STATIC ANALYSIS OF CONTINUOUS BEAM WITH NUMERICAL METHOD (FEM)

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ABSTRACT

Finite element method is a method of analysis and simulation of current real phenomena. This paper focuses on this method, applied through finite element analysis program Matlab, presenting a structural analysis application useful in the field of forest, mechanical and structural engineering.

Program designed by the authors using the finite element tool engineer put in hand work necessary to optimize the design, with positive effects on the complete analysis of stress and tensions in continuous beams.

1. INTRODUCTION

In the finite-element method, a distributed physical system to be analysed is divided into a number (often large) of discrete elements. The division into elements may partly correspond to natural subdivisions of the structure. Most or all of the model parameters have very direct relationships to the structure and material properties of the system [6], [7] [8].

2. MATERIALS AND METHODS

This paper presents the calculation of flat structures with rigid nodes using finite element method. In this case there are no inertial or damping effects, or at least they are negligible [3], [9], [10]. Flat structure is modeled as a simple continuous beam with simply supports and 1 articulated support located at the left end of the structure

This type of structure is composed of bars whit 2 nodes and 3 degrees of freedom on each node [1], [2] [6], [7].The three degrees of freedom per node are the horizontally and vertically displacements and also the rotated section. It aims to determine the nodals elastic equilibrium equations using the displacements method [4], [11], [14], [15]. The analysis requires two reference systems [12], [13], one local that is attached to each element of the bar and a global for the analysis of the entire structure of bars.

3. RESULTS AND DISCUSSION

Generalized displacement and generalized forces vectors (1) of a beam element are [3], [5], [8]:

$$\{d\} = \{u_i, v_i, \varphi_i, u_j, v_j, \varphi_j\}^T, \quad \{f\} = \{N_i, T_i, M_i, N_j, T_j, M_j\} \qquad (1)$$

Stiffness matrix elements are determined by applying displacements on each degree of freedom and blocking the corresponding the other remaining degrees of freedom.

At each applied displacement nodes produce at the ends of bar sectional efforts on the 6 degrees of freedom [9], [12]. By applying the 6 successive displacements and using the principle of superposition, determine the relationship between generalized displacement and generalized forces vector [4], [9]. Stiffness matrix contains terms that depend on the geometry of the beam and physical-mechanical properties of the material.

$$[k]=\begin{bmatrix} EA/l & 0 & 0 & -EA/l & 0 & 0 \\ 0 & 12EI_z/l^3 & 6EI_z/l^2 & 0 & -12EI_z/l^3 & 6EI_z/l^2 \\ 0 & 6EI_z/l^2 & 4EI_z/l & 0 & -6EI_z/l^2 & 2EI_z/l \\ -EA/l & 0 & 0 & EA/l & 0 & 0 \\ 0 & -12EI_z/l^3 & -6EI_z/l^2 & 0 & 12EI_z/l^3 & -6EI_z/l^2 \\ 0 & 6EI_z/l^2 & 2EI_z/l & 0 & -6EI_z/l^2 & 4EI_z/l \end{bmatrix} \quad (2)$$

Orthogonal matrix (4) that connects the components of a vector in global and local reference system is a transformation matrix and is of the form [8], [11]:

$$[K]=[T]^T\cdot[k]\cdot[T] \quad (3)$$

Where: $[K]$ – stiffness matrix, in global reference system.; $[k]$ – stiffness matrix, in local reference system; $[T]$ – transformation matrix.

$$[T]=\begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & 0 & 0 & -\sin\alpha & \cos\alpha \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The displacement and efforts at the ends of bars is determinate by applying conditions and solving the system equations of the nodal equilibrium. By applying the superposition principle [], [2],[4], we determined the relation between the sectional efforts to ends beam, when were applied nodal displacements (u,v,φ) in each node on the 3degrees of freedom. This is the equilibrum equation of beam elements in the local reference system.

