Bibliographie

K. B. Athreya—P. E. Ney, Branching processes (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 196), XI+287 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1972.

This book can be regarded as a continuation of T. E. Harris' book (Springer-Verlag, 1963). Chapter I gives a brief summary of classical results on Galton—Watson processes, and a more detailed treatise of its modern refinements, e.g. the decomposition of the supercritical branching process.

Chapter II develops the potential theoretical tools and their application to the discrete time case.

One of the most interesting theorems here is the sharp limit law due to the authors. Chapter III deals with the Markovian case using Kolmogorov's equations and martingale convergence theorems, and investigates the problem of imbedding a Galton—Watson process into a continuous time Markov branching process.

Chapter IV is devoted to the so called age-dependent process. It is not Markovian and the methods of the previous chapters do not apply. Such models were introduced and first studied by R. Bellman and T. E. Harris (1952). The key tool here is the renewal equation.

Chapter V generalizes results proved in previous chapters for the multitype case.

Chapter VI examines special processes, e.g. the branching diffusion, branching processes with random environments and the continuous state branching processes and processes with immigrations. The authors cite an abundance of literature. After every chapter there are complements and problems, including open ones.

The treatment is thorough, precise and easy to follow.

A. Krámli (Budapest)

N. L. Biggs, Interaction models, London Math. Soc. Lecture Notes Series 30, 101 pp., Cambridge University Press, Cambridge—London—New York—Melbourne, 1977.

Have you ever thought that the Four Colour Problem is in any connection with ferromagnetism? Well, this book shows such a connection!

To be less sensational but more specific, it has been the Four Colour Problem which has inspired most of the research done in connection with the so-called *chromatic polynomial.* This polynomial $p_G(x)$ expresses the number of colorations of the graph G in x colors. Graph theorists, primarily G. D. Birkhoff, H. Whitney and W. T. Tutte have developed several expansion formulas, generalizations, and other properties of this polynomial.

Meanwhile physicists, among others Ising, Mayer, Lieb etc. have studied "interaction models", which can be described in terms of a graph: the vertices are particles, and interaction exists between adjacent vertices only. Each particle has a finite number of possible states, and each interacting pair contributes to the Hamiltonian H of the system depending on their states. Important physical phenomena, like phase transitions, can be described in terms of the expression $\Sigma \exp H$ (summed over all states of the particles). This expression has properties very similar to those of the chromatic polynomial, in fact, the chromatic polynomial is a special case of it. Various properties and expansions of the chromatic polynomial have their analogues and generalizations in physics; most of them have been discovered independently but some discoveries have been inspired by this analogy.

"The lectures on which this book is based were intended for a 'mixed audience' ... some of the audience were basically physicists, and others were basically mathematicians... The desire to be intelligible to two classes of students has been my main preoccupation in preparing the lectures and writing the book." So the book may serve as a bridge between theoretical physicists and mathematicians working on two ends of the same problem, and who have now caught sight of each other.

The author also points out: "It would be idle to pretend that the material treated in this book is in its final form... If some of my errors stimulate other mathematicians to put things right, then the book will have served its purpose." Indeed, one feels the air of openness and the temptation to join the research when reading the book. And this, I think, is a good reccommendation.

L. Lovász (Szeged)

R. P. Burn, Deductive Transformation Geometry, XI+121 pages, Cambridge University Press, 1975.

There are two usual types of descriptions of the relationship between the Euclidean plane and the real number system. A geometric description was made by Hilbert under which the real number system emerged from postulated properties of points and lines. Algebraic descriptions explicitly start from the real number system, and construct points as ordered pairs, lines as linear subsets of ordered pairs of real numbers. The main aim of this book to show which properties of the real number system are required to establish particular theorems in geometry. The approach will be geometric in that the axioms will all be axioms of or about incidence, but the method will be algebraic, in that the existence and properties of a coordinate system will be obtained by exploring groups of transformations.

In Chapter 1 the affine incidence axioms in the plane are assumed, only to see what kind of geometry can be done without any algebraic properties of real numbers. The study of finite affine geometries gives rise to a great number of easily stated, but as yet unsolved problems. At the end of this chapter, by adjoining a line of new points to the affine plane, the projective plane is constructed. Chapter 2 goes back to the affine plane, defines parallel projections of lines, and affinities of lines as products of parallel projections. The affinities of a line onto itself which are products of two parallel projections are called (affine) permutations. Addition and multiplication of permutations can be defined in the usual way. By using the permutations, coordinate systems on lines can be introduced. Postulating that the permutations of a line form a group, the coordinate elements form an Abelian group under addition, and for the (not necessarily commutative) multiplication a one-sided distributive law follows. Chapter 3 is devoted to the study of those transformations of the plane which map each line onto a parallel line. There are two classes of these transformations; those with no fixed points, called translations, and those with just one fixed point, called enlargements. In Chapter 4 those planes are considered which admit all possible translations and enlargements. These are the Desarguesian planes. In these planes the underlying algebra of coordinates satifies both distributive laws. Chapter 5 deals with collineations of the plane, in particular with collineations having a line of fixed points, called axial collineations. The theorems obtained here are strikingly analogous to those obtained in Chapter 3 for translations and enlargements. In Chapter 6 a geometric description is given for those planes which have a field as their underlying algebra. It turns out that these are the Pappian planes. In Chapter 7 reflections are studied. Chapter 8 compares different systems of axioms given by Hilbert, Birkhoff, Moise and others.

