Bibliographie

M. Aigner, Kombinatorik, II. Matroide und Transversaltheorie (Hochschultext), XIII+324 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

The first part of this book (reviewed in these *Acta*, Vol. 38, p. 429) is an excellent introduction to modern "enumerative" combinatorics. This second part is an equally excellent treatment of matroid theory.

Matroids are a common abstraction of graphs, projective, affine and hyperbolic geometries, matrices (from the combinatorial point of view) and transversal systems. Accordingly, they can be described in many different but equivalent ways and one of the reasons of the strength of the theory is that each point of view yields a new insight to its problems. The book starts with formulating various systems of axioms for matroids and proving their equivalence, which is quite involved in some cases. This is followed by various examples of matriods and a description of the basic matroid operations (reduction, contraction, sum, extensions, duality). The second chapter deals with coordinatization and invariants like the chromatic and Tutte polynomials. In connection with graphic matroids, a considerably large part of graph theory is developed, among others planarity, flows, and chromatic number. The third chapter discusses transversal theory, including Menger's and Sperner's theorems, transversal matriods and gammoids.

The book is not only a rich, up-to-date account of this fast-growing and important field, but it is also very well-written. Exercises and references at the ends of the chapters help the reader in the further study of matroids. The book is warmly recommended to everyone learning, or doing research in, combinatorics.

L. Lovász (Szeged)

Tom M. Apostol, Introduction to Analytic Number Theory (Undergraduate Texts in Mathematics), xii+338 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

This textbook is a useful introduction to analytic number theory suitable for undergraduates with the knowledge of elementary calculus, but with no previous knowledge of number theory. The last four chapters require some background in complex function theory. The clarity of exposition is due to the fact that its material evolved from a course offered at the California Institute of Technology during the last 25 years. One of the goals of the author is to nurture the intrinsic interest of young mathematics students in number theory and to give some guide for them in the current periodical literature.

Chapters: 1. The Fundamental Theorem of Arithmetics, 2. Arithmetical Functions and Dirichlet Multiplication, 3. Averages of Arithmetical Functions, 4. Some Elementary Theorems on the Distribution of Prime Numbers, containing an elemantary proof sketch of the prime number theorem based on Selberg's asymptotic formula, 5. Congruences, 6. Finite Abelian Groups and Their Characters, 7. Dirichlet's Theorem on Primes in Arithmetic Progressions, 8. Periodic Arithmetical Functions and Gauss Sums, finishing with Pólya's inequality for the partial sums of primitive characters, 9. Quadratic Residues and the Quadratic Reciprocity Law, 10. Primitive Roots, 11. Dirichlet Series and Euler Products, 12. The Functions $\zeta(s)$ and $L(s, \chi)$ with a unified treatment of both functions by the Hurwitz zeta function, 13. Analytic Proof of the Prime Number Theorem, with applications to the divisor function, Euler's totient etc., 14. Partitions, as an introduction to additive number theory.

There are exercises at the end of each chapter.

A second volume is scheduled to appear in the Springer-Verlag Graduate Texts in Mathematics series under the title "Modular Functions and Dirichlet Series in Number Theory".

F. Móricz (Szeged)

Jon Barwise, Admissible Sets and Structures, An Approach to Definability Theory (Perspectives in Mathematical Logic series), XIV+394 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

Admissible set theory has been developing since the early sixties. It is a basic tool for studying definability theory over arbitrary structeres and it is also a basic source of interaction between model theory, recursion theory and set theory, theories all dealing in part with problems of definability and set existence. The book under review is the first monograph on the subject. It is written for graduate students, who are interested in mathematical logic, but because of its rich material, it can be considered as a handbook for specialists of admissible set theory, and on the other hand, because of its extremely clear, elegant, informal style, it is understandable and interesting even for all those mathematicians, who do not deal with the subject, but want to become acquainted with modern parts of mathematical logic.

