Bibliographie

M. Aigner, Kombinatorik I. Grundlagen und Zähltheorie (Hochschultext), XVII+409 pages. Springer-Verlag, Berlin—Heidelberg—New York, 1975.

The foundations of Combinatorics have developed very rapidly in the past years. A few decades ago combinatorics meant a collection of various enumeration problems, and there existed (as a separate discipline) several graph theoretical, statistical, geometrical results, problems and puzzles of combinatorial nature. We are witnessing the arousal of new notions, methods and theories of large unifying und theorem-proving power. Such are matroid theory (combinatorial geometries), the functional analysis treatment of generating functions, the theory of Moebius functions, categorial and lattice theoretical methods — just to mention those treated in the first volume of this nice book. In the light of these theories the enumerative and the "structural" parts of combinatorics turn out to be much closer related than thought before.

This book reflects these new changes. Although its subtitle is "Foundations and Enumeration", it treats parts of combinatorics which are of "structural" nature but play an important role in the enumerative theory as well (e.g. lattice theory or matroids). It is a first, and successful, attempt to present modern combinatorics and its relations to modern mathematics (algebra, functional analysis, category theory) in a textbook form. It goes into the material in a considerable depth (treating e.g. the Pólya Method), and remains easily readable and elegant. There are about 375 exercises, some of which contain further theoretical material.

It is the significance and novelty of this presentation that makes some criticism in order here. One, if not the most important, goal in deriving (sometimes rather complicated-looking) formulas and generating functions is to obtain asymptotic results. Pólya's famous paper, for example, carries through such a program: it derives generating functions and then, by the methods of function theory, obtains asymptotical formulas. The development in the methods for the first part of such an investigation has caused a tendency of forgetting the second, and I miss a mention of this in this book too.

L. Lovász (Szeged)

E. M. Alfsen, Compact Convex Sets and Boundary Integrals (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 56), IX+210 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1971.

In the preface of his book the author says that "the integral representation theorems of Choquet and Bishop—de Leeuw together with the uniqueness theorem of Choquet inaugurated a new epoch in infinite-dimensional convexity". Although it has long been clear that convexity arguments are very fruitful in functional analysis, only with the advent of Choquet's theory a couple of decades ago did a comprehensive theory of infinite dimensional convex sets begin to exist. Now the original proofs of the basic results, initially considered technically difficult, are very much simplified. "Choquet

Theory provides a unified approach to integral representations in fields as diverse as potential theory, probability, function algebras, operator theory, group representations and ergodic theory." The book under review is an up to date introduction to Choquet Theory. It can be used as a text book for graduate students as well as a reference book for the working mathematician. It also tries to stimulate further study of the finer structure of infinite dimensional compact convex sets.

The book consists of two chapters. Chapter I: "Representations of Points by Boundary Measures". The paragraphs are: Distinguished Classes of Functions on a Compact Convex Set; Weak Integrals, Moments and Barycenters; Comparison of Measures on a Compact Convex Set; Choquet's Theorem; Abstract Boundaries Defined by Cones of Functions; Unilateral Representation Theorems with Application to Simplicial Boundary Measures. Chapter II: "Structure of Compact Convex Sets". The paragraphs in this chapter are: Order-unit and Base-norm Spaces; Elementary Embedding Theorems; Choquet Simplexes; Bauer Simplexes and the Dirichlet Problem of the Extreme Boundary; Order Ideals, Faces, and Parts; Split-faces and Facial Topology; The Concept of Center for $A(K)$; Existence and Uniqueness of Maximal Central Measures Representing Points of an Arbitrary Compact Convex Set.

As prerequisite, only some basic knowledge of functional analysis and integration theory is assumed on the part of the reader.

József Szűcs (Szeged)

R. Alletsee, G. Umhauer, Assembler I, II, III, Springer-Verlag, Berlin—Heidelberg—New York, 1974. 126, 150, 170 pages.

The books are useful for teaching or learning the IBM Assembly programming language. The student has to have only a limited preliminary knowledge about computer's hardware. Decimal, binary, floating point arithmetical, logical and branching machine instructions, furthermore the data and storage definition statements are treated. The Assembler instructions and the logical input/output macro instructions are not fully described. When finishing the course the student can write programs of one segment and one section with simple input/output activity. Numerous examples and excercises help to understand the notions and language elements. Test controls in the paragraphs qualify the books for using in assembler courses as a teacher's manual.

Árpád Makay (Szeged)

William Arveson, An Invitation to C^*-Algebras (Graduate Texts in Mathematics, Vol. 39), X+106 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

This excellent book conveys to the reader the fundamentals of the representation theory of separable postliminal C^*-algebras, which are called by the author, after Kaplansky, GCR (generalized completely continuous representation) algebras. A GCR algebra is a C^*-algebra A having the following property: for every (two sided and closed) ideal J of A the quotient C^*-algebra A/J contains a non-zero C^*-algebra B such that the range of every irreducible $*$-representation of B on a Hilbert space consists of compact operators. It is known and proved in the book that the spectrum $\hat{A}$ of a separable GCR algebra A bears a standard Borel structure which makes it possible to uniquely decompose every separable, nondegenerate $*$-representation π of A as a direct integral of "orthogonal copies" of irreducible representations: $\pi\cong\int_{\hat{A}}^{\oplus}m(\xi)\xi\,d\mu(\xi)$, where μ is a finite positive Borel measure on $\hat{A}$ and m is an integral (possibly infinite) valued non-negative measurable function on $\hat{A}$ ($m(\xi)$ is the multiplicity of ξ in π and the decomposition is unique up to the equivalence class of μ). The complete proof of this last assertion is the main achievement of the book. It might seem so that the GCR property of A is a very strict stipulation. However, it is mentioned in the preface and text proper that "to this day no one has given a concrete parametric description of even the irreducible representations of any C^*-algebra which is not GCR" and "there is mathematical evidence which strongly suggests that no one ever will'. Thus, in spite of its specialization, the book is complete in this respect.