The book is written in a well readable way.

L. Gehér (Szeged)

Combinatorial Surveys, Proceedings of the Sixth British Combinatorial Conference, Edited by P. J. Cameron, 226 pages, Academic Press, London—New York—San Francisco, 1977

The book consists of lectures of invited speakers of the Combinatorial Conference held at Egham (London), 1977. The chapters, as the title shows, are survey-type, therefore the reader receives a good cross-section of advanced combinatorics. The index of the book shows both the unity of the subject (with designs, graphs, matroids, projective spaces as prevailing themes) and its diversity (campanology, the golden ration, parallelism, Ramanujan numbers, root systems, etc.). This Proceedings is a useful reference book.

Chapter 1 (F. Buekenhout, What is a subspace?) investigates the abstract concept of subspace and exhibits how to use it for graphs, matroids and block designs.

Chapter 2 (P. J. Cameron, Extensions of design: variations of a theme). It is an old result of the theory of designs that a projective plane can be extended by one point to a 3-design only if its order is 2, 4 or (possibly) 10. Generalizations in three different directions are given.

Chapter 3 (L. Lovász, Flats in matroids and geometric graphs) is devoted to show how to use the concept of a geometric graph to unify the theory of τ-critical graphs and to prove a conjecture of Gallai and some new Helly-type theorems on flats in geometries.

Chapter 4 (D. K. Ray-Chaudhuri, Combinatorial characterization theorems for geometric incidence structures) provides a thorough and deep survey of the theory of geometric incidence structures. It contains 29 theorems and embraces a great part of this theory. The emphasis is on theorems asserting that certain incidence structures are "coordinatizable", i. e. can be derived from geometries over finite fields.

Chapter 5 (N. J. A. Sloane, Binary codes, lattices, and sphere-packings) surveys a surprising connection between binary codes on the one hand, and sphere-packings and lattices in R^n on the other hand.

Chapter 6 (A. T. White, Graphs of groups on surfaces) deals with Cayley graphs of groups, embedding of them in a surface, voltage and current graphs, genus etc.

Chapter 7 (D. R. Woodall, Zeros of chromatic polynomials) provides an introduction to the theory of chromatic polynomials of graphs. The emphasis is on the location of real zeros, and on the multiplicities of the integer zeros.

A. Frank (Budapest)

R. D. Driver, Ordinary and delay differential equations (Applied Mathematical Sciences, Vol. 20), IX+501 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

The so-called functional differential equations arose a long time ago in the history of mathematics. These are differential equations, in which the unknown function and its derivatives occur with various different arguments. Nowadays such equations play a particularly important role in applications; they are motivated by problems in control theory, physics, biology, ecology, economics and the theory of nuclear reactors. During the past two or three decades a number of valuable monographs on this subject were published, but none of them can be considered as an introductory text.

This book is an excellent first course on *delay* differential equations (this means an equation expressing the highest order derivative of the unknown function x at time t in terms of x and its lower order derivatives at t and at *earlier* instants). One of its advantages is that the author gives simultaneously an introduction to ordinary differential equations also. The book is especially well-organized from the point of view of didactics. This is mostly due to the great number of examples worked out. They prepare the reader excellently for understanding the theorems and convince him of their applicability. Some of the examples are very interesting in themselves, e.g. population growth, electrical circuits, two-body problem of electrodynamics, control systems, and numerous mechanical examples. At the end of the chapters several problems of various difficulty are listed, with answers or hints.

The structure of the book is the following. First comes the uniqueness of the solution of ordinary differential equations satisfying a Lipschitz condition. The properties of the solutions of ordinary linear systems are treated. Further on existence and stability problems are discussed for ordinary and delay equations simultaneously. The final chapter is devoted to autonomous differential equations.

The book is recommended to lecturers wishing to introduce delay differential equations in the senior and beginning graduate level curricula. It is suitable also for students and users of mathematics interested in differential equations.

L. Hatvani—L. Pintér (Szeged)

Lars Gårding, Encounter with Mathematics, X+270 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

This book is meant mainly for people on the level of a junior student and aims to give a general but comparatively comprehensive picture of some of the central topics of mathematics. Roughly speaking, these are the branches initiated before the middle of the last century, but their development is mostly followed up to quite recent results. It is by no way an easy piece of reading for a beginner so that it seems to be more apt to inform students already interested in mathematics than to intrigue those maintaining a lukewarm relation to it.