In order to give an image of admissible sets to the reader of this review we quote from the introduction: "[In 1964] Kripke introduced admissible ordinals by means of an equation calculus. [In 1965] Platek gave an independent equivalent definition... by means of machines as follows: Let α be an ordinal. Imagine an idealized computer capable of performing computations involving less than α steps. A function F computed by such a machine is called α-recursive. The ordinal α is said to be admissible if, for every α-recursive function F, whenever $\beta<\alpha$ and $F(\beta)$ is defined, then $F(\beta)<\alpha$, that is, the initial segment determined by α is closed under F. The first admissible ordinal is ω. An ordinal like $\omega+\omega$ can not be admissible... The second admissible ordinal is, in fact, ω_1^c [the least non-recursive ordinal, i.e., the recursive analogue of ω_1]... Takeuti's work [in 1960—61] had shown that... the Kripke-Platek theory on an admissible ordinal α has a definability version on $L(\alpha)$, the sets constructible [in Gödel's sense] before the stage α... It leads us to consider *admissible sets,* sets A which, like $L(\beta)$ for α admissible, satisfy *closure conditions* which insure a reasonable definability theory on A."

The principles are formalized in a first order set theory KP. In order to study general definability over structures this theory is further generalized to a new theory KPU ("*K*ripke-*P*latek theory with *U*relements").

The book is supplemented by a list of references (consisting of about 150 items), a subject index, and a notation index.

A. P. Huhn (Szeged)

Christian Berg—Gunnar Forst, Potential theory on locally compact Abelian groups (Ergebnisse der Mathematik und ihrer Grenzgebiete 87), VII + 197 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

There are only few mathematical disciplines with so deep and vigorous connection to physics as potential theory has. It plays a central role not only in thermodynamics, electrostatics and gravitation, but also in mathematical analysis itself. Perhaps even today would physicists and mathematical analysts dispute over the proper place of potential theory, if, following a brakethrough in the mid-fifties, the whole theory had not been invaded by a new "enemy", probability theory. The probabilistic approach has not only led to new, very visual interpretations of the fundamental notions, but made many, formerly misteriously seeming relations transparent, and allowed to prove several new theorems.

The typical way of thinking in probabilistic potential theory is as follows. To any semigroup of contractive operators there belongs a potential theory, while any semigroup can be regarded as arising from some Markov process (the classical potential theory is associated with the Brownian semigroup, generated by the Laplace operator). The challenging fact for the analyst is that, though potential theory is linked with the transition semigroup only, the proofs generally use the whole Markov process, which is a much more complicated object. In order to recapture potential theory for analyis the proofs should be cleaned from arguments using sample paths of processes.

The present book is devoted to a partial solution of this problem, and presents a purely analytical treatment of the important class of transient convolution semigroups. The role of probability theory is degraded to support the reader by concrete examples only. Fourier-transform methods are systematically used as basic tools. Choosing locally compact Abelian groups for the fundamental space the authors seem to yield to the temptation of highest generality allowed by Fourier techniques. Debatable whether the gain on generality could compensate the loss of simplicity offered by the Euclidean space if the interesting noncommutative case cannot be included anyway.

The first two chapters making out about half of the book present the necessarry technical basis (I. Harmonic Analysis, II. Negative definite functions and semigroups), while the main topic is elaborated in the last chapter (III. Potential theory of transient convolution semigroups). On the reader's part a basic knowledge in functional analysis, Fourier-transform and group theory is assumed.

Besides its pioneering feature the book is distinguished by precise formulations, clear language and honest references. It will meet the interest of both analysts and probabilists.

D. Vermes (Szeged)

L. D. Berkovitz, Optimal Control Theory (Applied Mathematical Sciences, Vol. 12), IX + 304 pages, Springer-Verlag, New York— Heidelberg—Berlin, 1974.

Nowadays mechanization and automation of processes of production yield so complicated systems, the control of which needs methods scientifically well-founded. Control theory, one of the most successful and interesting branches of mathematics in the last twenty years, deals with such methods for systems having a mathematical model. This book is an introduction to the mathematical theory of optimal control of processes governed by ordinary differential equations.

