Bibliographie

J. P. Bickel—N. El Karoui—M. Yor, Ecole d'Eté de Probabilités de Saint-Flour IX—1979, IX + 280 pages;

J. M. Bismut—L. Gross—K. Krickeberg, Ecole d'Eté de Probabilités de Saint-Flour X—1980, X + 313 pages;

Edité par P. L. Hennequin (Lecture Notes in Mathematics, 876, 929), Springer-Verlag, Berlin—Heidelberg—New York, 1981, 1982.

These are the two new volumes of the now traditional Saint-Flour summer school series. Both volumes contain three longer survey articles of a subject area in probability or mathematical statistics. Bickel (72 pages) describes the recent flourishment of robust estimation theory concentrating in a very welcome way on the mathematical technique and not only on motivation as most authors do on this field. El Karoui (166 pages) gives a precise and unified account on stochastic control theory, represeting many results of various authors in the last three decades in the language of the French general theory of stochastic processes. Yor (42 pages) investigates a general stochastic filtration equation which containes most such equations in the literature. Bismut (100 pages) provides a shorter preliminary description of his "mecanique aléatoire" than in his later monograph (same *Lecture Notes*, 866, to be reviewed in the next volume of these *Acta*) which has come out earlier. Gross (104 pages) covers equilibrium thermodinamics, equilibrium statistical mechanics and random fields. Finally, Krickeberg (109 pages) overviews the statistical theory of point processes.

Sándor Csörgő (Szeged)

J. Bourgain, New Classes of $\mathscr{L}^p$-Spaces (Lecture Notes in Mathematics, 889), V+143, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

For normed linear spaces E and F let

$$d(E, F) = \inf \{\|T\| \|T^{-1}\| \mid T: E \to F \text{ is an onto isomorphism}\}$$

(in case E and F are not isomorphic, take $d(E, F) = \infty$). If $1 \leqq p \leqq \infty$ and $1 \leqq \lambda < \infty$ then a Banach space X is called a $\mathscr{L}^p_\lambda$-space provided for any finite dimensional subspace E of X there is a finite dimensional subspace F of X satisfying $E \subseteqq F$ and $d(F, l^p(\dim F)) \leqq \lambda$. Now a $\mathscr{L}^p$-space is a $\mathscr{L}^p_\lambda$ space for some $\lambda < \infty$.

This concept has turned out to be very useful in the local investigation of Banach spaces, although it has also many consequences on the global structure of the space.

In the book the author provides new constructions for $\mathscr{L}^p$-spaces which solve several open problems in the negative. The examples of $\mathscr{L}^p$-spaces $(1 < p < \infty)$ and $\mathscr{L}^1$-spaces are related and are constructed using trees on the integers.

Familiarity with the theory of Banach spaces, measures and universal algebras is necessary when reading the book, which is designed especially for research workers in this topic. Several open problems are also mentioned, so that Bourgain's work well illustrates the goal of the "Lecture Notes" program: "new developments in mathematical research and teaching-quickly, informally and at a high level".

V. Totik (Szeged)

K. L. Chung, Lectures from Markov Processes to Brownian Motion (Grundlehren der mathematischen Wissenschaften, 249), VIII+239 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This book begins at the beginning with the Markov property, followed quickly by the introduction of optimal times and martingales. These three topics in the discrete parameter setting are fully discussed in an earlier book of the author: A Course in Probability Theory (Academic Press, 1974, second edition). The Course may be considered as a general background. But apart from the material on discrete parameter martingale theory cited in § 1.4, the book is self-contained.

Chapter 2 serves as an interregnum between the more concrete Feller processes and Hunt's axiomatic theory. Strong and moderate Markov properties of a Feller process are established with certain measurability properties. Chapter 3 contains the basic theory as formulated by Hunt, including hitting times, recurrent and transient Hunt processes and the characterization of the hitting (balayage) operator. Properties of the Brownian motion are discussed in Chapter 4; the treatment of Schrödinger's equation by the Feynman—Kac method is new. In the last chapter a number of notable results in classical potential theory are established by the methods developed in the earlier chapters.

Each chapter ends with a section of historical remarks and a number of regrettably omitted topics are mentioned. The book contains a lot of exercises as proper extensions of the text. Graduate students and professional mathematicians will benefit from the clear, uncluttered treatment emphasizing fundamental concepts and methods.

