"CONVEX POLYHEDRON FEATURES AND THEIR UNFOLDING TO A CONNECTED NON-OVERLAPPING POLYGON" (PREPARING A CREATIVE PROVE OF THE DÜRER'S CONJECTURE)

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1. INTRODUCTION

Albrecht Dürer published around 1525 his conjecture: "All the polyhedrons can be unfolded by their suitable cutting edges to a plane so that we can receive a joined polygon-mesh with non-overlapping faces" [Dürer, 1]. The author of this article is dealing basically with the suitable positioning of cutting edges for unfolding the polyhedrons (to a plane), coding the polygon received and its modelling surface. His aim is to give tools for proving the Dürer's conjecture and/or to prepare a creative prove. The notion of the **finite convex polyhedrons** has a very large set of solids from tetrahedrons to the arbitrarily complicated polyhedrons —covered by **p≥3** sided convex polygons— which have less and less or nil symmetrics. In the case of the **analysed finite convex polyhedrons** 2 polygons meet in each edge, in their peak **q≥3** pieces of polygons meet where the angle is **alfa** $_{Pi}$**<360°** in consequence of convexity, otherwise it can be degraded to a plane and can become infinitely big, which was formerly excluded. The **spherical mosaic** ordered to the polyhedron can help us many times, which can be gained by **the projected polyhedron-peaks from an internal point to of an external sphere surface** which has only **mutual points (min2) with the polyhedron**.

They have many similar topological and geometrical properties, so the spherical mosaic **can help unfolding the facets** of the polyhedron and **defining the structure** of the unfolded polygon-mesh in a plane. The **spherical mosaic/globe notation system** is very useful:

- in the exposition of the performable operation on the polyhedron-surface in surface modelling,
- in marking out **cutting edges** of their polygons **marching via the "0 longitudinal circle"** and
- in the **joined polygon-chains: e.g.** (first of all) **via the "Equatorial (parallel) Circle"**. We can mention besides the **North (N) and South (S) Poles**, the **E**$_W$ starting point on the **Equatorial (E) Circle** and on the **"0 Longitudinal Circle"** walking round on the **E circle** from West to East to the arrival point **E**$_E$, which is identical with **E**$_W$ (**E**$_E$ ≡ **E**$_W$).

We would like to use also the notion of the **"Condensation-Points" (CP_i) on the Northern and Southern Hemispheres,** the **North (N) and South (S) Pole,** in which peaks

q_N>>3 and **q_S >>3** pieces of polygons run together, but in both of Hemispheres can be further "Condensation-points".

Naturally every globe point- (**N, S, E_W, E_E**, ...) notation can be used to the effect that every named globe point means **the nearest polyhedron-peak** (nodepoint). In the same way we mean by the **"Equatorial (E) Circle"** and **"Longitudinal Circles"** the **continuous zigzag edge-chains of the polyhedron, which are nearest to a circle** being discussed.

2. MODELLING CONVEX POLYHEDRONS BY BREP- AND WINGED EDGES-STRUCTURES

The Boundary REPresentation faces **(BREP)** modelling system was developed originally **for surface modelling** aim. But soon it was proved that BREP could be an efficient tool **for solid modelling too,** if a 3D region of the space is closed by boundary faces without holes, consequently a piece of the body is circumscribed by them. The **BREP** as a solid modeller **is an "object's present state"** describing system in 3D **by surface elements** in particular **by polygons** covering a polyhedron **or by free-form patches** covering a solid body or anyway a piece of the body **(Figure 1)**.

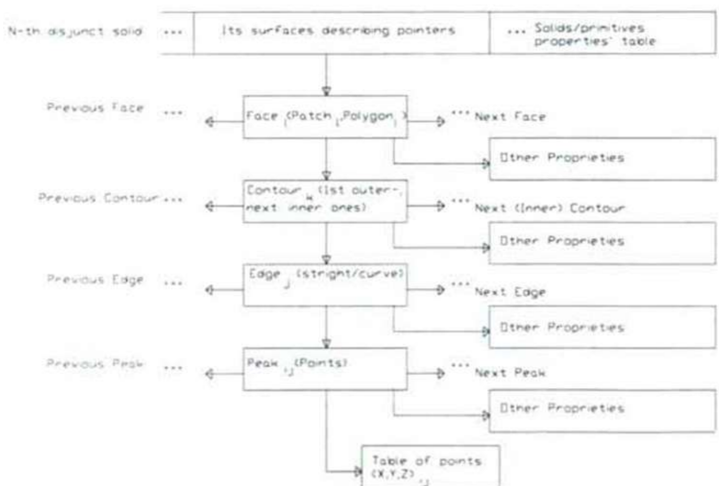

Figure 1 The main structure of the BREP surface model

In the BREP model all information is available in the traffic on the surface of a solid body connected with the neigbouring faces, their boundary contours, edges and peaks.

