Bibliographie

Arthur L. Besse, Manifolds all of whose geodesics are closed (Ergebnisse der Mathematik und ihrer Grenzgebiete, 93), IX+262 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

It had been a long-standing open problem, originating from W. Blaschke, whether an oriented "Wiedersehensfläche" is necessarily isometric to a sphere. (A Riemannian 2-manifold M is a "Wiedersehensfläche" if for every $x \in M$ there exists a $y \in M$ such that each geodesic starting from x passes through y. The question was solved affirmatively by L. Green in 1963, but many interesting problems, closely related to the previous one, were left open. The aim of this book is to give a detailed introduction and a comprehensive survey of the results and open questions in this topic.

In the first two chapters the author gives a short introduction to Riemannian geometry, geodesic flows and manifolds of geodesics. Chapter 3 presents a complete treatment of the geometric properties of compact symmetric Riemannian spaces of rank one, which are the basic examples of manifolds all of whose geodesics are closed. Chapter 4 deals with the geometry of Zoll and Tannery surfaces, which are further examples of such maifolds. Chapter 5 is devoted to the proof of Blaschke's "Wiedersehensfläche" conjecture and related questions. In Chapter 6 the geometry of geodesics in a harmonic manifold is studied. In Chapters 7—8 several results concerning topological invariants and the spectrum of the Laplace operator on a maifold all of whose geodesics are closed, are proved. The book closes with 5 Appendices written by D. V. A. Epstein, J. B. Bourguignon, L. B. Bergery, M. Berger, and J. L. Kazdan.

The book is well organized. It contains a detailed description of various Riemannian manifolds which are very close to the standard non-euclidean spaces from the geometric view-point and which have not been considered in earlier monographs. The book is highly recommended to anyone interested in the geometry of Riemannian manifolds.

P. T. Nagy (Szeged)

Eugen Blum—Werner Oettli, Mathematische Optimierung. Grundlagen und Verfahren (Ökonometrie und Unternehmensforschung, Bd. 20), XIV+413 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1975.

Der Band gibt einen sehr guten Überblick über dieses in dem letzten Jahrzehnt sich rapid entwickelnde Wissensgebiet. Einen bedeutenden Teil seines Umfangs nimmt die Beschreibung der Methoden und Verfahren der nichtlinearen Optimierung ein, wenn auch das zweite Kapitel die Zusammenfassung der grundlegenden Ergebnisse der linearen Programmierung enthält. Die Verfahren der nichtlinearen Programmierung, die auf gleichem Grundprinzip liegen, bilden je eine Verfahren-Familie. Jedes Kapitel des Buches stellt, nach der Mitteilung der diesbezüglichen theoretischen Kenntnisse, je eine Familie durch ein oder zwei konkrete Verfahren vor.

Die behandelten Verfahrenstypen (gleichzeitig Kapitelaufschriften) sind wie folgt: 5. Optimierung ohne Restriktionen; 6. Projektions- und Kontraktionsverfahren; 7. Einzelschrittverfahren; 8.

Schnittverfahren; 9. Dekompositionsverfahren; 10. Strafkostverfahren; 11. Verfahren der zulässigen Richtungen; 12. Das Verfahren der projizierten Gradienten.

Die Verfasser hatten besonderen Wert auf die theoretische Begründung der behandelten Verfahren gelegt und darauf, dass die Verfahren, die im Buch vorkommen, auch für Computers gut verwendbar sind. — Es gibt zwei Kapitel die ausschliesslich der theoretischen Begründung dienen: das eine Kapitel beschäftigt sich mit den Optimalitätsbedingungen, das andere mit der Dualitätstheorie.

Diese Monographie, die sowohl umfassende theoretische Kenntnisse als auch praktische Verfahren darbietet, kann jenen zum Studieren empfohlen werden, die sich im Themenkreis mathematische Optimierung weitläufige Kenntnisse erwerben möchten. Dem Textteil ist eine umfangreiche Bibliographie der nichtlinearen Optimierung beigefügt, die für die Spezialisten die weitere Orientierung ermöglicht.

L. Megyesi (Szeged)

J. C. Burkill, A First Course in Mathematical Analysis, Vi+186 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1978.

This is the first paperback edition of this textbook. The previous editions were published in 1962 (first edition), and in 1964, 1967, 1970, 1974 (reprints). The exposition is very clear and straightforward, the symbolism is very simple. The chapter on functions of several variables is quite short, it does not include the integration of such functions. There are many valuable exercises. The book can be recommended to undergraduate students of mathematics or to anyone who knows high school mathematics and wishes to start studying mathematical analysis.

József Szűcs (Szeged)

J. S. R. Chisholm, Vectors in three-dimensional space, XII+293 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1978.

This book deals with vector algebra and analysis and their application to three-dimensional geometry and to the analysis of fields in 3-space. Both the "pure" and "applied" aspects are considered. The text starts with the algebra of vectors based on the axioms of vector space algebra. When the axioms are introduced, their geometrical interpretation is given, so that they can be understood intuitively. The axiomatic scheme is extended to provide a definition of Euclidean space. The scalar and outer products in 3-space are also introduced in a geometric way. Descriptions of coordinate transformations, congruence and general linear transformations in terms of matrices are given and tensors in 3-space are defined in a classical manner by means of transformation laws. Another part of the text deals with vector analysis. This part begins with the definition and differentiation of curves and surfaces, and with a short account of the differential geometry of curves. Surface and volume integrals are also defined. At the end of the text the differential calculus of scalar and vector fields are investigated and two versions of Stokes' theorem are proved. All chapters contain a large number of problems, some of them are solved at the end of the book.

