A 5 state solution of the early bird problem in a one dimensional cellular space

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There exists a class of interesting problems for cellular automata characterized by their common property of decomposing some global behaviour into homogeneous parallel local transitions (VOLLMAR [6]). Well known representatives of this class are the firing squad synchronization problem (MOORE [2], VOLLMAR [4]) and the French flag problem (HERMAN [1]).

Another problem of this class was defined by ROSENSTIEHL et al. in [3] and named as the "early bird" problem.

1. The original definition of the early bird problem

To each of the n vertices of an elementary cyclic graph there is assigned an automaton. These automata may be "excited" (birds may come from the outside world) at different moments. The task is to distinguish between the first (early) and the later birds. More exactly the transition function must ensure the automaton excited first to be assumed a distinguished state while all the others a different state after some time interval. ROSENSTIEHL et al. [3] gave a $2n$ step solution on condition that at each moment maximally one excitation occurs.

2. The modified early bird problem

VOLLMAR in [5] defined the problem for a one-dimensional cellular space allowing more than one cell to be excited at a given time step. Only quiescent cells may be excited; before the first time step at least one cell should be excited. After a certain period the first bird(s) should be in a distinguished state while all the others in a different state.

The solution (VOLLMAR [5]) uses the "age of waves" concept: each bird sends out age signals that are compared (numerically). As a consequence elder bird(s) survive, while waves of the same age or waves reaching the border are reflected and mark the sender automata. After a certain number of time steps there remain(s) only early bird(s) marked from both directions.

3. A 5 state solution to the problem

The proposed solution uses the "age of waves" concept of VOLLMAR [5] but in a simplified manner. The age of a wave (i.e. of a bird) is modelled directly by the *length of the waves,* rather than by a counter which is hard to handle, especially, for the number of needed bits of a counter is dependent on the number of cells. Therefore the counter cannot be incorporated in cells' states, it is rather simulated by a group of cells.

The basic idea is to send L (left) and R (right) waves in the left and the right directions. At each time step the wave is growing by one cell thus modelling the age of the sender. When two waves are colliding, pairs of R and L states annihilate each other, and N (neutral) states will replace them.

An L or R wave reaching a bird (in state B) will cause the annihilation of it (state N will be generated instead of the state B).

Consequently, the needed cell-states are:

Q = quiescent (initial) state,
B = bird state (arises from state Q, spontaneously),
L = left wave, expanding to left,
R = right wave, expanding to right,
N = neutral state.

4. Construction of the transition function

In the following we construct the transition function on the basis of the above-described principle. The transition function will be described with "left, own, right $\to$ $\to$ new-state" terms.

First we assume only two birds with different ages (they were born in different time-steps). Each bird sends waves in both directions, this is ensured by terms

1. $BQQ \to R$,

2. $QQB \to L$.

The waves are growing in each step:

1/a. $RQQ \to R$,

2/a. $QQL \to L$.

It is clear, that the length of the waves is equal to the age of the sender, in each step. After a certain time the waves are colliding between the birds, then an annihilation process begins:

3. $RQL \to N$,
4. $RRL \to N$ }
5. $RLL \to N$ } These terms imply the transition $RRLL \to RNNL$ (that is, each section of cells with states $RRLL$ goes into $RNNL$).

From annihilation a neutral area arises, in which the R states step to the right, the L states to the left (the points mean arbitrary state):

$$\left.\begin{array}{lll} 6. & RN\ (\text{not } L) & \to R \\ 7. & \cdot RN & \to N \end{array}\right\}\ R \text{ steps right by the transition } \cdot RN\ (\text{not } L) \to \cdot NR \cdot$$

$$\left.\begin{array}{lll} 8. & (\text{not } R)\ NL & \to L \\ 9. & NL \cdot & \to N \end{array}\right\}\ L \text{ steps left by the transition } (\text{not } R)\ NL \cdot \to \cdot LN \cdot$$

$$\begin{array}{lll} 10. & RNL & \to N \quad \text{annihilation.} \end{array}$$

If the left bird is the earlier one, then after a certain time all the L states are annihilated between the birds, and the remained R states can go to the right and "kill" the right bird:

6/a. $RBR \to R$,
7/a. $\cdot RB \to N$.

