



SHM Strategy Optimization and Structural Maintenance Planning Based on Bayesian Joint Modelling

C. Xing, R. Caspeele and L. Taerwe

Department of Structural Engineering, Ghent University, Ghent, Belgium

Abstract: In this contribution, an example is used to illustrate the application of Bayesian joint modelling in optimizing the SHM strategy and structural maintenance planning. The model parameters were evaluated first, using the Markov Chain Monte Carlo (MCMC) method. Then different parameters including expected SHM accuracy and risk acceptance criteria were investigated in order to give an insight on how the maintenance planning and life-cycle benefit are influenced. The optimal SHM strategy was then identified as the one that maximizes the benefit.

1 Introduction

As SHM provides a way for collecting information to reduce uncertainties and to facilitate improved maintenance planning, it has received a lot of attention and is widely implemented in practice. The information collected comes at a cost, as a result of the installation of the SHM system, system maintenance, analysis of the data collected etc., which is not always justified by the received benefit. Therefore, the decisions whether to implement a SHM system and which strategy to implement should be based on the evaluation of their benefit, which should be done prior to their installation. The benefit provided by a certain SHM strategy is quantified by the value of information it provides, in monetary form. This can be calculated in the framework of decision theory as introduced in [13] as the difference between the expected life-cycle cost (or expected benefits versus costs) of performing SHM or not, as also presented in [1, 3, 6, 8, 9, 12].

A Bayesian framework of joint modelling of the time-dependent structural performance and the hazard function has been theoretically introduced in [18] in order to do the maintenance planning and to quantify the value of SHM (VoSHM). In that framework, there are several factors, including the precision of the SHM system, the risk acceptance criterion etc., that affect the assessed value of a SHM strategy. In this work, these influencing factors are evaluated, i.e. the risk acceptance criterion and the precision of the SHM strategy, as these influence the decision making with respect to an optimal SHM strategy. This is a practical problem that needs to be settled for the decision makers. Take the risk acceptance criterion as an example, a lower risk acceptance level often means more frequent inspection/maintenance, which usually leads to a relatively lower expected failure cost but higher investment in the inspection/maintenance. A balance needs to be sought. In order to do this, an illustrative ex-

ample is elaborated based on a reinforced concrete (RC) bridge girder subjected to corrosion. In the following sections, the joint modelling of the time-dependent measurements and hazard function is briefly introduced first and then the illustrative example is discussed. Finally the conclusions are given.

2 Joint Model and value of SHM

2.1 Joint Modelling of Time-dependent Measurements and Hazard Function

As introduced in [2], the hazard function, or hazard rate, is defined as the frequency of failure of a structure and is expressed in failures per unit time. In this regard, the probability of failure during a certain time interval can be calculated accordingly as the integral of the hazard function. As the failure rate is directly related to the structural performance, it is practical to model the hazard function based on the structural performance, i.e. the joint modelling of time-dependent measurements and the hazard function. Figure 1 shows an illustrative example of the joint modeling, where h_1^T, h_2^T are the threshold values, one of the risk acceptance criterion, of the hazard function. The joint model is formulated as:

$$\left\{ \begin{array}{l} y(t) = m(t) + \epsilon(t) \\ h(t) = \lim_{s \rightarrow 0} \frac{\Pr(t < T < t+s | T > t)}{s} = \sigma_2 t^{\sigma_2 - 1} \exp\{\alpha_1 m(t) + \alpha_2 m'(t) + \alpha_3\} \end{array} \right. \quad (1)$$