The book can serve as a textbook for undergraduate students.

L. Gehér (Szeged)

P. M. Cohn, Skew field constructions (London Mathematical Society Lecture Note Series, 27), XII+253 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1977.

This book is based on courses and lectures given by the author at numerous universities all over the world in the years 1971—1976. The purpose is to describe some methods of constructing skew fields (also called division rings), the starting point being the "coproduct construction", the author's famous result (1971) on the existence of a universal field of fractions of any coproduct of skew fields. This construction and the powerful coproduct theorems of G. M. Bergman (1974) form the background of the subsequent topics: a general discussion of skew field extensions in terms of presentations, the word problem for free fields and the solving of equations.

Only familiarity with the material of a standard algebra course is supposed on the reader's part, as the first three chapters summarize the classical results, e.g. Ore' s method of skew polynomials, skew power series and extensions of finite degree. The book is recommended first of all to research workers and postgraduate students, who want to get acquainted with this comparatively new branch of algebra which developed extremely rapidly during the past decade.

Á. Szendrei (Szeged)

Pierre Collet—Jean-Pierre Eckmann, A Renormalization Group Analysis of the Hierarchical Model in Statistical Mechanics (Lecture Notes in Physics, 74), 199 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

The present book deals with one of the most interesting methods of statistical physics, the so-called renormalizition group method. More precisely, the hierarchical models are investigated in detail. In this case the renormalization group method leads to a relatively simple non-linear integral equation. The investigation of this equation makes it possible to obtain a rigorous description of critical phenomena in the hierarchical models. Let us remark that the (rigorous) description of critical phenomena is one of the most difficult problems in statistical physics, and it is solved only in special cases.

The authors give a bibliography of the most important works on hierarchical models, and also prove several new results.

The book consists of two parts. In the first part the main definitions and theorems are given, and the different aspects of the renormalization group technique are discussed. The second part contains the detailed proofs.

The notation of hierarchical models is introduced in Sections 1.1 and 1.2, and a probabilistic interpretation of the renormalization group is given here also. The basic non-linear equation of the theory of hierarchical models is deduced, the critical models are investigated, and in particular the critical indices are computed. In Sec. 1.3 a very important theorem is given about the existence of non-gaussian solution of the basic non-linear equation.

Sections 1.4—1.6 contain several difficult theorems, connected with the computation of the critical indices. From a probabilistic point of view the problem is to determine the limit distribution of the average spin with an appropriate norming factor when the temperature is in a small neighbourhood of the critical temperature T_0. If the temperature T is above the critical temperature T_0 then the limit distribution is gaussian with variance $\sigma=\sigma(T)$ tending to infinity as $T\to T_0$. In case $T<T_0$ this distribution is the mixture of two gaussian distributions. If $T=T_0$, then the distribution is the solution of the basic nonlinear equation investigated in Sec. 1.3. We remark that the first results of this type were obtained in the works of Blecher and Sinai, but the proofs given in the second part of this book are considerably different. Sec. 1.7 contains a proof about the existence of the thermodynamical limit of free energy, of magnetisation, and other observations, and the critical indices of the hierarchical models are directly computed.

The book is written in a clear and concise form. It is an excellent introduction to this rapidly developing field. It may be very useful both for mathematicians and physicists.

Péter Major (Budapest)

B. Davis, Integral transforms and their applications (Applied Mathematical Sciences, 25), XII+411 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1978.

The book is intended to serve as introductory and reference material for the application of integral transforms to the solution of mathematical problems in the physical, chemical, engineering and related sciences. The material involved is rather selective than encyclopedic. There are many facets of subject, which are omitted or only outlined. On the other hand, the material is treated in various aspects and illustrated by appropriately chosen application examples.

The book is divided into four parts, supplemented by three Appendices, a Bibliography and an Index. Part I is devoted to the study of the Laplace transform. The inversion theorem is presented in detail, then various applications are made to the solution of ordinary differential equations, partial differential equations (diffusion, wave propagation, etc.) and integral equations (of Volterra type, equation for hard rods etc.).

Part II deals with the fundamental properties of the Fourier transform (up to the Kramers—König relations) and its application to potential and wave problems. Then the treatment proceeds to the theory of generalized functions. There is a considerable amount of "pure mathematics" associated with the understanding and use of generalized functions, because their use adds essentially to the power of the Fourier transform as a tool. Fourier transforms in two or more variables are also included.

Part III contains other important transforms: (*i*) Mellin transforms, (*ii*) Hankel transforms, and (*iii*) integral transforms generated by Green's functions. Special techniques are collected in Part IV. The Wiener—Hopf technique is developed in relation to some instructive problems like reflection and diffraction of waves, radiative processes in astrophysics etc. Then the presentation of the Laplace method for ordinary differential equations follows, by which one can produce integral transform solutions using Hermite-, Bessel- etc. functions. This part ends with different numerical inversion forms of Laplace transforms.

