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Weak dependecnies in the relational datamodel

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1 Introduction

One of the main concepts in relational database theory is the full family of functional dependencies, that was first axiomatized by W. W. Armstrong [1]. The full family of dual, strong and weak dependencies have also been introduced and axiomatized in [2,3,4]. The logical structures of them have also been investigated in [5,6,7,8,9,10].

In this paper, we give some results, that are related to weak dependencies. We give a necessary and sufficient condition for $W_R^+ = Y$, and then we construct an effective combinatorial algorithm to detemine irredundant relation R' for an arbitrary given relation R such that $R' \subseteq R, W_{R'}^+ = W_R^+$. Connections between dependencies are investigated also.

2 Definitions and axioms

Definition 2.1 *Let Ω be a finite set of attributes, and $R = \{h_1, \ldots, h_m\}$ be a relation over $\Omega, A, B \subseteq \Omega$. Then we say that B weakly depends A in R (denote $A \xrightarrow[R]{w} B$) if*

$$(\forall h_i, h_j \in R) \quad ((\forall a \in A) \quad (h_i(a) = h_j(a)) \longrightarrow (\exists b \in B) \quad h_i(b) = h_j(b)))$$

B functionally depends A in R (denote $A \xrightarrow[R]{f} B$) if

$$(\forall h_i, h_j \in R) \quad ((\forall a \in A) \quad (h_i(a) = h_j(a)) \longrightarrow (\forall b \in B) \quad (h_i(b) = h_j(b)))$$

B dually depends A in R (denote $A \xrightarrow[R]{d} B$) if

$$(\forall h_i, h_j \in R) \quad ((\exists a \in A) \quad (h_i(a) = h_j(a)) \longrightarrow (\exists b \in B) \quad h_i(b) = h_j(b))).$$

Let $W_R^+ = \{(A, B) : A, B \neq \emptyset \text{ and } A \xrightarrow[R]{w} B\}$ and $\overline{X} = \Omega \setminus X$ for any $X \subset P(\Omega)$.

$$F_R = \{(A, B) : A \xrightarrow[R]{f} B\} \text{ and } D_R = \{A, B) : A \xrightarrow[R]{d} B\}.$$

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Definition 2.2 *Let Ω be a finite set, and denote $P(\Omega)$ its power set, $P^+(\Omega) = P(\Omega) \setminus \{\emptyset\}$. Let $Y \subseteq P^+(\Omega) \times P^+(\Omega)$. Then we say that Y satisfies the w^+-axioms, if for all $A, B, C, D \in P^+(\Omega)$*

(w_1^+) $(A, B) \in Y,\ A \subseteq C, B \subseteq D \longrightarrow (C, D) \in Y$;

(w_2^+) $A, B \in P^+(\Omega), ((\forall X \in P^+(\Omega)) \quad (A \subseteq X \subseteq \overline{B} \longrightarrow (X, \overline{X}) \in Y)) \longrightarrow (A, B) \in Y$.

Let $Y \subseteq P(\Omega) \times P(\Omega)$. We say that Y satisfies the A-axiom if for all $A \subseteq \Omega$, there is an $E(A)$ such that

(f_1) $A \subseteq E(A)$, and $\forall B \subseteq E(A) \longrightarrow (A, B) \in Y$;

(f_2) $(C, D) \in Y, C \subseteq E(A) \longrightarrow D \subseteq E(A)$.

Y satisfies the B-axiom if for all $B \subseteq \Omega$, there is an $E(B)$ such that

(d_1) $B \subseteq E(B)$, and $\forall A \subseteq E(B) \longrightarrow (A, B) \in Y$;

(d_2) $(C, D) \in Y, C \not\subseteq E(B) \longrightarrow D \not\subseteq E(B)$.

Definition 2.3 *Let $Y \subseteq P^+(\Omega) \times P^+(\Omega)$. We say that Y is an w^+-family over Ω if Y satisfies the w^+-axioms.*

Let $Y \subseteq P(\Omega) \times P(\Omega)$. We say that Y is an $f-(d-)$family over Ω if Y satisfies the $A-(B-)$axiom.