If the idea of a proof is clear in a special case, then the generalization is relegated to the exercises. There are four chapters. Chapter 1 contains the rudiments of the theory of C^*-algebras. The second chapter deals with multiplicity theory, typé I von Neumann algebras, and type I repr, sentations of C^*-algebras. It gives the multiplicity theory of normal operations of C^*-algebras. It gives that all representations of a GCR algebra are type I. Chapter 3 is a nice introduction to polish spaces, standard and analytic Borel structures and cross sections. Chapter 4 uses the results of the preceding chapter to prove the decomposition theorem for representations of (separable) GCR algebras. It also contains a section on elementary reduction theory, just enough to prove the decomposition theorem. There is a bibliography and index.

The text tries to serve a large variety of readers: different subject matters are treated as independently as possible. Only the knowledge of the basic results of functional analysis, measure theory, and Hilbert space are assumed.

József Szűcs (Szeged)

Alan Baker, Transcendental Number Theory, X+147 pages, Cambridge University Press, 1975.

The book under review provides "a comprehensive account of the recent major discoveries" in the theory of transcendental numbers. At the beginning the author discusses the historical aspects of the theory and gives a survey of the subject as it existed around the turn of the century. The text includes among others the latest theories relating to linear forms in the logarithms of algebraic numbers, Schmidt's generalization of the Thue-Siegel-Roth theorem, Shidlovsky's work on Siegel's E-functions and Sprındžuk's solution to the Mahler conjecture. As proofs in the subject are usually long and intricate, the author felt necessary to select for detailed treatment only those that led to fundamental results and wide application.

"The test has arisen from lectures delivered in Cambridge, America and elsewhere, and it has also formed the substance of an Adams Prize essay."

József Szűcs (Szeged)

Raymond Balbes—Philip Dwinger, Distributive Lattices, XIII+294 pages, Columbia, Missouri, University of Missouri Press, 1974.

The theory of distributive lattices is one of the oldest branches of lattice theory. The connections of distributive lattices and other fields of mathematics, especially topology, algebra and logic are the sources of a number of deep and important results. However, for a long time the theory consisted of separate topics; the general methods to handle distributive lattices originated from universal algebra and category theory, and have been developed only in the last two decades. The authors of this book are among the eminent specialists in those researches leading to this development. Their book under review presents the theory of distributive lattices in the framework of a homogeneous theory based on topology, univeral algebra and category theory. The book is excellent and up-to-date.

From the Preface: "In Chapter I all those elements of univeral algebra and category theory which the reader will need — and in addition, some notions of set theory — are presented... The fundamental theory of distributive lattices is developed in Chapters II—VII. Some highlights in these chapters are the prime ideal theory, the representation theory, free algebras, coproducts and extension theorems... The special classes of distributive lattices which are discussed in this book are pseudocomplemented distributive lattices (Chapter VIII), Heyting algebras (Chapter IX), Post algebras (Chapter X), de Morgan algebras and Lukasiewicz algebras (Chapter XI). Finally Chapter XII is entirely devoted to complete and α-complete distributive lattices, which may satisfy a higher degree of distributivity."

There are numerous exercises scattered throughout the book. The book is addressed to graduate students and to those mathematicians who work in the field or want to become acquainted with it.

We may add that this book is useful and enjoyable for anybody who studies lattice theory or is interested in the applications of universal algebra and category theory.

A. P. Huhn (Szeged)

Anatole Beck, Continuous Flows in the Plane (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 201) X+462 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1974.

The study of continuous flows is an idealization of dynamical systems such as aerodynamics, hydrodynamics, electrodynamics etc. We imagine in the plane some sort of idealized particles which change position as time passes and after a time t, the particle which was at x will be transposed to the position $\varphi(t, x)$. After the definition of the moving points, fixed points, endpoints, stagnation points, regular and singular points, spirals, etc., the author gives a very geometrical description of the orbits. One of the basic results, the Gate Theorem, which simplifies the analysis of the orbits of any flow in the plane, is a generalization, in a sense, of the Jordan Curve Theorem.

If a flow φ is related to the flow ψ in such a way that every ψ-orbit is contained in a φ-orbit, we call ψ a reparametrization of φ. An important category of reparametrizations is the reparametrization by flow multiplers. In several chapters the author describes the important properties of these reparametrizations: canonical reparametrization, time measure of a quasi-reparametrization, algebraic combinations, etc. Every flow in the plane can be considered as a flow in the sphere which has ∞ as fixed point, every continuous flow in the sphere has at least one fixed point, thus the theories of flows in the plane and in the sphere are equivalent. In the chapters 6 and 7 the author concentrate on the problems: Given a flow φ on the boundary of a region, when does a continuous extension of this flow onto the given region exist? Let F be a compact subset of the sphere, and Y a subset of F. When does a continuous flow exist with fixpoints F and with stagnation points Y?

Let A and B be regions on the plane and φ a flow on A. Then for every homeomorphism f from A onto B this homeomorphism defines a flow $f\varphi$ on B. If $f\varphi$ is reparametrization of a flow ψ on B by a flow multiplier, then we say that φ and ψ are conjugate. It is examined in the last part of the book, when are the flows homeomorphic and when are they conjugate. The basic result of these analyses are the homeomorphism with an annular flow of standard type, the Theory of Kaplan and Markus, and the examination of the Kaplan diagramm.

The book only assumes a level of preparation equaivalent to first-year graduate courses, and it does not require any special knowledge of topology or differential equations. The work intended to serve as an introduction to the field of dynamics, particularly to readers with analytic training.

Z. I. Szabó (Szeged)

Norman Biggs, Finite Groups of Automorphisms (London Mathematical Society Lecture Notes Series 6), 117 pages, Cambridge University Press, 1971.

Since the beginnings of group theory, many important finite groups (especially, many simple ones) have been defined as automorphism groups of certain combinatorial structures. This book leads the reader through the main ideas of the development of this interrelation, starting with Galois and concluding with the quite recent discovery of new sporadic simple groups.