where equation 1 represents the process for modelling the structural performance indicators (PIs) varying with time, while equation 2 gives the survival model defining the hazard function. $y(t)$ denotes the value of the time-dependent observation outcome at any particular time point t , $m(t)$ is the underlying structural state which is a function of t and random effects and $\epsilon(t)$ the error term that is assumed independent of the random effects. We assume that $\epsilon(t) \sim N(0, \sigma_1^2)$ and $\text{CoV}[\epsilon(t), \epsilon(t')] = 0$ for $t \neq t'$, i.e. the error term is unbiased and the error values at different times are uncorrelated. The failure for an event is defined as the PI crossing a threshold value and is often expressed by a limit state function. As proposed in [18], a parametric proportional hazard model with Weibull baseline hazard is considered here, as in equation 2, where σ_2 is the shape parameter of the Weibull distribution and T is the failure time. This formulation postulates that the risk for a failure event at time t is associated with parameters α_1 and α_2 which quantify the sensitivity to the value of $m(t)$ and its derivative over time $m'(t)$. This is logical for degrading structures: a structure with high performance level $m(t)$ and low degrading rate $m'(t)$ often has a low failure rate. α_3 is another regression coefficient. Under the assumption that the hazard of the failure event is mainly based on the value of $m(t)$ and its decreasing rate $m'(t)$, α_3 can be predefined to a certain value.

As soon as the evaluated value of the parameters is available, the failure rate $h(t)$ at a time point and the failure probability during a period of time can be calculated as the integration of the hazard function and used for inspection time planning. The parameter estimation for the hazard function is based on the Bayesian theory and MCMC simulations. One can refer to [18] for detailed information and it will not be further discussed in this paper. The focus of this work is given to investigate how the risk acceptance criterion influences the decision or the optimal SHM strategy.

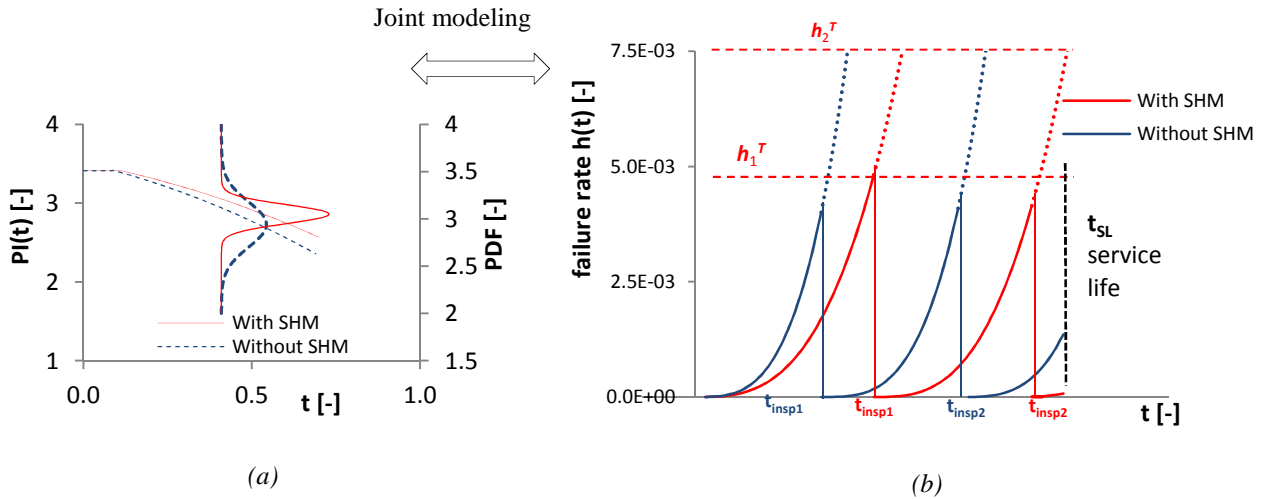


Figure 1. Joint modeling for (a) time-dependent structural performance and (b) hazard function

2.2 Hazard function based maintenance planning

2.2.1 Expected Total Life-Cycle Cost (TLCC) and Value of SHM

An event tree model of a structure (e.g. a bridge girder) with or without monitoring is presented in Figure 2. As illustrated previously in Figure 1, the implementation of SHM can provide more information of the structure which leads to a significant change of inspection/maintenance planning and as a result, the TLCC. Since the structural state and monitoring results are both uncertain, the decision problem can be described in terms of the following notations and events in a pre-posterior framework for Bayesian decision theory as developed in [13]:

- Θ : time-dependent structural state with prior PDF $f_{\Theta}'(\theta)$.
- Z : the inspection outcome which has an influence on the probability of detection and repair;
- e : the inspection decision (i.e. inspection date, type of inspection, etc.). Inspection decision e varies according to the value of threshold $h(t)^T$ that is applied;
- a : the maintenance action determined by the decision rule d and as a function of the inspection outcome z and inspection decision e , i.e. $a = d(e, z)$;
- X : the variable represents the monitoring result, which leads to an updating of the probability distribution of Θ to $f_{\Theta}'(\theta)$. M_0 denotes the case without taking SHM, and M_1 the case with a certain monitoring strategy implemented, as in the first node illustrated in Figure 2.

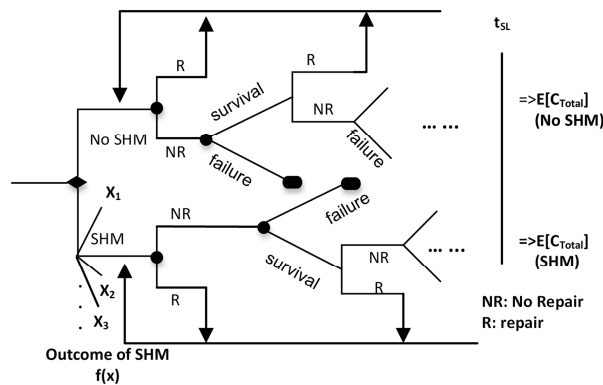


Figure 2 Decision tree model for inspection/repair planning with and without monitoring

For a monitoring strategy, calculation of the expected TLCC is needed based on the branches after the first node. The probability of occurrence of the branches after the first node in the decision tree is calculated based on the probabilities of detection of a certain deterioration state P_{det} and probabilities of taking repair action P_{rep} given a detected deterioration state with no repair before t as in equation 3, which is in accordance with [2, 7, 11].

$$P_{det} = \Phi\left(\frac{\delta(t) - \delta_{0.5}}{\sigma_{0.5}}\right), P_{rep} = \left(\frac{\delta(t)}{\delta_{max}}\right)^{r_a} \text{ where } \delta(t) \leq \delta_{max} \quad (3)$$

where Φ is the standard Gaussian cumulative distribution function; $\delta(t)$ and δ_{max} are the damage intensity at time t and the maximum acceptable damage intensity, respectively. The value of $\delta(t)$ represents the deterioration level and $\delta_{max}=1$. $\delta_{0.5}$ and $\sigma_{0.5}$ are parameters describing the quality of the inspection procedure, representing the damage intensity corresponding to a 50% probability of damage detection, and its standard deviation; r_a is a model parameter reflecting the attitude of the decision maker towards repair: $r_a < 1$ represents a proactive approach, $r_a = 1$ a linear approach and $r_a > 1$ a delayed one.

The branch of a failure event in the event tree requires the calculation of a failure probability $p_F(t)$ during time t given no repair before t . It can be calculated by integrating the hazard function.

In detail, the expected TLCC for an inspection plan is calculated based on the occurrence of each branch in the decision tree as well as the cost of the basic events, i.e. the expected cost of failure, inspection, repair and monitoring if undertaken. The expected cost for each branch is calculated as the result of all the events that happened in the branch. The expected TLCC for this inspection plan then is a weighted sum of the costs for all branches based on the occurrence probability of each branch. Specifically, the expected TLCC of a structural component during its design service life t_{SL} consists of the cost of failure, inspection, repair and monitoring (if undertaken):

$$E[C_T(e, d, t_{SL})] = E[C_F(e, d, t_{SL})] + E[C_I(e, d, t_{SL})] + E[C_R(e, d, t_{SL})] + E[C_M(e, d, t_{SL})] \quad (4)$$

where $E[C_T(e, d, t_{SL})]$ is the expected TLCC, and $E[C_F(e, d, t_{SL})]$, $E[C_I(e, d, t_{SL})]$, $E[C_R(e, d, t_{SL})]$, $E[C_M(e, d, t_{SL})]$ are the expected cost of failure, inspection, repair and monitoring respectively, which can be calculated in accordance with [6, 15].