Lajos Horváth (Szeged)

Combinatorial Mathematics VIII. Proceedings of the Eighth Australian Conference on Combinatorial Mathematics, Held at Deakin University, Geelong, Australia, August 25—29, 1980, XIV+359 pages. Edited by Kevin L. McAveney (Lecture Notes in Mathematics, Vol. 884) Springer-Verlag, Berlin—Heidelberg—New York, 1981.

These conference proceedings contain two expository papers, nine invited papers and twenty contributed papers. A great part of papers investigate symmetric combinatorial structures (vertex-transitive graph, finite projective plane, two-distance set, latin square, design). Many of the authors belong to the Australian school of combinatorics. The titles of expository papers are: R. G. Stanton and R. C. Mullin, Some properties of H-designs; R. G. Stanton and H. C. Williams, Computation of some number-theoretic coverings. The titles of invited papers are : B. Alspach, The search for long paths and cycles in vertex-transitive graphs and digraphs; C. C. Chen and N. F. Quimpo, On strongly hamiltonian abelian group graphs; R. L. Graham, Wen-Ching Winnie Li and J. L. Paul, Monochromatic lines in partitions of $\mathbf{Z}^n$; J. S. Hwang, Complete stable marriages and systems of I—M preferences; P. Lorimer, The construction of finite projective planes; R. C. Read, A survey of graph generation techniques; J. J. Seidel, Graphs and two-distance sets; J. Sheehan, Finite Ramsey theory is hard; R. G. Stanton, Further results on covering integers of the form $1+k2^n$ by primes.

L. A. Székely (Szeged)

Combinatorics and Graph Theory. Proceedings of the Symposium Held at the Indian Statistical Institute, Calcutta, February 25—29, 1980, VII+500 pages. Edited by S. B. Rao (Lecture Notes in Mathematics, Vol. 885), Springer-Verlag, Berlin—Heidelberg—New York, 1981.

These proceedings consist of 9 invited papers and 36 contributed papers. Eight of them investigate the degree sequence of several graphs. One can find many papers concerning designs. association schemes and enumeration problems.

The list of invited addresses is: C. Berge, Diperfect graphs; P. Erdős, Some new problems and results in graph theory and other branches of combinatorial mathematics; E. V. Krishnamurty, A form invariant multivariable polynomial representation of graphs; L. Lovász and A. Schrijver, Some combinatorial applications of the new linear programming algorithm; K. Balasubramanian and K. R. Parthasarathy, In search of a complete invariant for graphs; D. K. Ray-Chaudhuri, Affine triple systems; F. C. Bussemaker, R. A. Mathon and J. J. Seidel, Tables of two-graphs; S. S. Shrikhande and N. M. Singhi, Designs, adjacency multigraphs and embeddings: a survey; G. A. Patwardhan and M. N. Vartak, On the adjungate of a symmetrical balanced incomplete block design with $\lambda=1$.

L. A. Székely (Szeged)

E. B. Dynkin, Markov Processes and Related Problems of Analysis. Selected Papers (London Mathematical Society Lecture Note Series 54), VI+312 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1982.

It is widely acknowledged that Professor Dynkin's work in the last two decades has given a new shape to the theory of Markov processes. And as the role and importance of this theory within the whole theory of stochastic processes and in various appled branches cannot really be overemphasized, this is not a small thing. According to his own preface, Dynkin's new approach to Markov processes, and especially to the Martin boundary theory and the theory of duality, has the three distinctive features that the general non-homogeneous theory precedes the homogeneous one, that all the theory is invariant with respect to time reversion, and that the regularity properties of a process are formulated not in topological terms but in terms of behaviour of certain real-valued functions along almost all paths. This collection contains nine influential papers by him. The first seven of these, from 1960, 1964, 1969, 1971, 1972, 1973 and 1975, were originally published in the *Uspehi Matematicheskih Nauk* and translated into English in the *Russian Mathematical Surveys,* the eighth is his 1978 *Annals of Probability* paper, and the last one appeared in the *Transactions of the American Mathematical Society* in 1980. The author has revised the entire text of the English translations and corrected a few slips in the originals. Workers in Markov processes will find it very useful to have these papers in one volume.

Sándor Csörgő (Szeged)

P. D. T. A. Elliott, Probabilistic Number Theory I, Mean-Value Theorems, **II,** Central Limit Theorems (Grundlehren der mathematischen Wissenschaften 239, 240), XXXIII+359 pages, XXXIV+341 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979, 1980.