On the other hand, the **original solid modelling system CSG** —Constructive Solid Geometry— is a **"generative system"**, in this case using a **"historical method"** to describe a solid body from an initial state, via many stadia, up to the final state of the body [Braid, 3]. The tools for the solid-model modifying are **the Euler operators: OP {p, e, f, r, s, h}**, where **p**=peaks/vertricies, **e**=edges, **f**=faces, **r**=rings, **s**=shells, **h**=holes have the number of components. Every disjunct manifold solid, being in the real world, has the following topological condition: **p–e+ f–r=2*(s-h) (1st)**, which is the **Euler-Poincaré term**. In the case of the convex polyhedrons **Euler-Poincaré formula is** simplified as follows:

$$\mathbf{P - E - F = 2} \qquad \dots (\mathbf{2^{nd}}).$$

After using the solid model modifying operations the consistency of the body remains valid [Mäntylä, 7]. In practice we usually do not like to use the Euler operators, we should rather prefer to use the **BREP system operators** for manipulating the pieces of the surface-elements: extruding, rotating, wresting moving, unfolding and gluing the surface/solid elements, or the **generative CSG system operators** for the solid-Boole-algebraic operations: union, product and substract, etc.

In the early seventies the 3D geometrical modelling **systems had only polyhedron-modelling operations** such as describing, modifying polyhedral surfaces or solids, still in the cases of conic, cylindric and other curvilinear bodies, too. It was easier to start with modelling the surfaces or the bodies by the system which can only allow to describe and manipulate the boundaring by planes or the facetting bodies [see: M., Sabin, 2; I.,Braid, 3; M., Brun, 4]. This method could ensure many advantages when modelling rather complicated surfaces or bodies, too, like the hull of a ship-body, surface elements of a car-body or of an airplane-wing and designing these, and e.g. describing/modelling the surfaces and the movement of a very complicated airplane-landing ship. It offered many advantages when formerly using the polyhedral bodies and surfaces with quite a modest computer-throughput. **Most of the mechanical tasks can also be fulfilled by modelling the surfaces of the polyhedral bodies**: like designing parts-, tools-, envelop-surfaces, statical-, dynamical- and stress-analysing. **These facts drew the author's attention** to the polyhedral modelling and **unfolding the polyhedrons' surface.**

The BREP model became a really effective tool, when Baumgart published the **Winged edge modelling structure [Baumgart, 5]:** Node substructures. Each type of Node had a pointer-chain, and each edge, i.e. each ENOD has two wings: one "-FNOD" and one "+ FNOD", because each edge has two half-edges, both of which have one face, as a"wing".

Figure 4 shows the original Winged edge modelling structure, but for each node only the first few lines belong to it. We can use BREP model more effectively also as a solid modelling system because of its present state and locally object describing characteristic: surely in designing-, modifying- and describing tasks, one can be engaged more effectively with all of the little details, and it is not necessary to deal with or to modelize permanently the whole of the very big object in every case (see contrast with the CSG solid model, because of its global and generative characters). The surface-modelling properties of BREP became really better in the "BREP extended by Winged edge modelling structure", where we can determine the important properties of each point, e.g. on what face(s) is it,

- what kind of contour(s) are around every point also in the multiple contiguous surfaces,
- which are the characteristic-, boundary-, and mutual interfusing lines nearest to the actual point and
- which kind of peaks are crossed by the contour of each surface.
- we can determine in the case of a closed surface/body in what direction is the interior of the object,

in what direction is the normal vector pointed and where is the attendant trihedron (vectors).

The author augmented the original Winged edge modelling structure with many details to make it suitable for his special aims: for unfolding the surface of polyhedrons (see Figure 4). In BREP by the enlarged Winged edge model we can use each polyhedron edge as a "winged edge".

3. SOME PROPERTIES OF THE CONVEX POLYHEDRONS AND SEARCHING A SUITABLE EDGE-CUTTING STRATEGY

3.1 The homology of the convex polyhedrons and the spherical mosaics

There are some spherical mosaics simplier than the tetrahedron that can be produced via Euler operators beginning with the case of "Sphere and a point on it" where **the 2^{nd} formula is fulfilled (P=1, E=0, F=1)**. Via these Euler operators we can produce the simplest polyhedron, i.e. the tetrahedron (by its peaks) — to which a spherical mosaic can be ordered and vice-versa, thus they have a kind of homology. The BREP surface and solid modelling systems are using these properties, which are based on Euler operators [M. Mäntilä, 7].

This publication is dealing with a narrower set: with the convex polyhedrons. Naurally we can also declare that, it isn't possible to order any spherical mosaics to a convex polyhedron in a mutually unambiguous way, because each spherical mosaic can be projected to concave polyhedrons, too.