Chapter 1 is a brief introduction to permutation group theory.

Chapter 2 is devoted to the finite spaces and the finite linear groups. The simplicity of the projective linear groups and their relationship to projective geometries is shown. The symplectic, orthogonal and unitary groups are also introduced.

Chapter 3 introduces the $t-(v, k, \lambda)$ designs. For symmetric designs (when the numbers of points and blocks are equal), the Bruck-Ryser-Chowla theorem is derived. Then, transitive extensions of permutation groups and extensions of designs are studied. The Mathieu groups and the corresponding designs are introduced this way (following Witt's treatment).

Chapter 4 is concerned with automorphism groups of distance transitive graphs. (A graph $G=(V, E)$ is *distance-transitive* if, given $x_1, \ldots, x_4 \in V$ such that the distances $d(x_1, x_2)$ and $d(x_3, x_4)$ are equal, there is an automorphism $\alpha \in$ Aut G such that $\alpha x_1 = x_3$. $\alpha x_2 = x_4$. The "intersection matrix" contains the information on the numerical regularity properties of such a graph. A beautiful theory, provoding very restrictive necessary conditions on the existence of distance-transitive graphs with given intersection matrix in terms of eigenvectors of this matrix is developed. In the case when these conditions are fulfilled, the matrix is said to be *feasible*. The feasibility in the case of diameter 2 and the absence of triangles is studied in detail. Then, the problem of realizability of feasible matrices with small parameters is investigated. Finally, as a coronation of the material presented, a distance-transitive, triangle free graph of degree 22 with any two non-adjacent vertices having 6 common neighbors is constructed, hence the celebrated rank 3 simple group of Higman and Sims.

As an Appendix, a list of parameters of new sporadic simple groups and another list of the feasibility and of the status of realizability of intersection matrices of distance transitive graphs of diameter 2 and degree $\leqq 16$ is added. The literature mentions 10 books and 13 papers.

The book requires introductory linear algebra and group theory courses only. The selection of material as well as its presentation are excellent. It should be a pleasure for mathematicians interested in *combinatorics, linear algebra* and *group theory* to read the book, and to base (advanced) courses on it (as did the reviewer).

L. Babai (Budapest),

Norman Biggs, Algebraic Graph Theory (Cambridge Tracts in Mathematics, 67), vii+170 pages, Cambridge University Press, 1974.

The term "algebraic" in the title refers to classical algebraic techniques (determinants, matrices, polynomials, groups). The book exhibits some important areas of graph theory where applications of such techniques have proved particularly fruitful. Classical results of Kirchhoff, Cayley, Whitney as well as the striking development of the last few decades are represented in a unified treatment.

In Part I ("Linear algebra and graph theory"), the basic concepts are introduced (incidence and adjacency matrices, characteristic polynomial, spectrum of a graph Γ). The circuit- and cutset-spaces (the homology of Γ) and the complexity (the number of spanning trees) are discussed. Various expansions of determinants, related to Γ, in terms of certain subgraphs, conclude Part I.

Part II ("Colouring problems") starts with inequalities, bounding the chromatic number in terms of the spectrum of Γ. Among others a highly non-trivial lower bound, due to A. J. Hoffman, is derived.

The rest of Part II is devoted to the study of the *chromatic polynomial* of Γ. For u a positive integer this is the number of colorings of the vertices of Γ by colors chosen from the set $\{1, \ldots, u\}$

such that adjacent vertices have different colors, which turns out to be a polynomial in u. Several expansions in terms of various families of subgraphs are derived. The useful "logarithmic transformation" is introduced and applied to obtain a multiplicative expansion, depending on a restricted family of subgraphs. The deepest result of Part II is Tutte's identity, relating the *Tutte-polynomial* of Γ (defined in terms of certain spanning trees) to the *rank polynomial* (defined in terms of ranks and co-ranks of subgraphs). This is then applied to obtain another expansion of the chromatic polynomial, in terms of these trees.

The central concept investigated in Part III ("Symmetry and regularity of graphs") is that of *automorphisms* of Γ. Γ is *t-transitive* ($t \geqq 1$) if for any two paths of length t, and any directions given on them, there is an automorphism α of Γ mapping one onto an other. An elegant proof of Tutte's deep theorem is given, stating that *if Γ is a trivalent t-transitive graph, then* $t \leqq 5$. A 5-transitive trivalent graph is also exhibited. By a *covering graph* construction, infinitely many such graphs are obtained from a single one.

Next, *distance-transitive* graphs are introduced (see the above review on Biggs' "Finite Groups of Automorphisms"). Γ is called *distance-regular* if for any two vertices u and v, the number s_{hij} of vertices w having distance h from u and distance i from v depends only on the distance j between u and v. A distance-transitive graph is clearly distance-regular. Powerful matrix techniques are developed to handle distance-regularity. Part III ends with the beautiful theory of (k, g)-graphs, also known as Moore-graphs or cages (these are graphs of degree k and girth g, whose cardinality attains a certain trivial lower bound on the number of vertices). The main result, obtained by investigation of the multiplicities of eigenvalues of the adjacency matrix, is the following: *for* $k, g \geqq 3$, *a* (k, g)*-graph exists only if either* $g \in \{3, 4, 6, 8, 12\}$, *or* $g=5$ *and* $k \in \{3, 7, 57\}$.

The bibliography contains 80 items.

A great deal of material is included in the form of well-chosen examples and results at the end of each of the 23 chapters.

The most valuable feature of the book is the *concise, clear, exceptionally aesthetic presentation* of a really *exciting material*, almost no part of which has yet appeared in book form. Most proofs represent essential simplifications of the original ones.

The reader is assumed to have a moderate knowledge of matrix theory and the basic concepts of graph and group theory only. It appeals to mathematicians in any field, and probably it will soon become one of the fundamental works. Everyone interested in graph theory, combinatorics and applications of matrix techniques should read the book.