After an introductory chapter on the interrelation between mathematics and reality and a short account on the basic facts and problems of number theory, chapter 3 deals with algebra (equations, groups, rings, Galois theory). It includes Hilbert's Nullstellensatz with proof. The next section (Geometry and linear algebra) gives among others the foundations of Banach and Hilbert spaces, including the contraction theorem for Banach spaces with full proof and the spectral theorem for compact linear operators (proved for finite dimensions only). Speaking about continuity, the author presents Dedekind's theory of real numbers, uniform continuity and uniform convergence, and the notion of a topological space. After an interlude on the history of mathematics in the seventeenth century, chapters 7 and 8 deal with differentiation and integration and give a rather thorough picture of both fields. However to work with the Grassmann algebra is perhaps too formal at this level. — Fourier transform and the inversion formula are proved, to be used in the chapter treating probabilities. The section on series deals, among others, with the Weierstrass approximation theorems. The section on probability presents the basic classical results, a bit of statistics and of physical applications. In chapter 11 (Applications) mathematical modeling is illustrated on acoustics. The last section deals with sociology, psychology, and teaching of mathematics.

All in all, the material the book ranges over is rather large, even too large. This is partly compensated by a clear and well-considered treatment. It was also a good idea to give a few selected passages of classical works in the corresponding fields. There are a lot of misprints, several embarrassing ones.

G. Pollák (Szeged)

D. Gilbarg—N. S. Trudinger, Elliptic partial differential equations of second order (Grundlehren der mathematischen Wissenschaften 224), X+401 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

The book grew out of graduate courses held by the authors at Stanford University.

The theory of partial differential equations is now so large that a comprehensive treatment is impossible. Every book contains a small part of the theory only. The tools also vary considerably, there are books written in classical style with several applications while others are making use of concepts of modern mathematics. The present authors' aim was to write a book on second order elliptic partial differential equations for a broad spectrum of readers, interested in the various concepts, methods and techniques which have been developed in this theory.

The book consists of two parts. In Part One, Chapters 2—8, the linear theory is developed. Naturally, Laplace's and Poisson's equations are the starting points for the study of classical solutions. The Dirichlet problem for harmonic functions with continuous boundary conditions is investigated through the Perron method of subharmonic functions, emphasizing the maximum principle and the barrier concept for studying boundary behavior. The Hölder estimates for the solution of the Poisson equation are derived from an analysis of the Newton potential. Ch. 6 develops an extension of potential theory based on the fundamental observation that equations with Hölder continuous coefficients can be treated locally as a perturbation of constant coefficient equations. Ch. 8 on "Generalized solutions and regularity" shows that by Hilbert space methods a more general approach can be achieved to linear problems. Throughout the book — whenever it is possible — the authors emphasize the connections and applications to the nonlinear theory; thus the regularity theory and Hölder estimates of generalized solutions are fundamental to the nonlinear theory.

Part Two deals with the Dirichlet problem and related estimates for quasilinear equations. In Ch. 9 maximum and comparison principles are given for the solutions of quasilinear equations. Ch. 10 contains fixed point theorems of Schauder, Leray—Schauder and Brouwer, and some of their applications. Chapters 12, 13 and 14 present gradient estimates. Here we find the fundamental results of Ladyzhenskaya and Ural'tseva, the Jenkins and Serrin criterion for solvability of the Dirichlet problem for the minimal surface equation.

The work is almost entirely self-contained, some basic facts of real analysis and linear algebra are supposed only. Much of the material appears in a single volume for the first time. A number of interesting problems, historical material, and bibliographical references are added to each chapter.

Summing up, if one wishes to see a variety of modern methods and their applications to some classical problems of elliptic partial differential equations, examples to illustrate the developments a few up-to-date results, and references to further study, all gathered in a book which is written carefully and in an enjoyable style, then he should read this book.

L. Hatvani—L. Pintér (Szeged)

Computing Methods in Applied Sciences and Engineering, Second International Symposium, December 15—19, 1975, Iria Laboria, Institut de Recherche d'Informatique et d'Automatique: Edited by R. Glowinski and J. L. Lions (Lecture Notes in Economics and Mathematical Systems, 134), VIII+390 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

This book consists of a selected part of the lectures which were presented during the symposium indicated in the title. It has five parts: Numerical Algebra, Finite Elements, Dynamical Problems, Identification and Inverse Problems, and Integral Methods.

Part I treats the solution of large linear sparse systems of linear algebraic equations arising at the application of the finite element method (by Alan George), hypermatrix algorithms in connection with the solution of linear equations or eigenreduction (by K. A. Braun, G. Dietrich, G. Frik, Th. L. Johnson, K. Straub, and G. Vallianos), iterative methods for the solution of non-compatible systems of linear equations, occurring in connection with the solution of systems of difference equations approximating the Neumann boundary problem for elliptic differential equations (by Y. A. Kuznetsov), a generalized conjugate gradient method for nonsymmetric systems of linear equations (by P. Concus and G. H. Golub), and spline approximation in Euclidean spaces (by V. A. Vasilenko).

