

Bibliographie

A. C. Bajpai, I. M. Calus, and J. A. Fairley, Numerical methods for engineers and scientists: (A students' course book), XII+380 pages, Taylor & Francis Ltd, London, 1975.

The book comprises three 'Units': 1. Equations and Matrices, 2. Finite Differences and their Applications, 3. Differential Equations. The emphasis is on the practical side of the subject and the more theoretical aspects are omitted. The reader should be familiar with the items listed under the heading of Prerequisites at the beginning of each Unit. There are several references to the suitability of methods presented for programming on a computer. As different programming languages are in use, the various techniques discussed are not, with one exception, translated into computer programs, but a large number of flow diagrams are incorporated in the text.

The programmed method of presentation requires the active participation of the reader in many places where he is asked to answer a question or to solve, either partially or completely, a problem. The answers to these are always given so that the reader can check his attempt and thus obtain a continuous assessment of his understanding of the subject.

The book will certainly be useful as a textbook for both science and engineering students.

F. Móricz (Szeged)

H. Bühlmann—L. Loeffel—E. Neivergelt, Entscheidungs- und Spieltheorie. Ein Lehrbuch für Wirtschaftswissenschaftler (Hochschultext), XIII+311 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1975.

In everyday life, and especially in management praxis one often has to make decisions sequentially in a process in which some external effect modifies the evolution between two consecutive steps. The decision-maker wants, of course, to choose those decisions which ensure the most favourable evolution of the process, in other words, he wants to maximize his reward.

Decision and game theory deals with the mathematical analysis of such, so-called sequential, decision processes. If the influencing external effect is another decision-maker acting according to his own preference (reward) structure, then the corresponding process is called a game. If the disturbing effect is simply the chance, or, in other words, a non-interested decision-maker, then one faces a simple sequential decision problem. Risk theory, Wald's statistical decision theory and decision making under uncertainty are the most important sub-fields of decision theory.

The present book is a first introduction to decision and game theory. It was written for students in management science, and requires a mathematical education on secondary school level only.

The first part of the book deals with decision theory including utility theory. The second part is devoted to game theory, while the third one to statistical decision theory. Two mathematical appendices contain the more elaborate proofs, and a bibliography and an index close the volume. 121 figures and many explicitly solved examples help to understand the text.

D. Vermes (Szeged)

Surveys in Combinatorics, Proceedings of the 7th British Combinatorial Conference, ed. B. Bollobás, VII+261 pages, Cambridge University Press, Cambridge—New York—London—Melbourne, 1979.

These excellent surveys cover many basic areas in combinatorics and give a good picture of recent developments of the field. The papers are the following: N. L. Biggs: Resonance and reconstruction; A. Gardiner: Symmetry conditions in graphs; D. J. Kleitman: Extremal hypergraph problems; W. Mader: Connectivity and edge-connectivity in finite graphs; J. Nešetřil and V. Rödl: Partition theory and its applications; J. J. Seidel: Strongly regular graphs; J. A. Thas: Geometries in finite projective and affine spaces; C. Thomassen: Long cycles in digraphs with constraints on the degrees; D. Welsh: Colouring problems and matroids.

L. Lovász (Szeged)

Siegfried Brehmer, Hilbert-Räume und Spektralmaße, 224 Seiten, Akademie-Verlag, Berlin, 1979.

Der Hauptteil dieses Bändchens in der Reihe „Wissenschaftliche Taschenbücher“ ist der Theorie der beschränkten linearen Operatoren gewidmet. Im Mittelpunkt steht die Spektralzerlegung beschränkter selbstadjungierter Operatoren, die dann auch auf den Fall unbeschränkter selbstadjungierter Operatoren ausgedehnt wird. Der Rest bringt eine relativ elementare, aber gründlich ausgearbeitete Einführung in die Theorie der Spektralmaße und Spektralintegrale und gipfelt in der Bereitstellung der Funktionalkalküls für meßbare Funktionen von (nicht notwendig beschränkten) normalen Operatoren. Der Verf. stützt sich natürlich auf Standardwerken, macht aber gelegentlich auch Vereinfachungen und Erneuerungen, die teilweise seine Kollegen und Studenten gefunden haben.