Initial data of beam studies are: force is applied to the beam in node 8 having the coordinates $x=2000[mm]$, $y=0[mm]$; Section height is $h=100[mm]$; Young's modulus $E=2.1\cdot10^5[N/mm^2]$; Transverse modulus of the material $E=8\cdot10^4[N/mm^2]$;Tensile-compressive stiffness of the structure $E\cdot A=100^2$; Bending stiffness of the structure. $E\cdot I_z=100^4/12$, $F_y=10^3[N]$.

The numerical program.

```
% Continuous beam is considered. Required to determine the nodal displacements, stresses and sectional efforts at the ends of bars.
clear; clc;
%Cartesian coordinates of the nodes expressed in [mm]
%           x     y
nodes= [0  0
        200 0
        400 0
        600 0
        800 0
       1000 0
       1200 0
       1400 0
       1600 0
       1800 0
       2000 0]
% Finite element matrix
% elem nod1 nod2  h(section height)
elem=[ 1   2   100
       2   3   100
       3   4   100
```

```
        4  5  100
        5  6  100
        6  7  100
        7  8  100
        8  9  100
        9 10  100
       10 11  100]
% Young's modulus [N/mm^2]
E=2.1*10^5
% Transverse modulus of the material [N/mm^2]
G=8*10^4
% Tensile-compressive and bending stiffness of the structure.
ea=100^2
eiz=100^4/12
%Number of nodes of the structure
nnd=length(noduri(:,2))
% Number of elements of structure
nel=length(elem(:,2))
% Forces applied to the beam
%      node fx   fy   momz
forte=[ 8  0  -1000   0]
% Boundary conditions applied to the beam
%      node bx by  brz
cond=[ 1   1   1  0
       3   0   1  0
       5   0   1  0
       7   0   1  0
       9   0   1  0
      11   0   1  0]
% Determine the number of forces and boundary conditions applied to the structure
nnf=length(forte(:,1))
ncond=length(cond(:,2))
% Axes x and y coordinates of the node structure
cx=noduri(:,1)
cy=noduri(:,2)
%Number of degrees of freedom per node (ngn),element (nel) and the total number of degrees
of freedom (nec)
ngn=3
ngel=2*ngn
nec=nnd*ngn
% Initialization to zero for MR, F and index
MR=zeros(nec,nec)
F=zeros(nec)
index=zeros(2*ngn)
% The calculation of the beam with rigid nodes
for i=1:nel
nod1=elem(i,1)
nod2=elem(i,2)
h(i)=elem(i,3)
for ii=1:ngn
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```
index(ii)=ngn*(nod1-1)+ii
end
for iii=ngn+1:2*ngn
index(iii)=ngn*(nod2-2)+iii
end
% Length of beam finite elements
le=sqrt((cx(nod2)-cx(nod1))^2+(cy(nod2)-cy(nod1))^2)
% Cosines directors of beam elements.
c=(cx(nod2)-cx(nod1))/le
s=(cy(nod2)-cy(nod1))/le
length(i)=le'
% Vectors cosine directors
vc(i)=c
vs(i)=s
% Matrix elements stiffness
e1=ea/le
e2=12*eiz/le^3
e3=6*eiz/le^2
e4=4*eiz/le
e5=2*eiz/le
mrel=[e1 0 0 e1 0 0
      0 e2 e3 0 -e2 e3
      0 e3 e4 0 -e3 e5
      -e1 0 0 e1 0 0
      0 -e2 -e3 0 e2 -e3
      0 e3 e5 0 -e3 e4]
% Transformation matrix
c1=[c -s 0]'
c2=[s c 0]'
c3=[0 0 1]'
c0=[0 0 0]'
T=[c1 c2 c3 c0 c0 c0
   c0 c0 c0 c1 c2 c3]