3.2. The form-features of convex polyhedrons

We can classify the convex polyhedrons according to their form-features they can have

bar, sheet and **body characteristics**:

- a convex polyhedron has a **bar** character/nature at which the extent in **Z** direction is
  considerably bigger than in the direction **X** and **Y**: $\Delta Z >> \Delta X, \Delta Y$;
- one has a **sheet** character/nature at which the extent in **Z** direction is considerably less than
  in the direction **X** and **Y**: $\Delta Z << \Delta X, \Delta Y$;
- one has a **body** character/nature at which the extent in **X,Y** and **Z** direction is essentially
  the same: $\Delta Z \approx \Delta X \approx \Delta Y$;

Naturally we can define other form-features for convex polyhedrons on the base of extent in **X,Y** and **Z** directions.

3.3. Condensation-points and the successful edge cutting strategy

The convex polyhedrons and spherical mosaics can be classified also by their Condensation-points:

- the convex polyhedron can have **Cp_i=1** piece of Condensation-point, e.g. in the case of pyramidical or drop-shaped cut precious stone, when the **N (North) Pole** suits to that, where **q_N>>3** is and on the other end a **p** sided polygon closes the convex polyhedron. In this case unfolding the convex polyhedron we can get a joined polygon, which can have a **single running down star character form** (**Figure 2** and **3/a**) but it is possible unfolding this polyhedron to several other non-overlapping character forms, too. However, one may suitably indicate every time **the first cutting zigzag edge-chain on the "0 Longitudinal Circle"**;

- but the convex polyhedron can have **$Cp_i \geq 2$** pieces of Condensation-points and also of the spherical mosaics ordered to it (**Figure 2 and 3/b).** Using the spherical mosaics/globe notation **the two Condensation-Points (CP_N,CP_S) on the Northern and Southern Hemispheres** are the **North (N) and South (S) Pole,** where **q_N>>3** and **q_S>>3**. Further Condensation-Points (**CP_i, CP_j**) can be on the Northern and Southern Hemispheres, these can be stringed to the first cutting zigzag (CP_i**-N-E_W-S-CP_j**) edge-chain. In this classification 2 pieces of **CPi-s** are the most characteristic .

The author developed an algorithm of indicating and performing cutting edges **for unfolding the convex polyhedron-surface** to have one joined non-overlapping polygon in a plan.

a) **Before performing** each cutting-edge chain**, one must indicate** all of the edges:

- first we indicate one circle on the spherical mosaics/globe by its two endpoints,
- than we seek for each nearest peak-point of polyhedron to these two endpoints,
- one indicate all of the nearest peaks of polyhedron to this indicated circle on the spherical mosaics/globe, as a zigzag (cutting) edge-chain (Figure 5).

b) We have to indicate and to **perform "1 rowed joined polygon-chains"**,too. Before performing it each operation begins also with the indication, so indicating the joined polygon-chain as follows:

- first we can indicate one circle on the spherical mosaics/globe by its two endpoints, **by the beginning point** (e.g. first by **E_W being on the "Equatorial Circle" and on the "0 Longitudinal Circle"**) and walking round on the actual circle- **by the endpoint** (with first example on **E Circle West to East** by the endpoint **$E_E \equiv E_W$**), then

- follows indicating the **1 rowed polygon-chain nearest to the actual circle** (first nearest to the **E Circle**), which will also surround the actual circle in a zigzag line

Note: Only when we have indicated all of the 1 rowed polygon-chains, and **we have indicated all the polygons of the polyhedron completely, that time we could perform** the cutting edge-chains and the joined polygon-chains.

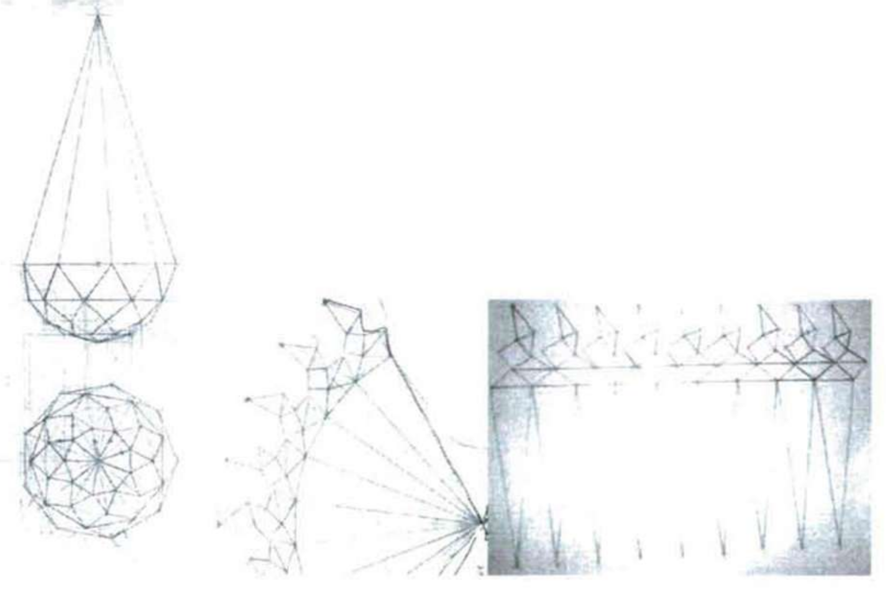

Figure 2 Unfolding the convex polyhedrons having some symmetries we can choose the algorithms which are suitable to the 1 and 2 CPi-s too (Figure 3/a and 3/b), for gainingthe joined non-overlapping polygon-meshes