Two more remarks on the presentation. (*i*) The author lays great stress on the use of complex variable techniques, which is frequently of great power. (*ii*) Each section is followed by a rich collection of appropriate problems serving as exercises for the reader.

The book is warmly recommended to everybody wishing to get acquainted with integral transforms, the applications of which outside mathematics, both directly and through differential equations, meet an ever increasing demand of natural sciences.

F. Móricz (Szeged)

W. G. Dixon, Special relativity. The foundation of macroscopic physics, VIII+261 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1978.

The macroscopic physics treated in special relativity seems irrelevant to many physicists, because these macroscopic phenomena under terrestrial conditions are described by the Newtonian theory in a simpler way and with a negligibly small error. This book proves that the macroscopic physics discussed in the framework of special relativity is significant not only from a theoretical point of view. It shows that an understanding of the basic laws of macroscopic systems can be gained more easily within relativistic physics than within Newtonian physics.

The first two chapters contain an introduction to Newtonian physics, to the spacetime structure of special relativity and to tensor algebra. After this introduction the book is devoted to three subjects of special relativity: dynamics, thermodynamics, and electromagnetism.

The theory of dynamics contains both the point particle dynamics and continuum dynamics. The most important topics of this theory are the momentum, angular-momentum and energy conservation laws. The part on thermodynamics describes the entropy law, the equilibrum thermodynamics of relativistic simple fluids and the thermodynamics of irreversible processes. The same techniques is applied to the study of the interaction of simple fluids with an electromagnetic field.

The corresponding Newtonian results of these theories are obtained by taking the Newtonian limit.

The book is not directed towards any particular university course, and it should be accessible to any undergraduate in mathematics or physics.

Z. I. Szabó (Szeged)

V. Dudley, Elementary Number Theory, 2^{nd} edition, IX+249 pages, W. H. Freeman and Co., Reading — San Francisco, 1978.

The second edition of this outstanding undergraduate textbook is a slightly extended version of the first one. Some errors have been removed (and as the author asserts in the preface, some new ones have been added). One of the main merits of the book is that — in contrast with most university textbooks — it can be used with success not only by the best students but also by the average ones, and indeed they can rather deeply understand the topic from it. This is achieved, beside a very clear treatment, by well-chosen exercises inserted in the basic text.

The first twelve chapters give a standard course on divisibility and on congruences, including quadratic reciprocity. Sections 14—15 deal with arithmetic in different place-value systems, sections 16—20 with different non-linear diophantine equations, and sections 21—22 with primes. The last section contains 100 additional problems of different levels. There are three appendices: one on proof by induction, another on problems for computers, and a factor table for integers $< 10\,000$.

There is a number of misprints (whether old or new) and sometimes they emerge at the most inconvenient moments — in numerical examples which ought to inform the reader; and puzzle him instead. This excellent book would have deserved a more careful printing.

G. Pollák (Szeged)

Herman H. Goldstine, A history of Numerical Analysis from the 16th through the 19th century (Studies in the History of Mathematics and Physical Sciences, 2.), XIV+348 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

The author worked together with Professors von Neumann and Murray on the problem of determining the eigenvalues and vectors of real symmetric matrices. They rediscovered, among other results, Jacobi's method. The author presented this in 1951 at a meeting on numerical analysis at UCLA. After the presentation Professor Ostrowski of Basel told him this had been done a century earlier by Jacobi. Partly this event had caused the author to get more comprehensive and thorough information from the history of ideas and methods of numerical analysis as a result of which this excellent book came into being.

The author attempts to trace the development of numerical analysis during the period in which the foundations of the modern theory were being laid. He chooses the most famous mathematicians of the period in question and concentrates on their major works in numerical analysis.

The book is divided into five chapters and ends with a rich Bibliography containing about 300 items and a detailed Index.

Chapter 1 is entitled „The Sixteenth and Early Seventeenth Centuries". During this period mathematical notation began to improve quite markedly and the reasonable symbolisms contributed greatly to the development of mathematics. One of the great discoveries of the sixteenth century was that of logarithms made independently by Bürgi and Napier.

Chapter 2 ("The Age of Newton") discusses Newton's contributions to numerical techniques such as his method for solving equations iteratively, his interpolation and numerical integral formulas as well as his ideas on calculating tables of logarithms and of sines and cosines. Newton's friends and contemporaries Halley, Cotes, Stirling, Maclaurin, Gregory, Moivre, and James Bernoulli, among others, quickly took up his ideas and published a great deal of work which is of interest in numerical analysis.

Chapter 3 ("Euler and Lagrange"): The invention of classical analysis is very largely due to Euler. Even a glance through a volume of his enormous collectedworks shows how Euler differed from Newton. There are no geometrical figures present, he worked with functions and studied their properties in the modern manner. He layed down at least the groundwork in virtually all topics of modern numerical analysis, especially the basic notions for the numerical integration of differential equations. Lagrange worked on linear difference equations and elaborated his famous method of variation of parameters in this connection. He was interested in interpolation theory and introduced some quite elegant formalistic procedures which enabled him to develop many important results.

