{k=1}^{n} x(k, R)e_O\right] + \left[\bigoplus_{k=1}^{n} y(k, R)\right],$$

where for each k

$$y(k, R) =$$

$$= \sum_{(j_1, j_2) \neq O} \left(\frac{x(k, (j_1 + r_1, j_2 + r_2))\beta((j_1 + r_1, j_2 + r_2), 1, r_1)}{x(s, R)\beta((r_1, r_2), 1, r_1)} \frac{\beta((j_1, j_2 + r_2), 2, r_2)}{\beta((0, r_2), 2, r_2)}\right) e_{(j_1, j_2)}.$$

Making use of (5), (I) and the facts that the net $\{w_{I,j}\}$ is monotonically decreasing and $\{w_{I,1}\}_I$ is square-summable, we have

$$\|y(k, R)\|^2 =$$

$$= \sum_{(j_1, j_2) \neq O} \left|\frac{x(k, (j_1 + r_1, j_2 + r_2))}{x(s, (r_1, r_2))}\right|^2 \left(\frac{\beta((j_1 + r_1, j_2 + r_2), 1, r_1)\beta((j_1, j_2 + r_2), 2, r_2)}{\beta((r_1, r_2), 1, r_1)\beta((0, r_2), 2, r_2)}\right)^2 \leqq$$

$$\leqq \sum_{(j_1, j_2) \neq O} \left(\frac{w_{(j_1 + r_1, j_2 + r_2), 1}}{w_{(1, 0), 1}}\right)^2 \left(\frac{\beta((j_1 + r_1 - 1, j_2 + r_2), 1, r_1 - 1)\beta((j_1, j_2 + r_2), 2, r_2)}{\beta((r_1, r_2), 1, r_1 - 1)\beta((1, r_2), 2, r_2)}\right)^2 \leqq$$

$$\leqq \frac{1}{w_{(1, 0), 1}^2} \sum_{(j_1, j_2) \neq O} w_{(j_1 + r_1, j_2 + r_2), 1}^2 \to 0 \quad \text{as} \quad |I| \to \infty.$$

Thus the vector $[x(s,R)]^{-1}\bigoplus_{k=1}^{n}x(k,R)e_O$ is in M. Now for each k, the net

$$\{x(k,R)/x(s,R)\colon R=R(I),\ I=(i_1,i_2)\}$$

is contained in the unit disc, and therefore contains a subnet convergent to a number z_k (say). It can be easily seen that there is a number k_0 $(1\leqq k_0\leqq n)$ such that k_0 will occur infinitely many times as a value of $s=s(I)$. Corresponding to this k_0, we have $z_{k_0}=1$. Thus $z_1e_O\oplus\ldots\oplus z_ne_O$ is a common eigenvector of $S_1^{(n)}$ and $S_2^{(n)}$ in M.

Lastly we make the following remarks:

1. The condition of monotonicity of the net $\{w_{I,j}\}$ in all our theorems can be replaced by the following less stringent requirements: if j_0 is the integer for which $\{w_{I,j}\}_I$ is square-summable, then

(i) $w_{(i_1,i_2,\ldots,i_N),j}\leqq w_{(i'_1,i'_2,\ldots,i'_N),j}$ for $1\leqq j\leqq N$ if $i_k\geqq i'_k$, $1\leqq k\leqq N$,

(ii) $w_{(i_1,\ldots,i_{j_0-1},i_{j_0}-1,i_{j_0+1},\ldots,i_N),j}\leqq w_{(i'_1,\ldots,i'_{j_0-1},i_{j_0},i'_{j_0+1},\ldots,i'_N),j}$ for $1\leqq j\leqq N$ if $i_k\geqq i'_k$, $1\leqq k\leqq N$, $k\neq j_0$, $i_{j_0}\geqq 1$ ($i_{j_0}=1$ in Theorem 2, Theorem 3).

2. We observe that Theorem 2 and Theorem 3 can be proved in a more general form on the lines of Theorem 1 in [10] and Theorem 1 in [11] respectively.

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