This monograph gives an excellent introduction to probabilistic number theory and summarizes its fundamental results. The first volume begins with some necessary results from measure theory and the theory of probability. The greatest part of these theorems are proved in the first chapter and the proofs of the remaining theorems can be found in every monograph on probability and measure theory.

After a discussion on the Selberg sieve method and the forms of the prime number theorem the author studies certain finite probability spaces, paying particular attention to a model of Kubilius which plays a crucial role in some later chapters. Chapter 4 contains the Turán—Kubilius inequality and its dual, and their connection with the inequality of the large sieve. New proofs of the classical Erdős and Erdős—Wintner theorems on the distribution of the values of additive arithmetic functions are presented. The first volume ends with the Halász method and the study of multiplicative arithmetic functions.

In the second volume the author studies the value distribution of arithmetic functions, allowing unbounded renormalisations. The methods involve a synthesis of probability and number theory, sums of independent random variables playing an important role. In particular, he investigates to what extent one can simulate the behaviour of additive arithmetic functions by that of suitably defined independent random variables. Subsequent methods involve both Fourier analysis on the line and the application of Dirichlet series.

Many additional topics are considered, a problem of Hardy and Ramanujan, local properties of additive arithmetic functions, the rate of convergence to the normal law and the arithmetic simulation of all stable laws. A number of conjectures is formulated in Chapter 17 and a list of unsolved problems is given in Chapter 23. The historical background of various results is discussed, forming an integral part of the text. The reader gets acquainted with further results on each topic and the references cover broad parts of the literature.

These very nice books may be recommended for everybody who is interested in probabilistic number theory. A graduate course may be based on a selection of results from the first volume.

Lajos Horváth (Szeged)

T. M. Flett, Differential Analysis, VIII+359 pages, Cambridge University Press, Cambridge—London—New York—New Rochelle—Melbourne—Sydney, 1980.

The book is concerned with the differential calculus of functions taking values in normed spaces.

In the first chapter such basic results on functions of one variable are treated as the mean value theorems, the increment inequality and monotonicity theorems. The second chapter is a good survey on the modern existence, uniqueness and continuation results in the theory of differential equations and inequilities even for infinite-dimensional systems. The third chapter deals with the Fréchet differential, which forms the basis of the calculus of functions of a vector variable. There are two other types of differentials for this purpose. The Gâteaux (or directional) differential is commonly discussed in the literature for this case. In the last chapter the author gives a detailed account also on the Hadamard differential which can be required for certain considerations in tangent spaces and in aspects of the theory of differential equations (for example, "differentiation along the curve").

A large part of the book is devoted to applications. Besides ordinary differential equations and inequalities, the author studies extremum problems for functions of a vector variable, Ljapunov stability, geometry of tangents, the Newton—Kantorovich method, etc. A great number of examples and exercises can be found in the book.

The chapters are concluded by very interesting long historical notes. For example, at the end of the first chapter the reader finds the adventurous history of the mean value and monotonicity theorems with the original proofs and methods. (It is interesting that Ampère, a pupil of Lagrange who later achieved fame for his researches in electricity and magnetism, attempted to show that every real-valued function has a derivative everywhere, and Chauchy's treatment of the mean value theorem stemmed from this paper.)

This nicely presented book is not only a very good monograph but also an excellent textbook in advanced calculus.

L. Hatvani (Szeged)

Functional Analysis in Markov Processes. Proceedings of the International Workshop held at Katata, Japan, August 21—26, 1981, and of the International Conference held at Kyoto, Japan, August 27—29, 1981. Edited by M. Fukushima (Lecture Notes in Mathematics, 923), V+307 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1982.

This volume comprises 15 original research papers based on lectures given at the above joint meetings. The first three longer papers (S. Kusuoka, Analytic functionals of Wiener process and absolute continuity; Y. Le Jan, Dual markovian semigroups and processes; M. Tomisaki, Dirichlet forms associated with direct product diffusion processes) are based on the main three-hour lectures and represent the main features of the meetings. The authors of the 12 shorter papers are Albeverio and Høegh—Krohn (2 papers), Fukushima, Getoor and Sharpe, Guang Lu and Minping, Gundy and Silverstein, Itô, Kanda, Kotani and S. Watanabe, Oshima, Pitman and Yor, and Stroock. Some of these papers apply the functional analytic theory of Markov processes to various branches of physics.