Chapter 4 ("Laplace, Legendre, and Gauss") begins with the presentation of Laplace's work, who used and developed the method of generating functions to study difference equations which came up in his study of probability theory. Using this apparatus, he was also able to develop various interpolation functions and to produce a calculus of finite differences. Gauss wrote much on numerical matters and obviously enjoyed calculating. The Method of Least Squares was published by Legendre in 1805 but had been used much earlier by Gauss. Also, Gauss took the Newton-Cotes method of numerical integration and showed that by viewing the position of the ordinates as parameters to be chosen one can materially improve convergence. Later Jacobi reconsidered this result and gave a very elegant exposition of it. Gauss wrote penetratingly on interpolation, and particularly on trigonometric interpolation. In fact he developed the entire subject of finite Fourier series, including what is now called the Cooley-Tukey algorithm or the fast Fourier transform.

After Gauss there were a considerable number of excellent mathematicians who either continued his ideas on numerical analysis or utilized their own discoveries to make more elegant what earlier mathematicians had done. Thus, for example, we find on the one hand Jacobi reconsidering some of Gauss' work and on the other hand Cauchy using his Residue Theorem to obtain polynomial approximations to a function. Chapter 5 ("Other Nineteenth Century Figures") is mainly devoted to the presentation of their work in this subject. Among others, Jacobi wrote a paper on finding the characteristic values of a real symmetric matrix mentioned at the beginning which has given rise to the modern Jacobi method and its variants. One of Cauchy's most significant discoveries was a method for finding a rational function which passed through a sequence of given points. This idea of approximation by rational, rather than polynomial, functions is still important and in another connection — Padé approximations — is also used today. Another great advance that Cauchy made was his method for showing the existence of the solutions of differential equations. This so-called Cauchy-Lipschitz method, as well as that of Picard, form the basis for some very important techniques for the numerical integration of such equations. These theoretical methods were exploited by Adams, Bashforth, and Moulton. In a quite different direction Heun, Kutta, and Runge developed a very pretty method for numerical integration of differential equations. One of the first problems run on ENIAC was done using Heun' s method. Their ideas are current today.

A listing of the contents could hardly give a right impression of the richness of the book. It will certainly be a very instructive and profitable reading for everyone interested in numerical analysis.

F. Móricz (Szeged)

H. B. Griffiths— P. J. Hilton, A comprehensive textbook of classical mathematics, XXIX + 637 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

This second edition differs from the original one published in 1970, by Van Nostrand Reinhold Co. in the elimination of some errors and the addition of a few calculus exercises. Since *these Acta* have not reviewed the first edition, we have a closer look at the book here. It is written to those already familiar, at a certain level, with the subject matter they choose from it. Roughly speaking, a one year specialized mathematical study is sufficient to read the material presented without difficulty. The main purpose of the authors has been "to encourage the reader to look at rather familiar ideas a second time, with a view to fitting them into the framework of present-day mathematical thought; and thus to enable the reader to see how certain key ideas recur again and again and give a real unity to apparently separate parts of his early mathematical experience". The presentation follows the "spiral approach": ideas are introduced informally; then precise proofs and definitions are given, after which informality comes again. To give the reader some information of the material covered in the book we list the titles of the eight parts as follows: The Language of Mathematics, Further Set Theory, Arithmetic, Geometry of R^3, Algebra, Number Systems and Topology, Calculus, Additional Topics in the Calculus, Foundations.

We recommend this book to anyone, even to the well-educated mathematician, who wishes to brush up on his basic mathematics.

József Szűcs (Szeged)

P. R. Halmos — V. S. Sunder, Bounded integral operators on L^2 spaces (Ergebnisse der Mathematik und ihrer Grenzgebiete, 96), XV + 132 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

The phrase "integral operator" used in this book is the natural "continuous" generalization of the operators induced by matrices, and the only integrals that appear are the Lebesgue-Stieltjes integrals on classical non-pathological measure spaces. To be more concrete, let X and Y be σ-finite and separable measure spaces. A kernel $k=k(x, y)$ is a complex-valued measurable function on the Cartesian product $X\times Y$. The domain of k is the set dom k of $g\in L^2(Y)$ that satisfies the following two conditions:

(i) $k(x,\cdot)g\in L^1(Y)$ for almost every x in X,

(ii) if $f(x)=\int k(x,y)g(y)\,dy$, then $f\in L^2(X)$.

It may happen that dom $k=0$ (the identically 0 function), for example, this is the case if $k(x,y)=$ $=1/(x-y)$ on $R\times R$ (the kernel that defines the Hilbert-transform). In any event, whatever its domain might be, a kernel always induces an operator, denoted by Int k, that maps dom k (in $L^2(Y)$) into $L^2(X)$; the image under Int k of a function g in dom k is the function f in $L^2(X)$ given by (ii). The integral operator Int k is linear but not necessarily bounded.

The book does not strive for maximum generality. The study is restricted mostly to bounded integral operators as indicated in the title. Even in this special setting the authors do not answer all the questions about integral operators. Frequently, when the systematic treatment encounters unanswered questions, the authors point out, where such questions arise, how they are connected with others, and what partial information about them is available. The emphasis in the treatment is on the basic implication relations on which the subject rests, rather than on its mechanical techniques.

The main prerequisite for an uninterrupted reading of the book is familiarity with the standard facts of measure theory and operator theory.