L. Babai and *P. Komjáth* (Budapest)

Kai Lai Chung, Elementary Probability Theory with Stochastic Processes (Undergraduate Texts in Mathematics), X+325 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1974.

This is the fir st volume of a new series and if the continuation will be so good as the beginning then this series will again be a new Springer-Verlag success. It is intended to be a very elementary introduction written by one of the outstanding experts of the field. A good deal of it does not even preassume calculus, but by brilliant organization, the author has succeeded in covering a wide range of topics, giving a real insight into the subject and preparing the reader for more advanced books. There are eight chapters: Set; Probability; Counting; Random variables; Conditioning and independence; Mean, variance and transformation; Poisson and normal distributions; From random walk to Markov chains; and three brief appendices: Borel fields and general random variables; Stirling's formula and DeMoivre-Laplace's theorem; Martingale. The body of each chapter also contains stimulating examples and at the end of each there are interesting classical and new problems for which solutions are also given at the end of the book. The emphasis is always on essential probabilistic reasoning, the style is inviting and at places humorous and all this is kept in good balance by the special intellectual power of the author. It can also stand up as a fine belletristic composition. Indeed, it is a book of great individuality.

S. Csörgő (Szeged)

N. S. M. Coxeter, Regular Complex Polytopes, X+185 pages, Cambridge University Press, Cambridge, 1974.

The very attentively constructed work gives a step by step introduction to the theme, beginning with plane and solid kinematics, through the geometrial description of the sixteen regular polytopes in four dimensional real Euclidean space and of finite multiplicative quaternion groups thereafter. (Chapters 1—7.) Meanwhile several devices and ideas which play central roles in the main Chapters are presented, such as free patterns, Cayley diagrams, the extended Schlöfli symbol, flags, Petrie polygons, Schwarz triangles, binary polyhedral groups, finite multiplicative quaternion groups etc.

In order to review the main sections of the book, let the corresponding part from the Preface be quoted: "The complete list of finite reflection groups in unitary n-space was complied in 1957 by Shephard and Todd, who found that there are many more of them in the plane than in any higher space. Chapter 10 checks their results (in the two dimensional case) by a new method: examining all the finite groups of unitary transformations and picking out those that are generated by reflections. In particular those that are generated by two reflections are the symmetry groups of the regular complex polygons. These are enumerated in Chapter 11. Somewhat surprisingly, it is possible to make real drawings of these imaginary figures, and in many cases such a drawing of one complex polygon serves as a *Cayley diagram* for the symmetry group of another. Chapters 12 and 13 deal with regular polytopes and honeycombs, using definitions suggested by Peter McMullen. There are interesting connections with certain projective configurations such as the 27 lines on the cubic surface. A remarkable presentation is found for the simple group of order 25920."

This book is an interesting and delectable reading both for research mathematicians and for students familiar with the material of the standard courses of elementary geometry and algebra. Most of the sections end with exercises; the solutions can be found at the end of the book. The beautiful presentation and the numerous figures also deserve special attention.

L. Stachó (Szeged)

Claude Dellacherie, Capacités et processus stochastiques (Ergebnisse der Mathematik und ihrer Grenzgebiete, 67) IX+155 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1972.

It is not an unfrequent opinion among mathematicians that the primary objects of probability theory are the distributions, and the sample space with its σ-fields constitutes only the necessary technical background. To avoid cumbrous measurability proofs some specialists prefer assuming sufficiently rich σ-fields to be given.

The author of the present book, a prominent member of the Strasbourg workshop of probability, does not share this opinion. On the contrary he shows that measurability properties of random processes with respect to some adequately defined σ-fields illuminate essential features of the processes and have deep connection with their sample path properties. The elegant general theory of stochastic processes elaborated in the book presents many classical questions (e.g. martingale decompositions) from a new unified view-pont. It can serve as a basis for a unified theory of stochastic integrals and can find important applications in statistics (filtration) of processes.

The book is divided into two parts. The first one contains the theory of Choquet capacity, the major tool of measurability proofs. This part bears interest not only for probabilists but for anyone working in measure theory. In the second part the main purpose of the book, the general theory of stochastic processes, is presented.

The whole exposition is brilliantly visual, its language is clear and easy-flowing.

D. Vermes (Szeged)

Joost Engelfriet, Simple Program Schemes and Formal Languages (Lecture Notes in Computer Science, Vol. 20), VI+254 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1974.

The aim of this book, as the author writes in the introduction, is "to fit a part of program scheme theory into a formal language theoretic framework in such a way that

(1) semantic properties of program schemes can be translated into syntactic properties of formal languages, and

(2) results from formal language theory need only to be 'rephrased' in order to be applicable to program schemes."

The book consists of three parts. In Part I formal languages are viewed as program schemes, called L-schemes. This is followed by the introduction of the following classes of program schemes: Ianov schemes, recursive systems of Ianov schemes, procedure parameter schemes, and μ-terms. The classes of L-schemes equivalent to these classes of program schemes are also given.

In Part II general properties of L-schemes, such as equivalence, semantic determinism and semantic regularity, are studied.

The general theory of L-schemes developed in Part II is used in Part III for investigating some specific problems concerning program schemes. Among the topics studied in Part III are the decidability of certain program scheme properties, translation of program schemes and program schemes with markers.

The book is self-contained with respect to the theory of program schemes. The reader is assumed to be familiar with the basic concepts of elementary set theory and elementary algebra as well as formal language theory.

The presentation of the material is very clear. The book is a valuable contribution to the literature of theoretical computer science.

Ferenc Gécseg (Szeged)

P. Erdős—J. Spencer, Probabilistic Methods in Combinatorics, 106 pages, Akadémiai Kiadó, Budapest, 1974.

This book describes a powerful method to prove theorems of combinatorial nature. The method, developed mainly by Erdős, is based on the following idea: often the existence of a certain structrure with some properties can be proved by selecting a structure at random and then showing that the probability that it has the desired property is positive. The method is, thus, non-constructive; somewhat surprisingly, it often gives much better results then any known constructive method.