The major steps are the following:

I)- we will indicate and perform **the first cutting zigzag edge-chain** on the **"0 Longitudinal Circle":** this will be the **S – E_w – N** zigzag cutting line, **with which we will open the closed surface covered by the polyhedron-body. Thus its surface becomes a closed two-dimensional (2D) surface in the space bordering by the cutted half-edges S – E_w – N — N– E_w– S, it becomes a closed, 2D surface in 3D completely filled by polygons;**

II)- indicating and **performing the first**, most important 1 rowed, **joined polygon-chain on the "Equatorial Circle"**, which consists of **m pieces of polygons;**

III)- then we can indicate and **perform the essential star-branches** from the polygon-edges of the joined polygon-chain **from the** Equatorial **(E) Circle** to the **North** and **South CPi. These essential star-branches** can have **max. n** pieces of polygons in the directions to the **N** or the **S CPi** (Figure 5).

From the edges of the Equatorial **(E)** joining **p>>3** sided polygons **max. p essential star-branches can branch off,** but generally **max.(p-2)** pieces can be started to the **N** and **S CPi-s** because 1-1 piece of engaged edges joins together polygon-pairs (**–FNOD** and **+FNOD**);

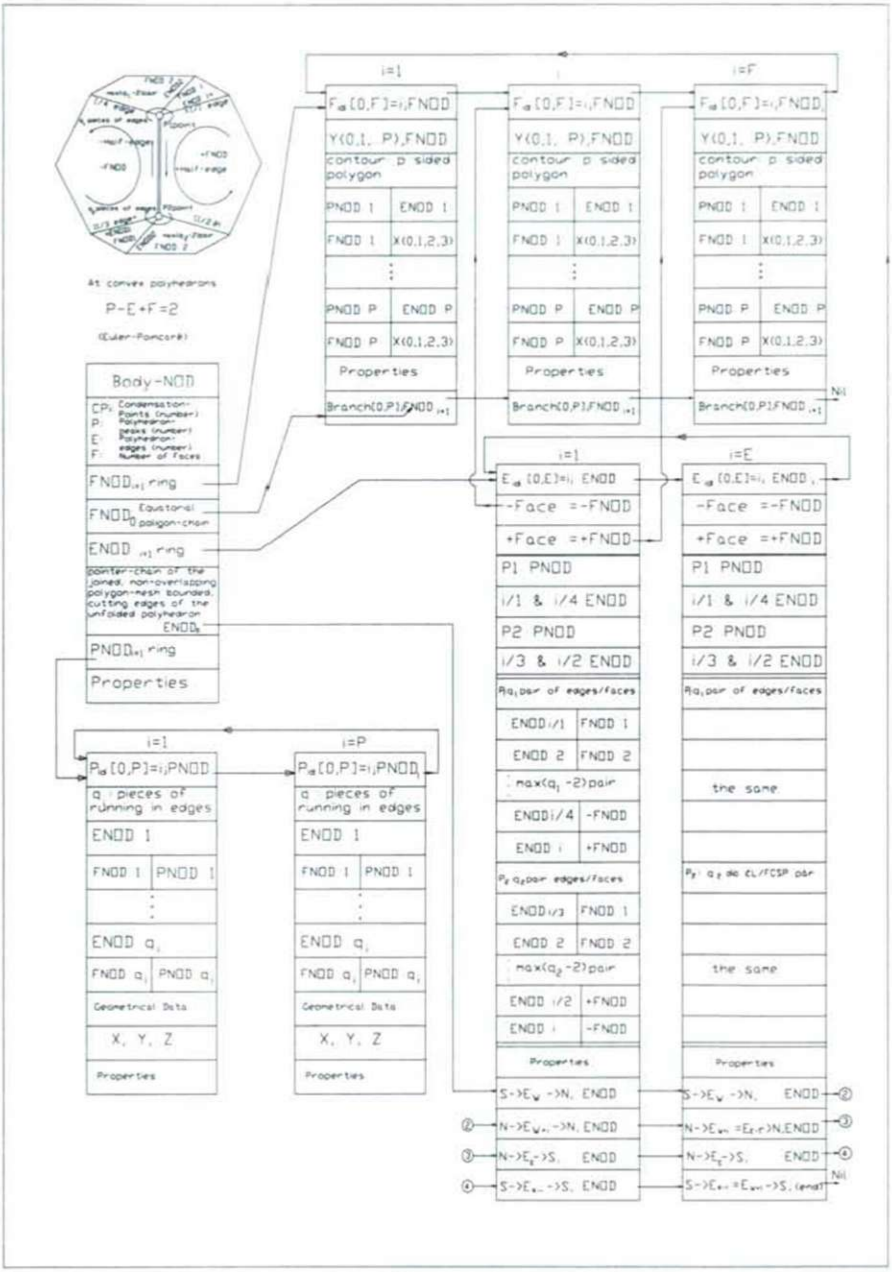

Figure 4 Augmented structure of the "Winged edge model" for unfolding the convex polyhedrons