$$E[C_F(e, d, t_{SL})]$$

$$= \sum_{t=1}^{t_{SL}} \left\{ (1 - \sum_{i=1}^{t-1} p_R(e, d, i)) \frac{1}{(1+r)^t} \cdot \left[\Delta p_F(e, d, t) \cdot (1 - p_F(e, d, t-1)) C_F + p_R(e, d, t) E[C_F(e, d, t_{SL} - t)] \right] \right\} \quad (5)$$

$$E[C_I(e, d, t_{SL})]$$

$$= \sum_{t=t_1}^{t_{nInsp}} \left[(1 - p_F(e, d, t)) (1 - \sum_{i=1}^{t-1} p_R(e, d, i)) \frac{1}{(1+r)^t} \cdot (C_{Insp} + p_R(e, d, t) E[C_I(e, d, t_{SL} - t)]) \right] \quad (6)$$

$$E[C_R(e, d, t_{SL})]$$

$$= \sum_{t=t_1}^{t_{nInsp}} \left[(1 - p_F(e, d, t)) (1 - \sum_{i=1}^{t-1} p_R(e, d, i)) \frac{1}{(1+r)^t} \cdot p_R(e, d, t) \cdot (C_R + E[C_R(e, d, t_{SL} - t)]) \right] \quad (7)$$

$$E[C_M(e, d, t_{SL})] = C_{M_{Iv}} + C_{M_{Is}} + \sum_{t=1}^{t_{SL}} \left[(1 - p_F(e, d, t_i)) \cdot (1 - \Delta p_F(e, d, t_i)) \right] \cdot C_{M_{Op}} \cdot \frac{1}{(1+r)^t} \quad (8)$$

where t_{nInsp} is the time for the n^{th} planned inspection, r is the discount rate. $C_F, C_I, C_R, C_M = (C_{M_{Iv}}, C_{M_{Is}}, C_{M_{Op}})$ are the expected cost of failure, inspection, repair and

monitoring, and C_M consists of system investment $C_{M_{Iv}}$, installation $C_{M_{Is}}$ and operation per year $C_{M_{Op}}$ respectively [6]. For more details, see [15, 17]. Then the VoSHM for the considered SHM strategy is then calculated as the difference between the expected TLCCs with and without this SHM strategy.

It should be mentioned that with a different value of $h(t)^T$, the planning of inspection times changes, leading to a change of the expected TLCC. Similarly, given a certain $h(t)^T$, the expected TLCC is different for M_0 and M_1 , since the planned inspection times are likely to be different in case the monitoring outcome leads to a different joint model. This will be discussed in the following example.

2.2.2 Risk acceptance criterion and decision rule

The risk acceptance criterion used in this contribution is denoted by $h(t)^T$, the threshold value of the hazard function. This is the maximum value given by h_{max} , the maximum acceptable annual failure rate. The latter is related to the failure consequences of a structure and can be obtained from the JCSS Probabilistic Model Code [5], where the target reliability index as a function of the consequence of failure and the risk reduction cost is defined. For existing structures of which the relative cost for increasing the safety is generally large, the acceptance criterion can be lowered as also suggested in [5], i.e. $\Delta p_f^{max} = 10^{-4} \text{ yr}^{-1}$ for large consequences, $\Delta p_f^{max} = 5 \times 10^{-4} \text{ yr}^{-1}$ for moderate consequences and $\Delta p_f^{max} = 10^{-3} \text{ yr}^{-1}$ for minor consequences, where Δp_f^{max} is the maximum acceptable annual probability of failure [15]. It is up to the decision makers to decide the use of other values than the latter ones depending on each case. According to the definition of the hazard function in section 2.1, the value of hazard $h(t)$ is the failure rate or the probability of failure per unit time, where in this work the unit time is defined as 1 year. Thus, the value of h_{max} and $h(t)^T$ can be specified in accordance with Δp_f^{max} .