Part II presents the mathematical foundations of hybrid and mixed finite element methods for plate bending problems described by fourth order elliptic equations (by F. Brezzi); the result that the strain energy of a shell is elliptic, using W. T. Koiter's linear model (by M. Bernadou and P. G. Ciarlet); the homogenization approach in engineering, where homogenization is meant as a method which studies the macrobehaviour of a medium by its microproperties (by Ivo Babuška); and finite element approximations for solving elastic problems (by J. Nitsche).

Part III deals with a finite element approximation for parabolic equations via an operator theoretical approach (by Hiroshi Fujita): a variational method for increasing the accuracy of the difference scheme in a close relation to the Richardson extrapolation (by G. I. Marchouk and V. V. Shaydourov); Runge—Kutta methods for the approximation of the evolution problem (by M. Crouzeix); and continuous and discontinuous finite element methods for solving the neutron transport equation (by P. Lesaint).

Part IV contains the survey papers of J. Cea (on domain identification problems), J. M. Boisserie and R. Glowinski (on optimization of rotational membranes), R. Glowinski and O. Pironneau (on optimal control), Masaya Yamaguti (on solidification), J. Galligani (on numerical problems of earth science), T. Dupont and H. H. Rachford Jr. (on a Galerkin method for liquid pipelines).

In Part V two papers give a glimpse into the integral equation methods applied to elasticity problems (by J. C. Lachet and J. O. Watson) and to fluid mechanics (by T. S. Luu), and a paper reviews curved finite element methods for the solution of singular integral equations on surfaces in R^3 (by J. C. Nedelec).

Of the above papers 13 are written in English, 9 in French. Each paper is followed by a rich and up-to-date bibliography.

The book is warmly recommended for use to research workers in numerical analysis as well as to experts in theoretical physics and engineering.

F. Móricz (Szeged)

H. Grauert—R. Rennert, Theorie der Steinschen Räume (Grundlehren der mathematischen Wissenschaften, 227), XX+249 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

The central results of complex function theory show that the fundamental difficulty in generalizing the classical theorems for a single variable to several variables is that in $\mathbf{C}^n$ there exist domains which are not domains of holomorphy. ($G\subset\mathbf{C}^n$ is called a domain of holomorphy if there is a holomorphic function on G which is singular in every boundary point of G). Since the main problems of complex analysis are solvable for domains of holomorphy, the natural question was raised how can one axiomatize intrinsically the complex spaces which are generalizations of domains of holomorphy and for which the classical results of complex analysis can be extended. A Stein space X is such a generalization of a domain of holomorphy, which in the singularity-free case (that is when X is a complex analytical manifold) can be characterized by the following properties:

(i) X is holomorphically separable, i.e. for every $x_0\in X$ there are holomorphic functions $f_1, \dots, f_m$ on X such that x_0 is isolated in the set $\{x\in X: f_1(x)=\dots=f_m(x)=0\}$.

(ii) X is holomorphically convex, that is, for every compact subset $K \subset X$ its holomorphically convex hull $\hat{K}$ is compact, where $\hat{K}$ is defined by

$$\hat{K} = \{x \in X: |f(x)| \leqq \sup_K |f|, \; f \text{ holomorphic on } X\}.$$

The theory of Stein spaces is developed using the methods of sheaf theoretic cohomology theory. A breaf survey on this subject is given in Chapters A and B. In Chapter I the coherence theorems on finite holomorphic maps are treated. Chapter II is devoted to the de Rham and Dolbeault cohomology theory. In Chapters III—IV the main theorems on Stein spaces (Theorems A and B) are proved, which are the generalizations of the Cartan—Serre theorems on singularity-free Stein mainifolds. Chapter V contains the fundamental applications of the main theorems to Cousin and Poincaré problems and to characterizations of Stein spaces. In Chapter VI the finite dimensionality theorem of Cartan and Serre is generalized to the complex spaces with singularities. Chapter VII treats the theory of compact Riemann surfaces, applying the preceding general results.

The book is a fundamental monography on the subject, it is well organized, the presentation is always clear. The reader is assumed to have a certain knowledge of complex analysis and sheaf theory.

P. T. Nagy (Szeged)

H. Grauert—K. Fritzsche, Several Complex Variables (Granduate Texts in Mathematics), VIII+207 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

This textbook is an excellent introduction to complex analysis in several variables, suitable for undergraduate students. The reader is only supposed to be familiar with elementary calculus, with the theory of complex functions of a single variable and with a few elements of algebra and general topology. In accord with its introductory character the book treats many examples and special cases in full detail and with numerous illustrative figures. At the end of the chapters the authors give a survey of the fundamental results of the theory, tempting the readers to further study.