Theorem 2.4 *[3]. Let $Y \subseteq P^+(\Omega) \times P^+(\Omega)$. If R is a relation over Ω, then W_R^+ satisfies the w^+-axioms. Conversely, if Y satisfies the w^+-axioms, then there is a relation R over Ω such that $Y = W_R^+$.*

3 The family of weak dependencies.

Definition 3.1 *Let Y be an w^+-family, and R be a relation over Ω. Then we say that R represents Y iff $W_R^+ = Y$.*

Definition 3.2 *Let Y be an w^+-family, over Ω, and $X \in P^+(\Omega)$. We say that $(X, \overline{X})$ is an Ω-dependency of Y if $(X, \overline{X}) \in Y$.*

Denote by $M(Y)$ the set of all Ω-dependencies of Y. We say that X is an Ω-left side of Y if $(X, \overline{X}) \in M(Y)$, and X is an Ω-right side of Y if $(\overline{X}, X) \in M(Y)$. Denote $GF(Y)$ the set of all Ω-left sides of Y, and $GD(Y)$ the set of all Ω-right sides of Y. It is obvious that $GF(Y)$ and $GD(Y)$ does not contain $\emptyset, \Omega$.

Theorem 3.3 *Let $G \subseteq P^+(\Omega) \setminus \{\Omega\}$. There exist exactly one w^+-family Y so that $GF(Y) = G$, where*

$$Y = \{(A, B) \in P^+(\Omega) \times P^+(\Omega) : (\forall X \in P^+(\Omega)) \quad (A \subseteq X \subseteq \overline{B} \longrightarrow X \in G)\}$$

Proof. In order to prove the theorem, we need the following lemma.

Lemma 3.4 *Let Y be an w^+-family over Ω. Then $(A, B) \in Y$ iff $(\forall X \in P^+(\Omega)) \quad (A \subseteq X \subseteq \overline{B} \longrightarrow (X, \overline{X}) \in M(Y))$.*

Proof. If $(A,B) \in P^+(\Omega) \times P^+(\Omega)$ satisfies

$$(\forall X \in P^+(\Omega)) \quad (A \subseteq X \subseteq \overline{B} \longrightarrow (X, \overline{X}) \in M(Y)),$$

then $(A,B) \in Y$ by (w_2^+). Conversely, if $(A,B) \in Y$, then

$$(\forall X \in P^+(\Omega)) \quad (A \subseteq X \subseteq \overline{B} \longrightarrow A \subseteq X, B \subseteq \overline{X} \longrightarrow (X, \overline{X} \in M(Y) \text{ by } (w_1^+)).$$

The lemma is proved.

□

We have to show that Y is an w^+-family. By the definition of Y, it is obvious that Y satisfies (w_2^+), where $M(Y) = \{(X,\overline{X}) \in Y\} = \{X,\overline{X}) : X \in G\}$, and $GF(Y) = G$. We have to prove that Y satisfies (w_1^+). For all $A,B,C,D \in P^+(\Omega), (A,B) \in Y, A \subseteq C, B \subseteq D, (\forall X \in P^+(\Omega)) \quad (C \subseteq X \subseteq \overline{D} \longrightarrow A \subseteq C \subseteq X \subseteq \overline{D} \subseteq \overline{B} \longrightarrow X \in G$ by $(A,B) \in Y)) \longrightarrow (C,D) \in Y$.

Now, we suppose that there is an w^+-family Y' so that $GF(Y') = G$, then $M(Y') = \{(X,\overline{X}) : X \in G\} = M(Y)$. Hence $Y' = Y$ by lemma 3.4. The proof is complete.

□

Corollary 3.5 *Let $G \subseteq P^+(\Omega) \setminus \{\Omega\}$. There exist exactly one w^+-family Y so that $GD(Y) = G$, where*

$$Y = \{(A,B) \in P^+(\Omega) \times P^+(\Omega) : (\forall X \in P^+(\Omega)) \quad (A \subseteq X \subseteq \overline{B} \longrightarrow \overline{X} \in G)\}$$

Definition 3.6 *[4]. Let $R = \{h_1, \dots, h_m\}$ be a relation over Ω. Let*

$$E_{i,j} = \{a \in \Omega : h_i(a) = h_j(a), 1 \leq i < j \leq m\}.$$

We call $E_{i,j}$ the equality set of R. Denote by E_R the family of all equality sets of R. Practically, it is possible that $\emptyset \in E_R$, and there are some $E_{i,j}$, which are equal to each other. According to the definition of relation, we have $\Omega \notin E_R$. Let

$$\begin{aligned} M_R &= \{E_{i,j} \neq \emptyset : \text{ if } E_{p,q}, E_{s,t} \in M_R, \text{ then } E_{p,q} \neq E_{s,t}\} \\ &= \{A_1, \dots, A_k : A_i \neq A_j \text{ for } i \neq j \text{ and } A_i \neq \emptyset \text{ for } i = \overline{1,k}\}. \end{aligned}$$

It is obvious that k is the number of elements of M_R, and all elements of M_R are not equal to each other. It is obvious that $A_i \notin \{\emptyset, \Omega\}$ for $i = \overline{1,k}$.