As the probabilities of detecting a deterioration state and the corresponding repair are both function of the structural state which is a time-dependent variable described by $PT(t)$, the planning of inspection times will have a large influence on the probability of occurrences of each branch in the decision tree. The threshold approach introduced in [16] is thereby implemented in such a way that inspection is carried out in the year before the threshold of failure rate $h(t)^T$ is crossed. The value $h(t)^T$ is a decision parameter set by the decision maker and it must remain lower than h_{max} . For decisions on repair, the specification of the parameters used in equation 9 is required:

$$\begin{cases} h(t) \leq h(t)^T & \text{for inspection} \\ \delta_{max} = \delta_0, r_a = r_0 & \text{for repair} \end{cases} \quad (9)$$

where δ_0 and r_0 are values assigned by the decision makers based on each case. δ_0 is the value of the maximum defect sizes that is allowed for a specific spot and r_0 is the value representing the repair approach, see equation 3.

3 The Influence of Risk Acceptance Criterion on the VoSHM

In the following example, the influence of $h(t)^T$ on the value of SHM is illustrated for a reinforced concrete (RC) bridge girder subjected to corrosion. The degradation of reinforcement in concrete structures is usually mainly due to corrosion of steel, caused by chloride penetration into concrete, concrete carbonation, alkali aggregate reactions, sulfate attack, or freeze-

thaw damage. Assuming that corrosion is mainly due to chloride penetration, the degradation process can be split up into corrosion initiation and corrosion propagation [4]. Here focus is given to the latter one. After the initiation of corrosion, localized pitting corrosion is considered as the predominant because it can be the cause of a high local reduction of cross-sectional area leading to a large reduction of flexural strength [11]. Therefore, the maximum pit depth $PT(t)$ is chosen as the time-dependent performance indicator $PI(t)$ of the beam in the following calculation. The time-dependent performance model is formulated based on [11, 14] as:

$$PT(\Delta t) = r_{corr}R \cdot \Delta t + \epsilon(t) \quad \Delta t > 0 \quad (10)$$

where Δt is the time interval since the corrosion initiation time, r_{corr} (mm/year) is the average instantaneous corrosion rate and R is the ratio of the maximum pit depth over the average one along a given length of a reinforcement bar. R follows a Gumbel distribution with location parameter μ and scale parameter β [14]. The expected value $E(R)$ is considered for this analysis (considered to be deterministic with a value of 5.67 in this case for simplicity). The probabilistic model of each parameter is listed in Table 1. In this case, the time-dependent PI, i.e. the pit depth, can be expressed as:

$$y(t) = PT(t) = r_{corr}R \cdot t + \epsilon(t) \quad (11)$$

where r_{corr} , in [mm/year], follows a lognormal distribution with mean 0.06 and CoV of 0.02.

3.1 Parameter estimation for the hazard function

In order to simulate the hazard function, the following limit state function is considered: $g(t) = PT_{max} - PT(t)$, where $PT_{max} = 4.43$ mm [10], and the structure is considered in “failure state” whenever $g(t) < 0$; For the example under consideration, the time-dependent structural performance parameters are available and the joint modeling can therefore be carried out by generating data sets from the known structural performance parameters, leading to an estimation of the parameters of the hazard function, assuming the hazard function is described by the following function:

$$h(t) = \sigma_2 t^{\sigma_2 - 1} \exp(\gamma \cdot r_{corr}R) \quad (12)$$

Based on the parameters listed in Table 1, 460 sample values of r_{corr} and $\epsilon(t)$ are drawn and accordingly the values of $PT(t)$ are calculated. The time-to-failure data is then obtained as well according to the limit state function. On the basis of this information, the estimation of the parameters of the hazard function is performed using the software WinBUGS and 100000 sampling iterations are used, from which the first 1000 are disregarded as a burn-in phase. The results are shown in Table 1 and Figures 3-4. As a result, the hazard function can be expressed as follows on the basis of the mean estimates for σ_2 and γ .