In the first chapter there are presented some examples of control problems drawn from different areas of application: problems of production planning, chemical and electrical engineering, flight mechanics and the classical brachistochrone problem. In the second chapter the precise and quite general formulation of the mathematical problem of optimal control is given. The treatment of the relationship between problems in the calculus of variations and control problems concludes the chapter. Then basic existence theorems follow for problems in which a certain convexity condition is present. The key theorem of the development is an improvement of Cesari's theorem invented by the author. The fourth chapter contains existence theorems without convexity assumptions. Such a result is proved for the minimization problem for inertial controllers. This is a mathematical idealization of systems in which the controls are assumed to possess inertia. We learn the method of replacing the original problem by a "relaxed problem" in which the convexity assumption is satisfied. At the end of the chapter problems linear in the state variable are studied, in which the constraint set is independent of the state variable. In such systems the so-called "bang-bang principle" is valid. The fifth chapter is devoted to the maximum principle and some of its applications. The author shows how to obtain some necessary conditions of the classical calculus of variation from the maximum principle, he takes up particularly linear time optimal problem. The sixth chapter is the proof of the maximum principle.

The treatment of the subject has the proper mathematical exactness and abstraction. The material is arranged so that the readers primarily interested in applications can omit the more advanced mathematical sections without loss of continuity. The book can be read by anyone familiar with the elements of Lebesgue integration and functional analysis.

Although it is impossible for such a book to be completely up to date as new developments are so rapid in this theory, we are sure that this book will enable its readers, students or professionals in mathematics and in areas of applications, to navigate the turbulent waters of control theory.

L. Pintér—L. Hatvani (Szeged)

T. S. Blyth—M. F. Janowitz, Residuation Theory (International Series of Monographs in Pure and Applied Mathematics, Vol. 102), IX+382 pages, Oxford—New York—Toronto—Sydney—Braunschweig, Pergamon Press, 1972.

The aim of this book is to contribute to the textbook literature in the field of ordered algebraic structures. From the Preface: "The fundamental notion which permeates the entire work is that of a residuated mapping and is indeed the first unified account of this topic".

An isotone mapping f between the ordered sets A and B is *residuated* if there exists an isotone mapping $h: B \to A$ such that $h \circ f \geqq \mathrm{id}_A$ and $f \circ h \leqq \mathrm{id}_B$. It can be proved that if such an h exists it is unique and called the *residual* of f. The residuated mappings on a bounded ordered set E form a semigroup with a zero and identity, and many important properties of E can be naturally characterized in this semigroup. Examples:

(i) There is a bijection between the binary relations on a set E and the residuated mappings on the power set of E.

(ii) Every bounded linear operator f on a Hilbert space H induces a residuated mapping on the lattice of closed subspaces of H, namely the mapping $M \mapsto \{f(m) | m \in M\}^{\perp\perp}$.

(iii) If A is a commutative ring with an identity element then, in the ordered semigroup of ideals of A, multiplication by a fixed ideal is a residuated mapping.

The text of this book is divided into three chapters. Chapter 1 is an introduction to residuated mappings and lattice theory. This chapter contains all the elemantary material which is required later. The lattice theoretic fundamentals are treated with the help of residuated mappings. Chapter 2 deals with the concept of the Baer semigroup and uses residuated mappings to show how these semigroups may be used to study lattices. It contains some of the important works of D. J. Foulis and

S. S. Holland Jr. on orthomodular lattices. In Chapter 3 the notion of residuated mappings is used for a discussion of residuated semigroups (an ordered semigroup is called *residuated* if each translation on it is a residuated mapping.)

This well-organized book "is designed to satisfy a variety of courses.... For example, Chapter 1 may be used as an advanced undergraduate course on ordered sets and lattice theory; Chapters 1 and 2 as a one-semester postgraduate course on lattice theory; and the whole text as an M. Sc. course on lattices and residuated semigroups."

The book only assumes that the reader is familiar with the elements of abstract algebra. There are a lot of exercises throughout the book (for a few of which some knowledge of general topology is advisable).

The book is well-readable and most of the results contained in it appear for the first time in book form and some of them are only just seeing the light of day. Most of the results have been developed in the last decade. The book certainly will inspire further research.

L. Klukovits (Szeged)

A. A. Borovkov, Stochastic processes in queuing theory (Applications of Mathematics 4), XI+280 pages, Springer-Verlag; New York—Heidelberg—Berlin, 1976.

Queuing theory was originated by the engineer Erlang in the early years of this century, as he first applied probabilistic methods in the design of telephone centers. Later his methods were successively extended to the analysis of more and more general mass service systems with random service times and requests arriving stochastically from the customers.