In Chapter I the notion and basic properties of holomorphic functions of several variables are introduced. In contrast to the one-variable theory, there exist domains G in $\mathbf{C}^n$ such that every holomorphic function on G has a holomorphic continuation beyond G. Domains $G \subset \mathbf{C}^n$ for which such a continuation of holomorphic functions on G do not exist are called domains of holomorphy. In Chapter II some characterizations of domains of holomorphy are given. Chapter III is devoted to the algebraic treatment of the ring of convergent power series and to its applications to the theory of analytic sets, which are locally the sets of zeros of holomorphic functions. Chapter IV is a brief introduction to sheaf theory. In Chapter V the notion of complex manifolds and Stein manifolds are introduced. (The latter is a natural generalization of the domain of holomorphy.) Here a lot of examples of complex manifolds are discussed. In Chapter VI the cohomology theory of sheaves is treated. It is a useful generalization of the Čech cohomology theory and provides a frame to express the main results for domains of holomorphy and Stein manifolds. Chapter VII is devoted to the analysis of real differentiability in complex manifolds. The notions of tangent spaces, differential forms and exterior derivation are introduced and the theorems of Dolbeault and de Rham are proved.

P. T. Nagy (Szeged)

James E. Humphreys, Linear Algebraic Groups (Graduate Texts in Mathematics), XIV+247 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1975.

The theory of linear algebraic groups has been studied extensively during the past twenty years following the fundamental work of A. Borel, Chevalley, Steinberg, Tits and others, and has made a significant progress in a number of areas: semisimple Lie groups and arithmetic subgroups,

p-adic groups, classical linear groups, finite simple groups, invariant theory, etc. This theory is hardly accessible for beginners, because in the fundamental monographs on the subject a substantial familiarity with the abstract methods of algebraic geometry is assumed. This book is to serve as a detailed textbook for graduate students on affine algebraic groups over an algebraically closed field K, containing a very useful introduction to algebraic geometry.

An affine algebraic group G is defined as an affine algebraic variety (i. e. a set of common zeros of a finite collection of polynomials in an affine space K^n), endowed with a structure of a group such that the group operations $(x, y) \to xy$ and $x \to x^{-1}$ are polynomial functions on G. The standard examples of affine algebraic groups are the classical linear groups: the general linear group $GL(n, K)$, the special linear group $SL(n, K)$, the symplectic group $Sp(n, K)$, the special orthogonal group $SO(n, K)$. It is true that any affine algebraic group is "linear" in the sense that it is isomorphic with an algebraic subgroup of some $GL(n, K)$.

The building of the theory of affine algebraic groups is considerably parallel to the theory of Lie groups, only the differential topological terms and methods in Lie group theory are replaced by the terms and methods of algebraic geometry. One can define an intrinsic algebraic notion of tangent space to an algebraic variety at a point, which in the case of an algebraic group can be endowed with an additional Lie algebra structure. This way a functor can be defined from the category of affine algebraic groups to the category of Lie algebras, and with the help of this functor the structure theory of Lie algebras can be applied to the theory of affine algebraic groups.

The basic concepts of algebraic geometry are introduced in Chapter I. The treatment is detailed only according to necessity and is not scheme-oriented. In Chapters II—V the basic facts about algebraic groups, their Lie algebras and homogeneous spaces are treated. In Chapters IV—IX special questions, essential tools for the structure theory are discussed: the Jordan—Chevalley decomposition, diagonizable groups, nilpotent and solvable groups, Borel subgroups, maximal tori etc. Chapter X is devoted to the study of structure theory of reductive groups, especially properties of the root systems, normal and parabolic subgroups. In Chapter XI the representations and classification of semisimple groups are treated. Chapter XII contains a survey, without proofs, of the rationality properties of algebraic groups.

The reader is supposed to be conversant with the standard results of commutative algebra and the structure theory of semisimple Lie algebras. The treatise provides a rich and up-to-date account of the theory of linear algebraic groups.

P. T. Nagy (Szeged)

J. Lindenstrauss—L. Tzafriri, Classical Banach Spaces. I. Sequence Spaces, 188 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1977.

The present volume deals with sequence spaces; the notion of a Schauder basis plays a central role here. The text is divided into four chapters. Chapter 1 contains a quite complete account of the main results on Schauder bases in general Banach spaces. Some notions related to Schauder bases, e.g. approximation properties, biorthogonal systems, Schauder decompositions are discussed in detail. Chapter 2 is devoted to the study of the classical sequence spaces $l^p (1 \leq p \leq \infty)$ and c_0. Subspaces and characterizations of these spaces among Banach spaces are studied. This chapter contains also a discussion of general results related to the approximation property. The last section deals with extension properties of c_0 and l^∞, the lifting property of l^1 and the automorphisms of these spaces. In Chapter 3 the special properties of symmetric bases and the relation between symmetric bases and unconditional bases are discussed. The final chapter deals with the study of the structure of some particular classes of spaces with symmetric bases, mainly Orlicz sequence spaces, and gives a detailed discussion of such spaces.

Familiarity with the basic results of real analysis and functional analysis is assumed. The book is highly recommended to anyone who is interested in Banach space theory.

L. Gehér (Szeged)

P. Medgyessy, Decomposition of superpositions of density functions and discrete distributions (Disquisitiones Mathematicae Hungaricae 8), 308 pages, Akadémiai Kiadó, Budapest, 1977.

The main problem discussed in this book reads as follows. Given the graph of the superposition of an unknown number, say N, of components (of unimodal density functions or discrete distributions) of a given type, how is it possible to determine N and obtain approximate values of some of the unknown parameters of the superposition? Problems of this nature arise e.g. in analysing spectra in spectroscopy or nuclear physics, in biology, in mathematical statistics etc. The book is a successful attempt to treat these problems in a unified way making use of rigorous mathematical tools.