The book illustrates the technique by solving a variety of combinatorial problems, some of very fundamental nature (e.g. Ramsey's Theorem, graph and hypergraph coloring etc.). In exercises several further results are listed, giving a good survey of the most recent status of these important researches. Several unsolved problems are stated as well. The treatment is elementary, it does not require any knowledge of probability theory, but it does require much computational skill in estimating binomial coefficients and in other techniques of "asymptotics".

It seems that the probabilistic method (with necessary modifications) may have a much wider range of application then found so far. Therefore, this nice book is most recommended to everyone learning, or working in, combinatorics or neighboring areas.

L. Lovász (Szeged)

Wendell H. Fleming—Raymond Richel, Deterministic and stochastic control theory (Applications of Mathematics, 1), 222 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1975.

Control theory is generally referred to as a modern discipline of applied mathematics though its fundamental problem "How to reach a goal in the best possible way?" is older then mankind itself. To have a well-posed problem clearly one has to define the goal of the activity and to say what is ment by the word "best" (i.e. to specify an expense function). But the very essence of the problem is determined by the possible ways of reaching the aim. The processes by which we can achieve our purpose determine our restricted freedom in the choice and we have to make the best possible compromise, i.e. to use the optimal strategy. Also the underlying processes serve as a basis for the classification of control problems into classes like deterministic, stochastic continuous, discontinuous problems, etc.

The first half of the present book contains a well-written self-contained exposition of deterministic control problems governed by ordinary differential equations. (Calculus of variations, Pontrjagin's principle, dynamic programming, existence and continuity of optimal strategies.) The proofs are detailed, many examples help understanding the presented material and its applications.

In contrast with the deterministic problems, no closed, rounded up theory exists as yet for stochastic control, not even for the control of diffusion processes, the subject of the second half of the book. So this part aims rather to introduce the reader into this rapidly developing field (up to its stage at about 1970), and to enable him to solve concrete problems. The authors start with a list of definitions and (in part rather deep) theorems from the theory of Markov processes and partial differential equations, necessary for the further development. Proofs are omitted but several examples and precise references support the reader not to get bored. The last chapter contains one, (the authors' own) approach to optimal control of diffusion processes via partial differential equations. It culminates in a sufficient optimality condition and an existence theorem, which enable them to solve the linear regulator problem, the permanent example in stochastic control. The Kalman-Bucy filter and the separation principle for linear systems are presented as well.

An extensive bibliography helps the orientation in recent literature.

D. Vermes (Szeged)

Dale Husemoller, Fibre Bundles (Graduate Text in Mathematics, 20), Second edition, 327 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1974.

This book contains important chapters of the theory of fibre bundles. The author concentrates on the work of Milnor, Hirzebruch, Bott, Adams, Hopf, Chern, Stiefel, Whitney, Grothendieck Atiyah, Toda, etc. In this second edition the author has added a section on the Adams conjuncture and an appendix on the suspension theorems.

The book consists of three parts. Part I contains the general theory of fibre bundles; the Milnor construction of a universal fibre bundle for any topological group is also given. Part II gives the elements of K-theory, namely stability properties of vector bundles, relative K-theory, Bott periodicity in the complex case, Clifford algebras, the Adams oprations and representations, representation rings of classical groups, the Hopf invariant, vector fields on the sphere and stable homotopy. The proof of Atiyah on the nonexistence of elements with Hopf invariant 1 is also presented and the proof of the vector field problem is sketched. A systematic development of characteristic classes and their applications to manifolds is given in Part III and is based on the approach of Hirzbruch as modified by Grothendieck.

Reading the book claims a certain knowledge from topology and the theory of differentiable manifolds. It is a very instructive reading due in part to the large number of exercises and examples.

Z. I. Szabó (Szeged)

John G. Kemeny—J. Laurie Snell, Finite Markov Chains (Undergraduate Texts in Mathematics), IX+210 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1976.

This book is a reprint of the 1960 edition published by D. Van Nostrand, Princeton, N. J., in the University Series in Higher Mathematics. No changes have been made of the first edition. It is a complete treatment of the theory of finite Markov chains and it has already proved its vitality in the last sixteen years. Suitable as an undergraduate introduction to probability theory or it can certainly replace a course in matrix calculus. Applications to learning theory and other socio-economic models (and to diffusion, genetics, sports, the Land of Oz and anything) are given. For a detailed review from such an authority as K. L. Chung see MR *22* (1961) #5998.

S. Csörgő (Szeged)

Rudolph Kurth, Elements of Analytical Dynamics (International Series in Pure and Applied Mathematics, Vol. 105), VIII+181 pages, Pergamon Press, Oxford—New York—Toronto—Sydney—Paris—Frankfurt, 1976.

This is a useful and easily readable textbook on analytical mechanics serving as a preparatory course to a profound study of topological dynamics for graduate students of mathematics. The reader is supposed to be familiar with some knowledge of calculus, general topology and differential geometry only. The mathematical structures occurring in the treatment of analytical dynamics are discussed in detail (e.g. the notion of differentiable manifold, elements of the theory of differential equations and of the calculus of variations). After the study of the Hamilton-Jacobi theory, Noether's theorem and the Liapunov stability theory the chapter "Jacobi's Geometric Interpretation of Dynamics" follows, which is a short introduction to Riemannian, Lagrangian and Finsler geometry.

P. T. Nagy (Szeged)

H. Elton Lacey, The isometric theory of classical Banach spaces (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 208), X+272 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1974.