Till now it has become an independent mathematical discipline, generally regarded as a subfield of applied probability.

But the phrase 'applied mathematical discipline' should be considered somewhat cautiously. If a pure mathematician reads e.g. the present book with the intention of learning what mathematics is good for, he would probably have a similar impression as he had studied lattice theory in fear of housebreakers. But he may not blame the book for his defect. Though queuing theory has its roots in applications, as a result of its development during the last half century it has reached the level of an axiomatic mathematical discipline, not less abstract than any other one. If someone really wants to apply it to a practical problem, the effort, generally necessary to connect theory with praxis, cannot be spared. But once this work has been invested, the present book will prove to be an extraordinarily useful aid.

The author presents queuing theory as a subfield of axiomatic probability theory, and this way he can cover an extremely broad class of problems within a reasonable space, and treat them by uniform methods. The first chapter is an introductory one dealing with the single server queue, but also pointing out all essential features of the theory. The following three chapters deal with functionals of sequences of i. i. d. (independent identically distributed) random variables. The original factorization method of the author allows to compute the distributions of several such functionals, a result with importance reaching far beyond queuing theory. The last three chapters deal with service systems with several and infinitely many servers and with refusals. Four appendices on renewal theory, ring factorizations, asymptotics of coefficients of series and estimates of distributions are included as well as a bibliography of 74 items. As prerequisite a basic knowledge of probability theory and stochastic processes, as well as some preknowledge in the motivation of the topic are assumed on the part of the reader.

Summing up, the book is a concise, uniform presentation of modern queuing theory in an exact form and clear language. It does not suggest problems in applications but helps anybody who is faced by such a problem.

D. Vermes (Szeged)

L. Fejes Tóth, Lagerungen in der Ebene, auf der Kugel und im Raum, XI+238 pages (Die Grundlehren der mathematischen Wissenschaften 65), 2nd edition; Springer-Verlag, Berlin—Heidelberg—New York, 1972.

Discrete geometry, the study of arrangements of various figures with certain extremality conditions, is one of the most vivid areas of geometry. This is due to a large extent to the first (1953) edition of this excellent monograph. This can be seen also from the fact that this second edition contains an Appendix of 29 pages, which surveys the most important developments in connection with problems formulated in the original edition. The riches of new results is really spectacular, and this Appendix may be very useful even for those who have read the first edition.

As pointed out in the preface, the author concentrates on arrangements in the best-known and most graphic spaces, the euclidean 2- and 3-spaces and the sphere. The first two chapters collect those results in elementary geometry and in the theory of convex bodies which play role in the sequel. It is, however, quite an informative reading even in itself, containing many interesting and not commonly known results. Chapters III and IV discuss optimal packings and coverings of planar regions by discs and other figures. Chapter V describes extremality properties of regular polyhedra. Certain quantities for polyhedra with a given number of vertices, edges and/or faces can be estimated so that equality stands for regular polyhedra only. For the case when no regular polyhedron with the given parameters exists, it is much more difficult to find the extrema. Problems of this type are discussed in Chapter VI. Chapter VII considers optimum packings and coverings in the space.

Let us finally remark that no knowledge of higher mathematics is required to read this book, and thus we may recommend it to everyone interested in geometry.

L. Lovász (Szeged)

P. J. Higgins, Introduction to Topological Groups (London Mathematical Society Lecture Note Series 15), V+106 pages, Cambridge, 1974.

Although important applications of topological groups require only a restricted part of it, textbooks generally cover the entire theory. This Introduction is designed to meet the needs of those who for the time being want to study topological groups for the sake of those applications only which utilize but a restricted part of the theory. Actually the author has given repeatedly introductory courses for first-year postgraduate students in algebra or number theory at the University of London and this Introduction is an amplified version of his lecture notes.

Chapter I contains such preliminaries as the fundamental concepts concerning groups and topological spaces. Topological groups are introduced in Chapter II where some basic facts concerning subgroups, quotient groups, connected groups and compact groups are presented. Chapter III is a concise account of integration on locally compact groups. Some examples and applications of the Haar integral are given in Chapter IV.