% Stiffness matrix in global reference system
mrel=T'*mrel*T
% Assembling the stiffness matrices of elements
for i1=1:ngel
j1=index(i1)
for i2=1:ngel
j2=index(i2)
MR(j1,j2)=MR(j1,j2)+mrel(i1,i2)
end
end
end
% Set up vector of nodal loads
for i=1:nnf
%Forces nodes
n=forte(i,1)
if forte(i,2)~=0
f=forte(i,2)
```

```
F(ngn*(n-1)+1)=F(ngn*(n-1)+1)+f
end
if forte(i,3)~=0
f=forte(i,3)
F(ngn*(n-1)+2)=F(ngn*(n-1)+2)+f
end
if forte(i,4)~=0
f=forte(i,4)
F(ngn*(n-1)+3)=F(ngn*(n-1)+3)+f
end
end
% Applying boundary conditions
for i=1:ncond
% Nodes with displacement zero
n=cond(i,1)
% Implementation of the conditions with zero displacement on  x direction
if cond(i,2)==1
MR(ngn*(n-1)+1,:)=zeros(1,nec)
MR(:,ngn*(n-1)+1)=zeros(nec,1)
MR(ngn*(n-1)+1,ngn*(n-1)+1)=1
F(ngn*(n-1)+1)=0
end
% Implementation of the conditions with zero displacement on  y direction
if cond(i,3)==1
MR(ngn*(n-1)+2,:)=zeros(1,nec)
MR(:,ngn*(n-1)+2)=zeros(nec,1)
MR(ngn*(n-1)+2,ngn*(n-1)+2)=1
F(ngn*(n-1)+2)=0
end
% Implementation of the conditions with zero rotations around  z axes
if cond(i,3)==1
MR(ngn*(n-1)+3,:)=zeros(1,nec)
MR(:,ngn*(n-1)+3)=zeros(nec,1)
MR(ngn*(n-1)+3,ngn*(n-1)+3)=1
F(ngn*(n-1)+3)=0
end
end
% Determination of initial unknowns represented by nodal displacements by solving the system of elastic nodal equations
% Format long e
depl=MR\F
for i=1:nnd
u(i)=depl(ngn*(i-1)+1)
v(i)=depl(ngn*(i-1)+2)
rotz(i)=depl(ngn*(i-1)+3)
end
% Display the primary unknowns (nodal displacements)
fprintf('nod u(mm) v(mm) rotz(rad)\n')
for i=1:nnd
fprintf(' %2.f %2.5f %2.5f\n',i,u(i),v(i),rotz(i))
end
```

```
fprintf('\n')
pause
%Determination of strains and tensions in ends of each beam finite element
 for i=1:nel
% Redefining nodes
nod1=elem(i,1)
nod2=elem(i,2)
% Calculation of beam lengths of all finite elements
le=sqrt((cx(nod2)-cx(nod1))^2+(cy(nod2)-cy(nod1))^2)
% Determine the cosine directors of each beam finite element
c=(cx(nod2)-cx(nod1))/le
s=(cy(nod2)-cy(nod1))/le
% Determination of global displacement of each beam finite element
ue1=depl(elem(i,1)*ngn-2,1)
ue2=depl(elem(i,1)*ngn-1,1)
ue3=depl(elem(i,1)*ngn,1)
ue4=depl(elem(i,2)*ngn-2,1)
ue5=depl(elem(i,2)*ngn-1,1)
ue6=depl(elem(i,2)*ngn,1)
%Determination of nodal displacements for each beam finite element in local reference system
ul1=c*ue1+s*ue2
ul2=(-s)*ue1+c*ue2
ul3=ue3
ul4=c*ue4+s*ue5
ul5=(-s)*ue4+c*ue5
ul6=ue6
% Calculation of stress from the first end of the beam
% Strain from tensile (compressive)
e11=(ul4-ul1)/le
% Strain from the bending deformation
e12=h(i)/(2*le^2)*(-6*ul2-4*ul3*le+6*ul5-2*ul6*le)
stress(1)=(e11-e12)*elem(i,3)
stress(2)=(e11+e12)*elem(i,3)
%The maximum stess to the first end of the beam
STRESS=max(abs([stress(1),stress(2)]))
% Calculation of stress at the second end of the beam
% Strain from tensile (compressive)
e21=(ul4-ul1)/le
% Strain from the bending deformation
e22=h(i)/(2*le^2)*(6*ul2+2*ul3*le-6*ul5+4*ul6*le)