The main purpose of this book is to investigate structural questions for classical Banach spaces. A Banach space is called classical, if it is either linearly isometric to an $L^1(\mu)$ space (real or complex) for some measure μ and some $1 \leq p \leq \infty$ or its dual space is linearly isometric to an $L^1(\mu)$ space; in the last case we say that the space is an L^1-predual space. Various necessary and sufficient conditions are given for a Banach space to be a classical one. They are framed in terms of conditions on the norm, conditions on the dual spaces and on subspaces. In the investigation the vector lattice structure of classical spaces plays a basic role. The book is divided into 7 Chapters. Chapters 1 and 2 summarize the fundamental definitions and theorems concerning partially ordered Banach spaces, topology and regular Borel measures. Chapter 3 deals with the algebraic and Banach space characterization of the space of continuous functions. Chapter 4 contains embedding theorems for classical sequence spaces into continuous function spaces. Chapter 5 is devoted to representation theorems for spaces of type $L^p(\mu)$. Chapter 6 contains characterizations of abstract L^p spaces and measure algebras (abstract L^p spaces are Banach lattices with p-additive norm). Chapter 7 gives characterizations of L^1-predual spaces.

All the chapters end with exercises and some open problems. General topology, Banach spaces, and measure theory are assumed as prerequisites.

L. Gehér (Szeged)

Ernst G. Manes, Algebraic Theories (Graduate Text in Mathematics, No. 26), 356 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

The following assignments are natural and often applied in mathematics: to each set S, assign its power set 2^S; to each element e, assign the one-element set $\{e\}$ (in this way one "inserts" S into 2^S); to each pair of relations, assign their relation-theoretical product (note that a relation between sets S and T may be considered as a mapping of S into 2^T). Formation of power set, insertion and product are connected by a few very simple laws; the same laws are observable, e.g., between formation of the free group $F(S)$ over S, insertion of free generators, and product of (homo)morphisms of free groups into one another (such a morphism of $F(S)$ into $F(T)$ may be considered as a mapping of S into $F(T)$). These assignments and laws lead to the notion of an *algebraic theory;* they furnish the "data" and "axioms" of this notion.

The above examples use the category of sets; however, algebraic theories can be defined over any category. The book we are concerned with develops a general theory of algebraic theories. This is the content of its main chapter, preceded by two big preparatory ones which are interesting also on their own right. The first of them presents a modern introduction to equational theory of algebras where infinitary operations are also allowed. The second chapter bears the attractive title "Trade Secrets of Category Theory", and, together with some paragraphs of the first chapter, it can serve as a mini-monograph on category theory for pure mathematicians. The last chapter deals with applications of algebraic theories to the following areas: topological dynamics, minimal realization of systems, theory of fuzzy automata. Since algebraic theories can be found in many further circumstances of algebra, topology and automata theory, the acquaintance with the third (main) chapter will be useful for everybody who is engaged in investigations in these fields.

The book is well-organized and well-readable; its style unites informality and exactness. The author helps the reader in several ways: every section is followed by historical notes and many exercises of various strength, while the entire book has useful indices and an abundant bibliography.

B. Csákány (Szeged)

P. McMullen—G. C. Shephard, Convex polytopes and the Upper Bound Conjecture (London Mathematical Society Lecture Notes Series 3), IV+184 pages, Cambridge University Press, 1971.

An outstanding problem in the theory of convex polytopes has been the Upper Bound Conjecture, describing which polytope (in d dimensions and with n vertices) has the largest possible number of faces. These notes were already in print when P. McMullen, one of the authors, succeeded to prove this famous conjecture. The solution was added to the book as a last chapter.

The book is devoted to the study of the combinatorial structure of convex polytopes. It describes the basic methods in this area: polarity, the Dehn-Sommerville questions, Gale diagrams, shelling. Many of these find their application in the solution of the upper bound conjecture. Although in great lines the presentation follows Grünbaum's well-known book "Convex Polytopes" (Wiley, 1967), there are several divergences, e.g. in the treatment of the support properties and in the proof of the Dehn-Sommerville equations. Also the authors manage to write up the material in a compact, and yet easily readable way. This book is well advised to all who want to learn, or do research in, the theory of convex polytopes.

L. Lovász (Szeged)

G. Pickert, Projektive Ebenen, Zweite Auflage (Die Grundlehren der mathematischen Wissenschaften, Band 80), IX+371 Seiten, Springer-Verlag, Berlin—Heidelberg—New York, 1975.

The first edition of this book in 1955 was the earliest in the mathematical literature giving a systematic treatment of the new domain of mathematics, called theory of projective planes, developed from the 1930's. The present book had a great effect encouraging the growth of the interest on this subject even beyond the area of foundation of geometries.

It is well known that the structure of projective planes has a greater variety then the structure of projective spaces, namely, Desargues's Theorem is not necessarily valid. Projective planes can be coordinatized by various not necessarily associative and distributive algebraic structures. Hence the projective planes provide models for algebraic structures, so they are useful in the study of questions of algebraic nature.

For the description of the structure of projective planes constructions and results from the geometry of webs (Geometrie der Gewebe) are used. This theory was introduced by Blaschke's school in the 1930's in connection with topological questions of differential geometry and developed later in algebraic and differential geometrical directions. A geometric web is three families of lines in the plane such that exactly one line of each family passes through each point. Very useful tools of the characterization of webs are the so-called "closure conditions", which are equivalent to identities for the coordinates of the plane.

The theory and classification of the finite and topological projective planes has made a very intensive progress in the last decades. The finite planes serve as standard models for combinatorial geometries, and the planes with topological and differentiable structures have a great interest in topological and differential geometry.

The book consists of 12 Chapters. The Chapters 1—2 serve as an introduction to the incidence structures and the theory of webs. Chapters 3—9 and 11 deal with planes satisfying various geometrical conditions and with algebraic investigations on the corresponding coordinate structures. In Chapters 10 and 11 a short introduction to the theory of topological and finite planes is given.

The book is recommended to mathematicians doing research in geometry, algebra or combinatorics and interested in problems connected with the theory of projective planes.

P. T. Nagy (Szeged)

G. Pólya—G. Szegő, Problems and Theorems in Analysis, Volumes I and II, (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Band 193 and 216), XIX+389 and XI+391 pages respectively, Springer-Verlag, Berlin—Heidelberg—New York, 1972 and 1976.