The presentation of this material is done with a perfection due both to a personal skill and to a deep familiarity with the literature.

J. Szenthe (Budapest)

John L. Kelley, General topology (Graduate Texts in Mathematics), XIV+298 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

This book is a reprint of the famous work of the author published by Van Nostrand in 1955. The text contains a systematic exposition of the most important topics of general topology. It is intended to provide the background material for modern analysis. It begins with a preliminary chapter (Chapter 0) which covers topics requisite to the main body of the work. The more serious results of this chapter are theorems from set theory. Chapter 1 introduces the concept of topological spaces, defines basic notions of topology and proves some simple theorems. Chapter 2 studies Moore—Smith convergence and characterizes those notions of convergence which can be described as convergence relative to some topology. The purpose of Chapter 3 is to investigate two methods of constructing new topological spaces from old ones. One of these is the standard method of topologizing the Cartesian product of spaces. The second method is based on the topological identification of the points of certain subsets of the spaces. This new topology is called the quotient topology. Both of these methods are defined by making certain functions continuous. Chapter 4 contains a systematic discussion of embedding and metrization theorems. In Chapter 5 the notions of compact and locally compact spaces are introduced. This chapter contains the most important theorems for compact spaces, and two methods of compactification of spaces: Alexandroff one point and Stone-Čech compactifications. Chapter 6 defines and discusses uniform spaces. In such spaces uniform continuity of functions and Cauchy nets can be defined. Conditions for the metrizability of uniform spaces are given and a proof can be found of the fact that any uniform space can be embedded in a complete uniform space, that is in a uniform space with the property that any Cauchy net has a limit point. This chapter ends with the Baire category theorem for metric spaces. Chapter 7 is devoted to the study of function spaces. The elements of these spaces are functions on a fixed set X to a fixed topological space Y. The various topologies of function spaces are discussed. Each chapter contains a rich collection of problems.

An Appendix deals with an axiomatic study of set theory.

L. Gehér (Szeged)

John L. Kelley—Isaac Namioka, Linear topological spaces (Graduate Tests in Matematics), XV+256 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976 (Second corrected printing).

The main purpose of this book is to give a detailed discussion of the theory of linear topological spaces, i.e. linear spaces with a topology such that scalar multiplication and addition are continuous. The text begins with an investigation of linear spaces (Chapter 1). The geometry of convex sets is the first topic which is peculiar to the theory of linear topological spaces. One section deals with the relation between orderings and convex cones. Both the algebraic and geometric forms of the Hahn—Banach theorem are proved. In Chapter 2, after establishing the geometric theorems on convexity, the elementary theory of linear topological spaces is developed. Most of the theorems of this chapter are specializations of basic theorems on topological groups or on uniform spaces, i.e. little use is made of scalar multiplication. The most serious results based on the full linear topological structure concern the criterion on normality. Chapter 3 is devoted to give a short glimpse into the fundamental category theorems. Chapter 4 deals with convex subsets of linear topological spaces and the closely related question of the existence of continuous linear functionals. The most powerful result of this chapter is the Krein—Milman theorem on the existence of extreme points of a compact convex set. Chapter 5 studies duality which is the central part of the theory of linear topological spaces. Duality can be defined only if the class of continuous linear functionals is large enough. This fact illuminates the role played by local convexity. Various topologies for a locally convex space and for its dual space are studied. The chapter concludes with a discussion of metrizable locally convex spaces. The text ends with an Appendix which deals with partially ordered locally convex spaces. The main result of the Appendix is the Kakutani characterization of Banach lattices which are of functional type or of L^1 type.

Familiarity of the reader with general topology is required.

L. Gehér (Szeged)

Peter Lax—Samuel Burstein—Anneli Lax, Calculus with Applications and Computing. Volume I (Undergraduate Texts in Mathematics), XI+513 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

"The traditional course (of calculus) too often resembles the inventory of a workshop, here we have hammers of different sizes, there saws, yonder planes; the student is instructed in the use of each instrument, but seldom are they all put together in the building of a truly worthwhile object" — say the authors in the preface. Their purpose is to emphasize the relation of calculus to science by devoting whole chapters to single, or several related, scientific topics. They intend to help the reader learn "how the notions of calculus are used to formulate the basic laws of science and how the methods of calculus are used to deduce consequences of those basic laws". Numerical methods are presented as organic parts of calculus. The treatment is intended to be rigorous without being pedantic.