Sándor Csörgő (Szeged)

Azriel Levy, Basic Set Theory (Perspectives in Mathematical Logic), XIV+391 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

The value of a good textbook on a subject can hardly be overestimated. I consider Levy's book such a good elementary-advanced work on a discipline which has just entered into the maturity age. The goal is to present basic set theory but the material is fairly up to date, several results from the late seventies are also incorporated. Many routine proofs are left to the reader but this only increases the legibility of the book. Also, a lot of exercises are presented — these range from almost trivial ones to advanced problems.

Basic Set Theory consists of two independent parts. The first one is devoted to the development of pure set theory. Here the material is common with many other books; the framework is with von Neumann's classes. Constructibility and forcing is excluded but the last chapter deals with the versions of the axiom of choice.

The second part is best named as selected topics. Although only a few topics are selected — the elements of description set theory; Boolean algebras and Martin's axiom and some infinite combinatorics — these enable the reader to get a sight of current research let alone their usefulness and applicability in other mathematical disciplines.

I recommend Levy's book both to lecturers on set theory and to students who may get acquainted with this challenging field through this well-written work.

V. Totik (Szeged)

Logic Symposia. Proceedings, Hakone 1979, 1980, Edited by G. H. Müller, G. Takeuti and T. Tugué (Lecture Notes in Mathematics, 891), XI+934 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

Two symposia on the foundations of mathematics were held at Gōra, Hakone, in Japan, on March 21—24, 1979 and February 4—7, 1980 mainly with Japanese participants. This book contains 15 papers read at these symposia. M. Hanazawa writes about Aronszajn trees, S. Hayashi about set theories in toposes, K. Hirose and F. Nakayasu about Spector second order classes, Y. Kakuda about precipitousness of ideals, T. Kawai about axiom systems of nonstandard set theory, S. Meahara about transfinite induction in an initial segment of Cantor's second number class, T. Miytake about proofs in recursive arithmetic, N. Motohashi about definability theorems, K. Namba about Boolean valued combinatorics, H. Ono and A. Nakamura about the connection of the undecidability of certain extensions with finite automata, I. Shinoda about sections and envelopes of type 2 objects, G. Takeuti and S. Titani about Heyting valued universes of intuitionistic set theory, S. Tugue and H. Nomoto about the independence of an elementary analysis problem, T. Uesu about intuitionistic theories and, finally, M. Yasugi writes about the Hahn—Banach extension theorem. Let us record here the result of S. Tugue and H. Nomoto: There are sets $A \subseteqq \mathbf{R}$ for which the statement "For any sequence $\{a_k\}$ of real numbers, if $\lim_{k\to\infty} e^{2\pi i a_k t}=1$ for every $t\in A$, then $\{a_k\}$ converges to 0" is independent of the axioms of ZFC.

V. Totik (Szeged)

Péter Major, Multiple Wiener—Itô Integrals. With applications to limit theorems (Lecture Notes in Mathematics, 894), VII+127 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

This monograph is about some recent and very deep results on the asymptotic behaviour of partial sums of discrete and generalised, "really" dependent random fields suggested by some important problems in statistical physics and in the theory of infinite particle systems. A modified version of multiple Wiener—Itô integrals, a notion originally designed for the study of nonlinear functionals over Gaussian fields, have proved to be a useful tool in the investigation of this renormalisation limit problem of random fields. Almost all results and proofs in this area are related to these integrals. This fact is what necessitated this clearly and elegantly written monograph. The section headings are: 1. On a limit problem, 2. Wick polinomials, 3. Random spectral measures, 4. Multiple Wiener—Itô integrals, 5. The proof of Itô's formula. The diagram formula and some of its consequences, 6. Subordinated fields. Construction of self-similar fields, 7. On the original Wiener—Itô integral, 8. Non-central limit theorems. The last, ninth section gives the history of the problems and poses a number of unsolved problems.

The volume is a self-contained exposition and is indispensable for anyone interested in the above problems and generally in self-similar processes.

Sándor Csörgő (Szeged)

Mathematics Tomorrow, Edited by L. A. Steen, 250 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1981.