$$h(t) = 4.054t^{3.054} \exp\{-10.14 r_{corr} \cdot R\} \quad (13)$$

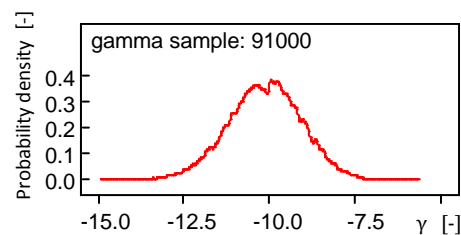
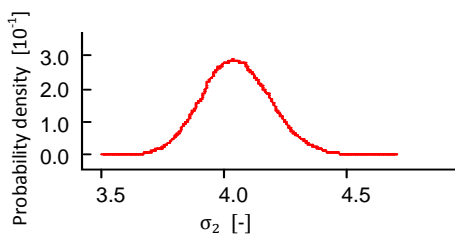


Figure 3. σ_2 posterior sample distribution Figure 4. γ posterior sample distribution

Table 1 Estimation of σ_2 and γ

Parameter	Mean	Standard deviation	Fractiles	
			2.5%	97.5%
σ_2	4.054	0.1398	3.792	4.341
γ	-10.14	1.067	-12.3	-8.11

As mentioned in section 2.2.1, as this hazard function $h(t)$ is available (see equation 13), the failure probability during a period of time can be calculated as the integration of the hazard function which can be used for inspection time planning as well as calculating the expected TLCC, which in this case is the one without implementing a SHM strategy.

3.2 Influence of $h(t)^T$ on the VoSHM

As the actual monitoring data for different SHM strategies are actually not available in the pre-posterior analysis, the information that may be collected through the SHM is simulated as a decrease of uncertainty in r_{corr} . In this case, the standard deviation of r_{corr} is reduced by multiplying M_e which is a lognormally distributed model parameter and represents the extent to which the monitoring strategy would reduce the uncertainty. The statistical distributions of M_e for different SHM strategies are listed in Table 2, in which 6 SHM strategies are considered with increasing accuracy from S1 to S6 represented by a decreasing mean value of M_e , considering that a low expected value of M_e results in a large reduction of the standard deviation of r_{corr} . Realizations of the outcome of structural health monitoring are now generated by random sampling from the random variable M_e and then inserted into equation 11. This step is repeated in a loop in which new sets of PI and time-to-failure data are generated facilitating the parameter estimations (as in section 3.1) of the hazard functions for S1 to S6, respectively. The parameters for the equivalent Weibull distributions and the relative value of the SHM investments are listed in Table 2 as well. For illustration, the parameters $\delta_{0.5} = 0.04$, $\sigma_{0.5} = 0.1\delta_{0.5}$ are considered for the inspection, and $\delta_{max} = 1$, $r_a = 2$ for the repair.