Real numbers are thought of as (i) entities that can be added, multiplied, etc; (ii) points of the real line; (iii) infinite decimals. The derivative is defined as the uniform limit of difference quotients. "This makes it evident that a function whose derivative is positive on an interval is an increasing function". After the mean value theorem, Taylor's theorem, and the characterization of maxima and minima a section is devoted to one-dimensional mechanics. Integral is introduced as an additive function of interval that has the lower-upper bound property. The exponential function is defined as modeling growth. There is an introduction to both discrete and continuous probability theory. Gauss' law of error is proved and applied to the diffusion process. Sine and cosine are treated through complex numbers. A brief discussion of two-dimensional mechanics is offered in terms of complex numbers. There is a whole chapter on vibrations and another one on populations dynamics. FORTRAN programs, instructions for their use, as well as an Index are appended.

József Szűcs (Szeged)

G. I. Marchuk, Methods of Numerical Mathematics (Applications of Mathematics, Vol. 2), xii+316 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1975.

This English translation of the Russian original is an adaptation of a series of lectures on numerical mathematics given by the author at the Novosibirsk State University. An attempt has been made to focus attention on those complicated problems of mathematical physics which can be reduced to simpler and theoretically better-developed problems allowing effective computer realization. Besides, the needs of scientists and engineers are also taken into account.

Chapter 1 is a brief survey of the fundamentals of the theory of difference schemes, used extensively in the following chapters. For differential equations with sufficiently smooth coefficients it is possible to obtain high-accuracy approximate schemes, providing approximate solutions with a given accuracy, at the expense of a formal increase in the dimensionality of the subspaces involved (for instance, by decreasing the mesh size). Since the class of problems which possess fairly smooth coefficients is somewhat small, the author pursues the idea of building a general framework for constructing the difference analogues of the equations which do not possess high smoothness properties. Even problems with discontinuous coefficients come up, e. g., when studying diffusion, heat conduction, and hydrodynamics. Ch. 2 begins with a detailed exposition of boundary problems of ordinary differential equations, and then turns to more or less general approaches to solving two- and multi-dimensional problems.

Ch. 3 treats methods for solving stationary problems given in the form $A\varphi=f$ where the operator A coincides with a matrix, φ and f are vectors. Among others, over-relaxation methods, gradient iterative methods, splitting-up methods are discussed. The main object is to present the methods for solving nonstationary problems $\partial A/\partial t+\varphi A=f$, including stabilization methods, predictor-corrector methods, component-by-component methods, etc. As an application, effective algorithms are given for equations of hyperbolic type.

Ch. 5 is devoted to numerical methods for two types of inverse problems. The first type involves determining past states of a process. In the second type of problems, one has to identify the coefficients of an operator with a known structure in terms of information provided by some functionals of the solution. Inverse problems of mathematical physics are often ill-posed in the sense that small perturbation in the observed functionals may imply large changes in the correspoding solutions. For a long time ill-posed problems had been considered uninteresting, however, a need to interpret geophysical data triggered intensive research into these problems. Broad classes of ill-posed problems, the so-called conditionally well-posed problems, are studied here.

As an illustration of the fundamental methods of numerical mathematics, the author gives in Ch. 6 an elegant summary about the simplest problems of mathematical physics, i.e., the Poisson equation, the heat equation, the wave equation, and the equations of "motion".

Ch. 7 deals with the application of the splitting-up method to one of the modern branches of mathematical physics, namely to the theory of radiative transfer, of great significance in reactor and nuclear physics.

Ch. 8 is an expanded version of the lecture held by the author at the International Congress of Mathematicians in Nice (1970). This chapter briefly reviews the fundamental directions in numerical mathematics.

The presentation is concise, but always clear and well-readable. At the end of the book there is a vast and almost complete bibliography of each chapter separately.

The book is primarily intended to benefit practicing scientists encountering truly complicated problems of mathematical physics and seeking help regarding rational approaches to their solution. It may not be an exaggeration to assert that this book is of basic importance for everybody who deals with problems of applied and numerical mathematics.