Three years ago the Joint Projects Committee on Mathematics and the Conference Board of Mathematical Sciences (USA) prepared a volume of essays: Mathematics Today: Twelwe Informal Essays. Mathematics Tomorrow continues the theme of Mathematics Today. It is written by individuals and it contains opinions and predictions about the direction that mathematics — research and education — should take in the future.

Mathematics Tomorrow is divided into four parts: What is Mathematics?; Teaching and Learning Mathematics; Issues of Equality and Mathematics for Tomorrow. No doubt, the most exciting section is the first one which tries to determine the relationship between "pure" and "applied" mathematics and the effect of this on mathematics teaching. Here some authors argue for radical reform, others express their concern because of the pragmatic trend in recent projects. Let us present here some valuable opinions:

"Pure mathematics can be practically useful and applied mathematics can be artistically elegant" (P. Halmos).

"Applied mathematics cannot get along without pure, as an anteater cannot get along without ants, but not necessarily the reverse" (P. Halmos).

"If the habit of understanding is lost at an elementary level, or never learned, it will not reappear when the problems become more complicated" (T. Poston).

"The new core for the mathematics major might consist of only one year calculus, one semester of linear algebra, and one semester of real analysis. ... we should even consider the extreme case that the new core might be the empty set" (W. F. Lucas).

"...if the overall enrollment decline in higher education reaches a point of serious cuts in departmental sizes, then many other groups will decide that they too can teach their own mathematics series courses. ... On the other hand many golden opportunities still exist ... The choice is up to the mathematics cummunity, but it must act quickly and in a meaningful way" (W. F. Lucas).

"The applications enthusiasts hold all the cards. They have behind them the power and influence of the natural organizations and commissions. They are reshaping the mathematics curriculum in their own image ... But I ask for a favor. Let one course, just one, remain pure ... And one day when the wind is right I'll do the Cauchy Integral Formula for the last time ... and the students will see the curve and the thing inside and the lazy integral that makes the function value appear as suddenly as my palm when I open my hand. They will see pure mathematics ... And we owe Paul Halmos a chance to see that some mathematics students know that his subject exists." (J. P. King).

V. Totik (Szeged)

N. H. McClamroch, State Models of Dynamic Systems, VIII+248 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

It often happens that results of pure mathematics do not come to applications even in such cases when they are undoubtedly applicable in general. This may be caused by the difficulties of synthetizing the physical argumentation and mathematical formalism. The first step toward this synthesis is finding an appropriate mathematical model for the dynamical system investigated. The model gives a bridge between the reality and the mathematical theory. Modeling is a very complicated phase of the investigation: it requires the knowledge not only of means of mathematics but also the proper theory of the object to be modelled. The book gives an excellent glance into this art. Its sub-title really characterizes its style: it is "a case study approach". At the beginning of every chapter there is a short abstract of the necessary concepts and results from system theory, which is followed by special cases of a very wide spectrum: temperature in a building, electrical circuits, DC motor, vertical ascent of a deep sea diver, automobile suspension system, magnetic loudspeaker, liquid level in a leaky tank, spread of an epidemic, continuous flow stirred tank chemical reactor, motion of a rocket near Earth, etc. The models are classified according to two independent points of view: linear-nonlinear and first order — higher order models are distinguished, respectively.

In many cases a detailed mathematical analysis is not possible for some reason or other. Then a computer simulation, based on the state equations may be required. To make it easier the book presents a special purpose simulation language (Continuous System Modeling Program).

We can recommend this excellent book for undergraduate students, users of mathematics, and for everybody interested in applications of mathematics.

L. Hatvani (Szeged)

Padé Approximation and its Applications. Proceedings, Amsterdam 1980, Edited by M. G. de Bruin and H. van Rossum, (Lecture Notes in Mathematics 888), VI+382 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

The book contains 29 papers delivered at the conference on "Padé and Rational Approximation, Theory and Applications" held at the Institute voor Propedeutische Wiskunde of the University of Amsterdam, October 29—31, 1980. This conference was the sixth in a series of conferences on the above subject in Europe (Canterbury 1972, Toulon 1975, Lille 1977 and 1978, Antwerp 1979), which well illustrates the interest in Padé approximation and related subjects.

The four invited lectures were: C. Brezinski: The long history of continued fractions and Padé approximants; P. R. Graves—Morris: Efficient reliable rational approximation; H. Werner: Nonlinear splines, some applications to singular problems; and L. Wuytack: The conditioning of the Padé approximation problem.