The book consists of 17 sections. The first five contain the definitions and the examples that are needed throughout. Sections 6—9 describe what can and what cannot be done with integral operators. The topics are the possibility of transforming integral operators by measure-preserving isomorphisms, the correspondence from kernels to operators, and the extent to which that correspondence preserves the algebraic operations on kernels. Sections 10 and 11 treat two important classes of kernels: absolutely bounded kernels and Carleman kernels.

Sections 12—14 provide some necessary tools from operator theory for the subsequent sections: a discussion of two different kinds of compactness, and the properties of the essential spectrum, culminating in the celebrated Weyl-von Neumann theorem on the possibility of a kind of generalized diagonalization for Hermitian operators on infinite-dimensional Hilbert spaces. The last three sections deal with the most interesting and up-to-date questions:

(i) Which operators can be integral operators? In precise terms, it asks for a characterization of those operators A on $L^2(X)$ for which there exists a unitary operator U on $L^2(X)$ such that UAU^* is an integral operator? This question has a complete answer.

(ii) Which operators must be integral operators? In precise terms: under what conditions on an operator A on $L^2(Y)$ does it happen that UAU^* is an integral operator for every unitary U on $L^2(X)$? This question has a satisfactory partial answer, but some special questions (e.g., about absolutely bounded kernels) remain open.

(iii) Which operators are integral operators? The problem is one of recognition: if an integral operator on $L^2(X)$ is given in some manner other than by its kernel, how do its operational and measure-theoretic properties reflect the existence of a kernel that induces it? Here various useful sufficient conditions are available, but none of them are necessary.

The writing of the book was mainly motivated by the fact, as the authors admit in Preface, that the theory of integral operators is the source of all modern functional analysis and remains to this day a rich source of non-trivial examples. Since the major obstacle to progress in many parts of operator theory is the dearth of concrete examples whose properties can be explicitly determined, a systematic theory of integral operators offers new hope for new insights. And the programme of the authors is completely materialized in this book, which makes a very essential contribution to the systematization of the theory of integral operators.

This book is indispensable for all specialists of functional analysis, but it is also warmly recommended to everybody who wants to keep pace with up-to-date developments in analysis.

F. Móricz (Szeged)

Herbert Heyer, Probability measures on locally compact groups, X+537 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

Probability measures on algebraic topological structures and especially on locally compact topological groups have become of increasing importance in recent years. The main purpose of the present book is to give a systematic presentation of the most developed part of the work done in this field. The text is divided into 6 Chapters. To make the book as self-contained as possible the first two chapters have been devoted to general tools from the harmonic analysis of locally compact groups and from the elementary convergence theory of convolution sequences of probability measures on the group. In Chapter 3 the general embedding problem is posed. The most important step on the way to central limit theorem is the embedding of an infinitely divisible measure into a continuous one parameter convolution semigroup. Since the embedding theorem does not hold in a general locally compact group, the question arises what classes of groups yield the validity of an embedding theorem. Establishing these classes of groups is the aim of this chapter. Chapter 4 includes an extensive discussion of the canonical representation of all continuous convolution semigroups in the sense of a Lévy — Khintchine formula. There is a natural connection between convolution semigroups on a group G and contraction semigroups of operators on certain function spaces E on G such that the problem of generating a convolution semigroup becomes a problem of determining the existence of the infinitesimal generator of the corresponding contraction semigroup on E. Using the solution of Hilbert's V. problem and the ideas of Bruhat a differentiable structure can be introduced in any locally compact group and the problem will be reduced via Lie projectivity to the Lie group case. The aim of Chapter 5 is twofold: to motivate the broad discussion of the central limit theorem in the special case of an Abelian group and to give certain auxiliary results which will be needed for the treatment of the problems for more general groups. Most of this material is applied to a detailed treatment of additive stochastic processes with values on a locally compact Abelian group having a countable topological basis. Chapter 6 is devoted to the central limit problem in the general case. Many facts discussed at an earlier stage will be combined here for a detailed study of Poisson and Gauss measures on arbitrary locally compact groups as well as for the study of the convergence behavior of triangular systems of probability measures in the sense of a Lindeberg-Feller central limit problem.

The book is highly recommended to research workers taking interest in modern probability theory and having certain knowledge of representation theory.

L. Gehér (Szeged)

M. Karoubi, *K*-theory (Grundlehren der mathematischen Wissenschaften — A Series of Comprehensive Studies in Mathematics, 226), XVIII+308 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

It's well-known that ordinary cohomology theory is defined uniquely (on the category of polyhedra) by the Eilenberg-Steenrod axioms. However, if we omit the so-called dimension axiom: $H^k(P)=0$ if P is a point and $k>0$, then there will exist infinitely many functors from the category of polyhedra into the category of abelian groups satisfying the other axioms. These functors are called extraordinary cohomology theories. One of the extraordinary theories is K-theory. The advantage of extraordinary theories is that they usually give much more information about the topological situation considered. The particular advantage of K-theory is that it appears very naturally in consideration of differential manifolds and fibre bundles, because its elements are — roughly speaking — the vector bundles themselves. The exact definition of the (topological) K-functor is the following: Let us consider all vector bundles over a space X. They form a semiring under the Whitney sum and the tensor product. This semiring — as well as any other one — defines a "minimal" ring (the Grothendick ring of the semiring). This ring is the $K(X)$ ring for the space X. Starting from the K-functor one can define a cohomology theory in the following way: For $i>0$ let $K^{-i}(X)$ equal $K(S^iX)$ where S^iX is the i-fold suspension on X. The sequence $K^{-i}(X)$ turns out to be periodic modulo 2 in the complex case and modulo 8 in the real case. This enables us to extend the definition of $K^i(X)$ for $i>0$. As well-known, the characteristic classes serve for the description of vector bundles by means of the classical cohomology theory. Characteristic classes can be defined in K-theory, too. The present book is the first monograph dealing with characteristic classes in K-theory, as well. There exist three ways to define characteristic classes:

1) the axiomatic way;
2) using the cohomology ring of the Grassman manifold;
3) by the Thom isomorphism theorem.