Table 2 Model parameters for 6 SHM strategies and relative costs

SHM Strategy	M_e (lognormal)		Equivalent Weibull distribution		C_F	C_{Insp}	C_R	$C_{M_{Iv}}$	$C_{M_{Is}}$	$C_{M_{Op}}$
	Mean	CoV	σ_2	γ	[-]	[-]	[-]	(10^{-2})	(10^{-2})	(10^{-3})
S1	0.99	0.1	4.110	17.615	1	0.02	0.10	2.66	2.00	1
S2	0.97	0.1	4.116	18.752	1	0.02	0.10	4.66	4.00	1.5
S3	0.95	0.1	4.156	20.352	1	0.02	0.10	6.66	5.60	2
S4	0.93	0.1	4.162	21.983	1	0.02	0.10	8.66	7.00	2.5
S5	0.91	0.1	4.163	23.123	1	0.02	0.10	10	8.00	10
S6	0.89	0.1	4.165	23.605	1	0.02	0.10	10	15	15

Since the value of $h(t)^T$ has a large influence on the planning of inspection times, the expected TLCC are expected to change with different $h(t)^T$ values. Figure 5 shows the influence of $h(t)^T$, changing from 10^{-3} to 10^{-1} , on the VoSHM. As can be seen in Figure 5, the values for these SHM strategies are positive and generally increase with the $h(t)^T$, from which it can be inferred that for the $h(t)^T$ value in this range the information provided by S1-S6 in monetary term overweighs their costs. Hence, the asset manager is encouraged to im-

plement SHM in this case. Moreover, the increase of VoSHM with $h(t)^T$ is mainly because the information provided by SHM are more valuable for cases with larger $h(t)^T$ values, since the probability of failure, and failure cost accordingly, would be high in such case if there was no SHM implemented. It also indicates that there is a potential to further increasing $h(t)^T$, which however requires additional calculations.

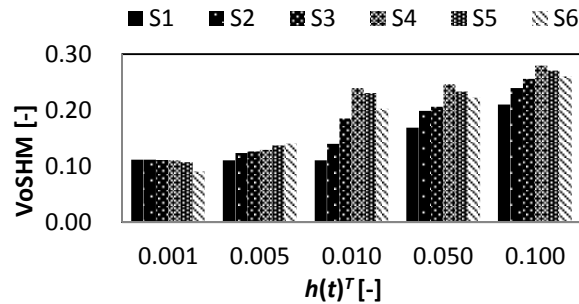


Figure 5. Influence of $h(t)^T$ on the VoSHM

It can be further noticed in Figure 5 that the VoSHM is higher than 0.2, which is significant, for most of the considered SHM strategies in case the threshold probabilities are larger than 10^{-2} . The high relative values are mainly caused by the reduction of the probability of failure due to SHM which is significant for $h(t)^T$ above 10^{-2} . However, for threshold probabilities $h(t)^T$ smaller than 10^{-2} the differences among SHM strategies are less significant since the cost of inspection contributes most, since the inspections would be planned frequently for small $h(t)^T$ values, as also can be seen from Figure 1 (b), regardless of the SHM strategies used.

It can be also noticed that for $h(t)^T = 10^{-3}$ the VoSHM of S6 is slightly smaller than that of S1 to S5. This is mainly due to the fact that the information provided by a higher accuracy SHM strategy, i.e. S6, is for that case not so cost effective compared to the other ones. The same situation holds for $h(t)^T = 10^{-2}$, where the maximum VoSHM is provided by S4 which is 0.239 in this case. For S1 to S4, the increase of SHM accuracy provides more value than that of implementing SHM, while the costs for increasing accuracy of S4 to S6 overweigh the value of information that they provided, which leads to a decrease of the VoSHM for S4 to S6. These cost-related observations should be taken into account by the decision maker prior to the implementation of SHM in order to maximize the VoSHM.

4 Conclusions

Based on a framework for joint modelling of the structural performance function and hazard function, the planning of structural inspection/maintenance and evaluating of VoSHM are introduced. In this framework, the evaluation of the VoSHM are related to several parameters, among which the influence of the risk acceptance criterion, i.e. the $h(t)^T$, is investigated with an example in this work. The results show that the VoSHM as well as the optimal SHM strategy varies with the change of $h(t)^T$, since it has a major influence on the inspection/maintenance time planning. Moreover, the VoSHM of different SHM strategies also varies for a specific value of $h(t)^T$, as the information provided by SHM strategies with different precision are closely related to the costs of their implementation. Therefore, a case specific balance needs to be sought by the asset manager prior to the implementation of SHM and the process of optimizing the SHM strategies and maintenance planning based on the Bayesi-

an joint model can be done by choosing the one that would maximize the VoSHM for that case.