Béla Sz.-Nagy (Szeged)

Shiing-shen Chern, Complex manifolds without potential theory (with an Appendix on the geometry of characteristic classes), V+152 pages. Second Edition, Springer Verlag, Berlin—Heidelberg—New York, 1979.

The new methods of complex manifold theory are very useful tools for investigations in algebraic geometry, complex function theory, differential operators and so on. The differential geometrical methods of this theory were developed essentially under the influence of Professor S.-S. Chern's works. The present book is a second edition; it was originally published by Van Nostrand in 1968. It can serve as an introduction to, and a survey of, this theory and is based on the author's lectures held at the University of California and at a summer seminar of the Canadian Mathematical Congress.

The methods of complex manifold theory have grown parallel to the Hodge—De Rham theory of harmonic integrals, which is an analogue of classical potential theory. The treatment of this book leaves out of consideration these analytical aspects of the theory; the title hints at this circumstance.

The text is illustrated by many examples. The reader is supposed to be acquainted with some differential geometry, fibre bundle and sheave theory. The book is warmly recommended to everyone interested in complex differential geometry.

P. T. Nagy (Szeged)

Shiing-shen Chern, Selected Papers, XXXII+476 pages. Springer Verlag, New York—Heidelberg—Berlin, 1978.

This book is a presentation of a fascinating personal Oeuvre and at the same time of the many-sided progress in differential geometry in the last 45 years. The volume contains approximately one third of Professor Chern's works, among them also some less known fundamental papers published in inaccessible journals.

The selection is introduced by three papers presenting Chern's mathematical and personal oeuvre written by André Weil, Phillip A. Griffiths and S.-S. Chern himself with the titles: "S.-S. Chern as Geometer and Friend", "Some Reflection on the Mathematical Contributions of S.-S. Chern" and "A Summary of My Scientific Life and Works", respectively.

Chern's investigations can be put into the following domains of differential geometry according to his own classification: projective differential geometry, euclidean differential geometry, geometric structures and their intrinsic connections, integral geometry, characteristic classes, holomorphic mappings, minimal submanifolds, webs. His results give programs for future research, and at the same time they pursue the geometric view of his masters: Wilhelm Blaschke and Elie Cartan.

This excellent book should not be missing in any mathematical library.

P. T. Nagy (Szeged)

P. Gänsler and W. Stute, Wahrscheinlichkeitstheorie (Hochschultext/Universitext), XII+418 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

The book is intended to serve as a graduate text in probability theory. No knowledge of measure or probability theory is presupposed, only a few notions and results from analysis, linear algebra and set theory are required. These prerequisites are collected in Ch. 0.

The text comprises the major theorems of probability theory and the measure theoretical foundations of the subject. The material of Chapters 1—6 may be considered as an introductory course in probability theory: 1. Measure theoretical tools and basic notions of probability theory, 2. Laws of large numbers, 3. Empirical distributions, 4. The central limit theorem, 5. Conditional expectations and distributions, 6. Martingales. The material of Chapters 7—10 may form the basis of an advanced course: 7. Stochastic processes, 8. Random elements in metric spaces, 9. Central limit theorems for martingale difference schemes, 10. Invariance principles.

There are exercises and remarks at the end of each chapter. The book is supplemented with a bibliography consisting of 154 items, a list of symbols and conventions, an author and subject index.

The textbook is written in a concise but always clear and well-readable way. We warmly recommend it to both students and lecturers at universities and technical colleges.

F. Móricz (Szeged)

I. I. Gihman—A. V. Skorohod, The theory of stochastic processes I, II, III (Grundlehren der mathematischen Wissenschaften 210, 218, 232), VIII+570, VII+441, VII+387 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1974, 1975, 1979.

Very few scientists show up in our age of specialization who would try to make an effort to penetrate in almost every important part of the whole branch of a mathematical field. This is what Professors Gihman and Skorohod do with the theory of probability and stochastic processes. That this is indeed so is recognized if the three-volume treatise under review is looked at as a part of

a larger series of books. This series consists of three more books by the same authors, another joint book by Professor Skorohod and N. P. Slobodenyuk, and five books by Skorohod alone.