stress(3)=(e21-e22)*elem(i,3)
stress(4)=(e21+e12)*elem(i,3)
% The  maximum stress to the first end of the beam STRESS2=max(abs([stress(3),stress(4)]))
%Maximum stress on each finite element beam
Stresselem=max(abs([STRESS1,STRESS2]))
tensmax(i)=Stresselem
% Display nodal unknowns tension and maximum stress on each beam finite elements
fprintf('\n')
fprintf('element  %2.f\n',i)
```

```
fprintf('node %2.f stressnod1 stressnod2 maxstressnode1 \n',elem(i,1))
fprintf(' %2.5f  %2.5f  %2.5f\n',stress(1),stress(2),STRESS1)
fprintf('node %2.f stressnod2 stressnod2   maxstressnode2 \n',elem(i,2))
fprintf(' %2.5f  %2.5f %2.5f\n',stress(3),stress(4),STRESS2)
fprintf('Maximum stress on element)
fprintf(' %2.5f \n', Stresselem)
end
  for i=1:nel
% Redefining nodes
nod1=elem(i,1)
nod2=elem(i,2)
% Calculation of beam lengths of all finite elements
le=sqrt((cx(nod2)-cx(nod1))^2+(cy(nod2)-cy(nod1))^2)
%Determine the cosine directors of each finite element
c=(cx(nod2)-cx(nod1))/le
s=(cy(nod2)-cy(nod1))/le
% Stiffness matrix of element
e1=ea/le
e2=12*eiz/le^3
e3=6*eiz/le^2
e4=4*eiz/le
e5=2*eiz/le
mrel=[e1  0   0  e1  0   0
        0  e2  e3  0  -e2  e3
        0  e3  e4  0  -e3  e5
      -e1  0   0  e1  0   0
        0 -e2 -e3  0   e2 -e3
        0  e3  e5  0  -e3  e4]
% Transformation matrix of elements
c1=[c -s 0]'
c2=[s c 0]'
c3=[0 0 1]'
c0=[0 0 0]'
T=[c1 c2 c3 c0 c0 c0
   c0 c0 c0 c1 c2 c3]
% Stiffness matrix in global reference system
mrel=T'*mrel*T
% The vector displacement for finite element beam
deplel=[u(nod1,v(nod1,rotz(nod1),u(nod2),v(nod2),rotz(nod2)]'
% Sectional efforts vector
ef=mrel*deplel
nx(1,i)=-ef(1) nx(2,i)=ef(4) ty(1,i)=-ef(2) ty(2,i)=ef(5) mz(1,i)=-ef(3) mz(2,i)=ef(6)
% Display the sectional efforts
fprintf('\n')
fprintf('elementul %2.f\n',i)
fprintf('nod %2.f Fx Fy Mz \n',elem(i,1))
fprintf(' %2.5f  %2.5f  %2.5f\n',ef(1),ef(2),ef(3))
fprintf('nod %2.f Fx Fy Mz \n',elem(i,2))
fprintf(' %2.5f  %2.5f  %2.5f\n',ef(4),ef(5),ef(6))
end
```

4. CONCLUSIONS

Numerical method has the advantage that the computer program developed by the author, leads to solutions of the problem that converge to the "exact" solution. The paper presented, is a novelty in terms of adapting to a full calculation of continuous beams regardless of physical-mechanical properties of materials they are made.

The main steps that were followed in this program by the author are:

-stiffness matrices-writing of the elements composing the structure of the continous beam;
-calculation of the cosine directors and transformation matrices;
-matrix assembly of each beam in the global stiffness matrix of the structure;
-establishment of nodal forces for the entire structure;
-application related conditions;
-determining the nodal equilibrium equations system;
-determining the efforts and the tension at each beam ends.

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