F. Móricz (Szeged)

A. R. Pears, Dimension Theory of General Spaces, XII+428 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1975.

The book is intended to serve as a reference work for mathematicians interested in general topology. The text starts (Chapters 1 and 2) with a summary of the most important notions and results of modern general topology which are indispensable for the main body of the book.

In Chapter 3 the author defines the principal concept of dimension, called covering dimension, as the least integer n such that every finite open covering has an open refinement of order not exceeding n, if such an integer exist; in the contrary case the space is said to have dimension ∞. Though this definition concerns general topological spaces, in most results the spaces are supposed to be normal. For normal spaces, covering dimension can be defined in terms of the order of finite closed refinements of finite open coverings. For Euclidean spaces the covering dimension turns out to coincide with the usual one. Two more characterizations of covering dimension for normal spaces can be obtained in terms of mappings from the space into Euclidean spheres. The concept of dimension would be different if based on arbitrary locally finite coverings instead of finite coverings. There are normal spaces of dimenson 0 which would have infinite dimension if locally finite coverings were permitted. Sum and monotonicity theorems for covering dimension are proved. Chapter 4 introduces the concepts of the small and large inductive dimensions. The main idea of definition is based on reducing the dimension of a space to the dimensions of the boundaries of open sets. The large inductive dimension satisfies sum and subset theorems for totally normal spaces. The small inductive dimension (called Menger dimension) has the greatest intuitive appeal and satisfies the subset theorem for arbitrary spaces. For separable metric spaces the three concepts of dimensions mentioned above coincide. In Chapter 5 the concept of local dimension is defined and theorems analogous to those in Chapters 3 and 4 are proved. Chapter 6 is devoted to the study of images of zero-dimensional spaces. In this chapter two further notions of dimension are introduced. Both of these definitions are in terms of families of locally finite closed coverings of a special type. Chapter 7 shows that a very satisfactory theory of dimension can be constructed for metrizable spaces, though there exists a metrizable space the small inductive dimension of which differs from its large inductive and covering dimensions (P. Roy's example). Chapter 8 mostly deals with the pathological dimension theory of compact Hausdorff spaces. An example (due to V. V. Filippov) shows that the small and large dimensions of such spaces need not coincide. Chapter 9 is devoted to the study of various connections between dimension and mappings in spheres, and relations between the dimension of the domain and range of a continuous surjection. The product theorems for covering and large inductive dimensions are proved. In Chapter 10 the concept of covering dimension is modified for non-normal spaces. Dimension-theoretical applications of the algebra of bounded continuous real functions on a topological space are given. For the dimension of a Tihonov space an algebraic characterization can be found.

There are notes at the end of every chapter (except Chapter 1), which contain references to the original sources. The notes also survey some recent developments which are not included in the book.

The book is highly recommended to anyone interested in general topology.

L. Gehér (Szeged)

R. von Randow, Introduction to the Theory of Matroids (Lecture Notes in Economics and Mathematical Systems), IX+102 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1976.

Matroid theory, originated by the pioneering paper of H. Whitney in 1935, has strongly developed in the last two decades. It relates basic concepts of various branches of mathematics (like linear algebra, graph theory, finite geometries, integer programming) and has applications, for example, in operations research or in electric network analysis.

However, the different terminology of various papers and the seemingly confusing situation of the existing (at least 7) different systems of axioms might discourage some readers. Even the professionals need sometimes a reference (both for research and teaching) containing the relations between the axioms (how to deduce one system from another).

The present book is therefore very useful for giving a clear introduction to the basic concepts and rigorous formal proofs for the fundamental properties of matroids. Chapter I presents five axiomatic definitions, introduces the concepts of independent set, basis, circuit, and rank. Chapter II treates further properties (span, hyperplane, dual and cocircuit), while Chapter III lists some important examples (collection of vectors, binary matriods, graphic and cographic matroids, transversal matroids and gammoids). In the third chapter most of the results are stated without proofs. The greedy algorithm is briefly presented in Chapter IV, while the last chapter is devoted to the exchange properties of the bases in a matroid.