A number of mathematicians has been brought up with the help of the famous and excellent problem-book *Aufgaben und Lehrsätze aus der Analysis*. The present book is not only an English translation of the German original. The original text has been enlarged by many new problems and there are some other changes. All the alterations amount to less than ten per cent of the text. The book also contains the solutions of the problems, which is of great help to the reader. These books are recommended to students and research workers who are interested in classical analysis problems.

L. Gehér (Szeged)

W. Rinow, Lehrbuch der Topologie, 724 pages, VEB Deutscher Verlag der Wissenschaften, Berlin, 1975.

The main text of this book is based on lectures in topology which have been held by the author since 1950 at Greifswald University. In accordance with this fact it is aimed to be a university textbook. The selection and style of the text show Professor Rinow's natural turn for the methods of instruction. In contrast with most modern topology books the text comprises general, combinatorial and algebraic topology. The book is divided into fifteen chapters. The first seven chapters lead the reader along the most significant parts of general topology, discussing all the usual concepts and problems like tracing and comparison of topologies, relativization, convergence, continuity of mappings, separation, compactness, metrization, uniform structures, etc. Chapter VIII gives a glance into combinatorial methods in topology and applies these to give a proof for the classical domain invariance theorem in Euclidean spaces. Chapter IX is devoted to a short survey of dimension theory. Chapter X introduces the concept of homotopy, studies mappings in spheres and proves the domain invariance theorem again. Jordan curve theorem and Schoenfliess theorem are also proved. The chapter ends with a short investigation into surface topology. The last five chapters deal with various homologies and cohomologies, with the connection between homologies and homotopy and with duality theorems.

The book is recommended to students and to anyone taking interest in topology.

L. Gehér (Szege)

C. P. Rourke and **B. J. Sanderson, Iutroduction to Piecewise-Linear Topology** (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 69), VIII+123 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1972.

This book is an excellent introduction to modern geometric topology, treating the continuous and smooth topology as a unified subject. The generalization of many results of smooth topology is made possible by the application of the new technique of geometric topology, called the piecewise-linear (p.l.) topology.

Chapters 1—5 (Polyhedra and p.l. maps; Complexes; Regular neighbourhoods; Pairs of polyhedra and isotopies; General position and applications) serve as an undergraduate introductory course to p.l. topology. Here familiarity with the elementary notions of point-set topology is assumed only.

Chapters 6—7 (Handle theory; Applications) give an account of Smale's handle theory in a piecewise linear setting and of its applications to the Poincaré conjecture and the h-cobordism theo-theorem. Originally, this theory was developed using the technique of differentiable topology, in spite of the fact that these problems are of continuous topological nature.

The results of algebraic topology which are used are collected in Appendices. A bibliography of research papers is also included.

P. T. Nagy (Szeged)

S. Buoncristiano, C. P. Rourke and B. J. Sanderson, A Geometric Approach to Society (London Mathematical Society Lecture Note Series 18), VI+149 pages. Cambridge University Press, Cambridge—London—New York—Melbourne 1976.

From the introduction: "The purpose of these notes is to give a geometrical treatment of generalised homology and cohomology theories. The central idea is that of a 'mock bundle', which is the geometric cocycle of a general cobordism theory, and the main new result is that any homology theory is a generalised bordism theory. Thus every theory has both cycles and cocycles; the cycles are manifolds, with a pattern of singularities depending on the theory, and the cocycles are mock bundles with the same 'manifolds' as fibres."

In Chapter I the transition from functor on cell complexes to homotopy functor on polyhedra is axiomatised, the mock bundles of Chapter II being the principal example. In Chapter II, the simplest case of mock bundles, corresponding to p.l. (piecewise linear) cobordism, is treated, but the definitions and proofs all generalise to the more complicated setting of later chapters. Chapter III gives the geometric treatment of coefficients, where again only the simplest case, p.l. bordism, is treated. A geometric proof of functoriality for coefficients is given in this case. Chapter IV extends the previous work to a generalised bordism theory and includes the 'killing' process and a discussion of functoriality for coefficients in general (similar results to Hilton's treatment being obtained). Chapter V extends to the equivariant case and discusses the Z_2 operations on p.l. cobordism in detail. Chapter VI discusses sheaves, which work nicely in the cases when coefficients are functorial (for 'good' theories or for 2-torsion free abelian groups) and finally Chapter VII proves that a general theory is geometric.

P. T. Nagy (Szeged)

Joe Rosen, Symmetry discovered. Concepts and Applications in Nature and Science, 138 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1975.

This book, written with an excellent sense of didactics, introduces the reader to the examination of symmetry of geometrical objects, nature and science in a very light and witty style. Rosen starts his voyage of discovering the world of symmetry by explaining what symmetry is, and where and how to find it.

In the first part of the book the author describes the symmetry groups of forms in planar and 3-dimensional spaces with many examples and figures. But symmetry is not restricted to geometrical constructions alone. The author shows that physical operations are often symmetrical in nature, and he also gives an insight into symmetry provided by science and technology.

Reading the present work requires no special mathematical preparation. The reader is playfully introduced into the basic concepts and terminology of symmetry. For the readers who wish to pursue specific topics the author has supplied many references.

Z. I. Szabó (Szeged)

G. Segal, New Developments in Topology, (London Mathematical Society Lecture Note Series 11), 128 pages, Cambridge University Press, 1974.

In June 1972 a Symposium in Algebraic Topology was held in Oxford. The main theme of this Symposium was the K-theory: The present book contains eleven treatises on K-theory written by participants, based on their lectures. The familiarity of the reader with modern algebraic topology is required.

L. Gehér (Szeged)

D. J. Simms—N. M. J. Woodhouse, Lectures on Geometric Quantization (Lecture Notes in Physics, Vol. 53), II+166 pages, Berlin—Heidelberg—New York, Springer-Verlag, 1976.