In these three volumes the authors endeavoured to present an exposition of the basic results, methods and applications of the theory of random processes. The various branches of the theory cannot, however, be treated in equal detail. A knowledge of basic probability and measure theory, as well as real and complex variable function theory and functional analysis (especially Hilbert space theory) is required from the reader. Therefore, these volumes are intended for professional mathematicians and graduate students rather than for undergraduates. A substantial number of the results are appearing in non-periodical literature for the first time, and there are results which have not been published even in periodicals. A great number of proofs of known results are also new. Since the authors are able to review the material in a long perspective, there is no doubt that this three-volume monograph will be one of the main references and sources of inspiration for research for a long time to come.

In what follows we can only try to indicate the contents by giving some key words.

Volume I. Chapter I (Basic notions of probability): axioms, independence, conditional expectation, random functions and mappings, Kolmogorov's fundamental theorem. *Chapter II* (Random sequences): martingales, semi-martingales, Markov chains, lattice random walk with vector jumps, stationary sequences, Birkhoff—Hinchin theorem. *Chapter III* (Random functions): Gaussian, Markov, independent increment processes, Doob's theorem on separable and measurable equivalents, criteria for the absence of second kind discontinuities, Kolmogorov's criterion for continuity. *Chapter IV* (Linear theory of random processes): second order random functions in a linear space, spectral decomposition of correlation functions of processes and fields, L^2 -continuity, -differentiability, -integrability, and -decomposability into orthogonal series. Stochastic measures and integrals, integral and spectral representations of second-order processes and fields. Linear transformations, admissible and physically realizable filters, filtering of stationary processes with minimal mean square error, forecasting. *Chapter V* (Probability measures on function spaces): Conditions for realizability of measures on function spaces endowed with metric or vector structure, positive definite functionals and measures on a Hilbert space X , characteristic, linear and quadratic functionals and Gaussian measures on X . *Chapter VI* (Limit theorems for random processes): weak compactness and convergence of probability measures in metric and Hilbert spaces, limit theorems for sums of independent variables in a Hilbert space, convergence of continuous processes and processes with no second kind discontinuities. *Chapter VII* (Absolute continuity of measures associated with random processes): densities of measures, admissible shifts of measures on a Hilbert space, absolute continuity under mappings, applications for Gaussian and Markov processes. *Chapter VIII*. (Measurable functions on Hilbert spaces): conditions for continuous approximation (in measure) of linear functionals, operators and mappings, orthogonal polynomials for Gaussian measures.

Volume II. Chapter I (Basic definitions and properties of Markov processes), *Chapter II* (Homogeneous Markov processes): semigroup theory, strong Markov property, local behaviour of sample paths, Feller processes, processes in locally compact spaces, cut-off and non-cut-off processes, multiplicative and additive functionals, excessive functions. *Chapter III* (Jump processes): structure of sample paths, homogeneous Markov processes with a countable set of states, semi-Markov jump processes, Markov processes with a discrete component. *Chapter IV* (Processes with independent increments): decomposition into discrete and stochastically continuous processes, conditions for the latter to be Poisson, Lévy representation for the characteristic function of the increments, distribution of functionals concerning fluctuations (supremum, arrival time, size of jumps), local behaviour, growth at infinity, vector-valued jump processes. *Chapter V* (Branching processes): branching Markov processes with a finite number of particles, infinitesimal characteristics of branching processes with a continuum of states, general Markov processes with branching.

Volume III. Chapter I (Martingales and stochastic integrals): quasi-martingales, stopping and random time substitution, decomposition of supermartingales, (local) square integrable martingales, continuous characteristics. Stochastic integrals over locally square-integrable martingales and martingale measures. Itô's formula, stochastic differentials, bounds on moments, representation of martingales by integrals over a Wiener measure, decomposition of locally square integrable martingales. *Chapter II* (Stochastic differential equations): the stochastic line integral, existence and uniqueness, finite-difference approximations, solutions of stochastic differential equations without an after-effect as a Markov process, differentiability with respect to initial data of solutions, limit theorems of stochastic differential equations. *Chapter III* (Stochastic differential equations for continuous processes and continuous Markov processes in R^m): Itô processes, processes of diffusion type, existence and uniqueness, diffusion processes in R^m , homogeneous processes with integrable kernel of a potential, local structure of continuous homogeneous Markov processes in R^m , M -functionals; the rank of a process, continuous processes in R^1 .