The greedy algorithm is certainly worth being included in any book about matroids — especially if the book is published in a series entitled "Lecture Notes in Economics and Mathematical Systems". However, in the opinion of the reviewer, the last chapter covers one of the less basic areas of matriod theory. Its theorems (and especially the more sophisticated counterexamples) can be presented in graduate courses very successfully, but are perhaps less essential in an introduction to the theory. The matroid partition and intersection theorems, mentioned "per tangentem" in the proof of Theorem 26, are perhaps more important than the whole fifth chapter.

Anyhow, the book gives a clear, up to date description of the fundamental concepts and results of the theory of matroids; it is recommended to everybody interested in this area of combinatorics.

A. Recski (Budapest)

Robert R. Stoll, Sets, Logic, and Axiomatic Theories, Second Edition, XI+233 pages, W. H. Freeman and Company, San Francisco, 1974.

This is the second edition of the author's highly popular textbook on the foundations of set theory and mathematical logic. The treatment is similar to that followed in the first edition but the material is more extensive. The book is intended to serve as a textbook for undergraduate students of mathematics and computer science. Its primary aim is "to bridge the gap between an undergraduate's initial conception of mathematics as a computational theory and the abstract nature of more advanced and more modern mathematics".

In Chapter I the elements of intuitive set theory are outlined. Specifically, this chapter discusses, within the framework of set theory, the following mathematical concepts: function, equivalence relation, ordering relation and natural number. A supplementary section deals with the axiom of choice. References to original works of Cantor, Frege, Russel and others make the reading of this chapter stimulating.

Chapter II presents the most basic notions and facts concerning the predicate calculus and first order logic.

After surveying the historical evolution of the axiomatic method, Chapter III deals with axiomatic theories. Among others consistency, completeness and independence of axiom system, metamathematics, recursive functions and Church's thesis are briefly discussed.

In Chapter IV the Lindenbaum algebra of a statement calculus is defined, whereafter a study of Boolean algebras completes the book.

A. P. Huhn (Szeged)

G. Szász, Théorie des treillis, IX+227 pages, Akadémiai Kiadó, Budapest, 1971.

After its Hungarian, German and English editions this is the French translation of the author's famous textbook on lattice theory. Since its first edition in 1959 this book has proved to be one of the most successful textbooks on algebra in general, and on lattice theory in praticular.

The book is intended to serve as a textbook for students wishing to study lattices and also for those mathematicians, especially algebraists, whose studies require some knowledge of lattice theory.

The chapter headings are: Partly ordered sets, Lattices in general, Complete lattices, Distributive and modular lattices, Special subclasses of the class of modular lattices, Boolean algebras, Semimodular lattices, Ideals of lattices, Congruence relations, Direct and subdirect decompositions.

A number of well-chosen examples and exercises help the reader understand the material. There is a bibliography consisting of 250 items and there are numerous references to this bibliography in the text bringing the mathematical research closer to the student.

This book can be recommended to anybody who is interested in abstract algebra.

A. P. Huhn (Szeged)

A. H. Stroud, Numerical Quadrature and Solution of Ordinary Differential Equations (A textbook for a beginning course in numerical analysis, Applied Mathematical Sciences, Vol. 10), XI+338 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1974.

This is a textbook for a one-semester course on the topics of numerical analysis mentioned in the title. It only requires from the reader knowledge of calculus; the occurring concepts are carefully defined and the results necessary to understand the subject are fully and exactly cited.

Chapter 1 (Background Information) contains statements of results from other branches of mathematics needed for numerical analysis. In Chapter 2 (Interpolation) the methods of interpolation used for the treatment of quadratures and differential equations are introduced. In Chapter 3 (Quadrature) three types of formulas for approximating definite integrals are discussed; these are the Newton—Cotes formulas, Gauss formulas and Romberg formulas. Chapter 4 (Initial Value Problems for Ordinary Differential Equations) contains classical methods of the numerical solution and some of their improved versions.

The book is well-written and well-organized. The methods are illustrated by interesting examples. For instance, one has the total numerical solution of the earth-moon-spaceship problem. Each of the paragraphs ends with problems. Marked sections serve as guides for further study.

The book contains Fortran-programs of the most important procedures, excellently running on computer in our experience.

Edith Huhn (Szeged)