In this book all the three definitions are presented. The particularity of K-theory is that in it the characteristic classes are the same as the cohomology operations. The cohomology operations are describe at the end of Chapter IV. Chapter V deals with interesting applications. First, it presents Adams' proof for the statement that on the sphere S^n there exists an H-structure iff $n=1$ or 3 or 7. (We remind that the H-structure is a generalization of the topological group structure.) The second application is the solution of the following question: How many continuous linearly independent vector fields do there exist on the u-sphere? The third interesting question in this chapter is the so-called Chern character which is an isomorphism between the groups $K_c(X)\ Q$ and $H^{even}(X; Q)$ for any compact space. At the end of the book are the most interesting questions: the Riemann-Roch theorem and the integrability theorems for the characteristic classes. The book is recommended to anybody wishing to study this new exciting and powerful mathematical theory.

András Szűcs (Szeged)

W. Klingenberg, Lectures on Closed Geodesics (Grundlehren der mathematischen Wissenchaften, 230), IX+227 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

The question about the existence of closed geodesics on a simply connected compact Riemannian manifold has been in the centre of investigations in global differential geometry since Jacobi's description of geodesics on an ellipsoid in 1842. In 1905 Poincaré claimed that this problem was closely related to the question whether there existed a periodic solution of the restricted three body problem.

The greatest advances in the theory of closed geodesics were the results of L. A. Lusternik and L. G. Schnirelmann in 1929 and of L. A. Lusternik and A. I. Fet in 1951. They showed that on a simply connected compact surface there exist at least three closed geodesics without self intersection and that at least one closed geodesic exists on every compact Riemannian manifold.

In the last 15 years Prof. Klingenberg worked out two very effective new approaches to the existence problem of closed geodesics: the Morse theory on an infinite dimensional Hilbert-Riemann manifold and the method of Hamiltonian systems and geodesic flows. The starting problem is completely solved at the present stage of investigations by the main theorem of this monograph: On a compact Riemannian manifold with finite fundamental group there exist infinitely many closed geodesics. This fundamental result of Klingenberg, published in detail here for the first time, gives essential new information even in the case of convex surfaces in euclidean 3-space.

The aim of this book is to give an up-to-date and detailed introduction to the new methods of investigation on the geometry of closed geodesics and to give self-contained proofs of the essential new results in this theory.

In Chapter 1 the notion of a Hilbert manifold is introduced and a canonical Hilbert manifold structure is defined on the space of closed curves in a compact Riemannian manifold. The question about the existence of closed geodesics can be translated into a question about the critical values of the energy function on the Hilbert manifold of closed curves.

Chapter 2 is devoted to the development of the Lusternik-Schnirelmann and Morse theory on the manifold of closed curves.

In Chapter 3 the theory of Hamiltonian systems is discussed from the aspects of geodesic flows on a Riemannian manifold. This proves to be a very effective tool for the study of periodic geodesics in a neighborhood of a given one.

Chapter 4 contains the main result on the existence of infinitely many closed geodesics on a manifold with finite fundamental group. It concludes with generic existence theorems derived from the properties of geodesic flows.

In Chapter 5 an n-dimensional generalization of the classical Lusternik-Schnirelmann theorem and a number of miscellaneous results about closed geodesics on special Riemannian manifolds are proved.

In an Appendix, an elementary treatment of the Lusternik-Schnirelmann theory is given independently of the previous parts of the book.

The book contains fundamental and new information about central problems of global differential geometry. Chapters 1—3 can serve as an excellent introduction into the new methods of investigation of geometry of geodesics, Chapters 4—5 contain the main results of the theory. The latter part is not very easily readable, because a great variety of analytical and topological methods is used. It is suggested to the reader that though the presented results solve the starting problems of the theory, a great many interesting questions are left open which can be studied with these new methods only.

The book is an indispensable monograph on the subject. It is warmly recommended to research workers in differential geometry, the global theory of dynamical systems, and nonlinear functional analysis.

P. T. Nagy (Szeged)

Wilhelm Klingenberg, A course in differential geometry (Graduate Texts in Mathematics, 51), XII + 178 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1978.

English translation of the original "Eine Vorlesung über Differentialgeometrie" (Heidelberger Taschenbücher, 1973; reviewed in *these Acta* **36** (1974)). It contains an excellent introduction to elementary differential geometry for undergraduate students. The present edition is more detailed and a number of figures is added.

P. T. Nagy (Szeged)

Hans Kurzweil, Endliche Gruppen. Eine Einführung in die Theorie der endlichen Gruppen (Hochschultext), XII + 187 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1977.