Theorem 3.7 *Let Y be a w^+-family, and R be a relation over Ω. Then R represents Y if and only if $GF(Y) = P^+(\Omega) \setminus (M_R \cup \{\Omega\})$.*

Proof. By theorem 3.3, it is easy to see that R represents Y iff $GF(W_R^+) = GF(Y)$. Consequently, we only must prove that $GF(W_R^+) = P^+(\Omega) \setminus (M_R \cup \{\Omega\})$.

It is obvious that $GF(W_R^+)$ does not contain $\emptyset$ and Ω. If $X \in P^+(\Omega) \setminus (M_R \cup \{\Omega\})$, then $X \notin \{\emptyset, \Omega\}$ and $X \neq E_{i,j}$ for $1 \leq i < j \leq m$. We have $(\forall h_i, h_j \in R) \quad ((\forall a \in X) \quad (h_i(a) = h_j(a)) \longrightarrow X \subset E_{i,j}, X \neq E_{i,j}$ and $E_{i,j} \neq \emptyset$ by $X \neq \emptyset \longrightarrow (\exists b \in \overline{X}) \quad (h_i(b) = h_j(b)))$.

Hence $(X,\overline{X}) \in W_R^+$ holds and we obtain $P^+(\Omega) \setminus (M_R \cup \{\Omega\}) \subseteq GF(W_R^+)$. Conversely, if $X \in GF(W_R^+)$, then $X \notin \{\emptyset, \Omega\}$ and $(X,\overline{X}) \in W_R^+$.

If $(h_i, h_j \in R)$ $((\exists a \in X)$ $(h_i(a) \neq h_j(a)))$, then $X \neq E_{i,j} \neq \emptyset$. If $(h_i, h_j \in R)$ $((\forall a \in X)$ $(h_i(a) = h_j(a)) \longrightarrow (\exists b \in \overline{X})$ $(h_i(b) = h_j(b)))$, then $X \neq E_{i,j}$.
Hence $X \neq E_{i,j}$ holds for $1 \leq i < j \leq m$, and we obtain

$$GF(W_R^+) \subseteq P^+(\Omega) \setminus (M_R \cup \{\Omega\}).$$

The theorem is proved.

□

Definition 3.8 *[10]. Let $R = \{h_1, \ldots, h_m\}$ be a relation over Ω. Let*

$$N_{i,j} = \{a \in \Omega : h_i(a) \neq h_j(a), 1 \leq i < j \leq m\}.$$

We call $N_{i,j}$ the non-equality set of R. Denote by N_R the family of all non-equality sets of R. Practically, it is possible that $\Omega \in N_R$, and there are some $N_{i,j}$, which are equal to each other. According to the definition of relation, we have $\emptyset \notin N_R$. Let

$$\begin{aligned} S_R &= \{N_{i,j} : \text{ if } N_{p,q}, N_{s,t} \in S_R, \text{ then } N_{p,q} \neq N_{s,t}\} \\ &= \{B_1, \ldots, B_k : B_i \neq B_j \text{ for } i \neq j\}. \end{aligned}$$

It is obvious that k is the number of elements of S_R, and all elements of S_R are not equal to each other. It is obvious that $B_i \neq \{\emptyset\}$ for $i = \overline{1, k}$.

Corollary 3.9 *Let Y be an w^+-family, and R be a relation over Ω. Then R represents Y if and only if $GD(Y) = P^+(\Omega) \setminus (S_R \cup \{\Omega\})$.*

The next proposition shows that from given any w^+-family Y, we can construct one simple non-empty relation R such that $W_R^+ = Y$.