V. Totik (Szeged)

N. U. Prabhu, Stochastic Storage Processes. Queues, Insurance Risk, and Dams (Applications of Mathematics, 15), VI+140 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

The main classes of stochastic models investigated in this book are queueing, insurance risk and dams. The stochastic processes underlying these models are usually (but not always) Markovian, in particular, random walks and Lévy processes. In order to answer important questions concerning these models we have to study various aspects of these processes such as the maximum and minimum functionals and hitting times.

The Introduction contains the definitions of single-server queueing systems, inventory models, storage models, insurance risk and continuous time inventory and storage models. The book is in two parts. In part I the author presents the theory of single-server queues with the first come, first served discipline. This part is based on the close connection between random walks and queueing problems. In the first three sections of Chapter 1, the author proves some important theorems on ladder processes, renewal functions and the maximum and minimum of random walks. The results described here provide answers to most of the important questions concerning this general system, but in special cases of Poisson arrivals or exponential service time, or systems with priority queue disciplines, there still remain some questions. These latter are more appropriately formulated within the framework of continuous time storage models, which is developed in part II.

In part II the author considers a model in which the input is a Lévy process and the output is continuous and is at a unit rate except when the store is empty. In spite of its simplicity, the concepts underlying this model and techniques used in its analysis are applicable in a wide variety of situations, for example, in insurance risk and queueing systems with first come, first served discipline, or priority disciplines of the static or dynamic type.

The book is clearly written, supplied with exercises at the end of sections, but the references are old and not enough to orient in recent developments in stochastic storage processes. It is recommended for every graduate student who has a background in elementary probability theory and wishes to begin studies in this part of applied probability.

Lajos Horváth (Szeged)

Recent Results in Stochastic Programming. Proceedings, Oberwolfach 1979, Edited by P. Kall and A. Prékopa, Lecture Notes in Economics and Mathematical Systems, VI+237 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

This volume contains the papers presented at a meeting on stochastic programming, held at Oberwolfach, January 28—February 3, 1979. It is divided into two parts.

The first, theoretical part consists of the papers by Bereanu, Bol, Brosowski, Groenewegen and Wessels, Heilmann, Haneveld and Rinott about the topics: (stochastic-) parametric programs, multi-stage SLP, minimax rules for SLP, convexity problems, etc.

The second part is devoted to the applications concerning water resources, portfolio selection, asphalt mixing, network planning, etc. Here the authors are: Deák, Dupacová, Kall, Kallberg and Ziemba, Karreman, Kelle, Marti, Prékopa.

An important note from the preface of the volume: "during the last two decades knowledge, theoretical and computational, on stochastic programming, and practical experience with it, have been developed so far, that neglecting a priori the stochastic nature of parameters ... can no longer be justified."

V. Totik (Szeged)

Robert B. Reisel, Elementary Theory of Metric Spaces (A Course in Constructing Mathematical Proofs), 120 pages, Universitext, Springer-Verlag New York, Inc., 1982.

It is only the second goal of this book to teach the elementary theory of metric spaces, the first goal is to teach an understanding of proofs and their constructions on an appropriate field of mathematics.

The book is offered to junior students having a level of mathematical maturity, e.g. a complete course in calculus. The author requires a lot of self-contained work from his students: he gives all the definitions (including the set-theoretical ones), shows examples, states theorems and leaves proofs to the reader. A number of hints is given and the more difficult proofs can be found in an appendix. He says "I think that the best way to use this book is in a seminar ... it could, however, be used in a lecture course where many of the proofs would be assigned to the students. It would be suitable as the text or a supplementary text in courses in general topology, real analysis or advanced calculus." A solitary student studying this book needs a teacher who criticizes his proofs.

The material of the book worked well in the author's seminar at Loyola University of Chicago in the past fifteen years.

L. A. Székely (Szeged)

S. H. Saperstone, Semidynamical Systems in Infinite Dimensional Spaces (Applied Mathematical Sciences, 37), VII+474 pages, Spriger-Verlag, New York—Heidelberg—Berlin, 1981.

Since 1927, when G. D. Birkhoff published his classical monograph on dynamical systems the so-called topological dynamics has given a framework for the qualitative theory of solutions of differencial equations of certain types. Nowadays many attempts have been made to extend results of this theory to new types of equations.