- these **max. m** of **essential star-branches** start from the **E polygon-chain** to the **N** and **S CPi**-s, but they can die off on the way, and new essential star-branches can be born because of the recursive branching out of themselves. Thus not definitely the same number of **essential star-branches** march into the **CPi-s** as many of them could start from the **E polygon-chain**;

- **vica versa**, from the **N** and **S CPi**-s **q_N pieces ($q_N >> 3$)** hence **not m** pieces of **cutting edges** start to the polygons of the **E** polygon-chain, but the same number of **$q_N >> 3$ and $q_S >> 3$ pieces of polygons running into the N** and **S CPi**-s, that is **q_N** pieces of essential star-branches/1 rowed, joined **polygon-chain start to the "E**quatorial **(E) Circle".**

IV)- after indicating the essential star-branches **we must indicate all the recursive branching out of themselves** to less star-branches, up to where they could reach – marching from **W**est to East- the border of the next essential star-branch or of any other star-branch.

Surely all of the star-branches are 1 rowed and they **couldn't cut** -only can touch- another polygon of another **neighbouring star-branch**.

Notes: The essential star-branches are **indicated by the "Longitudinal Circles"** running from the **N**orth and South **CPi**-s to the forthcoming polygon-peaks of the Equatorial (**E**) **Circle** joined polygon-chain. These **"Longitudinal Circles", in fact** the **continuous zigzag edge-chains of the polyhedron, which are nearest to these circles,— make regions for the essential star-branches and the recursive branching out star-branches**.

We can observe that **all the peaks of the polyhedron have min. 1 cutting edge** — in this cutting edges strategies proposed (see the double lines, namely half-edges in Figure 5).

Affirmation (1): By this strategy for cutting out the edges **"one can always be able unfolding the convex polyhedron-surfaces to a star-shaped, one-rowed joined polygon-chain"**, which has a general **tree structure** with **n** levels and **m** branches (Figure 5).

4. AUGMENTED STRUCTURE OF THE "WINGED EDGE MODEL" FOR UNFOLDING THE CONVEX POLYHEDRONS

We can unfold the surface of each convex polyhedron to a **star-shaped tree structure formed from joined polygon-chains suitably for Affirmation (1), but it is a much more modest affirmation, than the Dürer's conjecture was made about 1525**.

He stated: **"all the convex polyhedron-surface can be unfolded without exception —to a joined non-overlapping polygons— by cutting their suitable edges"** [Dürer, 1].

This conjecture was proved by O'Rourke at al. about 1992 [O'Rourke, 8] -and in the information/knowledge of the present author- via a creative mood. O'Rourke was the first who asserted that it is possible, namely to a joined non-overlapping polygon, suitably for the condensation points following manifold running down,- to a complex star-shaped polygon.

The **"Winged edge model"** with some **augmented structure is** effectively suitable —according to the author— not only for modelling the convex polyhedron body and surface as well as marching on its surface, but for also **the following operations** (Figure 4):
- for unfolding the polyhedron-surfaces, namely indicating-, after performing the cutting edges,
- tracing and debugging the performing process, moreover
- describing and coding the unfolded polyhedron-surface to a mesh of convex polygons which can be the above mentioned 1 rowed joined, star-shaped, non-overlapping or overlapping a concave polygon.

Thus the **winged edge can be an excellent tool for the proof** of the non-overlapping character. The enlarged winged edge structure contains Point NODe (PNOD)-, FaceNODe (FNOD) EdgeNODe (ENOD) substructures, and among those Body NODe Body-NOD gives relation.

Some data in the **augmented winged edge for FNOD** about the Faces **of the convex polyhedron:**

2^{nd} data: Y{0,1,...P} gives the characteristic of **i^{th} actual polygon/Face**-state, where its meaning is:

Y{0,1,...P}=0 if not one single **(0)** side of the **i^{th}** actual polygon are prepared that is we don't know anything about edges of the i^{th} actual polygon then its edges are cutting or joining/winged edges;

Y{0,1,... P}=j, (max. P) if **j pieces of edges are already prepared** partially or wholly (see the state of edge describing **X{0,1,2,3}** state-characteristic, the **5^{th}** data of the actual (**i^{th}**) Node;

5^{th} data: to all the **P pieces of edges** of the **i^{th}** actual polygon belonging to **4-4 data**, there are:

a) the starting **point/PNOD** pointer of the **actual k[1,P]th edge** ;

b) the **ENOD** pointer of the **actual k^{th} edge**;

c) neighbouring **polygon's FNOD** pointer **being on the other side** of the **actual k^{th} edge**

d) the state characteristic X{0,1,2,3}of the **actual k^{th} edge, which means:**

X{0,1,2,3}=0, if **actual k^{th} edge** hasn't been yet analysed **in view of cutting** and not in having a role as a winged edge **joining the polygons,** thus we haven't analysed in the process of unfolding to a plane;

X{0,1,2,3}=1 or 2, if the **actual k^{th}** edge is a **cutting edge** and **X{0,1,2,3}=1**, if this **actual edge could have a role only ones** in the process of **marching around the unfolded, joined polygon-mesh**, and **X{0,1,2,3}=2**, if on the **actual k^{th}** edge we marched already forwards and backwards, too.