A little monograph "Decomposition of superpositions of distribution functions" (Akadémiai Kiadó, Budapest, 1961) was already published by the present author. As far as we know, this was the first systematic elaboration on such problems. Unfortunately, several problems that needed to be treated were numerically incorrect (ill-posed) thus their treatment was unsatisfactory there. A systematic investigation of handling ill-posed problems started in 1962. On the other hand, the author also found new ideas and methods in connection with the decomposition problems, as a result of which the whole discipline has taken a more coherent form.

The present book is not a revised or enlarged edition of the earlier work. Naturally, the main problem and certain results are the same in both books. However, they are restricted here to a narrower area: to superpositions of density functions and of discrete distributions, while the treatment in the earlier book was excessively general. As to the rest, however, this work is thoroughly new. The fundamental scope of the present book essentially belongs to numerical analysis, and not to probability theory or mathematical statistics. Only methods that can be realized numerically are considered, and several former procedures analytically elegant in themselves but useless in practice are omitted. In spite of this the majority of tools come from probability theory.

The book consists of five chapters, divided and subdivided into paragraphs, sections and subsections. A Postscript summarizes the possible tasks of future research. Remarks, historical comments, unanswered questions etc. are collected at the end of each paragraph under the title Supplements and problems. They may also point out the future tasks in this field.

Chapter I is an introduction. It provides the basic concepts and formulates the so-called decomposition problem. For density functions this reads as follows. Let $f(x; \alpha, \beta)$ be a two-parameter density function of known analytical form, and let a superposition

$$g(x) = \sum_{k=1}^{N} p_k f(x; \alpha_k, \beta_k)$$

be given, where $N, p_k, \alpha_k, \beta_k$ $(k=1, 2, \dots, N)$ are unknown parameters; there are no identical pairs (α_k, β_k) and $p_k>0$. We have to estimate these parameters or a part of them on the basis of the knowledge of $g(x)$.

Chapter II summarizes the mathematical tools applied in the book. Many of them are due to the author, e.g., a new characterization of the shape (of the "narrowness") of the graph of a density function or discrete distribution (§§ 4 and 5).

Chapter III is devoted to the decomposition of superpositions of density functions essentially by means of the unimodality preserving, narrowness-increasing transformations. Historically the first decomposition problem: the decomposition of superpositions of normal density functions, investigated by N. Sen in 1922, appears here as a particular case within a group of problems. This chapter also includes the decomposition of superpositions of exponential density funcions, which is more difficult than the former one. This problem is important in many fields of natural sciences, too.

Chapter IV deals with the decomposition of superpositions of discrete distributions. The methods here differ from those in the previous chapter essentially in their discrete character. From the viewpoint of numerical analysis the situation is much easier here; in the most important cases ill-posed problems do not appear and every theoretically good method can be adopted in practice.

Chapter V surveys those numerical methods which are of use in decomposition problems, e.g. the solution of integral equations of the first kind, numerical computation of convolution transforms, etc. The most important part of this chapter deals with the numerical treatment of ill-posed problems. Among others, the so-called regularization method invented by A. N. Tihonov in 1963 is given in detail, as well as several methods of the author, which have been proved useful in certain special cases.

The reading of the book requires only a few notions from probability theory and numerical analysis. A great number of figures helps the reader to understand the text. There is an abundance of numerical examples taken from practice.

At the end References, complete up to 1975, list the papers comprising nearly 400 items. Certain items from the References were picked out to compose a Chronological Bibliography.

To sum up, this well-written book fills in a gap in the literature. It provides a rich and up-to-date material of the fast-growing discipline indicated in its title, whose significance is becoming crucial for practice.

It is no exaggeration to say that the book is indispensable for everyone, either mathematician or specialist in a field of sciences, dealing with decomposition problems. It is also very useful for all those mathematicians whose interest is in probability theory, mathematical statistics or numerical analysis.

F. Móricz (Szeged)

N. Rouche—P. Habets—M. Laloy, Stability theory by Liapunov's direct method (Applied Mathematical Sciences, Vol. 22), XII+396 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1977.

In 1892 A. M. Ljapunov invented a new method — called by himself a direct method — for the study of stability and asymptotic properties of solutions of ordinary differential equations. By the aid of this method, based upon the study of the behaviour of certain scalar auxiliary "energy-like" functions along the solutions, he solved numerous important problems in theoretical mechanics and in the qualitative theory of differential equations. The direct method was further developed by N. G. Cetajev and his school in the 1930—40's. In the 1950's the development was even more rapid since the method proved to be most useful in the study of stability problems in control systems and in biological, physical, technical and económical systems described by ordinary differential equations.