Dieses Buch, das durch seine leichte Lesbarkeit und seinen didaktisch guten Aufbau überwiegend für Studenten zusammengestellt ist, "möchte — nach seiner Zielsetzung — den Leser mit den Grundlagen und Methoden der Theorie der endlichen Gruppen vertraut machen und ihn bis an aktuelle Ergebnisse heranführen". Sein Lesen benötigt nur elementare Kenntnisse der linearen Algebra. Für diejenigen, die sich späterhin mit dem Problem der Bestimmung einfacher Gruppen eingehend befassen möchten, ist das Lesen dieses Lehrbuches besonders vom Nutzen, denn sie finden hier zahlreiche grundlegende Kenntnisse, Begriffe, die zum Studieren des genannten Themas unentbehrlich sind.

L. Megyesi (Szeged)

W. S. Massey, Algebraic topology: An introduction (Graduate Texts in Mathematics, 56) XXI + 261 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

The book is the 4th corrected printing of the excellent textbook, the earlier printings of which were published by Harcourt Brace and World, New York, 1967. It is a very elegant introduction to algebraic topology, concerning three classical subjects: 2-dimensional manifolds, fundamental groups and covering spaces.

Chapter 1 discusses 2-dimensional manifolds with numerous examples and exercises. The classification theorem for compact surfaces is also proved.

Chapters 2—4 deal with fundamental groups. Besides their basic properties, the Brouwer fixed point theorem and the Seifert — Van Kampen theorem on the fundamental group of the union of two spaces are discussed.

The covering spaces and the relationship between covering spaces and the fundamental groups are described in Chapter 5. Chapters 6—7 present topological proofs of several well-known theorems of group theory, namely, the Nielsen-Schreier theorem on subgroups of a free group, the Kurosh theorem on subgroups of a free product, and the Grushko theorem on the decomposition of a finitely generated group as a free product. Chapter 8 gives an outlook to algebraic topology.

Z. I. Szabó (Szeged)

Th. Meis und U. Marcowitz, Numerische Behandlung partieller Differentialgleichungen (Hochschultext), VIII+452 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1978.

A good introduction to the study of numerical methods of partial differential equations. The authors confine themselves chiefly to methods of solving linear second-order partial differential equations with two independent variables. However, differential equations with more variables than two as well as non-linear partial differential equations are also treated.

The book consists of three parts and an Appendix. Part I is devoted to the study of initial value problems in differential equations of hyperbolic or parbolic type. Part II proceeds to the solution of boundary value problems in differential equations of elliptic type. The concepts of consistency, stability and convergence of a method are in the central place of the treatment. The most widely used numerical methods of solving partial differential equations are the finite difference methods, and they are presented in detail. The use of the Fourier method for standard problems in mathematical physics, the variational methods and collocation methods of solving boundary value problems are dealt with also in detail.

Part III systematically develops a substantial portion of the theory of iterative methods for solving systems of (linear or non-linear) algebraic equations that arise in the numerical solution of boundary value problems by finite difference methods. The focal point is an analysis of the convergence properties of the successive overrelaxation method (SOR method) in the linear case, and that of the Newton-Raphson method with some of its variants in the non-linear case. Some techniques for solving large systems of linear algebraic equations with sparse matrices are also included.

The forth part called Appendix contains the FORTRAN programs of certain well-chosen problems with the necessary explanation and documentation.

The presentation is always clear and well-readable. The theoretical background is given in detail, the methods are illuminated in a many-sided manner. The textbook is highly recommended to students in numerical analysis as well as to experts in physics, chemistry and engineering interested in the solution of partial differential equations.

F. Móricz (Szeged)

David Mumford, Algebraic Geometry. I. Complex Projective Varieties (Grundlehren der mathematischen Wissenschaften, 221), X+186 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

From the thirties on, Algebraic Geometry turned from Geometry towards Algebra. The algebraic methods enriched the machinery with extremely powerful tools, but they need a very long, systematic, and — at least for the beginners — boring foundation. On the other hand, even if one works with schemes, a good geometric intuition is needed in order to "see" the problem. However, most recent books on algebraic geometry emphasise the algebra part, and say very little (if any) about the geometric sources.

The present book provides an introduction to the subject, emphasising the geometric part. It shows the deep connections between algebraic and analytic geometry and topology. First the analytic structure of an algebraic variety is investigated. Then the Zariski and Euclidean topologies are compared, and Chow's theorem is proved which characterises the projective algebraic varieties as closed analytic submanifolds of complex projective spaces. Another fascinating characterisation is given later when projective varieties are described as compact oriented differentiable submanifolds of the complex projective spaces having minimal volume in a certain natural sense. Finally the Euclidean topology of curves is determined.

The use of topological methods throughout the book enables the author to make considerable shortcuts in the proofs and makes the definitions clearer (this concerns mainly the notions connected with multiplicity). The last paragraph deals with the 27 lines on a cubic surface, one of the most astonishing facts in geometry.

The algebraic part of the theory is also developed in a very efficient way that leads quickly to interesting theorems.

The whole book is written in a very clear and concise style. All this makes the book an excellent introduction, especially suitable for mathematicians who are not primarily interested in algebraic geometry.