These lectures are written in the spirit of the geometric quantization programme of B. Kostant and J.-M. Sourian. The aim of this programme is to formulate the procedure of quantization in differential geometric language. The systems of classical mechanics are modelled by symplectic geometries and Hamiltonian systems. The procedure of quantization is a construction of a Hilbert space H on which each classical observable (that is, each smooth function on the symplectic manifold M) is represented as an Hermitian operator in such a way that the Poisson bracket of classical observables is represented by the commutator of the corresponding operators. In the simplest case, the Hilbert space H consists of complex valued functions on the manifold M. In the case of more complicated systems (e.g. particles with internal degrees of freedom) H is constructed from the sections of a certain Hermitian line bundle over M. The described process of quantization is illustrated by very interesting examples.

The treatment assumes an experience in differential geometrical technique, especially in exterior calculus. In appendices a brief survey of the underlying mathematical theory is given: fibre bundles, Chern characteristic classes, and Lie algebra cohomology theory.

P. T. Nagy (Szeged)

Frank Spitzer, Principles of Random Walk (Graduate Texts in Mathematics 34), second edition, XIII+408 pages, New York—Heidelberg—Berlin, Springer-Verlag, 1976.

This is the second edition of a book (the first one was published by D. Van Nostrand, Princeton, N.J., in the University Series in Higher Mathematics, 1964) which can be safely called a classic. Classic, not in the sense that it would be old, but that it is fundamental and belongs to the group of best books ever published in probability theory. For an extensive and through-going review on the real mathematical content of the first edition we refer to MR 30(1965)#1521 by T. Watanabe. The book presents a complete and nearly self-contained treatment of random walk and certainly covers almost all major topics in the theory up to 1964. From the author's preface: "In this edition a large number of errors have been corrected, an occasional proof has been streamlined, and a number of references are made to recent progress". These new references (placed in brackets and footnotes) are to a supplementary bibliography, which contains 26 new items, and make the book again up-to-date. It is written mainly for probabilists and the prerequisite is, as described in the preface to the first edition, "some solid experience and interest in analysis, say, in two or three of the following areas: probability theory, real variables and measure, analytic functions, Fourier analysis, differential and integral operators". It has served as the main source for research in this area in the last twelve years, and it certainly will maintain this role for a long time to come.

S. Csörgő (Szeged)

Zhe-Xian Wan, Lie Algebras (International Series of Monographs in Pure and Applied Mathematics, Vol. 104), VIII+228 pages, Pergamon Press, Oxford—New York—Toronto—Sydney—Braunschweig, 1975.

This book is based on a series of lectures given in the seminar on Lie groups at the Institute of Mathematics of Academia Sinica (Peking) during the years 1961—1963. The purpose of the book "is to supply an elementary background to the theory of Lie algebras, together with sufficient material to provide a reasonable overview of the subject". In accord with its introductory character the book deals only with algebras over the complex field.

Chapters 1—4 present an introduction to the general theory of Lie algebras (nilpotency and solvability, Cartan subalgebras, Cartan's criterions). Chapters 5—8 deal with the structure and classification theory of semisimple Lie algebras and with their automorphisms. Chapters 9—11 serve as an introduction to the representation theory of semisimple Lie algebras. Chapters 12—15 contain selected topics on representation theory. Chapter 15 is devoted to the real forms of complex semisimple Lie algebras.

The book is well organized, the presentation is concise but always clear and well-readable, its format is nice.

P. T. Nagy (Szeged)

Bertram A. F. Wehrfritz, Infinite linear groups (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 76), XIV+229 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1973.

A linear group is a group of invertible matrices with entries in a commutative field. Their study started in the early years of this century with the work of Burnside and Schur. In the last twently years infinite linear groups have been used increasingly in the theory of abstract groups. On the one hand, much of the work on linear groups is hard to read for group theorists, and on the other hand, many results on linear groups appeared under purely group-theoretic titles. The book under review is the first to gather all this material together.

Infinite linear groups are useful in group theory in several ways. First of all, they arise via the automorphism groups of certain types of abelian groups: free abelian groups of finite rank, torsion-free abelian groups of finite rank and divisible abelian p-groups of finite rank. Thanks to Mal'cev, infinite linear groups play, in these days, a central role in the theory of soluble groups satisfying various rank conditions and in the theory of the automorphism groups of these groups. It is a recent result, that "the automorphism groups of certain finitely generated soluble (in particular finitely generated metabelian) groups contain significant factors isomorphic to groups of automorphisms of finitely generated modules over certain commutative Noetherian rings". Linear groups also arise via the following theorem of Mal'cev: a group G is isomorphic to some linear group of degree n if and only if each of its finitely generated subgroups is isomorphic to a linear group of degree n. If one has some information about which linear groups are isomorphic to the finitely generated subgroups of G, then one can sometimes find a concrete linear group that is isomorphic to G. "This led to very important characterizations of certain groups such as PSL(2, F) over locally finite fields F, which now play a crucial role in the theory of locally finite groups". In the author's opinion "to date we have only scratched the surface of the applications of infinite linear groups to locally finite groups."

Linear groups are also important in that they form a relatively accessible class of highly non-trivial, highly non-soluble groups, and, consequently, it is relatively easy to test conjectures on them. Moreover, it is quite common to solve a general problem for the linear case first. On the other hand, it sometimes happens that one ad-hoc knows that a group is isomorphic or related to a certain linear group.

The arrangement of the book is the following: the fundamentals are given in chapters 1, 5, 6, and, to some extent, 2. The basic material is split into two parts in order to present the theories of soluble linear groups and finitely generated linear groups in Chapters 3 and 4, before the reader gets bored. Roughly speaking, Chapter 1 is the ring theoretic and Chapters 5 and 6 are the geometric introduction. The rest of the 14 Chapters is devoted to the study of Jordan decomposition in linear groups, structure theorems for locally nilpotent linear groups, upper central series, locally super-soluble linear groups, periodic linear groups, groups of automorphisms of finitely generated modules over commutative rings, algebraic groups over algebraically closed fields. "Suggestions for Further Reading", a Bibliography, and Index close the book.

József Szűcs (Szeged)