Although this development is still going on, no book wholly devoted to the subject was published since 1967, when W. Hahn's book appeared. So an up-to-date monograph was needed to synthetize modern results of the theory, to describe its present state, and to make users of mathematics acquainted with the latest interesting applications taken from various fields. This excellent book answers these purposes in every respect. It is a collective work based on the material of a seminar held at the University of Louvain during the academic year 1971—72. Besides the three authors, C. Risito, K. Peiffer, R. J. Ballieu, Dang Chan Phien and J. L. Corne also worked on some chapters.

In the first two chapters the authors give a compact but intelligible introduction to the basic concepts, theorems and topics of stability theory, which are already considered classical.

Chapter III is a pearl in the book. As Lagrange stated and Dirichlet proved, a mechanical equilibrium of a conservative system is stable at each point where the potential function has a strict minimum. In this chapter, the authors first study some versions of this theorem. Next, they consider the inversion of this theorem, a classical incompletely solved problem of theoretical mechanics. In the two final sections the effect of dissipative and gyroscopic forces on stability properties of an equilibrium position is treated. The chapter is very valuable also because this topic was not considered in earlier monographs.

The following three chapters are: IV. Stability in the presence of first integrals; V. Instability; VI. A survey of qualitative concepts.

A set of the phase space is called attractive if every solution starting near the set tends to the set as $t\to\infty$. Chapters VII and VIII deal with this concept for autonomous and nonautonomous equations, which have been recently studied in addition to asymptotic stability.

Chapter IX is a splendid review of the comparison method, which can be considered as the combination of the classical Ljapunov method with the theory of differential inequalities.

The subject-matter of the book is fortunately selected so that the reader is informed of what happened in the Ljapunov theory of stability during the last decade. The style is always clear, precise, but not too abstract even for users of mathematics. For them the latest significant and characteristic applications of Ljapunov's direct method will especially be useful, e.g. stability and instability of the betatron, nonlinear electrical networks, the ecological problem of interacting populations, stability of composite systems.

This book is indispensable for specialists in stability problems, or more generally, in the qualitative theory of differential equations, but it is also useful for students and for everybody interested in applications of differencial equations.

L. Hatvani—L. Pintér (Szeged)

The State of the Art in Numerical Analysis, Proceedings of the conference held at The University of York, England, April 12—15, 1976; organized by The Institute of Mathematics and its Applications; edited by D. A. H. Jacobs; XIX+978 pages, Academic Press, London—New York—San Francisco, 1977.

The book surveys those areas of numerical analysis in which considerable advance has been achieved during the last ten years (1965—1975). It provides descriptions of theories, comparisons of methods, computational techniques and even algorithms, while indicating in some cases where current research efforts are being concentrated, and in others where future research might profitably be directed.

The book is divided into seven Parts, each of which comprises several chapters. Alternatively each Part of the book can be consulted to obtain a survey of one branch of numerical analysis, or all the Parts together to obtain an up-to-date overview of the many different branches and topics of numerical analysis.

Part I: *Linear Algebra.* Very valuable contributions are made here by J. H. Wilkinson (*Some Recent Advances in Numerical Linear Algebra* giving, among others, a concise account of the square-root-free Givens transformations, the *QZ* algorithm), P. E. Gill and W. Murray *(Modification of Matrix Factorizations after a Rank-one Change)*, and J. K. Reid *(Sparse Matrices)*.

Part II: *Error analysis.* It begins with the explanation of the ideas of error analysis made by C. G. Broyden, then follow contributions by N. Metropolis *(Methods of Significance Arithmetic)* and K. Nickel *(Interval-Analysis)*.

Part III: *Optimization and Non-Linear Systems.* Here the most remarkable chapters are by K. W. Brodlie *(Unconstrained Minimization)* and J. E. Dennis, Jr. *(Non-Linear Least Squares and*

Equations). The first gives a rather comprehensive account of the quasi-Newton methods from the original conception to modern implementations and new ideas, while the second treats unconstrained minimization, non-linear squares, and simultaneous solution of non-linear equations as a trilogy of problems. There follow contributions by P. E. Gill and W. Murray *(Linearly-Constrained Problems including Linear and Quadratic Programming),* R. Fletcher *(Methods for Solving Non-Linearly Constrained Optimization Problems),* and E. M. L. Beale *(Integer Programming).*

Part IV: *Ordinary Differential Equations and Quadrature.* The initial value problem (including the problem of stiff systems) is discussed by J. D. Lambert and the boundary value problem by J. Walsh. J. E. Lyness *(Quid, Quo, Quadrature)* deals in detail with the principles of automatic quadrature routines and multidimensional quadrature.

Part V: *Approximation Theory.* It contains three chapters: *Numerical Methods for Fitting Functions of Two Variables* by M. J. D. Powell, *Recent Results in Approximation Theory* by D. Kershaw, and *A Survey of Numerical Methods for Data and Function Approximation* by M. G. Cox.

Part VI: *Parabolic and Hyperbolic Problems* consisting of the following chapters: *Finite Element Methods in Time Dependent Problems* by A. R. Mitchell, *Initial-Value Problems by Finite Difference and Other Methods* by K. W. Morton with special emphasis on stability problems and convergence, *Splitting Methods for Time Dependent Partial Differential Equations* by A. R. Gourlay.