Apart from the bibliography, each volume ends with a section of historical and bibliographical remarks and a (not too rich) subject index. Also, an Appendix is included in Vol. III, correcting some errors in the first two volumes. All three volumes were translated by Samuel Kotz who has done a superb job.

Sándor Csörgö (Szeged)

S. A. Greibach, Theory of program structures: Schemes, Semantics, Verification (Lecture Notes in Computer Science, 36), 364 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

Investigations concerning semantics play a fundamental rôle in computer science. This book contains the material of a first course on schematology, dealing with one approach to formalizing the elusive notion of the "semantics of programming languages". It is a nice introduction intending to make the reader familiar with the theory of program schemes and related topics.

In accordance with the introductory feature of the book, numerous examples are included to illustrate each new construction and many of the proofs, while in some cases the formal proofs are given in outline only. The book concludes with a large number of exercises. All these greatly help the reader to understand the main ideas.

As familiarity with formal languages and finite state machines makes the understanding of some chapters easier, we recommend this book first of all to students with this background.

G. Maróti (Szeged)

Maurice Holt, Numerical Methods in Fluid Dynamics (Springer Series in Computational Physics), VIII + 253 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

At the present time the majority of unsolved problems in fluid dynamics are governed by non-linear partial differential equations and can only be treated by a numerical approach. The development of large-scale computers have formed a basis for algorithmic constructions and extensive mathematical experiments in this area, too, as a result of which a lot of principal advances have been recently made in numerical methods.

The first part of this monograph describes two recent finite difference methods, both developed in the USSR. The first is due to Godunov (Ch. 2) originally presented in 1960 and revised in 1970. The second method was developed principally by Rusanov in 1964 in collaboration with Babenko, Voskresenskii and Liubimov, and is familiarly known as the BVL method (Ch. 3). Both the Godunov and BVL methods have their origins in the method of characteristics (in two dimensions). Ch. 4 contains the method of characteristics for three-dimensional problems.

The second part treats the methods of integral relations (Ch. 5) introduced by Dorodnitsyn in 1950 and extended in 1960, the method of lines and Telenin's method (Ch. 6) developed from 1964 onwards. The objective of all these methods is to eliminate finite difference calculations in one or more coordinate directions by using interpolation formulae, especially polynomials or trigonometric functions, to represent the unknowns in selected directions.

The presentation is made for graduate students in engineering or applied mathematics with basic knowledge of fluid mechanics, partial differential equations and numerical analysis. Many applications and samples of numerical solutions of model problems are presented.

The book is warmly recommended to everyone practicing numerical analysis in industry or teaching at universities and technical colleges. It will certainly stimulate some of the readers to look for further effective numerical methods to attack the rather difficult problems of fluid dynamics.

F. Móricz (Szeged)

E. H. Lockwood and R. H. Macmillan, *Geometric Symmetry*, X+228 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1978.

This large-scale summarizing work retrieves a long-time missing unified basic collection of discrete symmetry groups and present them not only for the specialists of this discipline, but also for artists and for the interested general public.

The book is divided into a "Descriptive" part and a part on "The mathematical structure". (Both parts discuss discrete symmetries of spaces of dimension not higher than 3 and this splitting of themes is no benefit for the user who wishes to find all information about say, the frieze-groups or the plane-groups in the same place.)

The book consists of the following chapters: Part I. 1. Reflexions and rotations, 2. Finite patterns in the plane, 3. Frieze patterns, 4. Wallpaper patterns, 5. Finite objects in three dimensions, 6. Rod patterns, 7. Layer patterns, 8. Space patterns, 9. Patterns allowing continuous movement, 10. Dilation symmetry, 11. Colour symmetry, 12. Classifying and identifying plane patterns, 13. Making patterns; — Part II. 14. Movements in the plane, 15. Symmetry groups. Point groups, 16. Line groups in two dimensions, 17. Nets, 18. Plane groups in two dimensions, 19. Movements in three dimensions, 20. Point groups in three dimensions, 21. Line groups in three dimensions, 22. Plane groups in three dimensions, 23. Lattices, 24. Space groups I, 25. Space groups II, 26. Limiting groups, 27. Colour symmetry.