X{0,1,2,3}=3, if the **actual k^{th}** edge is **winged edge**, thus the to neighbouring, joining faces on the

(**–FNOD** and **+FNOD** at Figure 5) **will be already indicated for joining.** This state generally can rise only after the whole unfolding process to the plane,- for all of the edges.

7^{th}data: Branch[0,P] gives it how many sides of the **actual k^{th}** edge give branches for starting element of the 1 rowed joining polygon-chains: (for the **Equatorial** joining polygon-chains, for the **essential star branches or for** the recursive branching **star-branches**)

8^{th}data: it is the **next FNOD** pointer of the Equatorial joining polygon-chain in the direction E_W->E_E

Notes: All the other 1 rowed joined polygon-chains are coded by the **FNOD** beads **on the base of the state characteristics** and **of the pointers** of the **P** sided polygons. In this way the unfolded joined polygon-mesh can be projected very effectively from the polyhedron-surface, which will form a generally **n level in (m or p) branching tree structure**

Some data in the augmented winged edge structure **for ENOD** (Edges) **of the convex polyhedron:**

4^{th} data: the actual **i^{th} winged edge** boundered by **P1** and **P2 has q_1 and q_2** pairs of **ENOD/FNOD** pointers, which are coded in the CCW direction on the structure (Figure 4).

6^{th} data: via the tools of the suitably cutting edges —unfolding the convex polyhedron-surfaces— got a **joined star-shaped polygon-mesh** consisting of **one rowed-polygon-chains. We could describe by pointer-chains the marching process around this concave polygon.** The sections of this pointer chain can be found at the end of the **ENOD** substructure.

5. THE WINGED EDGE IS THE TOOL FOR PROVING THE NON-OVERLAPPING BY COMPLETE INDUCTION

This chapter gives a proof for the non-overlapping polygon-mesh. The convex polyhedron's **surface cut near to "0 Longitudinal Circles"** can be decomposed to one rowed polygon-chains.

AFFIRMATION: The surface of the convex polygon can be unfolded to a **joined non-overlapped** polygon-mesh, **if marching through** all the polygon-chains **by a "piece of surface"** in the **i=1,2,...,m/n** cases with suitable cutting edges (Figure 5). It is provable, that the polygons of each **"piece of surface"** of all the polygon-chains— will move off from each other and move off from the earlier unfolded polygons, too. **The "piece of surface" is "the winged edges" joined polygons.**

All the peaks have one or more cutting edges at the proposed cutting edge-algorithm.

The bigger steps of proving the NON-OVERLAPPING is the following:

I) it was already indicated by the **beginning cutting edge-chain,** a closed 2D zigzag line in the space **from half–edges (S – E_w – N — N– E_w– S)**, which **opened the polyhedron's space-portion** and transformed its surface **into a closed 3D surface** completely filled by polygons;

II) then was indicated the Equatorial **(E) joined one-rowed polygon-chain**, so with that the steps will be introduced in details **i=1,2,...,(m-1),m** of the proof **with complete induction;**

III) later had to be indicated all of the **"essential star-branches"** branching off the **E polygon-chain,** which were marching from West to East: **E_W -> $E_E \equiv E_W$ (i=1,2,...,m).** At all of the **"essential star-branches"** we must apply **the proof with complete induction** from the North and South **Pole/CPi** backwards in the steps **j,k=n,(n-1),...,2,1** up to the **E Circle** or up to the **borders** of the next **"essential star branch"**/recursive **star-branch.** This proof with complete induction can be introduced **in II) case** may apply also to this **III) case** but with opposite direction.

IV) finally at each polygon-chain of all the **star-branches recursively** branching off the "essential star-branches", ought to be applied the **"proof with complete induction",** always from the dying off peak-point to the arrival point of the higher level **"star-branches" or** upto the **E Circle.** Otherwise it is **sufficient thinking over recursively this proving process.**

II) The proof *with complete induction* of affirmation concerning the E joined polygon-chain

1st step: The "P1$^{i=1}$–P2$^{i=1}$" edge is joining together (Figure 5) the **i=1st polygon** to the **i=2nd polygon**.