For the state L similarly:

8/a. $LBL \to L$,
9/a. $BL \cdot \to N$.

The described process is presented on Listing 1 generated by computer-simulation. The cell-states are displayed with the conversion Q="·", B="B", L="<", R=">" and N="*". On the edges of the cellular space dummy cells are used with the state N.

Listing 1

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STEP  0: * . . . . B . . . . . . . . . . . . . *
STEP  1: * . . . < B > . . . . . . . . . . . . *
STEP  2: * . . < < B > > . . . . . . . . . . . *
STEP  2: * . . < < B > > . . . . . . . . B . . *
STEP  3: * . < < < B > > > . . . . . . < B > . *
STEP  4: * < < < < B > > > > . . . . < < B > > *
STEP  5: * * < < < B > > > > > . . < < < B > * *
STEP  6: * < * < < B > > > > > > < < < < B * > *
STEP  7: * * < * < B > > > > > * * < < < B * * *
STEP  8: * < * < * B > > > > * > < * < < B * * *
STEP  9: * * < * * B > > > * > * * < * < B * * *
STEP 10: * < * * * B > > * > * > < * < * B * * *
STEP 11: * * * * * B > * > * > * * < * * B * * *
STEP 12: * * * * * B * > * > * > < * * * B * * *
STEP 13: * * * * * B * * > * > * * * * * B * * *
STEP 14: * * * * * B * * * > * > * * * * B * * *
STEP 15: * * * * * B * * * * > * > * * * B * * *
STEP 16: * * * * * B * * * * * > * > * * B * * *
STEP 17: * * * * * B * * * * * * > * > * B * * *
STEP 18: * * * * * B * * * * * * * > * > B * * *
STEP 19: * * * * * B * * * * * * * * > * > * * *
STEP 20: * * * * * B * * * * * * * * * > * > * *
STEP 21: * * * * * B * * * * * * * * * * > * > *
STEP 22: * * * * * B * * * * * * * * * * * > * *
STEP 23: * * * * * B * * * * * * * * * * * * > *
STEP 24: * * * * * B * * * * * * * * * * * * * *
STEP 25: * * * * * B * * * * * * * * * * * * * *
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The terms described above represent only the typical situations in the case of two birds. If *more then two birds* are allowed and all special cases are respected (e.g. two neighbouring birds, a bird killed from both direction at the same time, etc.), then the following extended transition function called as *"early bird function"* is needed (in the following terms an expression (B, R) means "state B or state R"):

1. $(B, R)\ Q\ (Q, N) \to R$	wave-growing
2. $(Q, N)\ Q\ (B, L) \to L$	
3. $(B, R)\ Q\ (B, L) \to N$	wave-growing with annihilation
4. $\cdot\ R\ L \to N$	annihilation by the transition
5. $R\ L\ \cdot \to N$	$\cdot RL \cdot \to \cdot NN \cdot$
6. $R\ (B, N)\ (\text{not } L) \to R$	R steps right by the transition
7. $\cdot\ R\ (B, N) \to N$	$\cdot R\ (B, N)\ (\text{not } L) \to \cdot NR \cdot$
8. $(\text{not } R)\ (B, N)\ L \to L$	L steps left by the transition
9. $(B, N)\ L\ \cdot \to N$	$(\text{not } R)\ (B, N)\ L \cdot \to \cdot LN \cdot$
10. $R\ (B, N)\ L \to N$	annihilation by the transition $\cdot R\ (B, N)\ L \cdot \to \cdot NNN \cdot$

11. In all other cases the new state must be equal to the old own state.

5. Exact proof of the algorithm

It is easy to prove that for two birds the "early bird function" works right. For the *general case,* where in each step any quiescent cell can change into the bird-state, an exact proof is given in the following.

Theorem. A one dimensional 5-state cellular space consisting of m cells is considered, where

— in the initial configuration (at $t=0$) each cell is in state Q, and the dummy cells on the edges are in state N,

— between any two steps (so to say, at $t+1/2$) any quiescent cell can alter into state B.

Statement. Using the "early bird function" in this cellular space, after a finite time (it seems that maximum $3m$ steps) only the "early birds" (the birds arisen at first) are existing, all other cells have the state N.