It must be noted that the references are incomplete. For instance there is no reference to the works of Coxeter or Fejes Tóth in discrete geometry. Perhaps this is the cause of some mistake in the historical introduction: it were not Pólya or Niggli who first enumerated the 17 wallpaper groups in 1924, but Fedorov in 1891 and later, independently of him, Fricke and Klein in 1897.

A particular virtue of the book is the Notation and Axes supplementary chapter which symbolizes the beneficent endeavour of the authors to unify the notation system of the groups studied. Perhaps because the main user of these symmetry groups is crystallography, the authors' efforts aim to generalize the crystallographic notation, although that is not ideal owing to its redundancy. As far as we know this book is a pioneering work not only in summarizing geometric symmetry but in the unification of its notation, too.

Its clear structure, neat way of exposition and abundant illustrations in color make this excellent book an attractive reading, a valuable and useful help for teachers on all levels (in secondary or high schools, or at universities), and even for artists, textile designers, architects, etc.

Dénes Nagy and Szaniszló Bérczi (Budapest)

László Lovász, *Combinatorial Problems and Exercises*, 551 pages, Akadémiai Kiadó and North Holland Publishing Company, Budapest, 1979.

Though the roots of combinatorics go back to the 18th and 19th centuries, it has become a coherent discipline in the last twenty years only. Mostly isolated theorems were known beside the earlier developed enumeration techniques. The recent extremely rapid development of combinatorics was influenced by the occurrence of combinatorial problems connected with computer science, operation research, statistics, coding theory etc. The enormous quantitative increase has been accompanied by the appearance of several new methods, techniques and theories. This development is manifested also by the increase in the number of books from no more than a dozen in the middle of this century to several hundreds today. Lovász' book is a masterpiece among them.

In spite of the modest title it is not just a collection of problems but it builds up more than a dozen "theories" and techniques in combinatorics, some of them presented here for the first time as a coherent topic.

It is a three-level version of the classical book of Pólya—Szegő: *Aufgaben und Lehrsätze aus der Analysis*, containing parts as Problems, Hints and Solutions. These cover classical theorems and the latest results as well. A large part of the text has appeared previously in research papers only. In many cases the proofs are much simpler than the original ones.

The first four chapters are devoted to enumeration; generating-function techniques (the first developed techniques in Combinatorics), the famous Pólya method (used for some classical problems on partitions), sieve methods, a large part of the latter in probabilistic setting such as M. Hall's and Rényi's method for coding permutations, enumeration of trees and one-factors.

§ 5 is on duality and parity. Here the nature of the solutions unifies the material more than the problems themselves. § 6—§ 7 deal with connectivity, Menger—König—Hall—Tutte—Edmonds type factorization theorems, the 'max-flow — min cut' theorem, a subject which is strongly connected with linear programming.

Chromatic number is a concept whose origin goes back to the last century (Four Colour Conjecture). It is now of completely independent interest; e.g. chromatic polynomials and the problem of characterization of critical graphs concerning the chromatic number are considered in § 8.

§ 9 deals with independent sets, characterization of critical graphs concerning maximal independent sets and their applications; e.g. game-theory.

§ 10 contains extremal problems characterized by Turán's theorem, and several problems on Hamiltonian lines.

§ 11 and § 12 deal with algebraic graph theory, spectra of graphs and automorphisms of graphs.

§ 13 contains hypergraph theory, including intersection theorems like the Sperner and the Erdős—Ko—Rado theorems, fractional and integer matching and covering, and it ends with Lovász' perfect graph theorem. Various proof techniques are demonstrated, some of them developed, partly or fully, by the author himself.