As is known a dynamical system is formed by a group of transformations of a Hausdorff topological space into itself. The family of solutions of an autonomous system of differential equations forms such a group of transformation, but the more general systems (nonautonomous ones, functional-differential equations etc.) do not have this property. But some representations of solutions can be embedded into an appropriate function space where they generally form no longer a group but only a semigroup, in other words, a semidynamical system. Typically the space is not locally compact. The author makes it clear which properties of dynamical systems can be generalized to semidynamical ones, and what special kinds of properties the semidynamical systems have.

Titles of the chapters describe well the topics involved: I. Basic definitions and properties, II. Invariance, limit sets and stability, III. Motions in metric space, IV. Nonautonomous ordinary differential equations, V. Semidynamical systems in Banach space, VI. Functional differential equations, VII. Stochastic dynamical systems, VIII. Weak semidynamical systems and processes.

Each chapter is followed by exercises, notes and comments, and an extensive bibliography. Most of the source material is from the 1960's and 1970's and was previously available only in journals.

This book will be very useful for both mathematicians and users of mathemtics interested in the qualitative theory of differential equations and its applications.

L. Hatvani (Szeged)

Vjačeslav V. Sazonov, Normal Approximation — Some Recent Advances (Lecture Notes in Mathematics, 879), VII+105 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1981.

Although there are many central themes in modern probabilistic research, the historically first such theme, the central limit theorem, will undoubtedly always remain one. These notes, based on a series of lectures the author gave in 1979 at the University of California, Los Angeles, and at Moscow State University, are devoted to the study of the rate of convergence in the central limit theorem for independent and identically distributed random elements in finite dimensional Euclidean spaces and in real separable Hilbert spaces. The aim of the monograph is to outline the main directions and methods in the recent progress of the field. There are basically two main methods here. Quite naturally the author has chosen to emphasize the direct method of convolutions, initiated by Bergström and developed by the author himself, rather than the method of characteristic functions.

Sándor Csörgő (Szeged)

Séminaire de Probalilités XV, 1979/80. Avec table générale des exposés de 1966/67 à 1978/79, IV+704 pages;

Séminaire de Probabilités XVI, 1980/81, V+622 pages;

Séminaire de Probabilités XVI, 1980/81. **Supplément: Géométrie Differentielle Stochastique**, III+285 pages;

Edité par J. Azéma et M. Yor (Lecture Notes in Mathematics, 850, 920, 921), Springer-Verlag, Berlin—Heidelberg—New York, 1981, 1982, 1982.

These three volumes of the traditional seminar notes, centred originally in Strasbourg and now, beginning with the volume XIV, in Paris, show the French school of probability in its best again. The main theme is of course the traditional "general" theory of stochastic processes, but there are many other topics dealt with. It would be impossible to list these here since there are altogether 109 papers in the three volumes (27 in English), but almost everybody working in stochastic processes will find at least one indispensable for him. Even so, the martingale approach to some Wiener—Hopf problems in a two-part longer paper by London, McKean, Rogers and Williams in volume XVI deserves special mention. It is a great help for the readers of this series, and especially for those who do not travel often enough to France, that volume XV contains a complete list of contents of the first fourteen volumes with editorial notes on the correction, rectification, extension, or improvements of many papers in subsequent volumes. The supplement volume on stochastic differential geometry, containing six longer papers by Schwartz, Meyer, Emery, Darling and Azencott, appears to be very important. It provides "the present state of art" of a new and vigorously developing branch of stochastics.

Sándor Csörgő (Szeged)

E. B. Dynkin—A. A. Yushkevich, Controlled Markov Processes (Grundlehren der mathematischen Wissenschaften, 235), XVII+289 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

Optimal control of a stochastic process means the optimal choice of some free parameters influencing the future evolution of the underlying random process, in order to achieve optimal dynamics (minimal costs or maximal rewards). The selection of the control parameters is made on the basis of observations of the past of the process. Evidently this model describes the situation in many engineering and economic problems. Hence stochastic control is by right considered as an applied mathematical discipline.

The original Russian version of the present book was written in the time (1975) when the authors, both of them outstanding mathematicians, worked with the Central Institute of Mathematical Economics in Moscow. They accomplished a work, very rare in the literature, which is equally exciting for mathematicians and specialists motivated by economics. The volume actually deals with the optimal control of discrete-time Markov chains, or, in other terminology, with multi-stage Markov decision problems. In this respect the title is somewhat misleading as the theory of continuous-time processes, where entirely different mathematical problems arise, is not included. Neither are considered computational aspects of the determination of the optimal strategy.