János Kollár (Budapest)

R. K. Sachs — H. Wu, General Relativity for Mathematicians (Graduate Texts in Mathematics, 48) XII+291 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

Many recent monographs on general relativity treat the subject in the frames of modern differential geometry. The present book also gives a clear and geometrical description of general relativity, using this terminology. The reader is only supposed to have familiarity with tensor algebra, differential topology, and the rudiments of Riemannian geometry.

After some mathematics and physics background on Lorentzian manifolds the book gives a systematic description of particle dynamics, electromagnetism and several matter models. In the second part it treats several cosmological questions: Einstein's field equation, the theory of photons and photon gases, the Einstein — de Sitter and Schwarzschild model of space-time, black holes, etc.

The book is a fundamental monograph on the subject. It is especially well-organized; its didactical value is greatly enhanced also by the great number of examples worked out.

Z. I. Szabó (Szeged)

Mathematics Today. Twelve Informal Essays, Edited by Lynn Arthur Steen, VI+367 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1978.

This book is intended to be a popularizing work for the intelligent non-mathematicians, particulary for those who have already met mathematics in their scientific research. It contains 12 well-readable essays by 12 authors extraordinarily well selected from distinct fields of pure and applied mathematics. The essays are the following: Mathematics—Our Invisible Culture (Allen L. Hammond), Number Theory (Ian Richards), Groups and Symmetry (Jonathan Alperin), The Geometry of the Universe (Roger Penrose), The Mathematics of Meteorology (Philip Thompson), The Four Color Problem (Kenneth Appel and Wolfgang Haken), Combinatorial Scheduling Theory (Ronald Graham), Statistical Analysis of Experimental Data (David S. Moore), What is a Computation? (Martin Davis), Mathematics as a Tool for Economic Understanding (Jacob Schwartz), Mathematical Aspects of Population Biology (Frank C. Hoppensteadt), The Relevance of Mathematics (Felix E. Browder and Saunders Mac Lane).

The book is printed in an aesthetic format. It is highly recommended to professional mathematicians as well as to laymen.

L. Gehér (Szeged)

Şerban Strătilă—László Zsidó, Lectures on von Neumann Algebras, 478 pages, Editura Academiei (Bucureşti, Romania) — Abacus Press (Tunbridge Wells, Kent, England), 1978.

This book is a revised and updated English version of the original Roumanian "Lecţii de algebre von Neumann" published in 1975 by the above Roumanian publisher. Both authors made important contributions to the theory of von Neumann algebras. This theory was initiated by J. von Neumann and F. J. Murray in the thirties in connection with infinite group representations and theoretical physics, etc. The first systematic treatment of the subject was given by J. Dixmier in 1957, which followed I. Kaplansky's lecture notes "Rings of operators" published in 1955 (reprinted in 1968). It was Dixmier who used the term "von Neumann algebras" as an equivalent of "rings of operators" or "operator algebras". His monograph included almost all the important results known in the field by then. The theory of von Neumann algebras has developed very rapidly and extensively since that time. Besides the many research papers, a number of expository works have also been published, such as Sakai's excellent monograph "C^*-algebras and W^*-algebras" (1971), which followed his lecture notes "The theory of W^*-algebras" (1962), J. R. Ringrose's "Lecture notes on von Neumann algebras" (1967), "Lectures on Banach algebras and spectral theory", and "Lectures on the trace in finite von Neumann algebras" (1972), Takesaki's lecture notes "The theory of operator algebras" (1970), and "Lecture notes on operator algebras" (1973/1974), Topping's "Lectures on von Neumann algebras" (1971), a new edition of Dixmier's classic in 1969, and the Roumanian version of the present book in 1975.

A turning point in the development of the theory of von Neumann algebras came with M. Tomita's discovery of modular Hilbert algebras in 1967. His results were published in Takesaki's lecture notes "Tomita's theory of modular Hilbert algebras and its applications" in 1970. Tomita devised canonical forms for arbitrary von Neumann algebras.

At the present stage of development it cannot be expected that a single volume expounds all features of the existing theory. The book under review develops the theory of standard von Neumann algebras (or, in other words, Tomita's theory), but it does not discuss reduction theory, the isomorphism theory of factors, non-commutative harmonic analysis and ergodic theory, applications to operator theory and theoretical physics, the generation of von Neumann algebras. C^*-algebras are only incidentally referred to. Just as Dixmier's classic, it develops the spatial theory of von Neumann algebras, i.e., von Neumann algebras are considered sub-algebras of the full operator algebra, in contrast with Sakai's abstract treatment, where they are considered as C^*-algebras with a predual. The material presented covers the results contained in M. Takesaki's work on Tomita's theory and in Dixmier's classic, except reduction theory and examples of factors. The contributions of van Daele and the second author of this book made it possible to simplify the proof given by Takesaki in his lecture notes. Following I. Cuculescu and S. Sakai, the commutation theorem for tensor products is proved independently of Tomita's theory.

The book is clearly written. It only assumes knowledge of the rudiments of functional analysis. There are many valuable exercises, some of them borrowed from Dixmier's classic. Very valuable comments supplement the text proper at the end of each chapter. A very thorough 20 page bibliography on operator algebras and related topics is included. We very warmly recommend this book to beginners in operator algebras or to research workers who will find that this book gives very thorough bibliographical information or serves very well as a reference book.

József Szűcs (Szeged)