§ 14 contains the Ramsey theory. This includes Ramsey-type theorems for systems of finite sets and also other structures (as integers, vector spaces, arithmetic progressions). The last chapter is devoted to reconstruction problems.

Throughout the book there is a strong emphasis on "good characterization", on algorithmic aspects, on the connection of combinatorics with integral linear programming, on the use of linear algebra, and on probabilistic setting.

The exposition is extremely clear and elegant. The author seems always to find the simplest way to prove the deepest theorems.

The book is highly recommended not only to young researchers but also to the specialists in combinatorics and the mathematical public in general.

It will undoubtedly not only "help in learning existing techniques in combinatorics" but will also stimulate new ideas.

"Some fields have had to be completely omitted: random structures, integer programming, matroids, block designs, lattice geometry, etc. I hope eventually to write a sequel to this volume covering some of these latter topics." Having an outstanding book like this we are looking forward to the next volume.

Vera T. Sós (Budapest)

J. D. Monk, *Mathematical Logic* (Graduate Texts in Mathematics, 37), 531 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

This book is based on the author's lectures given at various universities.

After a survey of recursive function theory and the elements of logic, the reader is made familiar with the concept of first order languages and the basic facts concerning them. This part of the book serves as a preparation for the following chapters, dealing with decidable and undecidable theories and other topics in model theory. The book concludes with touching upon several other kinds of logics, e.g., many-sorted logic, second-order logics, etc.

At the end of each chapter the reader finds references and a rich variety of interesting exercises.

We recommend this excellent work to everyone interested in, or dealing with, mathematical logic. First or second year graduate students can study sentential logic and its relationship to Boolean algebras by reading chapters 8 and 9 only. Because of the very abstract nature of the subject we suggest reading the whole book first of all to postgraduate students, as well as young logicians, who thereby can be helped efficiently in preparing the material of their lectures on mathematical logic

G. Maróti (Szeged)

H. Rademacher, *Lectures on Elementary Number Theory*, IX+146 pages, Robert E. Krieger Publishing Co., Huntington, New York, 1977.

Number theory is full of problems and results that most mathematicians know, but the general feeling about their proof is that it is very difficult and technical. We all know about the prime number theorem, quadratic reciprocity, Dirichlet's theorem on primes in arithmetic progressions, Brun's theorem on twin primes, just to mention some of the most classical examples. But very few of us have had the possibility of learning the proofs of these facts, although these proofs are not as inaccessible as believed. This, of course, is a pity, because a major contribution of number theory to mathematics is in the powerful methods which emerge from the solutions of its simple yet very difficult, challenging problems. Some of the "elementary" proofs in number theory may contain the kernel of other more general mathematical theories. This is why it is great to be able to read accounts of some of these classical difficult problems in number theory in a form accessible to non-specialists, in particular students.

This classic book, whose second printing is reviewed here, discusses some of these well-known but not generally well-understood problems in "elementary" number theory ("elementary" only means that no complex function theory is used: real calculus is used and the field is full with complicated, ingenious arguments). It is not a textbook but it does start with the basics: unique factorization, Farey fractions, linear Diophantine equations, congruences. It gets to quadratic reciprocity through an interesting detour to constructing regular heptadecagons, Lagrange resolvents and Gaussian sums. After discussing lattice point techniques and some results on prime distribution

like Chebyshev's theorem, the book gives the proof of Dirichlet's theorem on primes in arithmetical progressions, and of Brun's theorem on the convergence of the sum of reciprocals of twin primes.

This book is indeed recommended to everyone, in particular to students: its material belongs to what may be regarded as "basic mathematical intelligence", its presentation is easy to follow and yet it leads the reader to the deepest "elementary" results in number theory.

L. Lovász (Szeged)

R. D. Richtmyer, Principles of advanced mathematical physics. I (Texts and Monographs in Physics), XV+422 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978. — DM 44,—.

As the author points out in the preface, nowadays physics cannot apply intuitive methods as earlier, it needs a high level adequate mathematics of a wide range. However, branches of mathematics are used from a specific physical point of view, i.e., some of the mathematical theories are irrelevant to physics while some results marginal to the mathematical theories have great importance in physics. The aim of the book is to collect mathematics from this special physical point of view. The title is somewhat misleading because the book does not concern any principles; it concentrates on Hilbert and Banach spaces and distributions, linear operators and their spectra, with special attention to operators that emerge from differential equations in physics.