Presenting the material the authors approach step by step from simpler to more advanced problems, always keeping an eye on applications. This way the reader never looses contact with practice, and the necessity of each step towards increasing abstraction is sufficiently motivated. As prerequisites only basic probability and measure theory are required.

Summing up, the present book can be recommended both to mathematicians wishing to cultivate applied probability and to economists intending to solve their problems by mathematical methods. The level of applied mathematical literature would considerably increase if everyone cast a glance at Dynkin and Yuskevich's presentation before sitting down to write his own monograph.

D. Vermes (Szeged)

N. V. Krylov, Controlled Diffusion Processes (Applications of Mathematics, 14), XII+308 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

What can hope a mathematician moving from discrete models to continuous ones? Gain on conformity with reality? More aesthetic simplified theory? Challenging new difficulties? The step between optimal control theory of discrete-time random processes and their continuous-time analogues can offer all the three sorts of rewards. The author seems to prefer confrontation of difficulties to conformity and simplicity. For him the theory of continuous-time stochastic processes is a source of hard enough mathematical problems most of which he masters triumphally.

The general object of stochastic control theory was outlined in the previous review. In order to understand the particularities of continuous-time control problems, the origins and the practical motivations of their theory and to estimate correctly the arising difficulties, the non-specialist should first consult the excellent introduction: W. H. Fleming and R. W. Rishel, Deterministic and Stochastic Optimal Control (Springer-Verlag 1975). Together with the monograph under review, the two books constitute indeed an excellent, high-level presentation of the control theory of continuous processes.

Roughly speaking the determination of the optimal strategy for a controlled diffusion process is equivalent to the solution of a possibly degenerated non-linear parabolic PDE with non-continuous right-hand side — the so-called Bellman equation. But in what sense does this equation have solutions and in what class of functions is there a unique solution? These are the central questions of the theory and once the author answers them, he arrives at a unified theory of classical calculus of variations and of the control of continuous deterministic and stochastic processes. But the technical machinery yielding this theory is not less significant. Some techniques of estimation will perhaps find as much applications in mathematics itself as the whole theory will do in practice.

The style of the book resembles an extended research article on a vividly developing field rather than an explanation of an applications-oriented mathematical theory. In order to make the author's important results available to English reading specialists in a shortest possible time, Springer-Verlag translated the Russian original without any change. (Not even bibliographical hints to meanwhile published proofs of stated results were included.)

The chapter-headings are: 1. Introduction; 2. Auxiliary propositions; 3. General properties of a playoff function; 4. The Bellman equation; 5. The construction of ε-optimal strategies 6. Controlled processes with unbounded coefficients; Appendices, Notes, Bibliography, Index. As "auxiliary propositions" the author presents L_p estimates for stochastic integrals, existence of diffusions with measurable coefficients, Markov property and parameter dependence of solutions of stochastic equations, Itô's formula in Sobolev spaces.

D. Vermes (Szeged)

I. I. Gihman—A. V. Skorohod, Controlled Stochastic Processes, VII+237 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979. (Translated from the Russian by S. Kotz.)

Is it necessary to recommend to readers the most recent monograph of the well-known authors of 12 previous volumes covering most of the theory of stochastic processes, initiators of several important mathematical theories? The 13$^{\text{th}}$ of their books shows once again the brilliant talent of its authors as, seemingly without any serious reading of the existing literature on stochastic control theory, they arrive with a single blow at the proximity of the results achieved in the last decade on this rapidly developing field. Specialists, familiar with the alternative ways leading to the same results, will find interesting the authors' approach via weak approximation which has several advantages as pointed out in H. J. Kushner, Probability Methods for Approximation in Stochastic Control and for Elliptic Equations (Academic Press, 1977).

As prerequisites, rudiments of measure theory and functional analysis and a knowledge of the theory of stochastic processes are supposed, the latter at the approximate level of the three-volume treatise by the same authors. Besides specialists of stochastic control theory the book will turn out to be useful to any mathematician learning Russian. As the English translation preserves the typically Russian structures and English terms are chosen much nearer to their Russian originals than the commonly used English terminology, the text suits ideally to translation exercises (from English to Russian).

D. Vermes (Szeged)