Proposition 3.10 *Let Y be an w^+-family over $\Omega, GF(Y)$ be a set of all Ω-left sides of Y, and let $M = P^+(\Omega) \setminus (GF(Y) \cup \{\Omega\})$.*

If $|M| = 0$ then R is relation for any one-element. If $|M| \geq 1$ then we assume that $M = \{A_1, \ldots, A_k\}$, we set $R = \{h_1, h_2, \ldots, h_{2k-1}, h_{2k}\}$ as follows:

$$\text{for } i = 1, \ldots k : \forall a \in \Omega \quad h_{2i-1}(a) = 2i - 1$$

$$h_{2i}(a) = \begin{cases} 2i - 1 & \text{if } a \in A_i \\ 2i & \text{otherwise} \end{cases}$$

Then R represents Y.

Proof. If $|M| = 0$ then $GF(Y) = P^+(\Omega) \setminus \{\Omega\}$. So $(X, \overline{X}) \in Y$ for all $X \in P^+(\Omega) \setminus \{\Omega\}$ and we have $Y = P^+(\Omega) \times P^+(\Omega)$ by (w_2^+). Thus $W_R^+ = Y$ stands for any one-element relation and $R \neq \emptyset$. If $|M| \geq 1$ then it is obvious that $R \neq \emptyset$ holds. Clearly, $E_R = M \cup \{\emptyset\}$. Hence $M = M_R$ holds and we have $GF(Y) = P^+(\Omega) \setminus (M_R \cup \{\Omega\})$. By Theorem 3.7 we obtain $W_R^+ = Y$. The proposition is proved.

□

We say that R is w^+-irredundant relation if $R' \subset R$ imply $W_{R'}^+ \neq W_R^+$. We give an effective algorithm, which determines for a given arbitrary relation R a relation R' such that $R' \subseteq R, W_{R'}^+ = W_R^+$ and R' is irredundant.

Algorithm 3.11 *Let $R = \{h_1, \ldots, h_m\}$ be a relation over Ω.*

Step 1. From given relation R we construct $E_R = \{E_{i,j}$ is an equality set of $R, 1 \leq i < j \leq m\}$.

Step 2. From E_R we construct $M_R = \{E_{i,j} \neq \emptyset$: if $E_{p,q}, E_{s,t} \in M_R$, then $E_{p,q} \neq E_{s,t}\}$. Assume that $M_R = \{A_1, \ldots, A_k\}$.

We construct sets of index pairs, as follows: Let

$$I_1 = \{(i,j) : E_{i,j} = A_1\}, \ldots, I_k = \{(i,j) : E_{i,j} = A_k\}.$$

Denote by l_i the number of elements of I_i, where $i = 1, \ldots, k$.

Denote $I_q^1(p)$ and $I_q^2(p)$ the first and second indicies of p-th pair in I_q, where $q = 1, \ldots, k$ and $1 \leq p \leq l_q$. After that we perform the program IRREDUNDANT. Then $R' = \{h_i : i \in C\}$ is an w^+-irredundant relation such that $R' \subseteq R$, and $W^+_{R'} = W^+_R$.

Proof. It is obvious that $R' \subseteq R$. According to the construction of the algorithm, it can be seen that after we perform the program, $I_t \neq \emptyset$ holds for $t = 1, \ldots, k$. On the other hand, by theorem 3.7 we have $W^+_{R'} = W^+_R$. By procedure delete (i,j), program deletes all redundant rows of R. Thus, R' is an w^+-irredundant relation. The proof is complete.