a) At the **P1$^{i=1}$** peak of the convex polyhedron: **originally** there is **no edge cut,** so the polygons marching into this peak have the angle (sum of their plane-angle): **alfa$_{P1}^{i=1}$ <360°;** if at the unfolding process **one edge of P1$^{i=1}$ is indicating to cut,** this angle will be **alfa$_{P1}^{i=1}$ =360°.** So **the i=1st (actual) "essential star branch" bordered P1$^{i=1}$≡P2$^{j=1}$** polyhedron peak and the **P2$^{j=2}$** (i=1) peak joining actual "essential star branch" having P2$^{j=1}$–P2$^{j=2}$ will be opened. So it will be cut to half-edges, then the **i= 2nd "essential star branch's"actual polygon** will move off the **i= 1st "essential star branch's"actual j=2 polygon.**

b) At the **P2$^{i=1}$** peak of the convex polyhedron: **originally** there is also **no edge cut,** so the polygons marching into this peak have the angle (sum of their plane-angle): **alfa$_{P2}^{i=1}$ <360°;** if at the unfolding process **one edge of P2$^{i=1}$ is indicating to cut,** this angle will be **alfa$_{P2}^{i=1}$ =360°.** So each polygons marching into the **P2$^{i=2}$** can be unfolded and will move off the polygon being in the cutting edge's other endpoint being the **k= 2nd** polygon of the **i= 2nd "essential star branch".** Thus we can state, that the joining edge between the **i= 1st and i= 2nd** polygons, the P1$^{i=1}$–P2$^{i=1}$ **winged edge has** both of two endpoints **cutting edge**, this way **the polygons** —earlier touched one another— being in **P1$^{i=1}$ and P2$^{i=1}$ will move off each other.**

2ndstep: The "P1$^{i=2}$–P2$^{i=2}$" edge is joining together(Figure 5) the **i=2nd polygon** to the **i=3rd polygon**.

a) At the **P1$^{i=2}$** peak of the convex polyhedron: **originally** there is **no edge cut,** so the polygons marching into this peak have the angle (sum of their plane-angle): **alfa$_{P1}^{i=2}$ <360°;** if at the unfolding process **one edge of P1$^{i=2}$ is indicating to cut,** this angle will be **alfa$_{P1}^{i=2}$ =360°.** So **the i=2nd (actual) "essential star branch" bordered P1$^{i=2}$≡P2$^{j=1}$** polyhedron peak and the **P2$^{j=2}$** (i=2) peak joining actual "essential star branch" having P2$^{j=1}$– P2$^{j=2}$ will be opened. Thus it will be cut to half-edges, then the **i= 3rd "essential star branch's"actual polygon** will move off the **i= 2nd "essential star branch's"actual j=2nd polygon.**

b) At the **P2$^{i=2}$** peak of the convex polyhedron: **originally** there is also **no edge cut,** so the polygons marching into this peak have the angle (sum of their plane-angle): **alfa$_{P2}^{i=2}$ <360°;** if at the unfolding process **one edge of P2$^{i=2}$ is indicating to cut,** this angle will be **alfa$_{P2}^{i=2}$ =360°.** So each polygons marching into the **P2$^{i=2}$** can be unfolded and will move off the polygon being in the cutting edge's other endpoint being the **k= 3rd** polygon of the **i= 3rd "essential star branch".** Thus we can state, that the joining edge between the **i= 2nd and i= 3rd** polygons, the P1$^{i=2}$–P2$^{i=2}$ **winged edge has** both of two endpoints **cutting edge**, this way **the polygons** —earlier touched one another— being in **P1$^{i=2}$ and P2$^{i=2}$ will move off each other.**

3rd step: The "$P1^{i=(m-1)}$–$P2^{i=m}$" edge is joining together the **$i=(m-1)^{st}$ polygon** to the **$i=m^{th}$ polygon**.

a) At the **$P1^{i=(m-1)}$** peak of the convex polyhedron: **originally** there is **no edge cut,** so the polygons marching into this peak have the angle (sum of their plane-angle): **$alfa_{P1}^{i=(m-1)}<360°$;** if at the unfolding process **one edge of $P1^{i=(m-1)}$ is indicating to cut,** this angle will be **$alfa_{P1}^{i=(m-1)}=360°$.**