The *proof* is based on the notion "route of the wave-states". To define this notion some investigations are needed for the behaviour of wave-states. The following properties can be found:

— A wave-state (i.e. L or R) may arise only from state Q, by terms 1 and 2.

— L states move to the left, R states to the right. More exactly, if in front of a wave-state there is a state N or B, then the wave-state steps forward (see terms 6—9). If in front of a wave-state there is the same wave-state or state Q, then the wave-state remains on its place (by "term 11").

— If an R and an L are colliding, then they annihilate each other (see terms 4, 5, 10). A wave-state reaching the border of the cellular space is annihilated by the dummy cell (see terms 7, 9).

— The behaviour of a wave-state is always independent from the state occuring behind it.

These properties show, that a wave-state *arises* on a certain point of the cellular space, it *goes* left or right depending on its type, and it is *annihilated* on another point of the cellular space. The section of cells, determined by the point of origin and the point of annihilation of a wave-state, will be called as the *route of the wave-state.*

If a cell contained in a route of a wave-state has been excited, then obviously this bird cannot survive. This fact gives special importance for the routes of the states R and L, which can be characterized in the following lemma.

Lemma. (i) If a state L and a state R arose at the same time on the both ends of a quiescent section $Q \dots Q$, then after a finite time they will meet and annihilate one another.

(ii) If a wave-state arose on the end of an outside quiescent section (bounded by a dummy cell on its other end), then the wave-state will go to the left or to the right until it reaches the border, and will be annihilated by the dummy cell.

Proof. First the statement (i) will be proved, using induction for the length n of the quiescent section $Q \dots Q$.

For $n=2$ the statement (i) is obvious, because we have the transition $QQ \to RL \to NN$ in this case.

Now the statement (i) is assumed for any section with length less then n, and a quiescent section of length n is considered, on the both ends of which an $R-L$ pair was arisen at time t (hereby the length of the section was reduced to $n-2$). Between t and $t+1$ (so to say, at $t+1/2$) a number of birds may be excited in this section, hereby the section may be divided into more subsections, each having a length less then n. At time $t+1$ all quiescent sections of length 1 have disappeared (see term 3), and on the both ends of all other sections states R and L are arising. By the induction assumption these $R-L$ pairs must annihilate each other. So the original R and L — arisen on the ends of the section of length n — cannot meet with any other wave-state, therefore they will annihilate each other.

The statement (ii) can be proved in a similar way. □

Applying these results it is easy to prove the original theorem.

Assume, that the early birds are excited at time $t_0+1/2$, the configuration at this time-point consists from bird sections and quiescent sections alternating one another. At time t_0+1 on the ends of each quiescent section an $R-L$ pair arises. These pairs — according to the lemma — will annihilate each other, so their routes cover all the space between the early birds. Similarly, the routes of the wave-states, arisen on the ends of the outside quiescent sections, cover the space between the outside early birds and the dummy cells. This fact implies, that *all later birds will be killed.* On the other hand, *the early birds must survive,* because the route of any wave-state (arising after t_0) is contained by one of the quiescent sections at t_0+1.

With these notes the proof of the theorem is complete.

6. Simulation examples

The presented solution of the early bird problem is demonstrated below using computer-simulation. The cell-states are displayed with the conversion $Q=$"•", $B=$"B", $L=$"<", $R=$">" and $N=$"∗". On the edges of the cellular space the dummy cells are displayed, too.

In the case of Listing 2 four birds come from the outside world (at $t=4{,}5$ two birds at the same time). After $t=15$ only the early bird lives, in the further (not displayed) steps the remained wave-states will be annihilated by the dummy cells.