Acknowledgement

The financial support from the China Scholarship Council (No. 201306320169) is gratefully acknowledged.

References

- [1] J.O. Almeida, P.F. Teixeira, R.M. Delgado, *Life cycle cost optimisation in highway concrete bridges management*, Structure and Infrastructure Engineering, 2013, 1-14.
- [2] G. Barone, D.M. Frangopol, *Hazard-Based Optimum Lifetime Inspection and Repair Planning for Deteriorating Structures*, Journal of Structural Engineering 139:12, 2013, 04013017 (12 pp.)-04013017 (12 pp.).
- [3] G. Barone, D.M. Frangopol, *Reliability, risk and lifetime distributions as performance indicators for life-cycle maintenance of deteriorating structures*, Reliability Engineering & System Safety 123, 2014, 21-37.
- [4] L. Bertolini, *Steel corrosion and service life of reinforced concrete structures*, Structure and Infrastructure Engineering 4:2, 2008, 123-137.
- [5] M. Faber, A. Vrouwenvelder, *Probabilistic model code*, Joint Committee on Structural Safety, 2001.
- [6] M.H. Faber, S. Thons, *On the value of structural health monitoring*, Safety, Reliability and Risk Analysis: Beyond the Horizon, 2014, 2535-2544.
- [7] D.M. Frangopol, K.Y. Lin, A.C. Estes, *Life-cycle cost design of deteriorating structures*, Journal of Structural Engineering-Asce 123:10, 1997, 1390-1401.
- [8] S. Kim, D.M. Frangopol, *Cost-Based Optimum Scheduling of Inspection and Monitoring for Fatigue-Sensitive Structures under Uncertainty*, Journal of Structural Engineering-Asce 137:11, 2011, 1319-1331.
- [9] S. Kim, D.M. Frangopol, *Cost-Effective Lifetime Structural Health Monitoring Based on Availability*, Journal of Structural Engineering-Asce 137:1, 2011, 22-33.
- [10] S. Kim, D.M. Frangopol, B. Zhu, *Probabilistic optimum inspection/repair planning to extend lifetime of deteriorating structures*, Journal of Performance of Constructed Facilities 25:6, 2010, 534-544.
- [11] P.S. Marsh, D.M. Frangopol, *Reinforced concrete bridge deck reliability model incorporating temporal and spatial variations of probabilistic corrosion rate sensor data*, Reliability Engineering & System Safety 93:3, 2008, 394-409.
- [12] A.D. Orcesi, D.M. Frangopol, *Optimization of bridge maintenance strategies based on structural health monitoring information*, Structural Safety 33:1, 2011, 26-41.
- [13] H. Raiffa, R. Schlaifer, *Applied statistical decision theory*, Harvard university press, Boston (Mass.), 1961.
- [14] M.G. Stewart, *Spatial variability of pitting corrosion and its influence on structural fragility and reliability of RC beams in flexure*, Structural Safety 26:4, 2004, 453-470.
- [15] D. Straub, *Generic approaches to risk based inspection planning for steel structures*, vdf Hochschulverlag AG 2004.
- [16] D. Straub, *Generic Approaches to Risk Based Inspection Planning for Steel Structures*, Chair of Risk and Safety, Institute of Structural Engineering, ETH Zürich, Zürich, 2004.

- [17] S. Thöns, *Monitoring based condition assessment of offshore wind turbine support structures*, ETH, 2011.
- [18] C. Xing, R. Caspeele, L. Taerwe, *Evaluating the value of structural health monitoring with longitudinal performance indicators and hazard functions using Bayesian dynamic predictions*, Workshop on Quantifying the value of structural health monitoring, 2015, pp. 27-37.