□

We have $I_1[1 : l_1], \ldots, I_k[1 : l_k]$.

```
Program IRREDUNDANT;
begin
C := ∅;
for q := 1 to k do
     for p := 1 to l_q do
        for s := 1 to 2 do
           if I_q^s(p) ∉ C then
              begin
                 t := q;
                 while t ≤ k do
                    begin
                       r := 1;
                       while (I_t^1(r) = I_q^s(p) or I_t^2(r) = I_q^s(p)) and r ≤ l_t do
                             r := r + 1;
                       if r = l_t + 1 then
                                 begin C := C ∪ I_q^s(p); t := k + 2
                                 end
                                 else t := t + 1;
                    end;
                 if t = k + 1 then
                    for t := q to k do
                       for r := 1 to l_t do
                          begin
                             if I_t^2(r) = I_q^s(p) then begin
                                                        delete (I_t^1(r), I_q^s(p));
                                                        l_t := l_t - 1
                                                     end;
                             if I_t^1(r) = I_q^s(p) then begin
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                    delete $(I_q^s(p), I_t^2(r))$;
                    $l_t := l_t - 1$
                end;
            end;
        end;
end;
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Remark 3.12 *It can be seen that each Step of algorithm 3.11 requires time polynomial in the number of rows and columns of R. Consequently, the time complexity of Algorithm 3.11 is polynomial in $|R|$ and $|\Omega|$, where by $|R|$ and $|\Omega|$ the number of elements of R and Ω.*

It is easy to see that R', which is constructed in Algorithm 3.11 is an w^+-irredundant relation, and has maximal cardinality.

It can be seen that if R is any w^+-irredundant relation, and R represents Y, then $\sqrt{2|M|} < |R| < 2|M|$, where $M = P^+(\Omega) \setminus (GF(Y) \cup \{\Omega\}) \neq \emptyset$, and $|R| = 1$ when $M = \emptyset$.

4 Connections between dependencies.

Claim 4.1 *[3]. Let R be a relation over Ω and let $A, B \subseteq \Omega$. Then we have*

$A \xrightarrow[R]{f} B$ *iff* $(\forall b \in B)\ (A \xrightarrow[R]{w} \{b\})$;

$A \xrightarrow[R]{d} B$ *iff* $(\forall a \in B)\ (\{a\} \xrightarrow[R]{w} B)$.

We have obtained that W_R^+ uniquely determines F_R and D_R.

Definition 4.2 *Let F be an f-family over Ω, and $(A, B) \in F$. Then we say that (A, B) is a maximal right side dependency of F if $\forall B' : B \subseteq B', (A, B') \in F \longrightarrow B' = B$. Denote by $M(F)$ the set of maximal right side dependencies of F. We say that $B (B \subseteq \Omega)$ is a maximal right side of F iff there is an A so that $(A, B) \in M(F)$. Denote $G(F)$ the set of maximal riht sides of F. A family G of subset of Ω is closed under intersection iff $A, B \in G$ imply $A \cap B \in G$. Denote M^+ the set $\{\cap M' : M' \subseteq M\}$. We say that M generates G iff $M^+ = G$.*

Theorem 4.3 *[1]. Let F be an f-family over Ω. The $G(F)$ is closed under intersection. Conversely, if G is any family of subset of Ω, which is closed under intersection, then there exists exactly one f-family F such that $G(F) = G$, where $F = \{(A, B) : \forall C \in G : A \subseteq C \longrightarrow B \subseteq C\}$.*

Definition 4.4 *Let D be a d-family over Ω, and $(A, B) \in D$. Then we say that (A, B) is a maximal left side dependency of D if $\forall A' : A \subseteq A', (A', B) \in D \longrightarrow A' = A$. Denote by $M(D)$ the set of maximal left sides dependencies of D. We say that $A (A \subseteq \Omega)$ is a maximal left side of D iff there is an B so that $(A, B) \in M(D)$. Denote $G(D)$ the set of maximal left sides of D. A family G of subset of Ω is called d-semilattice iff G contains $\emptyset, \Omega$ and $A, B \in G$ imply $A \cap B \in G$.*