Listing 2

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STEP  0: * . . . . . . . . . . . . . . B . . . . . . . . . . . . . . . . . . *
STEP  1: * . . . . . . . . . . . . . < B > . . . . . . . . . . . . . . . . . *
STEP  2: * . . . . . . . . . . . . < < B > > . . . . . . . . . . . . . . . . *
STEP  2: * . . . . . . . . B . . . < < B > > . . . . . . . . . . . . . . . . *
STEP  3: * . . . . . . . < B > . < < < B > > > . . . . . . . . . . . . . . . *
STEP  4: * . . . . . . < < B > * < < < B > > > > . . . . . . . . . . . . . . *
STEP  4: * . . . . . . < < B > * < < < B > > > > . . . . B . B . . . . . . . *
STEP  5: * . . . . . < < < B * * * < < B > > > > > . . < B * B > . . . . . . *
STEP  6: * . . . . < < < < B * * < * < B > > > > > > < < B * B > > . . . . . *
STEP  7: * . . . < < < < < B * < * < * B > > > > > * * < B * B > > > . . . . *
STEP  8: * . . < < < < < < B < * < * * B > > > > * > < * B * B > > > > . . . *
STEP  9: * . < < < < < < < < * < * * * B > > > * > * * * B * B > > > > > . . *
STEP 10: * < < < < < < < < < < * * * * B > > * > * > * * B * B > > > > > > . *
STEP 11: * * < < < < < < < < < * * * * B > * > * > * > * B * B > > > > > > > *
STEP 12: * < * < < < < < < < < * * * * B * > * > * > * > B * B > > > > > > * *
STEP 13: * * < * < < < < < < < * * * * B * * > * > * > * > * B > > > > > * > *
STEP 14: * < * < * < < < < < < * * * * B * * * > * > * > * > B > > > > * > * *
STEP 15: * * < * < * < < < < < * * * * B * * * * > * > * > * > > > > * > * > *
STEP 16: * < * < * < * < < < < * * * * B * * * * * > * > * > > > > * > * > * *
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In the case of Listing 3 six birds come from the outside world (three birds at $t=0{,}5$ and three birds at $t=2{,}5$). During 22 steps all late birds are killed.

Listing 3

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STEP  0: * . . . . . . . . . . . . . . . . . . . . . . B B . B . . . . . . . *
STEP  1: * . . . . . . . . . . . . . . . . . . . . . < B B * B > . . . . . . *
STEP  2: * . . . . . . . . . . . . . . . . . . . . < < B B * B > > . . . . . *
STEP  2: * . . . . B . . . . . . . . B B . . . . . < < B B * B > > . . . . . *
STEP  3: * . . . < B > . . . . . . < B B > . . . < < < B B * B > > > . . . . *
STEP  4: * . . < < B > > . . . . < < B B > > . < < < < B B * B > > > > . . . *
STEP  5: * . < < < B > > > . . < < < B B > > * < < < < B B * B > > > > > . . *
STEP  6: * < < < < B > > > > < < < < B B > * * * < < < B B * B > > > > > > . *
STEP  7: * * < < < B > > > * * < < < B B * > * < * < < B B * B > > > > > > > *
STEP  8: * < * < < B > > * > < * < < B B * * * * < * < B B * B > > > > > > * *
STEP  9: * * < * < B > * > * * < * < B B * * * < * < * B B * B > > > > > * > *
STEP 10: * < * < * B * > * > < * < * B B * * < * < * * B B * B > > > > * > * *
STEP 11: * * < * * B * * > * * < * * B B * < * < * * * B B * B > > > * > * > *
STEP 12: * < * * * B * * * > < * * * B B < * < * * * * B B * B > > * > * > * *
STEP 13: * * * * * B * * * * * * * * B < * < * * * * * B B * B > * > * > * > *
STEP 14: * * * * * B * * * * * * * * < * < * * * * * * B B * B * > * > * > * *
STEP 15: * * * * * B * * * * * * * < * < * * * * * * * B B * B * * > * > * > *
STEP 16: * * * * * B * * * * * * < * < * * * * * * * * B B * B * * * > * > * *
STEP 17: * * * * * B * * * * * < * < * * * * * * * * * B B * B * * * * > * > *
STEP 18: * * * * * B * * * * < * < * * * * * * * * * * B B * B * * * * * > * *
STEP 19: * * * * * B * * * < * < * * * * * * * * * * * B B * B * * * * * * > *
STEP 20: * * * * * B * * < * < * * * * * * * * * * * * B B * B * * * * * * * *
STEP 21: * * * * * B * < * < * * * * * * * * * * * * * B B * B * * * * * * * *
STEP 22: * * * * * B < * < * * * * * * * * * * * * * * B B * B * * * * * * * *
STEP 23: * * * * * < * < * * * * * * * * * * * * * * * B B * B * * * * * * * *
STEP 24: * * * * < * < * * * * * * * * * * * * * * * * B B * B * * * * * * * *
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