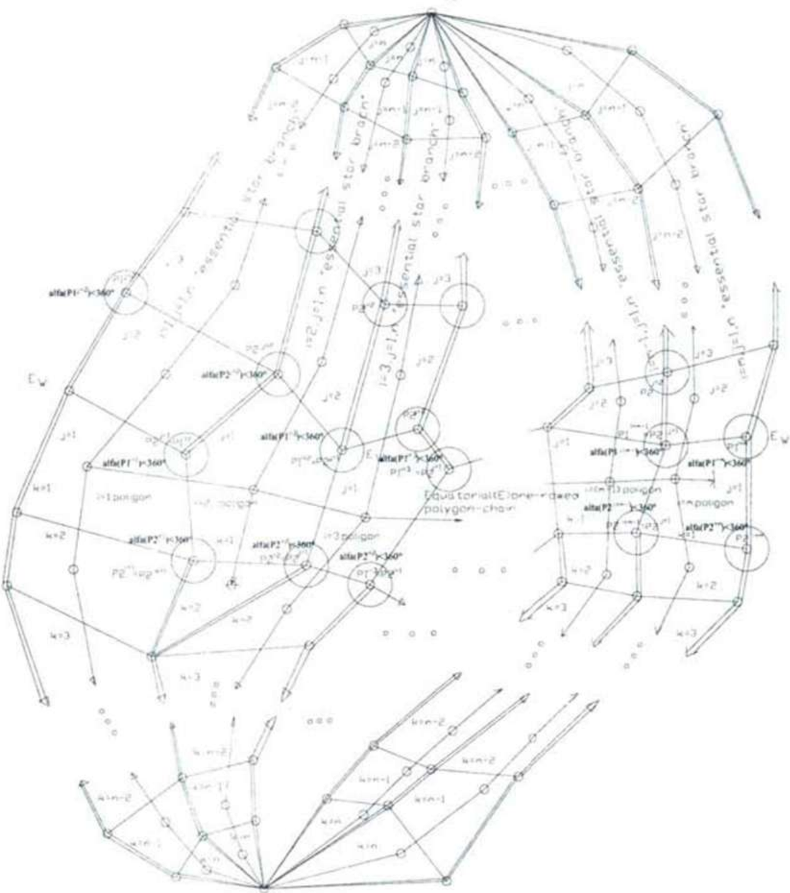

Figure 5 The Application of the Winged Edges Model

Thus the **$i=(m-1)^{st}$ actual "essential star branch" bordered $P1^{i=(m-1)} \equiv P2^{j=1}$** polyhedron peak and the **$P2^{j=2}$** ($i=m-1$) peak joining actual "essential star branch" having **$P2^{j=1}$–$P2^{j=2}$** will be opened. Thus will be cut to half-edges, then the **$i=m^{th}$ "essential star branch's"actual polygon** move off the **$i=(m-1)^{st}$ "essential star branch's"actual $j=2$ polygon.**

b) At the **$P2^{i=(m-1)}$** peak of the convex polyhedron: **originally** there is also **no edge cut,** so the polygons marching into this peak have the angle (sum of their plane-angle **$alfa_{P2}^{i=(m-1)}<360°$**; if at the unfolding process **one edge of $P2^{i=(m-1)}$ is indicating to cut,** this angle will be **$alfa_{P2}^{i=(m-1)}=360°$.** So each polygons marching into the **$P2^{i=(m-1)}$** can be unfolded and will move off the polygon being in the cutting edge's other endpoint being the **$k=2^{nd}$** polygon of the **$i=m^{th}$ "essential star branch".** Thus we can state, that the joining edge between the **$i=(m-1)^{st}$ and $i=m^{th}$** polygons, the **$P1^{i=(m-1)}$–$P2^{i=m}$ winged edge has** both of two endpoints **cutting edge**, this way **the polygons** — earlier touched one another— being in **$P1^{i=(m-1)}$ and $P2^{i=m}$ will move off each other.**
Conclusion: On the base of the above mentioned **"proof with complete induction" that the polygons of** the Equatorial **one-rowed joined polygon-chain** —having **m** polygons marching **E_W-> E_E— can be unfolded to a plane, and at their cutting out edges the earlier touching polygons move off each other at all of the analysed convex polyhedrons.** ■
III) Affirmation: The polygons of the "essential star branches" chosen arbitrarily **move off each other** and form any earlier neighbouring unfolded polygons — **after the suitable edges cut at all of the finite convex polyhedrons.** In this main step **one had to be indicated to all of the "essential star-branches"** branching off the **E polygon-chain,** which were marching from **W**est to East: **E_W -> $E_E \equiv E_W$ ($i=1,2,\ldots,m$).** At all of the **"essential star-branches"** we must apply **the proof with complete induction** from the **N**orth and **S**outh **Pole/CPi** backwards in the steps **$j,k=n,(n-1),\ldots,2,1$** upto the **E Circle** or upto the **borders** of the next **"essential star branch"**/recursive **star-branch. This proof with complete induction can be** introduced **in II) case** may apply also to this **III) case** but with opposite direction. In this manner we proved **with complete induction at all of the one-rowed joined polygon-chains/essential star-branches**, that in their environment —unfolding the convex polyhedrons by cutting the suitable edges to half-edges,— **the polygons move off each other and all the earlier unfolded neighbouring polygons.** ■

6. SUMMARY

The author developed and introduced a modified winged edge structure solid body/surface modelling tool, which was applied by him for unfolding the surface of the finite convex polyhedrons. He gave an creative proof for the Dürer's conjecture pulished about 1525: "all the convex polyhedrons can be unfolded to a plane for a joined, non-overlapping polygon by their suitable cutting edges [Dürer, 1]. First O'Rourke at al. said, that this conjecture is true and they gave probably a proof in a creative way –as the author knows–about 1992 [O'Rourke, 9].

The author would like to draw attention to unfolding the concave polyhedrons and to the free form surfaces covered bodies, contained **$p=3,4,\ldots,6,\ldots$sided polygons/patches** to reach less overlapping and deformation during the unfolding process, which tasks are very important in the engineering applications.

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