Theorem 4.5 *[2]. Let D be an d-family over Ω. Then $G(D)$ is a d-semilattice over Ω. Conversely, if G is any d-semilattice, then there exists exactly one d-family D such that $G(D) = G$, where $D = \{(A, B) : \forall C \in G : A \not\subseteq C \longrightarrow B \not\subseteq C\}$.*

Theorem 4.6 *Let Y be an w^+-family over Ω. Then $D(Y) = \{(A,B) : \forall a \in A, (\{a\},B) \in Y\}$ is an d-family over Ω and $G(D) = (P^+(\Omega) \setminus GD(Y) \setminus \{\Omega\})^+ \cup \{\emptyset\}$. $(D = D(Y))$*

Proof. It is easy to see that $D(Y)$ satisfies B-axiom. So $D(Y)$ is an d-family over Ω. Clearly, $G(D)$ is an d-semilattice over Ω. It is obvious that $G(D)$ contains $\emptyset, \Omega$. Set $\overline{GD(Y)} = P^+(\Omega) \setminus GD(Y)$, clearly, $(\overline{GD(Y)} \setminus \{\Omega\})^+$ contains Ω (by convention $\cap\emptyset = \Omega$). Now, we assume that $X \neq \emptyset, \Omega$ and if $X \in \overline{GD(Y)}$ then $(\overline{X}, X) \notin Y$. Set $X_1 = \{a \in \Omega : (\{a\}, X) \in D\} = \{a \in \Omega : (\{a\}, X) \in Y\}$. We have $X \subseteq X_1$, if we suppose that $X \neq X_1$ and choose an element a from $(X_1 \setminus X)$ then $(\{a\}, X) \in Y$ and $a \notin X$. So $(\overline{X}, X) \in Y$ by (w_1^+), this contradicts $(\overline{X}, X) \notin Y$. Hence, $X = X_1$ and $X \in G(D)$. We obtain $(\overline{GD(Y)} \setminus \{\Omega\})^+ \cup \{\emptyset\} \subseteq G(D)$.

Conversely, if $X \in G(D)$ and $X \notin \{\emptyset, \Omega\}$, then $X = \{a \in \Omega : (\{a\}, X) \in Y\}$. If we assume that $\forall Z \in \overline{GD(Y)} : X \supset Z$ then $X = \Omega$ by (w_2^+). So this contradicts $X \neq \Omega$. Consequently, there is an $Z \in \overline{GD(Y)}$ such that $X \subseteq Z$.

If there is an $Z \in \overline{GD(Y)}$ such that $X = Z$, then $X \in (\overline{GD(Y)} \setminus \{\Omega\})^+$.

Conversely, we set $H = \{Z \in \overline{GD(Y)} : X \subset Y\} = \{Z_1, \ldots, Z_k\}$. We have $X \subseteq \bigcap_{i=1}^{k} Z_i$. Let us choose an element a from $\bigcap_{i=1}^{k} Z_i$ then $(\{a\}, X) \in Y$ by (w_2^+). So, we have $\bigcap_{i=1}^{k} Z_i \subseteq X$. Thus, $X = \bigcap_{i=1}^{k} Z_i$ and we obtain $X \in (\overline{GD(Y)} \setminus \{\Omega\})^+$.

The theorem is proved.

□

Corollary 4.7 *Let Y be an w^+-family over Ω, $\overline{GF(Y)} = P^+(\Omega) \setminus GF(Y)$, $C = \bigcap_{X \in \overline{GF(Y)}} X$. Then $F(Y) = \{(A,B) : \forall b \in B, (A, \{b\}) \in Y\} \cup \{(\emptyset, D) : D \subseteq C\}$ is an f-family over Ω and $G(F) = (GF(Y) \setminus \{\Omega\})^+$.*

Remark. It is easy to see that $F(Y)$ satisfies A-axiom and we have $E(\emptyset) = C$.

References

[1] W. W. Armstrong; Dependency Structures of Data Base Relationships, Information Processing 74, North-Holland Publ. Co. (1974) pp. 580-583.

[2] G. Czédli; Függőségek relációs adatbázis modellben, Alkalmazott Matematikai Lapok 6 (1980) pp. 132-143.

[3] G. Czédli; On dependencies in the Relational Model of Data, Elektronische Informationsverarbeitung und Kybernetik, EIK 17(1981) 2/3, pp. 103-112.

[4] J. Demetrovics; Relációs adatmodell logikai és strukturális vizsgálata, MTA-SZTAKI Tanulmányok, Budapest, 114 (1980) pp. 1-97.

[5] J. Demetrovics, Gy. Gyepesi; On the functional dependency and some generalizations of it, Acta Cybernetica V/3 (1988) pp. 295-305.

[6] J. Demetrovics, V. D. Thi: Some results about functional dependencies, Acta Cybernetica VIII/3 (1988) pp. 273-278.

[7] J. Demetrovics, V.D. Thi; Relations and minimal keys, Acta Cybernetica VIII/3 (1988) pp. 279-285.

[8] V. D. Thi; Minimal keys and antikeys, Acta Cybernetica Tom 7, Fasc. 4 (1986) pp. 361-371.

[9] V. D. Thi; Strong dependencies and *s*-semilattices, Acta Cybernetica VIII/2 (1987) pp. 175-202.

[10] V. D. Thi; Logical dependencies and irredundant relations, Computers and Artificial Intelligence, Vol. 7, 1988, No. 2, pp. 165-184.

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