There is a great demand for such books which can serve as basic ones for students. That is why it is a pity that measure theory is not treated thoroughly, hence probability theory cannot be set forth in its natural way and the spectral theories of self-adjoint and unitary operators are formulated by spectral families (resolutions of the identity) instead of projection valued measures, involving thus more complicated tools.

The treated material is essential for general understanding of physics (except perhaps the last chapter: non-linear problems; fluid dynamics); the presentation of the subject is clear and suitable for the purpose of the author. The book will certainly prove very useful for students in physics.

T. Matolcsi (Budapest)

A. N. Shiryaev, Optimal Stopping Rules (Applications of Mathematics, Vol. 8), X+217 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1978.

What is the secret of a successful life? Perhaps nothing but the ability to stop each activity at the right moment. The vital applicability of the theory of optimal stopping is consequently beyond doubt. But there was one more reason why to publish the present volume in the series "Applications of Mathematics". Namely the theory of optimal stopping itself serves as an interesting field of application for other deep mathematical disciplines.

The present book is a well-written, concise presentation of the beautiful round theory of optimal stopping of Markov processes. Although it is shown that all interesting non-Markovian stopping problems can be reduced to equivalent Markovian ones, the decision of the author to restrict himself to Markov processes was a step off the applications in favour of the methodological closedness of the theory. The emphasis of the volume lies on the demonstration how potential- and martingale-theoretical results can be applied to solve the mathematical problem of optimal stopping. Possible applications of optimal stopping theory are only outlined. But this incompleteness from the side of applications does not lessen the value of the book. On the contrary, it has the effect of forcing the reader to think it over and fill up the gaps by himself. This way the passive reader is converted into an

active partner in research. Besides conciseness and theoretical clarity, this is the very property which makes the book extremely fitting to serve as a basis for a half-year course for advanced students in probability. The presented material can also be regarded as a first non-trivial introduction to the theory of filtration and control of stochastic processes.

The Russian original was substantially improved and enlarged before translation. The chapter-headings are 1. Random Processes: Markov Times; 2. Optimal Stopping of Markov Sequences; 3. Optimal Stopping of Markov Processes; 4. Some Applications to Problems of Mathematical Statistics. A detailed bibliography and an index close the volume.

D. Vermes (Szeged)

Dietrich Stoyan, Qualitative Eigenschaften und Abschätzungen stochastischer Modelle, X+198 pages, Akademie-Verlag, Berlin, 1977.

The theories of queues, inventories, dams, risks and reliability belong to the oldest spheres of applied probability, and even non-specialists know that they are merely different interpretations of the same mathematical discipline. It is not the lack of a common language that gives rise to the very non-homogeneous outlook of these theories, but rather the dissimilarity of the applied techniques.

The situation very much resembles to the early decades of the theory of differential equations, when the explicit form of the exact solutions was of primary interest. At that time the necessary approaches varied from equation to equation. Only Liapunov's direct method and the monotonicity methods (differential inequalities and fixed-point theorems) have opened the fundamentally new prospects of the uniform, so called qualitative, theory of differential equations.

The stochastic theory of queues, inventories, etc. now stands at the beginning of a similar vigorous development. The aim of the present booklet is to awake interest in this new field. Most of the book deals with monotonicity methods, based mainly on the author's own results. Although these techniques are far not as powerful at the present stage as their deterministic analogues (they are used only for obtaining some estimates), they appear to be a first step towards a uniform qualitative theory of queues, reliability etc. The last chapter of the book gives a short glimpse into the modern but already well-developed stability theory of stochastic models.

The purpose of the author is to give a first introduction and therefore the more laborious proofs are only sketched. The reader is supposed to have some pre-knowledge in the theory of queues, inventories, etc. The language of the book is clear, but, due to some long definitions and complicated formulations, it is not very easy-flowing. Some open problems, an extensive bibliography and an index supplement the volume.

D. Vermes (Szeged)