Part VII: *Elliptic Problems and Integral Equations.* An exhaustive study is given by L. Fox *(Finite-Difference Methods for Elliptic Boundary-Value Problems),* presenting new developments in economic direct methods, strongly implicit iterative and factorization methods, mechanization of some acceleration devices, etc. The further chapters are by R. Wait *(Finite Element Methods for Elliptic Problems)* and Ben Noble *(The Numerical Solution of Integral Equations).*

It may be anticipated that a large number of those practicing numerical analysis in industry or at universities and technical colleges will find great value in reading this book. Their knowledge and appreciation of the different aspects of numerical analysis should be greatly increased. It will also be of great value for teachers, as a source book of up-to-date information. Useful references for further study and a rich bibliography are added to this valuable work.

F. Móricz (Szeged)

André Weil, Elliptic functions according to Eisenstein and Kronecker (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 88), 93 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

In the first part of the book the author successfully presents Eisenstein's approach to elliptic functions. One of the most important virtues of this approach is that it supplies directly (without recourse to function theory) many formulas on elliptic functions, in the explicit form, appropriate for their use in number theory.

The principal chapters of this part are: Trigonometric functions, The basic elliptic functions. Basic relations and infinite products.

The second part gives a systematic exposition of applications of Eisenstein's approach. It also reinterpretes some of the results of Kronecker using the theory of distributions.

The principal chapters of this part are: Kronecker's double series, Pell's equation and the Chowla—Selberg formula.

J. Németh (Szeged)

Alan I. Weir, General Integration and Measure, vol. 2, XI+298 pages, Cambridge University Press, 1974.

The present volume (comprising Chapters 8—17 of the work) is written as "self-contained" as possible. Chapter 8 introduces the notion of the Daniell integral. The fundamental Monotone and Dominated Convergence Theorems are established. Measurable functions are defined in terms of integrable functions by using the idea of truncation of a function by another function. The notion of measure follows naturally. Stone's theorem relating measure and integration is proved. In Chapter 9 Lebesgue—Stieltjes integrals are defined as Daniell integrals on spaces $L^1(R^k)$, which contain all the step functions (or equivalently all the continuous functions on R^k which vanish outside a compact set). Various forms of Riesz's Representation Theorem for bounded linear functionals on the space of continuous functions on a compact topological space provide the subject matter of Chapter 10. In Chapter 11 the general notion of measure is introduced, the extension of a measure on a ring to a measure on a σ-algebra is done by means of the Daniell integral. Chapter 12 contains a classical approach of the problem of integration with respect to a measure on a σ-algebra. Chapter 13 is devoted to the study of uniqueness of extensions of measures in the case where the universal space is σ-finite with respect to the measure. In Chapter 14 product measures are defined as extensions of a measure on elementary product sets to a measure on a σ-algebra. Chapter 16 introduces the notions of real and complex measures. The Jordan Decomposition Theorem shows the connection between positive and real or complex measures. In Chapter 17 the Radon—Nikodým theorem is proved both in measure theoretic form and for Daniell integrals, and used for a study of dual spaces of L^p spaces. A short Appendix gives a summary of the most important topological notions used in the text. All the chapters end with exercises; the solutions can be found at the end of the book.

An elementary knowledge of topological spaces is assumed. The book is offered for students.

L. Gehér (Szeged)

Livres reçus par la rédaction

A. Baker—D. W. Masser (editors), **Transcendence theory: Advances and applications.** Proceedings of a conference held in Cambridge in 1976, X+236 pages, London—New York—San Francisco, Academic Press, 1977. — £ 11.00.

M. S. Bartlett, An introduction to stochastic processes with special reference to methods and applications, Third edition, XVII+388 pages, Cambridge University Press, 1977. — £ 12.50.

N. L. Biggs, Interaction models (London Mathematical Society Lecture Note Series, 30), VI+101 pages, Cambridge University Press, 1977. — £ 4.50.

B. Boos, Topologie und Analysis. Einführung in die Atiyah—Singer—Indexformel (Hochschultext), XVI+352 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1977. — DM 38,—.

Complex analysis and algebraic geometry. A collection of papers dedicated to K. Kodaira. Edited by W. L. Baily, Jr. and T. Shioda, XII+401 pages, Cambridge University Press, 1977. — £ 28.00.

R. H. Crowell—R. H. Fox, Introduction to knot theory (Graduate Texts in Mathematics, Vol. 57), X+182 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1977. — DM 27,90.

Á. Császár, General topology (Disquisitiones Mathematicae Hungaricae, 9), 488 pages, Budapest, Akadémiai Kiadó, 1977.

W. Felscher, Naive Mengen und abstrakte Zahlen. I, 260 Seiten, Mannheim—Wien—Zürich, Bibliographisches Institut, 1978.

P. Ganssler—W. Stute, Wahrscheinlichkeitstheorie (Hochschultext), XII+418 Seiten, Berlin—Heidelberg—New York, Springer-Verlag, 1977. — DM 36,—.